Particle creation due to tachyonic instability in relativistic stars

A. G. S. Landulfo,[1,*] W. C. C. Lima,[1,†] G. E. A. Matsas,[2,‡] and D. A. T. Vanzella[1,§]

[1]*Instituto de Física de São Carlos, Universidade de São Paulo, Caixa Postal 369, 13560-970 São Carlos, São Paulo, Brazil*
[2]*Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271 - Bloco II, 01140-070 São Paulo, São Paulo, Brazil*
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Dense enough compact objects were recently shown to lead to an exponentially fast increase of the vacuum energy density for some free scalar fields properly coupled to the spacetime curvature as a consequence of a tachyonic-like instability. Once the effect is triggered, the star energy density would be overwhelmed by the vacuum energy density in a few milliseconds. This demands that eventually geometry and field evolve to a new configuration to bring the vacuum back to a stationary regime. Here, we show that the vacuum fluctuations built up during the unstable epoch lead to particle creation in the final stationary state when the tachyonic instability ceases. The amount of created particles depends mostly on the duration of the unstable epoch and final stationary configuration, which are open issues at this point. We emphasize that the particle creation coming from the tachyonic instability will occur even in the adiabatic limit, where the spacetime geometry changes arbitrarily slowly, and therefore is quite distinct from the usual particle creation due to the change in the background geometry.

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I. INTRODUCTION

It was recently shown that relativistic stars may become unstable due to quantum-field effects [1,2]. The so-called *vacuum awakening effect* occurs for a free scalar field Φ properly coupled to the spacetime curvature [1]. This effect is characterized by an exponential point-dependent increase and decrease of the vacuum expectation value of the stress-energy-momentum tensor $\langle \hat{T}_{ab} \rangle$. This is caused by a tachyonic-like instability, which induces an exponential growth of $\langle \hat{\Phi}^2 \rangle$. Once the effect is triggered and the scalar field exits its initial quiescent regime, few milliseconds would be enough for the vacuum to overwhelm the energy density of the compact object. The star destiny is presently uncertain because it depends on how scalar field and spacetime geometry evolve in the unstable phase to reach a final stable configuration. As recently argued in Ref. [3], in some cases the appearance of a proper scalar field could restabilize the star, a phenomenon usually called *spontaneous scalarization* [4]. This would typically change the star gravitational mass by a few percent. However, depending on how the star enters the unstable phase, it seems possible that the scalar field does not react fast enough, leading to some dramatic implosion/explosion event. Whatever turns out to be the final configuration, being the star somehow rebalanced or destroyed, the unstable phase must be detained and the vacuum must evolve to some new stationary regime. This observation alone allows us to extract quite important information about the final state of the scalar field as the vacuum "falls asleep" again. In particular, we show that the vacuum fluctuation built up during the unstable epoch leads to particle creation in the final stationary state. The amount of created particles will depend mostly on the duration of the unstable epoch and final stationary configuration.

The paper is organized as follows. In Sec. II, we discuss the quantization procedure for a free scalar field in a curved spacetime in the presence of tachyonic-like modes. In Sec. III, we apply the previous-section results to review the vacuum awakening effect in relativistic stars. In Sec. IV, we probe the unstable phase using Unruh-DeWitt detectors. In this period, no natural particle content can be ascribed to the scalar field and the use of detectors is particularly useful to investigate the behavior of the vacuum fluctuation. We show in this section that even assuming a static spacetime in the unstable phase when the vacuum is "awake," particle detectors following orbits of the time-like isometry will copiously excite. This is possible according to co-static observers because each detector excitation is accompanied by a corresponding decrease in the energy stored in the field due to the excitation of a nonstationary (tachyonic) mode which contributes with negative energy. Then, in Sec. V, we show that at least part of the quantum fluctuations built up in the awaken phase eventually draws to particle creation as the unstable period ends and the vacuum falls dormant again. We emphasize that the particle creation occurs even assuming that the spacetime change is arbitrarily slow and, thus, differs, e.g., from the well-known phenomenon of particle creation in evolving universes induced by the change of the background geometry [5–7]. A toy model is also offered to illustrate in a concrete scenario the conclusions above. We close the paper with our final remarks in Sec. VI.

*andre.landulfo@ufabc.edu.br
†wccl@ift.unesp.br
‡matsas@ift.unesp.br
§vanzella@ifsc.usp.br

II. PRELIMINARIES: FREE SCALAR FIELD QUANTIZATION WITH TACHYONIC-LIKE MODES

A. Standard field quantization in globally hyperbolic spacetimes

In this section, we review the standard quantization of free scalar fields in curved spaces [8,9] giving particular attention to the case of static spacetimes with tachyonic-like modes. Let us begin by considering a globally hyperbolic spacetime $(\mathcal{M}, g_{ab})$ foliated with Cauchy surfaces Σ_t labeled with a parameter t. Now, let us cover the Σ_t surfaces with x^i $(i = 1, 2, 3)$ coordinates satisfying $n^a \nabla_a x^i = 0$, where n^a is the future-directed unit vector field normal to Σ_t. In terms of coordinates $x = (t, x^i)$, we can write the spacetime line element as

$$ds^2 = N^2(-dt^2 + h_{ij}(x) dx^i dx^j), \tag{1}$$

where $N = N(x) > 0$ is the lapse function and ${}^{(3)}g_{ij} = N^2 h_{ij}$ is the three-dimensional spatial metric induced on each Cauchy surface Σ_t.

We define the dynamics of a real scalar field Φ with mass m in $(\mathcal{M}, g_{ab})$ through the action

$$S \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}(\nabla_a\Phi\nabla^a\Phi + m^2\Phi^2 + \xi R\Phi^2), \tag{2}$$

where $g \equiv \det(g_{ab})$ and $\xi \in \mathbb{R}$ determines the nonminimal coupling between the scalar field and the scalar curvature R. This leads to the following Klein-Gordon equation:

$$-\nabla_a\nabla^a\Phi + m^2\Phi + \xi R\Phi = 0. \tag{3}$$

Next, we define the Klein-Gordon inner product between any two solutions u and v of Eq. (3) as

$$(u, v)_{\rm KG} \equiv i\int_{\Sigma_t} d\Sigma n^a[u^*\nabla_a v - v\nabla_a u^*], \tag{4}$$

where $d\Sigma$ is the proper-volume element on Σ_t, and we recall that Eq. (4) does not depend on the choice of Σ_t.

The conjugate-momentum density $\Pi(x)$ is defined as

$$\Pi \equiv \delta S/\delta\dot{\Phi} = \sqrt{{}^{(3)}g}\, n^a\nabla_a\Phi, \tag{5}$$

where "$\cdot$" $\equiv \partial_t$ and ${}^{(3)}g \equiv \det({}^{(3)}g_{ij})$. The canonical quantization procedure consists of promoting field and momentum density to operators $\hat{\Phi}$ and $\hat{\Pi}$, respectively, satisfying canonical commutation relations:

$$[\hat{\Phi}(t, \mathbf{x}), \hat{\Phi}(t, \mathbf{x}')]_{\Sigma_t} = [\hat{\Pi}(t, \mathbf{x}), \hat{\Pi}(t, \mathbf{x}')]_{\Sigma_t} = 0, \tag{6}$$

$$[\hat{\Phi}(t, \mathbf{x}), \hat{\Pi}(t, \mathbf{x}')]_{\Sigma_t} = i\delta^3(\mathbf{x}, \mathbf{x}'), \tag{7}$$

where $\mathbf{x} \equiv (x^1, x^2, x^3)$.

In order to realize a representation of these commutation relations, consider positive- and negative-norm solutions of Eq. (3), $u_\alpha^{(+)}$ and $u_\alpha^{(-)} \equiv (u_\alpha^{(+)})^*$, respectively, which together form a complete set of normal modes satisfying

$$(u_\alpha^{(+)}, u_\beta^{(+)})_{\rm KG} = -(u_\alpha^{(-)}, u_\beta^{(-)})_{\rm KG} = \delta(\alpha, \beta), \tag{8}$$

$$(u_\alpha^{(+)}, u_\beta^{(-)})_{\rm KG} = 0. \tag{9}$$

Here, $\delta(\alpha, \beta)$ is the delta function associated with the quantum numbers formally described by α, β. Then, we construct the field operator using $\{u_\sigma^{(+)}, u_\sigma^{(-)}\}$ as

$$\hat{\Phi} = \int d\mu(\sigma)[\hat{a}_\sigma u_\sigma^{(+)} + \hat{a}_\sigma^\dagger u_\sigma^{(-)}], \tag{10}$$

where μ is a measure defined on the set of quantum numbers and in order to satisfy Eqs. (6) and (7), the annihilation $\hat{a}_\sigma$ and creation $\hat{a}_\sigma^\dagger$ operators must satisfy the usual commutation relations $[\hat{a}_\alpha, \hat{a}_\beta^\dagger] = \delta(\alpha, \beta)$, $[\hat{a}_\alpha, \hat{a}_\beta] = 0$. The vacuum state $|0\rangle$ of this representation is defined by requiring $\hat{a}_\sigma|0\rangle = 0$ for all σ.

B. Quantum fields in static spacetimes

Now, we restrict our analysis to static spacetimes in which case the line element (1) is cast as

$$ds^2 = N^2(-dt^2 + h_{ij}(\mathbf{x}) dx^i dx^j), \tag{11}$$

where $N = N(\mathbf{x}) > 0$. Under this condition, we write the field equation (3) in the form

$$-\frac{\partial^2\tilde{\Phi}}{\partial t^2} = [-\Delta + V_{\rm eff}(x)]\tilde{\Phi}, \tag{12}$$

where $\tilde{\Phi} \equiv N\Phi$, Δ is the Laplace operator associated with h_{ij}, and

$$\begin{aligned} V_{\rm eff}(\mathbf{x}) &= N^{-1}\Delta N + N^2(m^2 + \xi R) \\ &= (1 - 6\xi)N^{-1}\Delta N + N^2 m^2 + \xi K \end{aligned} \tag{13}$$

is the effective potential with $K = K(\mathbf{x})$ being the scalar curvature associated with h_{ij}.

