# New cosmic accelerating scenario without dark energy

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We propose an alternative, nonsingular, cosmic scenario based on gravitationally induced particle production. The model is an attempt to evade the coincidence and cosmological constant problems of the standard model ( $\Lambda$ CDM) and also to connect the early and late time accelerating stages of the Universe. Our space-time emerges from a pure initial de Sitter stage thereby providing a natural solution to the horizon problem. Subsequently, due to an instability provoked by the production of massless particles, the Universe evolves smoothly to the standard radiation dominated era thereby ending the production of radiation as required by the conformal invariance. Next, the radiation becomes subdominant with the Universe entering in the cold dark matter dominated era. Finally, the negative pressure associated with the creation of cold dark matter (CCDM model) particles accelerates the expansion and drives the Universe to a final de Sitter stage. The late time cosmic expansion history of the CCDM model is exactly like in the standard ACDM model; however, there is no dark energy. The model evolves between two limiting (early and late time) de Sitter regimes. All the stages are also discussed in terms of a scalar field description. This complete scenario is fully determined by two extreme energy densities, or equivalently, the associated de Sitter Hubble scales connected by  $\rho_I/\rho_f = (H_I/H_f)^2 \sim 10^{122}$ , a result that has no correlation with the cosmological constant problem. We also study the linear growth of matter perturbations at the final accelerating stage. It is found that the CCDM growth index can be written as a function of the  $\Lambda$  growth index,  $\gamma_{\Lambda} \simeq 6/11$ . In this framework, we also compare the observed growth rate of clustering with that predicted by the current CCDM model. Performing a  $\chi^2$  statistical test we show that the CCDM model provides growth rates that match sufficiently well with the observed growth rate of structure.

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# I. INTRODUCTION

The existence of a cosmological constant  $\Lambda$ , which can be used in order to explain the recent cosmic acceleration, has brought the following major theoretical problem: within the framework of the quantum field theory the vacuum energy density is more than 120 orders of magnitude larger than the observed  $\Lambda$  value measured by the current cosmological data. This is the so called ''old'' cosmological constant problem  $[1,2]$  $[1,2]$  $[1,2]$ . The "new" problem  $[3]$  $[3]$  asks, why is the vacuum density so similar to the matter density just now? Many solutions to both theoretical problems have been proposed in the literature [\[4–](#page-9-3)[6\]](#page-9-4). An easy way to overpass the above problems is to replace the constant vacuum energy with a dark energy (DE) that evolves with time. However the nature of DE is far from being understood. Indeed a main caveat of this methodology is the fact that the majority of the DE models that appeared in the literature are plagued with no physical basis and/or many free parameters.

Nevertheless, there are other possibilities to explain the present accelerating stage. In particular, the inclusion of the backreaction in the Einstein field equations (EFE) via an effective pressure (which is negative for an expanding space-time) opened the way for cosmological applications. In these models, the gravitational production of radiation or cold dark matter provides a mechanism for cosmic acceleration as earlier discussed in Refs. [[7–](#page-9-5)[9](#page-9-6)]. As a consequence, several interesting features of cosmologies where the dark sector is reduced due to the creation of CDM matter have been discussed in the last decade [\[10–](#page-9-7)[13](#page-9-8)].

In brief, the merits of the particle creation scenario with respect to the usual DE ideology are (a) the former has a strong physical basis namely nonequilibrium thermodynamics, while the latter (DE) has not and (b) the particle creation mechanism unifies the dark sector (dark energy and dark matter), since a single dark component (the dark matter) needs to be introduced into the cosmic fluid and thus it contains only one free parameter. Interestingly, from the viewpoint of a statistical Bayesian analysis models which include only one free parameter should be preferred along the hierarchy of cosmological models [[14](#page-9-9)]. We would like to emphasize here that the only cosmological models (to our knowledge) which satisfy the above statistical condition are

- (i) The concordance  $\Lambda$ CDM which however suffers from the coincidence and fine-tuning problems  $[4-6]$  $[4-6]$  $[4-6]$ .
- (ii) The braneworld cosmology of Ref. [\[15\]](#page-9-10) which however does not fit the  $SNIa + BAO + CMB$ (shift-parameter) data (see Ref. [\[16\]](#page-9-11)).