The existence of a time-like Killing field $\kappa^b = (\partial_t)^b$ associated with spacetime (11) suggests that we look for solutions of Eq. (12) in the form $\tilde{u}_\sigma^{(+)} \propto F_\sigma(\mathbf{x})\exp(-i\omega_\sigma t)$ corresponding to solutions $u_\sigma^{(+)} \propto N^{-1}F_\sigma(\mathbf{x})\exp(-i\omega_\sigma t)$ for Eq. (3). In this case, $F_\sigma(\mathbf{x})$ will satisfy

$$[-\Delta + V_{\rm eff}(\mathbf{x})]F_\sigma(\mathbf{x}) = \lambda_\sigma F_\sigma(\mathbf{x}), \qquad \lambda_\sigma = \omega_\sigma^2, \tag{14}$$

with proper boundary conditions. At this point, the only restriction on λ_σ is the one imposed by Hermiticity of the operator $-\Delta + V_{\rm eff}(\mathbf{x})$, which demands $\lambda_\sigma \in \mathbb{R}$.

Let us consider first solutions of Eq. (14) with positive eigenvalues: $\lambda_\sigma \equiv \varpi_\sigma^2 > 0$. Then, the corresponding positive-norm solutions satisfying Eq. (3) will be the usual oscillatory modes:

$$v_\sigma^{(+)} = \frac{e^{-i\varpi_\sigma t}}{\sqrt{2\varpi_\sigma}N(\mathbf{x})}F_\sigma(\mathbf{x}), \qquad \varpi_\sigma > 0, \tag{15}$$

where we demand

$$\int_{\Sigma_t} d^3x\sqrt{h}F_\alpha(\mathbf{x})^* F_\beta(\mathbf{x}) = \delta(\alpha, \beta) \tag{16}$$

in order to guarantee that modes (15) are properly normalized according to Eqs. (8) and (9).

Now, we note that in some cases Eq. (14) also allows for solutions with negative eigenvalues: $\lambda_\sigma \equiv -\Omega_\sigma^2 < 0$. These solutions are associated with solutions of Eq. (12) with exponentially increasing and decreasing $\exp(\pm\Omega_\sigma t)$ time dependence. Under such circumstances, $\{v_\sigma^{(+)}, v_\sigma^{(-)}\}$ must be supplemented by an extra set of modes $\{w_\sigma^{(+)}, w_\sigma^{(-)}\}$ in order to generate a basis for the solution space of Eq. (3). Normalized positive-norm modes $w_\sigma^{(+)}$ can be found and read

$$w_\sigma^{(+)} = e^{i\beta_\sigma}\frac{(e^{\Omega_\sigma t - i\alpha_\sigma} + e^{-\Omega_\sigma t + i\alpha_\sigma})}{\sqrt{4\Omega_\sigma \sin(2\alpha_\sigma)}N(\mathbf{x})}F_\sigma(\mathbf{x}), \qquad \Omega_\sigma > 0, \tag{17}$$

where $\alpha_\sigma \in]0, \frac{\pi}{4}]$ and, for the sake of convenience, we did not vanish the arbitrary global phase β_σ yet. By choosing, e.g., $\alpha_\sigma = \beta_\sigma = \pi/4$, Eq. (17) would assume the simple form

$$w_\sigma^{(+)} = \frac{(e^{\Omega_\sigma t} + ie^{-\Omega_\sigma t})}{2\sqrt{\Omega_\sigma}N(\mathbf{x})}F_\sigma(\mathbf{x}), \qquad \Omega_\sigma > 0 \tag{18}$$

but we shall adopt here the same choice as in Ref. [1], where $\beta_\sigma = 0$ and $\alpha_\sigma = \pi/12$:

$$w_\sigma^{(+)} = \frac{(e^{\Omega_\sigma t - i\pi/12} + e^{-\Omega_\sigma t + i\pi/12})}{\sqrt{2\Omega_\sigma}N(\mathbf{x})}F_\sigma(\mathbf{x}), \qquad \Omega_\sigma > 0, \tag{19}$$

in order to make $w_\sigma^{(+)}$ look "as similar as possible" to $v_\sigma^{(+)}$. Because $w_\sigma^{(+)}$ and $w_\sigma^{(-)}$ grow exponentially in time, we borrow from cosmology the "tachyonic" term (see, e.g., Ref. [10]) and refer to these modes accordingly. (It should be noted, however, that in the cosmological context the scalar field is *self-interacting*, as ruled by some interacting potential, in contrast to our present case where it is *free*.)

As a result, the field operator $\hat{\Phi}(x)$ can be constructed using $\{v_\sigma^{(+)}, v_\sigma^{(-)}\}$ and $\{w_\sigma^{(+)}, w_\sigma^{(-)}\}$ as

$$\hat{\Phi} = \int d\mu(\sigma)[\hat{b}_\sigma v_\sigma^{(+)} + \hat{b}_\sigma^\dagger v_\sigma^{(-)}] + \sum_\sigma [\hat{c}_\sigma w_\sigma^{(+)} + \hat{c}_\sigma^\dagger w_\sigma^{(-)}], \tag{20}$$

where $[\hat{b}_\alpha, \hat{b}_\beta^\dagger] = \delta(\alpha, \beta)$, $[\hat{c}_\alpha, \hat{c}_\beta^\dagger] = \delta(\alpha, \beta)$ (with the other commutators vanishing), and we have used the summation symbol in the right-hand side of Eq. (20) because the tachyonic modes will be labeled later with quantum numbers assuming discrete values. We recall that the vacuum state $|0\rangle$ satisfies $\hat{b}_\sigma|0\rangle = \hat{c}_\sigma|0\rangle = 0$ for all σ. We note that in contrast to the $v_\sigma^{(+)}$ and $v_\sigma^{(-)}$ modes, $w_\sigma^{(+)}$ and $w_\sigma^{(-)}$ are not frequency eigenstates of $i\partial/\partial t$. As a result, the vacuum $|0\rangle$ and the other Fock states do not have in general any natural particle-content interpretation (see Refs. [11,12] for a field-theoretic discussion on the Fock space in the presence of tachyonic modes). This feature will lead us to use Unruh-DeWitt detectors to probe vacuum fluctuations of the scalar field in Sec. IV.

Nevertheless, important pieces of information are directly provided through the (formal) stress-energy tensor operator:

$$\hat{T}_{ab} = (1-2\xi)\nabla_a\hat{\Phi}\nabla_b\hat{\Phi} + \xi R_{ab}\hat{\Phi}^2 - 2\xi\hat{\Phi}\nabla_a\nabla_b\hat{\Phi} + (2\xi - 1/2)g_{ab}[\nabla_c\hat{\Phi}\nabla^c\hat{\Phi} + (m^2 + \xi R)\hat{\Phi}^2] \tag{21}$$

and the corresponding Hamiltonian:

$$\hat{H} \equiv \int_{\Sigma_t} d\Sigma_a \kappa_b \hat{T}^{ab} = \int_{\Sigma_t} d\Sigma \hat{\rho}, \tag{22}$$

where $d\Sigma_a \equiv d\Sigma n_a$, $\kappa_b = (\partial_t)_b$,

$$\hat{\rho} \equiv n_a \kappa_b \hat{T}^{ab} \tag{23}$$

is the energy-density operator in Σ_t associated with the time-like isometry, and we recall that Eq. (22) does not depend on the Σ_t choice because $\nabla_a(\kappa_b \hat{T}^{ab}) = 0$. Thus, the total energy is conserved. By using Eq. (20) in the Hamiltonian operator (22), we obtain

$$\hat{H} \equiv \int_{\Sigma_t} d\Sigma N^{-1}\hat{T}_{00} = \frac{1}{2}\int d\mu(\sigma)(\hat{b}_\sigma^\dagger\hat{b}_\sigma + \hat{b}_\sigma\hat{b}_\sigma^\dagger)\varpi_\sigma + \sum_\sigma \hat{\mathcal{H}}_\sigma, \tag{24}$$

where

$$\hat{\mathcal{H}}_\sigma \equiv -[\sqrt{3}/2(\hat{c}_\sigma^\dagger\hat{c}_\sigma + \hat{c}_\sigma\hat{c}_\sigma^\dagger) + \hat{c}_\sigma\hat{c}_\sigma + \hat{c}_\sigma^\dagger\hat{c}_\sigma^\dagger]\Omega_\sigma. \tag{25}$$

In contrast to the first term in the right-hand side of Eq. (24), associated with the oscillatory $v_\sigma^{(\pm)}$ modes, which always gives a positive-definite contribution to the energy expectation value for *every* state choice, the second term, associated with the tachyonic modes $w_\sigma^{(\pm)}$, gives a negative contribution to the energy expectation value for *some* states. This can be easily seen by rewriting Eq. (25) as

$$\hat{\mathcal{H}}_\sigma = (1 - \sqrt{3}/2)\hat{p}_\sigma^2 - (1 + \sqrt{3}/2)\Omega_\sigma^2\hat{q}_\sigma^2, \tag{26}$$

where we have defined the position- and momentum-like operators

$$\hat{q}_\sigma \equiv \frac{1}{\sqrt{2\Omega_\sigma}}(\hat{c}_\sigma + \hat{c}_\sigma^\dagger), \tag{27}$$

$$\hat{p}_\sigma \equiv i\sqrt{\frac{\Omega_\sigma}{2}}(\hat{c}_\sigma^\dagger - \hat{c}_\sigma), \tag{28}$$

respectively, satisfying $[\hat{q}_\sigma, \hat{p}_\sigma] = i\hat{I}$ with $\hat{I}$ being the identity operator. Equation (26) is formally identical to the Hamiltonian of a nonrelativistic particle in a harmonic potential turned upside down [13]. It is clear, then, that the "potential" term gives a negative contribution to the energy expectation value. In particular, for states $|\Psi\rangle$ satisfying

$$\hat{c}_\sigma^\dagger \hat{c}_\sigma |\Psi\rangle = \Xi|\Psi\rangle, \qquad \Xi \in \mathbb{N}, \tag{29}$$

which include the vacuum state, it is easy to see that $\langle\Psi|\hat{c}_\sigma\hat{c}_\sigma + \hat{c}_\sigma^\dagger\hat{c}_\sigma^\dagger|\Psi\rangle = 0$ and, thus,

$$\langle\Psi|\hat{\mathcal{H}}_\sigma|\Psi\rangle = -\sqrt{3}(1/2 + \Xi)\Omega_\sigma < 0. \tag{30}$$

Hence, for these states the negative contribution coming from the "potential" in Eq. (26) dominates over the corresponding positive one coming from the "kinetic" term. (We shall return to this point when we discuss the excitation of Unruh-DeWitt detectors in Sec. IV.) The fact that the right-hand side of Eq. (30) may be arbitrarily negative for sufficiently large Ξ reflects the fact that $\hat{\mathcal{H}}_\sigma$ is unbounded from below.

III. AWAKING THE VACUUM IN RELATIVISTIC STARS DUE TO TACHYONIC INSTABILITY

Now, we shall see how tachyonic modes can appear in neutron-like stars and discuss their consequences [1,2]. Let us assume the case in which (A) classical matter initially scattered throughout space with very low density eventually collapses to form (B) a static and stable star according to general relativity. Spacetimes associated with situations A-B are well described by the line elements [see Eq. (11)]

$$ds^2 = \begin{cases} -dt^2 + d\mathbf{x}^2 & \text{(A)} \\ N_{(B)}^2(-dt^2 + h_{ij}^{(B)}dx^i dx^j) & \text{(B)} \end{cases}. \tag{31}$$

We note that for the time being we will restrict our investigation to the static regions A and B of the spacetime. Comments about how our present analysis can be completed as one takes into account the time evolution between the static eras are made along the text. For the sake of obtaining explicit results, we make an extra simplification in this section and consider spherically symmetric stars in which case Eq. (31) is replaced by

$$ds^2 = \begin{cases} -dt^2 + d\mathbf{x}^2 & \text{(A)} \\ -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta d\varphi^2) & \text{(B)} \end{cases}, \tag{32}$$

where $f = f(\chi) > 0$ and $r = r(\chi) \geq 0$ satisfy $f(\chi) \to 1$ and $r(\chi)/\chi \to 1$ for $\chi \to \infty$, and $dr/d\chi > 0$ so that no trapped light-like surface is present.

We construct the field operator similarly as in Eq. (10):

$$\hat{\Phi} = \int d^3k[\hat{a}_\mathbf{k} u_\mathbf{k}^{(+)} + \hat{a}_\mathbf{k}^\dagger u_\mathbf{k}^{(-)}], \tag{33}$$

where we choose here $u_\mathbf{k}^{(\pm)}$ such that they assume the usual flat-space stationary form in the asymptotic past (region A):

$$u_\mathbf{k}^{(\pm)} \overset{\text{(A)}}{=} (16\pi^3\omega_\mathbf{k})^{-1/2}\exp[\mp i(\omega_\mathbf{k} t - \mathbf{k}\cdot\mathbf{x})] \tag{34}$$

with $\mathbf{k} \in \mathbb{R}^3$ and $\omega_\mathbf{k} \equiv \sqrt{\mathbf{k}^2 + m^2}$. This choice is motivated by the fact that we shall assume hereafter the scalar field to be in the no-particle state $|0\rangle_{\text{in}}$ as described by static observers in the asymptotic past: $a_\mathbf{k}|0\rangle_{\text{in}} = 0$.

Now, let us represent $\hat{\Phi}$ in terms of positive- and negative-norm modes in region B, when the star has settled down, as [see Eq. (20)]

$$\hat{\Phi} = \sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}v_{\varpi l\mu}^{(+)} + \hat{b}_{\varpi l\mu}^\dagger v_{\varpi l\mu}^{(-)}] + \sum_{\Omega l\mu}[\hat{c}_{\Omega l\mu}w_{\Omega l\mu}^{(+)} + \hat{c}_{\Omega l\mu}^\dagger w_{\Omega l\mu}^{(-)}], \tag{35}$$

where [14]

$$v_{\varpi l\mu}^{(+)} \overset{\text{(B)}}{=} e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}r(\chi)}Y_{l\mu}(\theta,\phi), \qquad \varpi > 0, \tag{36}$$

$$w_{\Omega l\mu}^{(+)} \overset{\text{(B)}}{=} (e^{\Omega t - i\pi/12} + e^{-\Omega t + i\pi/12}) \times \frac{F_{\Omega l}(\chi)}{\sqrt{2\Omega}r(\chi)}Y_{l\mu}(\theta,\phi), \qquad \Omega > 0. \tag{37}$$

Here, $Y_{l\mu}(\theta,\phi)$ are the usual spherical harmonics ($l = 0, 1, 2, \ldots$ and $\mu = -l, -l+1, \ldots, l$), $F_{\varpi l}(\chi)$ and $F_{\Omega l}(\chi)$ satisfy

$$-\frac{d^2}{d\chi^2}F_{\varpi l} + V_{\text{eff}}^{(l)}F_{\varpi l} = \varpi^2 F_{\varpi l} \tag{38}$$

and

$$-\frac{d^2}{d\chi^2}F_{\Omega l} + V_{\text{eff}}^{(l)}F_{\Omega l} = -\Omega^2 F_{\Omega l}, \tag{39}$$

respectively, and

$$V_{\text{eff}}^{(l)} \equiv f\left(m^2 + \xi R + \frac{l(l+1)}{r^2}\right) + \frac{1}{r}\frac{d^2 r}{d\chi^2} \tag{40}$$

is the effective potential. For perfect-fluid stars, the effective potential (40) can be cast as

$$V_{\text{eff}}^{(l)} = f\left[m^2 + \frac{l(l+1)}{r^2} + \left(\xi - \frac{1}{6}\right)R + \frac{8\pi G}{3}(\bar{\rho} - \rho)\right], \tag{41}$$

where $\rho = \rho(\chi)$ is the mass-energy density of the stellar fluid and

$$\bar{\rho}(\chi) \equiv \frac{3M(\chi)}{4\pi[r(\chi)]^3} \tag{42}$$

is the corresponding average density up to the radius coordinate $r(\chi)$, which encompasses a mass $M(\chi)$. We remind that according to general relativity $R = 8\pi G(\rho - 3p)$, where p is the hydrostatic pressure which bears the star up against its weight.

As discussed in Sec. II B, the appearance of tachyonic modes in the present context will depend on the existence of nontrivial solutions for Eq. (39). They are expected to exist for negative enough effective potentials satisfying

$$|V_{\rm eff}^{(l)}|R_s^2 \gtrsim 1, \tag{43}$$

where $r = R_s$ is the star radius. Because the centrifugal barrier in Eq. (41) is positive, we look for tachyonic solutions of Eq. (39) with $l = 0$ which are the most likely ones to exist (if any). By taking $f \sim 1$ and assuming $\bar{\rho} \sim \rho$, we obtain from Eq. (41) that

$$V_{\rm eff}^{(0)} \sim m^2 + (\xi - 1/6)R.$$

Clearly, only the second term in the right-hand side can be negative. Then, Eq. (43) implies that a necessary condition for the existence of tachyonic modes with $\xi \approx 1$ is

$$\frac{\rho}{10^{15}\ {\rm g/cm^3}}\left(\frac{R_s}{7\ {\rm km}}\right)^2 \gtrsim 1 \tag{44}$$

and

$$\frac{m^2/(3.5 \times 10^{-11}\ {\rm eV})^2}{\rho/(10^{15}\ {\rm g/cm^3})} \ll 1, \tag{45}$$

where we have set $\rho \sim p$. Equations (44) and (45) show that the appearance of tachyonic modes for small ξ values in the spacetime of typical neutron stars requires the scalar field to be light: $m \lesssim 10^{-11}$ eV.

Although light scalars are widely considered in astrophysics and cosmology, there is the issue about how much extra mass they could acquire from Planck-scale radiative corrections. For axions, e.g., a general expression for the mass shift can be cast as $\delta m_a^2 \sim K_a f_a^{n+2}/M_P^n$, where K_a is some unknown coupling constant, $f_a \sim 10^{12}$ GeV is the energy scale of the Peccei-Quinn symmetry breaking, $M_P \sim 10^{19}$ GeV is the Planck energy, and n is a model-dependent positive integer (associated with the dimension of the symmetry-breaking operators appearing in the effective Lagrangian) [15]. We see, then, that δm_a can easily exceed, e.g., 10^{-5} eV (ruling out axions as a dark matter candidate) unless K_a and n turn out to be small and large enough, respectively (see Ref. [16] and references therein). Fortunately, explicit models showing how scalar fields can be protected from acquiring large mass due to quantum gravity effects have already been worked out (see, e.g., Ref. [17]). In our context, assuming the electroweak symmetry breaking of the standard model which has an energy scale of $\Lambda_{\rm ESM} \sim 100$ GeV, the corresponding mass shift would be $\delta m^2 \sim K\Lambda_{\rm ESM}^{n+2}/M_P^n$, where again K and n are unknown. Here, we pragmatically *assume* that Planck-scale effects will not shift the mass of our scalar field beyond 10^{-12} eV. At this point, it is difficult to say how strong this assumption is because of our lack of understanding of the Planck-scale physics. Still, this is much less demanding than what is usually required for quintessence fields where the mass shift cannot typically exceed the mass scale defined by the Hubble constant 10^{-33} eV [18]. A detailed analysis of this issue would be welcome but goes far beyond the scope of this paper. For computational purposes, we take our scalar field to be massless.

By assuming stars with uniform and parabolic density profiles and suitable ξ values (typically $\xi > 1/6$ and $\xi \lesssim -2$), it was shown in Ref. [2] that tachyonic modes do appear for M/R_s ratios compatible with neutron-like stars.

Now, let us proceed by recalling that the positive-norm in-modes $u_{\mathbf{k}}^{(+)}$, which in region A look like as exhibited in Eq. (34), will emerge, in general, as a combination of positive- and negative-norm modes $\{v_{\varpi l\mu}^{(+)}, v_{\varpi l\mu}^{(-)}\}$ and $\{w_{\Omega l\mu}^{(+)}, w_{\Omega l\mu}^{(-)}\}$ in region B [see Eqs. (36) and (37)]. Hence, not only the in-vacuum will not coincide in general with the out-vacuum but also at least some of the in-modes will certainly go through a phase of exponential growth provided, of course, the existence of tachyonic modes $w_{\Omega l\mu}^{(\pm)}$. This leads to what was denominated *vacuum awakening effect in relativistic stars*, i.e., an exponential amplification of the vacuum fluctuations [1,2]. In order to see this, we use Eq. (35) to calculate

$$ {}_{\rm in}\langle 0|\hat{\Phi}^2|0\rangle_{\rm in} \overset{\rm (B)}{\sim} \kappa \frac{e^{2\bar{\Omega}t}}{8\pi\bar{\Omega}}\left(\frac{F_{\bar{\Omega}0}(\chi)}{r(\chi)}\right)^2[1 + \mathcal{O}(e^{-\epsilon t})], \tag{46}$$

where $F_{\bar{\Omega}0}(\chi)$ denotes the solution of Eq. (39) with the most negative eigenvalue, $-\bar{\Omega}^2$ (taking $l = 0$, which is the most favorable case), $\epsilon = {\rm const} > 0$, and κ is a constant of order unity whose value depends on (i) projections of modes $u_{\mathbf{k}}^{(\pm)}$ on $w_{\bar{\Omega}l\mu}^{(\pm)}$ and (ii) the quantum state, assumed here to be the in-vacuum $|0\rangle_{\rm in}$. It is worthwhile to emphasize that ultraviolet divergences, which should be renormalized to obtain $\langle\hat{\Phi}^2\rangle$, are associated with the $\varpi \to \infty$ sector of the oscillatory modes [see Eq. (35)] and does not concern the tachyonic modes ($\Omega^2 \leq \bar{\Omega}^2 < \infty$), which are the ones giving the dominant contribution in Eq. (46) (because of the $\exp(2\bar{\Omega}t)$ term). Accordingly, the expectation value of the vacuum energy density (23), namely,

$$ {}_{\rm in}\langle 0|\hat{\rho}|0\rangle_{\rm in} \equiv n^a\kappa^b\, {}_{\rm in}\langle 0|\hat{T}_{ab}|0\rangle_{\rm in},$$

experiences an exponential amplification:

$$ {}_{\rm in}\langle 0|\hat{\rho}|0\rangle_{\rm in} \overset{\rm (B)}{\sim} \kappa\frac{\bar{\Omega}e^{2\bar{\Omega}t}}{16\pi\sqrt{f}}\left\{\frac{1-4\xi}{2r^2}\frac{d}{d\chi}\left(r^2\frac{d}{d\chi}\left(\frac{F_{\bar{\Omega}0}}{\bar{\Omega}r}\right)^2\right) + \frac{\xi}{\bar{\Omega}^2r^2}\frac{d}{d\chi}\left(\frac{F_{\bar{\Omega}0}^2}{f}\frac{df}{d\chi}\right)\right\}[1 + \mathcal{O}(e^{-\epsilon t})]. \tag{47}$$

The time scale which rules how fast the vacuum energy density increases is given by $\bar{\Omega}^{-1} \sim |V_{\rm eff}^{(0)}|^{-1/2} \sim R_s$ [see Eq. (43)]. By using this, we rewrite Eq. (47) as

$$ {}_{\rm in}\langle 0|\hat{\rho}|0\rangle_{\rm in} \overset{\rm (B)}{\sim} \bar{\Omega} h(\bar{r}) e^{2\bar{\Omega} t}/R_s^3 \overset{\rm (B)}{\sim} h(\bar{r}) \times \exp\left[\frac{t/(10^{-5}\ {\rm s})}{R_s/(10\ {\rm km})}\right] \frac{10^{-62}\ {\rm g/cm^3}}{R_s^4/(10\ {\rm km})^4}, \quad (48) $$

where $h(\bar{r})$ is a dimensionless function of $\bar{r} \equiv r/R_s$ (which vanishes asymptotically and is of order unity for $\bar{r} \sim 1$). We see from Eq. (48) that once the effect is triggered by a neutron-like star with $R_s \approx 10$ km, few milliseconds would be enough for the vacuum energy density to become dominant over the star classical mass-energy density (which can be as high as 10^{14}–10^{17} g/cm^3). We must emphasize that at some point the spacetime must backreact against the growth of the vacuum energy density, affecting the field and ceasing the instability by taming the tachyonic modes. Eventually, field and spacetime must reach a new stable configuration. As argued in Ref. [3], one possibility would be that for some values of ξ the spontaneous scalarization mechanism [4] could restabilize the star. In our context, this would correspond to a symmetry breaking which would lead $\langle\hat{\Phi}\rangle$, which is null as calculated in the $|0\rangle_{\rm in}$ vacuum state, to acquire a nonzero large value compatible with the exponentially amplified $\langle\hat{\Phi}^2\rangle$ [see Eq. (46)]. Whether the star will end up destroyed or somehow rebalanced is unknown at this point.

We close this section explaining how the vacuum energy density amplification is consistent with energy conservation discussed below Eq. (23). For this purpose, let us note that $\nabla_a(\kappa_b \hat{T}^{ab}) = 0$ can be rewritten as

$$ \partial_t \hat{\rho} + \frac{1}{\sqrt{^{(3)}g}} \partial_i\left(\sqrt{^{(3)}g}\,\hat{j}^i\right) = 0, \quad (49) $$

where $\hat{j}^i \equiv -\sqrt{f}\hat{T}^i_0$ is the energy-current density. The corresponding vacuum expectation value can be calculated and reads

$$ {}_{\rm in}\langle 0|\hat{j}^i|0\rangle_{\rm in} \overset{\rm (B)}{\sim} -\bar{\Omega} V^i(\bar{r}) e^{2\bar{\Omega} t}/R_s^3, \quad (50) $$

where ${}^{(3)}\nabla_i V^i \equiv ({}^{(3)}g)^{-1/2}\partial_i(\sqrt{^{(3)}g}\,V^i) = 2\bar{\Omega} h(\bar{r})$. Thus, the total energy is conserved because the gravitational field redistributes the vacuum energy density in such a way that an amplification of ${}_{\rm in}\langle 0|\hat{\rho}|0\rangle_{\rm in}$ somewhere with positive magnitude must be compensated elsewhere by a corresponding amplification with negative magnitude.

IV. PROBING THE AWOKEN PHASE USING DETECTORS

Now, in order to probe the building up of the vacuum energy density in region B, where the vacuum is awake by the presence of tachyonic modes, we will look directly at the response of Unruh-DeWitt detectors. We shall do so because, as discussed in Sec. II B, the Fock-space states have no natural particle-content interpretation [see discussion below Eq. (20)]. Here, we relax the spherical symmetry assumption of the previous section and consider regions A and B as described by the line elements (31). Because we want to avoid any contributions in the response coming from the motion of the apparatus, the detector (with proper time τ) is made to lie static following an integral curve $x = x(\tau)$ of the time-like isometry.

We consider here a two-level Unruh-DeWitt detector represented by a Hermitian operator $\hat{m}_0$ acting in a Hilbert space spanned by unexcited and excited energy eigenstates $|E_0\rangle$ and $|E\rangle$ ($E > E_0$), respectively. The detector is prepared to be initially unexcited. For our purposes, it is convenient to switch it on in the very beginning of region B, where we set $\tau \equiv 0$. At the tree level, the excitation probability as a function of the proper-time interval T is given by [7]

$$ P_{\rm exc} = |\langle E|\hat{m}_0|E_0\rangle|^2 \mathcal{F}(\Delta E), \quad (51) $$

where $\Delta E \equiv E - E_0$ and the response function is

$$ \mathcal{F}(\Delta E) = \int_0^T d\tau \int_0^T d\tau' e^{-i\Delta E(\tau-\tau')} G^+_{\rm in}[x(\tau), x(\tau')] \quad (52) $$

with

$$ G^+_{\rm in}[x(\tau), x(\tau')] \equiv {}_{\rm in}\langle 0|\hat{\Phi}[x(\tau)]\hat{\Phi}[x(\tau')]|0\rangle_{\rm in}. $$

The two-point function is written throughout the spacetime in terms of the in-modes as

$$ G^+_{\rm in}[x, x'] = \int d^3k\, u^{(+)}_{\mathbf{k}}(x) u^{(-)}_{\mathbf{k}}(x'), \quad (53) $$

where we recall that in the asymptotic past $u^{(\pm)}_{\mathbf{k}}(x)$ take the simple form given in Eq. (34). Next, we suitably decompose $u^{(+)}_{\mathbf{k}}$ in terms of $\{v^{(\pm)}_\sigma\}$ and $\{w^{(\pm)}_\Omega\}$ as

$$ u^{(+)}_{\mathbf{k}} = \alpha^*_{\Omega\mathbf{k}} w^{(+)}_\Omega - \beta_{\Omega\mathbf{k}} w^{(-)}_\Omega + \int d\mu(\sigma)(\alpha^*_{\sigma\mathbf{k}} v^{(+)}_\sigma - \beta_{\sigma\mathbf{k}} v^{(-)}_\sigma), \quad (54) $$

where, for the sake of simplicity, we have assumed that the scalar field is made unstable in region B by the existence of a single tachyonic mode [see Eq. (19)]:

$$ w^{(+)}_\Omega \overset{\rm (B)}{=} \frac{(e^{\Omega t - i\pi/12} + e^{-\Omega t + i\pi/12})}{\sqrt{2\Omega} N_{\rm (B)}(\mathbf{x})} F^{\rm (B)}_\Omega(\mathbf{x}), \quad \Omega > 0. \quad (55) $$

The tachyonic mode above will be labeled by Ω (with the other quantum numbers being omitted to simplify notation), while the oscillatory modes are cast here as [see Eq. (15)]

$$ v^{(+)}_\sigma \overset{\rm (B)}{=} \frac{e^{-i\varpi_\sigma t}}{\sqrt{2\varpi_\sigma} N_{\rm (B)}(\mathbf{x})} F^{\rm (B)}_\sigma(\mathbf{x}), \qquad \varpi_\sigma > 0. \quad (56) $$

The Bogoliubov coefficients in Eq. (54) are

$$\alpha_{\sigma\mathbf{k}} = (u_{\mathbf{k}}^{(+)}, v_{\sigma}^{(+)})_{\rm KG}, \qquad \beta_{\sigma\mathbf{k}} = -(u_{\mathbf{k}}^{(-)}, v_{\sigma}^{(+)})_{\rm KG},$$
$$\alpha_{\Omega\mathbf{k}} = (u_{\mathbf{k}}^{(+)}, w_{\Omega}^{(+)})_{\rm KG}, \qquad \beta_{\Omega\mathbf{k}} = -(u_{\mathbf{k}}^{(-)}, w_{\Omega}^{(+)})_{\rm KG}$$

as calculated in *any* Cauchy surface.

Now, we must proceed and evaluate the response function (52). For this purpose, it is convenient to use Eqs. (54)–(56) to calculate

$$\int_0^T d\tau e^{-i\Delta E\tau} u_{\mathbf{k}}^{(+)}[x(\tau)] = \int d\mu(\sigma)\Big[\alpha_{\sigma\mathbf{k}}^* \frac{F_\sigma^{(\rm B)}}{N_{(\rm B)}}\psi_{\sigma+} - \beta_{\sigma\mathbf{k}}\frac{F_\sigma^{(\rm B)*}}{N_{(\rm B)}}\psi_{\sigma-}^*\Big]_{\mathbf{x}=\mathbf{x}_d} + \Big[\alpha_{\Omega\mathbf{k}}^*\frac{F_\Omega^{(\rm B)}}{N_{(\rm B)}}\Psi_+ - \beta_{\Omega\mathbf{k}}\frac{F_\Omega^{(\rm B)*}}{N_{(\rm B)}}\Psi_-^*\Big]_{\mathbf{x}=\mathbf{x}_d}, \tag{57}$$

where we have defined

$$\Psi_\pm \equiv \int_0^T d\tau \frac{e^{\mp i\Delta E\tau}(e^{\Omega\tau/N_{(\rm B)}-i\pi/12} + e^{-\Omega\tau/N_{(\rm B)}+i\pi/12})}{\sqrt{2\Omega}} = \frac{1}{\sqrt{2\Omega}}\Big[\frac{e^{i\pi/12}(1-e^{-(\Omega/N_{(\rm B)}\pm i\Delta E)T})}{\Omega/N_{(\rm B)}\pm i\Delta E} + \frac{e^{-i\pi/12}(e^{(\Omega/N_{(\rm B)}\mp i\Delta E)T}-1)}{\Omega/N_{(\rm B)}\mp i\Delta E}\Big], \tag{58}$$

$$\psi_{\sigma\pm} \equiv \frac{1}{\sqrt{2\varpi_\sigma}}\int_0^T d\tau e^{-i(\varpi_\sigma/N_{(\rm B)}\pm\Delta E)\tau} = \frac{2e^{-i(\varpi_\sigma/N_{(\rm B)}\pm\Delta E)T/2}}{\sqrt{2\varpi_\sigma}}\frac{\sin[(\varpi_\sigma/N_{(\rm B)}\pm\Delta E)T/2]}{\varpi_\sigma/N_{(\rm B)}\pm\Delta E}, \tag{59}$$

and $\mathbf{x} = \mathbf{x}_d$ is the detector's spatial position. Then, we write the detector response function (52) with the help of Eq. (57) as

$$\mathcal{F}(\Delta E) = [\mathcal{F}_0 + \mathcal{F}_1 + \mathcal{F}_2]_{\mathbf{x}=\mathbf{x}_d}, \tag{60}$$

where

$$\mathcal{F}_0 = \int d^3k \Big|\alpha_{\Omega\mathbf{k}}^*\frac{F_\Omega^{(\rm B)}}{N_{(\rm B)}}\Psi_+ - \beta_{\Omega\mathbf{k}}\frac{F_\Omega^{(\rm B)*}}{N_{(\rm B)}}\Psi_-^*\Big|^2, \tag{61}$$

$$\mathcal{F}_1 = \int d^3k\int d\mu(\sigma) \times 2{\rm Re}\Big[\Big(\alpha_{\Omega\mathbf{k}}^*\frac{F_\Omega^{(\rm B)}}{N_{(\rm B)}}\Psi_+ - \beta_{\Omega\mathbf{k}}\frac{F_\Omega^{(\rm B)*}}{N_{(\rm B)}}\Psi_-^*\Big) \times\Big(\alpha_{\sigma\mathbf{k}}\frac{F_\sigma^{(\rm B)*}}{N_{(\rm B)}}\psi_{\sigma+}^* - \beta_{\sigma\mathbf{k}}^*\frac{F_\sigma^{(\rm B)}}{N_{(\rm B)}}\psi_{\sigma-}\Big)\Big], \tag{62}$$

$$\mathcal{F}_2 = \int d^3k\Big|\int d\mu(\sigma)\Big(\alpha_{\sigma\mathbf{k}}^*\frac{F_\sigma^{(\rm B)}}{N_{(\rm B)}}\psi_{\sigma+} - \beta_{\sigma\mathbf{k}}\frac{F_\sigma^{(\rm B)*}}{N_{(\rm B)}}\psi_{\sigma-}^*\Big)\Big|^2. \tag{63}$$

The physical meaning of the response $\mathcal{F}(\Delta E)$ is more easily grasped in the case where the proper time interval T is "large", i.e., $T \gg \Delta E^{-1}$ (in addition to $T \gg \Omega^{-1}$ whenever the tachyonic mode is present) and we will assume this hereafter up to the end of this section. In this case, Eqs. (58) and (59) can be written as

$$\Psi_\pm \approx \frac{e^{-i(\pi/12\pm\Delta E)T}e^{T\Omega/N_{(\rm B)}}}{\sqrt{2\Omega}(\Omega/N_{(\rm B)}\mp i\Delta E)} \tag{64}$$

and

$$\psi_{\sigma\pm} \approx \sqrt{\frac{2\pi^2}{\varpi_\sigma}}e^{-i(\varpi_\sigma/N_{(\rm B)}\pm\Delta E)T/2}\delta(\varpi_\sigma/N_{(\rm B)}\pm\Delta E), \tag{65}$$

respectively. For the sake of comparison, we shall discuss separately the situations where the tachyonic mode $w_\Omega^{(+)}$ is present from the one where it is absent.

In the case where the tachyonic mode $w_\Omega^{(+)}$ is absent, $F_\Omega^{(\rm B)} = 0$ and, thus, the detector response becomes simply

$$\mathcal{F}(\Delta E) = \mathcal{F}_2|_{\mathbf{x}=\mathbf{x}_d}.$$

This is the usual result whose interpretation is straightforward: assuming that the detector stays switched on for an arbitrarily long time T, we have from Eq. (65) that $\psi_{\sigma+} \approx 0$ and according to Eq. (63) the detector excites by the absorption of particles created due to the spacetime transition from regions A to B ($\beta_{\sigma\mathbf{k}} \neq 0$). [Note that when no restriction is posed on T, the detector excitation will also have a contribution coming from the process of switching it on and off ($\psi_{\sigma+} \neq 0$) [19].]

It is also interesting to note that in the absence of the tachyonic mode, the response will not grow faster than T. By recalling from Eq. (65) that $\psi_{\sigma+} \approx 0$, we write the response as [see Eq. (63)]

$$\mathcal{F}(\Delta E) \approx \int d^3k\Big|\int d\mu(\sigma)\frac{F_\sigma^{(\rm B)*}}{N_{(\rm B)}}\beta_{\sigma\mathbf{k}}\psi_{\sigma-}^*\Big|^2_{\mathbf{x}=\mathbf{x}_d} \leq \int d^3k\Big(\int d\mu(\sigma)|\beta_{\sigma\mathbf{k}}|^2\Big) \times\Big(\int d\mu(\sigma)|\psi_{\sigma-}|^2\frac{|F_\sigma^{(\rm B)}|^2}{N_{(\rm B)}^2}\Big)_{\mathbf{x}=\mathbf{x}_d}, \tag{66}$$

where we have used above the Cauchy-Schwarz inequality. Now, by using Eq. (59) [or directly Eq. (65)] in conjunction with the identity $\lim_{A\to\infty}\sin^2(\omega A)/\omega^2 A = \pi\delta(\omega)$, we obtain

$$|\psi_{\sigma-}|^2 \approx (\pi T/\varpi_\sigma)\delta(\varpi_\sigma/N_{(\rm B)} - \Delta E). \tag{67}$$

This is used in Eq. (66) to conclude indeed that the response will not grow faster than T:

$$\mathcal{F}(\Delta E) \lesssim C_2 T \tag{68}$$

with

$$C_2 = \int d^3k \left(\int d\mu(\sigma) |\beta_{\sigma\mathbf{k}}|^2 \right) \times \left(\frac{\pi}{\Delta E} \int d\mu(\sigma) \frac{|F_\sigma^{(\mathrm{B})}|^2}{N_{(\mathrm{B})}^2} \delta(\varpi_\sigma - N_{(\mathrm{B})} \Delta E) \right)_{\mathbf{x}=\mathbf{x}_d}. \quad (69)$$

On the other hand, assuming that the tachyonic mode $w_\Omega^{(+)}$ is present, the detector response will be dominated by Eq. (61) and, thus, $\mathcal{F}(\Delta E) \approx \mathcal{F}_0|_{\mathbf{x}=\mathbf{x}_d}$. Thus, we use Eq. (64) in Eq. (61) to obtain

$$\mathcal{F}(\Delta E) \approx Z \exp[2T\Omega / N_{(\mathrm{B})}(\mathbf{x}_d)] \times \int d^3k \left| \alpha_{\Omega\mathbf{k}}^* \frac{F_\Omega^{(\mathrm{B})}}{N_{(\mathrm{B})}} e^{-i\pi/12} - \beta_{\Omega\mathbf{k}} \frac{F_\Omega^{(\mathrm{B})*}}{N_{(\mathrm{B})}} e^{i\pi/12} \right|^2_{\mathbf{x}=\mathbf{x}_d}, \quad (70)$$

where

$$Z = \frac{1}{2\Omega[\Delta E^2 + \Omega^2 / N_{(\mathrm{B})}^2(\mathbf{x}_d)]}.$$

The exponential increase in the detector response reflects the growth of the vacuum fluctuations and will continue as long as the unstable phase is not forced to terminate. This is possible because each excitation of the detector is accompanied by a corresponding decrease of the energy stored in the field due to the excitation of a tachyonic mode $\hat{c}_\Omega^\dagger |0\rangle_{\mathrm{in}}$ with negative energy expectation value [see Eq. (30) and corresponding discussion]. The copious excitation of the detector realizes the fact that in the unstable phase the scalar field functions as an energy reservoir only limited by backreaction effects.

V. FALLING ASLEEP OF THE VACUUM AND PARTICLE CREATION

A. General discussion

As already mentioned, at some point the unstable phase must cease, leading the system back to some stationary configuration. This will be represented by the static region C in Eq. (71), which completes the scenario presented by Eq. (31):

$$ds^2 = \begin{cases} -dt^2 + d\mathbf{x}^2 & (\mathrm{A}) \\ N_{(\mathrm{B})}^2(-dt^2 + h_{ij}^{(\mathrm{B})} dx^i dx^j) & (\mathrm{B}) \\ N_{(\mathrm{C})}^2(-dt^2 + h_{ij}^{(\mathrm{C})} dx^i dx^j) & (\mathrm{C}) \end{cases}. \quad (71)$$

Here, $N_{(J)} = N_{(J)}(\mathbf{x}) > 0$, $J \in \{\mathrm{B}, \mathrm{C}\}$, are smooth functions and $h_{ij}^{(J)} = h_{ij}^{(J)}(\mathbf{x})$ $(i, j = 1, 2, 3)$. We note that for the sake of simplicity we are using the same coordinate notation $(t, \mathbf{x})$ for the three epochs.

Because the field is assumed to be deprived of tachyonic modes in region C, we expand $\hat{\Phi}$ as

$$\hat{\Phi} = \int d\mu(\sigma) [\hat{d}_\sigma \nu_\sigma^{(+)} + \hat{d}_\sigma^\dagger \nu_\sigma^{(-)}], \quad (72)$$

where $\{\nu_\sigma^{(+)}, \nu_\sigma^{(-)}\}$ are normal modes which in region C satisfy

$$\nu_\sigma^{(+)} \overset{(\mathrm{C})}{=} \frac{e^{-i\varpi_\sigma t}}{\sqrt{2\varpi_\sigma} N_{(\mathrm{C})}(\mathbf{x})} F_\sigma^{(\mathrm{C})}(\mathbf{x}), \qquad \varpi_\sigma > 0, \quad (73)$$

and analogously for $\nu_\sigma^{(-)}$. All symbols in Eq. (73) can be inferred from Eq. (56) by replacing "B" by "C." Moreover, because region C is also static, we may wonder what will be the particle content of the scalar field in this region. The key point consists in realizing that the in-vacuum fluctuations which were exponentially amplified during the unstable phase cannot, in general, be accommodated as mere fluctuations of the out-vacuum state (the one defined by $d_\sigma |0\rangle_{\mathrm{out}} \equiv 0$ for all σ and which represents absence of particles according to static observers in region C). In conclusion, a burst of particles is expected as the field exits the unstable phase B.

Let us estimate the expectation number of created particles in the simplified case where the spacetime is symmetric by time reflection with respect to some Cauchy surface Σ_{t_S} in region B. Hence, we consider a particular case of Eq. (71), namely,

$$ds^2 = \begin{cases} -dt^2 + d\mathbf{x}^2 & (\mathrm{A}) \\ N_{(\mathrm{B})}^2(-dt^2 + h_{ij}^{(\mathrm{B})} dx^i dx^j) & (\mathrm{B}) \\ -dt^2 + d\mathbf{x}^2 & (\mathrm{C}) \end{cases}, \quad (74)$$

although we emphasize that we have chosen regions A and C to be flat only for the sake of simplicity; the same reasoning presented below may be straightforwardly applied to other static spacetimes. Now, we focus on the normal modes $u_{\mathbf{k}}^{(\pm)}$ and $\nu_{\mathbf{k}}^{(\pm)}$ with respect to which asymptotic observers in regions A and C define their no-particle states, respectively. In the past and future regions, they assume the following forms:

$$u_{\mathbf{k}}^{(\pm)} \overset{(\mathrm{A})}{=} (16\pi^3 \omega_{\mathbf{k}})^{-1/2} \exp[\mp i(\omega_{\mathbf{k}} t - \mathbf{k} \cdot \mathbf{x})], \quad (75)$$

$$\nu_{\mathbf{k}}^{(\pm)} \overset{(\mathrm{C})}{=} (16\pi^3 \omega_{\mathbf{k}})^{-1/2} \exp[\mp i(\omega_{\mathbf{k}} t - \mathbf{k} \cdot \mathbf{x})]. \quad (76)$$

We are interested in $u_{\mathbf{k}}^{(\pm)}$ and $\nu_{\mathbf{k}}^{(\pm)}$ evolved forward and backward to the beginning and end of region B, namely, $u_{\mathbf{k}}^{(\pm)}(t_0, \mathbf{x})$ and $\nu_{\mathbf{k}}^{(\pm)}(t_0 + T, \mathbf{x})$, respectively. Here, $t = t_0 \equiv t_S - T/2$ determines the beginning of the unstable phase B and T represents its coordinate-time duration. Because of our assumption that the spacetime is symmetric by time reflection with respect to Σ_{t_S}, we have (up to global phases)

$$u_{\mathbf{k}}^{(+)}(t_S - t, \mathbf{x}) = \nu_{-\mathbf{k}}^{(-)}(t_S + t, \mathbf{x}).$$

In particular, for $t = T/2$:

$$\nu^{(-)}_{-\mathbf{k}}(t_0 + T, \mathbf{x}) \overset{(B)}{=} u^{(+)}_{\mathbf{k}}(t_0, \mathbf{x}).$$

Then, we use Eq. (54) to decompose $u^{(+)}_{\mathbf{k}}$ in terms of $v^{(\pm)}_{\sigma}$ and $w^{(\pm)}_{\Omega}$, obtaining

$$\nu^{(-)}_{-\mathbf{k}}(t_0 + T, \mathbf{x}) \overset{(B)}{=} \alpha^*_{\Omega\mathbf{k}} w^{(+)}_{\Omega}(t_0, \mathbf{x}) - \beta_{\Omega\mathbf{k}} w^{(-)}_{\Omega}(t_0, \mathbf{x}) + \int d\mu(\sigma)[\alpha^*_{\sigma\mathbf{k}} v^{(+)}_{\sigma}(t_0, \mathbf{x}) - \beta_{\sigma\mathbf{k}} v^{(-)}_{\sigma}(t_0, \mathbf{x})], \quad (77)$$

where we assume again the existence of a single tachyonic mode for the sake of simplicity.

In order to investigate particle creation in region C, we must, e.g., project $u^{(+)}_{\mathbf{k}}(t_0 + T, \mathbf{x})$ into $\nu^{(-)}_{\mathbf{k}}(t_0 + T, \mathbf{x})$, where [see Eq. (54)]:

$$u^{(+)}_{\mathbf{k}}(t_0 + T, \mathbf{x}) = \alpha^*_{\Omega\mathbf{k}} w^{(+)}_{\Omega}(t_0 + T, \mathbf{x}) - \beta_{\Omega\mathbf{k}} w^{(-)}_{\Omega}(t_0 + T, \mathbf{x}) + \int d\mu(\sigma)[\alpha^*_{\sigma\mathbf{k}} v^{(+)}_{\sigma}(t_0 + T, \mathbf{x}) - \beta_{\sigma\mathbf{k}} v^{(-)}_{\sigma}(t_0 + T, \mathbf{x})]. \quad (78)$$

It is easy to see that

$$v^{(\pm)}_{\sigma}(t_0 + T, \mathbf{x}) = \exp(\mp i\varpi_\sigma T) v^{(\pm)}_{\sigma}(t_0, \mathbf{x}), \quad (79)$$

while we obtain from Eq. (55) that

$$w^{(\pm)}_{\Omega}(t_0 + T, \mathbf{x}) = \pm 2i \sinh(\Omega T \mp i\pi/6) w^{(\pm)}_{\Omega}(t_0, \mathbf{x}) \mp 2i \sinh(\Omega T) w^{(\mp)}_{\Omega}(t_0, \mathbf{x}). \quad (80)$$

Then, by using Eqs. (77) and (78), we obtain for large enough ΩT that

$$(\nu^{(-)}_{\mathbf{k}'}, u^{(+)}_{\mathbf{k}})_{\mathrm{KG}} \sim e^{\Omega T} \zeta_{\mathbf{k}\mathbf{k}'},$$

where the Klein-Gordon inner product was realized on the Σ_{t_0+T} Cauchy surface and

$$\zeta_{\mathbf{k}\mathbf{k}'} = i[(\alpha^*_{\Omega\mathbf{k}} e^{-i\pi/6} - \beta_{\Omega\mathbf{k}})\alpha_{\Omega-\mathbf{k}'} - (\alpha^*_{\Omega\mathbf{k}} - \beta_{\Omega\mathbf{k}} e^{i\pi/6})\beta^*_{\Omega-\mathbf{k}'}].$$

This leads to an expectation number of created particles with quantum numbers $\mathbf{k}'$ given by

$$\langle N_{\mathbf{k}'} \rangle \sim e^{2\Omega T} \int d^3k |\zeta_{\mathbf{k}\mathbf{k}'}|^2,$$

which grows exponentially as scaled by the product ΩT. In particular, even if the transitions from regions A to B and from regions B to C were made arbitrarily slow in order to minimize any particle creation due to background change ($\beta_{\sigma\mathbf{k}} \approx 0$), this would not alter the fact that a large amount of particles would be eventually created as the vacuum falls asleep (at least in the present scenario; see additional comments at the end of Sec. V B).

Next, we shall show that the burst of particles calculated above does not rely on phase B being static; it will occur as long as the in-vacuum fluctuations get significantly amplified.

B. A toy model

In order to illustrate our general conclusion above, let us make an explicit calculation assuming a concrete scenario complying with the asymptotic static regions A and C considered in Eq. (74) but assuming some time evolution in the intermediate region B. This is in agreement with the idealized situation where initially spread out matter collapses to form a compact object and eventually disperses back to infinity. Instead of calculating the particle production over the whole space, we shall restrict attention to the interior of a small cubical box with coordinate volume L^3 (oblivious to the matter forming the star), initially empty (of Φ particles), placed in the very beginning at the spatial position where the star core will form. The convenience of introducing a small box is that we can cover its interior with approximately Cartesian spatial coordinates $\tilde{\mathbf{x}}$ (in which first-order spatial derivatives of the metric are negligible), writing the line element as

$$ds^2 \approx a^2(-dt^2 + d\tilde{\mathbf{x}}^2). \quad (81)$$

Here, $a = a(t) > 0$ is introduced to reflect the background time evolution at the star's center ($a = 1$ in regions A and C) and we have omitted the second-order spatial dependence of the metric (which, nevertheless, contribute to the scalar-curvature term). The background evolution is chosen such that at some point the vacuum in the box is awaken by the presence of (sixfold degenerate) tachyonic modes [20]. After the unstable phase is terminated, we calculate the number of massless scalar particles which were created inside the box.

Using Eq. (81), we write Eq. (3) as

$$\frac{1}{a^4}\frac{\partial}{\partial t}\left(a^2 \frac{\partial \Phi}{\partial t}\right) - \frac{1}{a^2}\nabla^2\Phi + \xi R \Phi = 0, \quad (82)$$

where $\nabla^2 \equiv \sum_j \partial^2/\partial \tilde{x}^{j^2}$ is the usual Laplace operator. Assuming, for the sake of simpilicity, periodic boundary conditions, we look for solutions of Eq. (82) in the form

$$\phi_{\mathbf{k}}(t, \tilde{\mathbf{x}}) = \frac{\chi_{\mathbf{k}}(t)}{a(t)\sqrt{L^3}} e^{i\mathbf{k}\cdot\tilde{\mathbf{x}}}, \quad (83)$$

where $\mathbf{k} \equiv 2\pi\mathbf{n}/L$ with $\mathbf{n} \in \mathbb{Z}^3$ ($\mathbf{n} \neq \mathbf{0}$). By using Eq. (83) in Eq. (82) we find that

$$\left[-\frac{d^2}{dt^2} - V_{\mathrm{eff}}(t)\right]\chi_{\mathbf{k}} = \mathbf{k}^2\chi_{\mathbf{k}}, \quad (84)$$

where

$$V_{\mathrm{eff}}(t) = a^2\xi R - a^{-1}d^2a/dt^2. \quad (85)$$

Equation (83) makes explicit another neat feature of introducing the small box: the boundary condition which it imposes locks the spatial dependence of the modes so that the time evolution can only mix modes with the same $\tilde{\mathbf{x}}$ dependence. This property will be used later to simplify the Bogoliubov-coefficient calculation.

Now, we assume that energy density and pressure of ordinary matter at the center of the star drives R in Eq. (85) to induce the following simple form for the effective potential:

$$V_{\rm eff}(t) = \begin{cases} 0 \ \text{ for } \ t \le 0 \ \text{ and } \ t \ge \eta_0 \\ 4V_0(t/\eta_0)(t/\eta_0 - 1) \ \text{ for } \ 0 < t < \eta_0 \end{cases}, \qquad (86)$$

where η_0, $V_0 = \text{const} > 0$. We see that $V_{\rm eff}(t)$ has a parabolic form in the region $0 < t < \eta_0$ and reaches its minimum, $-V_0$, at $t = \eta_0/2$ (see Fig. 1).

Convenient sets of in-modes $\{U_{\mathbf{k}}^{(\pm)}\}$ for $t \le 0$ and out-modes $\{V_{\mathbf{k}}^{(\pm)}\}$ for $t \ge \eta_0$ are exhibited below:

$$U_{\mathbf{k}}^{(\pm)} \stackrel{t\le 0}{=} \frac{e^{\mp i(\omega_{\mathbf{k}} t - \mathbf{k}\cdot\tilde{\mathbf{x}})}}{\sqrt{2L^3\omega_{\mathbf{k}}}}, \qquad V_{\mathbf{k}}^{(\pm)} \stackrel{t\ge \eta_0}{=} \frac{e^{\mp i(\omega_{\mathbf{k}} t - \mathbf{k}\cdot\tilde{\mathbf{x}})}}{\sqrt{2L^3\omega_{\mathbf{k}}}}, \qquad (87)$$

where $\omega_{\mathbf{k}} \equiv \| \mathbf{k} \|$ The general expression of $U_{\mathbf{k}}^{(\pm)}$ which complies with Eq. (83) and fits with its form (87) in region A is

$$U_{\mathbf{k}}^{(\pm)}(t, \tilde{\mathbf{x}}) = \frac{\chi_{\mathbf{k}}^{(\pm)}(t)}{a(t)\sqrt{L^3}} e^{\pm i\mathbf{k}\cdot\tilde{\mathbf{x}}} \qquad (88)$$

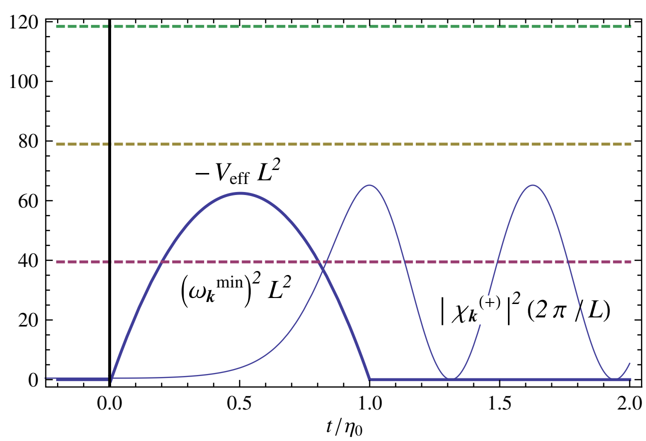

FIG. 1 (color online). We plot $-V_{\rm eff}$ as a function of t for $V_0 = 1.6/(L/2\pi)^2$ with the three smallest $\omega_{\mathbf{k}} \equiv 2\pi \| \mathbf{n} \| /L$ possible values (see horizontal dashed lines). We also plot $|\chi_{\mathbf{k}}^{(+)}|^2$ for $\| \mathbf{k} \| = \omega_{\mathbf{k}}^{\rm min}$ assuming $\eta_0 = 5L/2\pi$. Initially $|\chi_{\mathbf{k}}^{(+)}|^2$ equals $L/4\pi$, then grows exponentially in the unstable region, where $-V_{\rm eff} - (\omega_{\mathbf{k}}^{\rm min})^2 > 0$, and eventually oscillates around $(|\beta_{\mathbf{k}}|^2 + 1/2)(L/2\pi)$. The large amplitude which characterizes $|\chi_{\mathbf{k}}^{(+)}|^2$ at the end of the unstable phase reflects the fact that the in-vacuum fluctuations do not evolve into mere fluctuations of the out-vacuum state.

with $\chi_{\mathbf{k}}^{(\pm)}(t \le 0) = e^{\mp i\omega_{\mathbf{k}} t}/\sqrt{2\omega_{\mathbf{k}}}$. From the spatial dependence of the modes, we readily see that [recall the discussion below Eq. (85)]

$$U_{\mathbf{k}}^{(+)}(t, \tilde{\mathbf{x}}) = \alpha_{\mathbf{k}} V_{\mathbf{k}}^{(+)}(t, \tilde{\mathbf{x}}) + \beta_{-\mathbf{k}} V_{-\mathbf{k}}^{(-)}(t, \tilde{\mathbf{x}}), \qquad (89)$$

where the Bogoliubov coefficients between the bases $\{U_{\mathbf{k}}^{(\pm)}\}$ and $\{V_{\mathbf{k}'}^{(\pm)}\}$ are

$$\alpha_{\mathbf{k}\mathbf{k}'} = \alpha_{\mathbf{k}'}\delta_{\mathbf{k}\mathbf{k}'}, \qquad \beta_{\mathbf{k}\mathbf{k}'} = \beta_{\mathbf{k}'}\delta_{\mathbf{k}-\mathbf{k}'}.$$

For $\chi_{\mathbf{k}}^{(+)}$ in region C, Eqs. (87)–(89), imply

$$\chi_{\mathbf{k}}^{(+)}(t) \stackrel{t\ge\eta_0}{=} \alpha_{\mathbf{k}} \frac{e^{-i\omega_{\mathbf{k}} t}}{\sqrt{2\omega_{\mathbf{k}}}} + \beta_{-\mathbf{k}} \frac{e^{i\omega_{\mathbf{k}} t}}{\sqrt{2\omega_{\mathbf{k}}}},$$

from where $\alpha_{\mathbf{k}}$ and $\beta_{\mathbf{k}} = \beta_{-\mathbf{k}}$ can be easily obtained in terms of $\chi_{\mathbf{k}}^{(+)}$ and $\dot{\chi}_{\mathbf{k}}^{(+)} \equiv d\chi_{\mathbf{k}}^{(+)}/dt$ evolved into region C:

$$\alpha_{\mathbf{k}} = \left[\frac{e^{i\omega_{\mathbf{k}} t}}{\sqrt{2\omega_{\mathbf{k}}}}(\omega_{\mathbf{k}}\chi_{\mathbf{k}}^{(+)} + i\dot{\chi}_{\mathbf{k}}^{(+)})\right]_{t\ge\eta_0},$$

$$\beta_{\mathbf{k}} = \left[\frac{e^{-i\omega_{\mathbf{k}} t}}{\sqrt{2\omega_{\mathbf{k}}}}(\omega_{\mathbf{k}}\chi_{\mathbf{k}}^{(+)} - i\dot{\chi}_{\mathbf{k}}^{(+)})\right]_{t\ge\eta_0}.$$

[It can be easily verified using Eqs. (84) and (86) that the expressions for $\alpha_{\mathbf{k}}$ and $\beta_{\mathbf{k}}$ above do not depend on the value of $t \ge \eta_0$.] Therefore, assuming that the field is initially in the no-particle state $|0\rangle_{\rm in}$ with respect to asymptotic past observers as defined by the in-modes $U_{\mathbf{k}}^{(\pm)}$, the expectation value of created particles in region C [7,21]

$${}_{\rm in}\langle 0|\hat{N}_{\rm out}|0\rangle_{\rm in} = \sum_{\mathbf{k},\mathbf{k}'} |\beta_{\mathbf{k}\mathbf{k}'}|^2$$

is given by

$$\begin{aligned} {}_{\rm in}\langle 0|\hat{N}_{\rm out}|0\rangle_{\rm in} &= \sum_{\mathbf{k}} |\beta_{\mathbf{k}}|^2 \\ &= \sum_{\mathbf{k}} \left[\frac{|\dot{\chi}_{\mathbf{k}}^{(+)}|^2}{2\omega_{\mathbf{k}}} + \frac{\omega_{\mathbf{k}}|\chi_{\mathbf{k}}^{(+)}|^2}{2} - \frac{1}{2}\right]_{t\ge\eta_0}, \end{aligned} \qquad (90)$$

where $\hat{N}_{\rm out} \equiv \sum_{\mathbf{k}} \hat{a}_{\mathbf{k}}^{\rm out\dagger}\hat{a}_{\mathbf{k}}^{\rm out}$ with $\hat{a}_{\mathbf{k}}^{\rm out}$ and $\hat{a}_{\mathbf{k}}^{\rm out\dagger}$ being the annihilation and creation operators defined with respect to the out-modes $V_{\mathbf{k}}^{(\pm)}$, respectively. This provides an expression for obtaining the expectation number $|\beta_{\mathbf{k}}|^2$ of created particles with quantum numbers $\mathbf{k}$ once the oscillatory in-mode $\chi_{\mathbf{k}}^{(+)}$ is (numerically) evolved until region C.

The assumption to illustrate the appearance of tachyonic modes consists in choosing a star which becomes dense enough and a coupling ξ such that $-V_{\rm eff} - (\omega_{\mathbf{k}}^{\rm min})^2 > 0$ for the least energetic (sixfold degenerate) modes allowed in the box, $\omega_{\mathbf{k}}^{\rm min} = 2\pi/L$, for some time interval. As a result, the corresponding $\chi_{\mathbf{k}}^{(+)}$ solutions satisfying Eq. (84) are verified to exponentially grow for some time rather than to oscillate, triggering the vacuum awakening effect (see Fig. 1). In Fig. 2, we plot $|\beta_{\mathbf{k}}|^2$ as a function of η_0, which

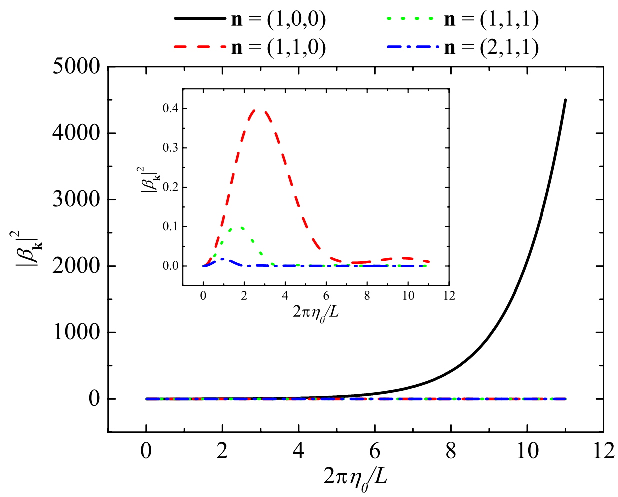

FIG. 2 (color online). The expectation value of created particles $|\beta_{\mathbf{k}}|^2$ with quantum numbers $\mathbf{k} = 2\pi\mathbf{n}/L$ is exhibited as a function of η_0, where we have assumed that for some time interval the star becomes dense enough such that $-V_{\rm eff} - (\omega_{\mathbf{k}}^{\rm min})^2 > 0$, while for $\omega_{\mathbf{k}} > \omega_{\mathbf{k}}^{\rm min}$ we always have $-V_{\rm eff} - \omega_{\mathbf{k}}^2 < 0$ (see Fig. 1). Here, we have chosen $V_0(L/2\pi)^2 = 1.6$.

scales with the time interval during which the vacuum stays awakened.

Clearly, the final state is dominated by modes with $\omega_{\mathbf{k}}^{\rm min} = 2\pi/L$, which have experienced a phase of exponential growth. The intensity of the particle burst is strongly influenced by how long the vacuum remains awake. The inset of Fig. 2 focuses on modes with $\omega_{\mathbf{k}} > 2\pi/L$, which are not exponentially enhanced, and stresses the usually modest particle creation observed in time-varying spacetimes with asymptotic flat regions [7,21]. We note that in the adiabatic limit, where the background geometry changes arbitrarily slowly ($\eta_0 \to \infty$), particle creation for modes with $\omega_{\mathbf{k}} > 2\pi/L$ goes to zero as expected, in contrast to the ones for $\omega_{\mathbf{k}}^{\rm min} = 2\pi/L$, which diverges. For a 10 m side box, an awakening time interval corresponding to $\eta_0 \sim 10^{-6}$ s would eventually lead to a massive creation of particles, with energy $2\pi/L$, engendering densities of 10^{14} g/cm^3, which is the typical density for some compact stars. If we relax our small box assumption and take $L \sim 10$ km, the same density would be reached for $\eta_0 \sim 10^{-3}$ s. Interestingly enough, this corresponds to the time interval for the vacuum energy density to take control over the evolution of the compact star once the vacuum awakening effect is triggered [see discussion below Eq. (48)].

We stress that the conclusions above are derived by assuming the effective potential (86), which is symmetric by time reflection, and can change depending on the star evolution. In order to show this, let us discuss the energetics of particle creation in the context of our toy model. For this purpose, it is useful to make the transformation $\Phi \to \tilde{\Phi} = a\Phi$ to write Eq. (82) for $\tilde{\Phi}$ as

$$[\partial^2/\partial t^2 - \nabla^2 + V_{\rm eff}(t)]\tilde{\Phi} = 0. \qquad (91)$$

Thus, we have translated the problem into the simpler one of a scalar field $\tilde{\Phi}$ in a flat spacetime ($\mathbb{R}^4$, η_{ab}) subject to an external time-dependent potential $V_{\rm eff}(t)$.

The action which gives rise to Eq. (91) and its corresponding stress-energy tensor are

$$S \equiv -\frac{1}{2}\int_{\mathbb{R}^4} d^4x\sqrt{-\eta}(\partial_a\tilde{\Phi}\partial^a\tilde{\Phi}^* + V_{\rm eff}\tilde{\Phi}\tilde{\Phi}^*) \qquad (92)$$

and

$$T_{ab} = \partial_{(a}\tilde{\Phi}\partial_{b)}\tilde{\Phi}^* - \frac{1}{2}\eta_{ab}[\partial_c\tilde{\Phi}\partial^c\tilde{\Phi}^* + V_{\rm eff}\tilde{\Phi}\tilde{\Phi}^*], \qquad (93)$$

respectively. Next, it can be shown from Eqs. (91) and (93) that

$$\partial_a(T^a_b(\partial_t)^b) = -\frac{1}{2}\frac{dV_{\rm eff}}{dt}|\tilde{\Phi}|^2, \qquad (94)$$

which can be rewritten as

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = \frac{1}{2}\frac{dV_{\rm eff}}{dt}|\tilde{\Phi}|^2, \qquad (95)$$

where

$$\rho \equiv \frac{1}{2}\left(\frac{\partial\tilde{\Phi}}{\partial t}\frac{\partial\tilde{\Phi}^*}{\partial t} + \nabla\tilde{\Phi}\cdot\nabla\tilde{\Phi}^* + V_{\rm eff}|\tilde{\Phi}|^2\right)$$

and

$$\mathbf{j} \equiv -\frac{1}{2}\left(\frac{\partial\tilde{\Phi}^*}{\partial t}\nabla\tilde{\Phi} + \frac{\partial\tilde{\Phi}}{\partial t}\nabla\tilde{\Phi}^*\right).$$

We see from Eq. (95) that the energy stored in the scalar field is not locally conserved whenever $dV_{\rm eff}/dt \neq 0$. The extra energy pumped into or out of the field is accounted by the "external agent" responsible to change $V_{\rm eff}$. Moreover, notice from Eq. (95) that even if $V_{\rm eff}(t)$ is symmetric under time reflection, the decrease in the field energy when it enters the unstable phase ($dV_{\rm eff}/dt < 0$ and small vacuum fluctuations $\langle\hat{\Phi}^2\rangle$) is more than compensated by the increase in the field energy when it exits the unstable phase ($dV_{\rm eff}/dt > 0$ and large vacuum fluctuations $\langle\hat{\Phi}^2\rangle$). In fact, the latter can be overwhelmingly larger than the former, with the net extra energy being responsible for the particle burst. This analysis implies that in a physical situation, the final verdict concerning the amount of created particles will depend on a more detailed understanding on the spacetime evolution in the unstable phase, which would inform us about how long the vacuum would stay awake, and on the final classical configuration reached by the gravitational and scalar fields, which would tell us how much energy would turn out available for particle creation.

A closing remark for this section is in order. In our calculations the expectation value of the field remains zero throughout the background evolution. However, this field configuration $\langle\hat{\Phi}\rangle = 0$ is obviously unstable during the intermediate phase when the vacuum is awake. Thus, one may speculate whether during the transition to the intermediate phase the classical field profile $\langle\hat{\Phi}\rangle$ could continuously adjust itself to nonzero values ("continuous spontaneous scalarization") in order to stabilize the system, in which case tachyonic-like modes would never really be present. Unfortunately, a definite verdict to this question is beyond the scope of semiclassical gravity since it involves the subtleties of decoherence of a *free* field, initially in a state which is symmetric by the exchange $\Phi \leftrightarrow -\Phi$, to a symmetry-broken phase in which $\langle\hat{\Phi}\rangle \neq 0$. Notwithstanding, a reasonable conjecture seems to be that coherence can be sustained for as long as the background geometry (which can be regarded as the *sole* classical "apparatus" with which the field interacts) is oblivious to Φ. In other words, it seems quite possible that $\langle\hat{\Phi}\rangle = 0$ until backreaction becomes important. But when that happens, the fluctuations $\langle\hat{\Phi}^2\rangle$ will already be amplified to the point where they cannot be accommodated as mere vacuum fluctuations. A burst of particles should follow, regardless whether $\langle\hat{\Phi}\rangle$ remains null or spontaneous scalarization takes place. Another important point is that spontaneous scalarization takes place only for negative values of ξ [3]. Therefore, for $\xi > 0$ the whole scenario of a gradually changing $\langle\hat{\Phi}\rangle$ ensuring the stability of the system seems even more unlikely.

VI. FINAL REMARKS

After a review of the vacuum awakening effect in relativistic stars, we have probed the exponential increase of the quantum field fluctuations using Unruh-DeWitt detectors. The fast increase of these fluctuations may lead eventually to an important burst of free-field particles after the vacuum is forced to fall asleep again. This burst of particles would draw a significant amount of energy from the initial system. The amount of created particles will depend on the duration of the unstable epoch and on the final gravitational and scalar field configuration, which are open issues at this point. A possible signal favoring the vacuum awakening effect for a free field would be the unveiling of astrophysical events outpouring less amounts of *visible* energy than would be expected. This may also provide an efficient way of converting energy initially stored in the form of ordinary matter (forming the star) into a "dark" component which couples only to gravity.

Simultaneous to the completion of this article, it was posted a classical analysis showing that during the scalarization process a strong emission of scalar radiation should occur [22], which is in line with the conclusions presented here, especially with the discussion at the end of the previous section.

ACKNOWLEDGMENTS

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