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(iii) The current particle creation model which simultaneously fits the observational data and alleviates the coincidence and fine-tuning problems [\[12,](#page-9-12)[13\]](#page-9-8).

In this paper, we are proposing a new cosmological scenario which is complete in the following sense: all the accelerating stages of the cosmic evolution are powered uniquely by the gravitational creation of massless (at the very early stage) and massive cold dark matter particles (at the late stages).

In our scenario, the Universe starts from an unstable de Sitter dominated phase ( $a \propto e^{H_1 t}$ ) powered by the production of massless particles, and, as such, there is no the horizon problem. Subsequently, it deflates and evolves to the standard radiation phase ( $a \propto t^{1/2}$ ) thereby ending the creation of massless particles. Due to expansion, the radiation becomes subdominant with the Universe entering in the cold dark matter (CDM) dominated era, in which the linear growth of matter fluctuations is taking place in a natural way. Finally, the negative pressure associated with the creation of cold dark matter particles accelerates the expansion and drives the Universe to a final de Sitter stage. In addition, the transition from Einstein-de Sitter ( $a \propto t^{2/3}$ ) to a de Sitter final stage ( $a \propto e^{H_f t}$ ) guarantees the consistence of the model with the supernovae type Ia data and complementary observations, including the growth rate of clustering. A transition redshift of the order of a few (exactly the same value predicted by  $\Lambda$ CDM) is also obtained.

The paper is structured as follows. In Sec. [II,](#page-1-0) we discuss the basic ideas underlying the particle production in an expanding universe and set up the basic equations whose solutions describe the complete evolution of our model. In Sec. [III](#page-4-0), we study the linear growth of perturbations, whereas in Sec. [IV,](#page-6-0) we constrain the growth index through a statistical analysis involving the latest observational results. In Sec. [V,](#page-7-0) a possible scalar field description for all stages is discussed, and, finally, in Sec. [VI](#page-8-0) we summarize the basic results.

# II. A COMPLETE COSMOLOGICAL SCENARIO WITH GRAVITATIONAL PARTICLE PRODUCTION

<span id="page-1-0"></span>The microscopic description for gravitationally induced particle production in an expanding universe began with Schrödinger's [\[17\]](#page-9-13) seminal paper, which referred to it as an alarming phenomenon. In the late 1960s, this issue was rediscussed by Parker and others [[18](#page-9-14)–[20](#page-9-15)] based on the Bogoliubov mode-mixing technique in the context of quantum field theory in a curved space-time described by the general relativity theory  $[21]$  $[21]$  $[21]$ . Physically, one may think that the (classical) time varying gravitational field works like a ''pump'' supplying energy to the quantum fields.

In order to understand the basic approach, let us consider a real minimally coupled massive scalar field  $\phi$  evolving in a flat expanding Friedman-Robertson-Walker (FRW) geometry. In units where  $\hbar = k_B = c = 1$ , the field is described by the following action:

$$
S = \frac{1}{2} \int \sqrt{-g} d^4x [g^{\alpha \beta} \partial_{\alpha} \phi \partial_{\beta} \phi - m^2 \phi^2].
$$
 (1)

In terms of the conformal time  $\eta$  ( $dt = a(\eta)d\eta$ ), the metric tensor  $g_{\mu\nu}$  is conformally equivalent to the Minkowski metric  $\eta_{\mu\nu}$ , so that the line element is  $ds^2 =$  $a^2(\eta)\eta_{\mu\nu}dx^{\mu}dx^{\nu}$ , where  $a(\eta)$  is the cosmological scale factor. Writing the field  $\phi(\eta, x) = a(\eta)^{-1} \chi$ , one obtains from the above action

$$
\chi'' - \nabla^2 \chi + \left( m^2 a^2 - \frac{a''}{a} \right) \chi = 0, \tag{2}
$$

<span id="page-1-1"></span>where the prime here denotes derivatives with respect to  $\eta$ . Notice that the field  $\chi$  obeys the same equation of motion as a massive scalar field in Minkowski space-time, but now with a time dependent *effective mass*,

$$
m_{\text{eff}}^2(\eta) \equiv m^2 a^2 - \frac{a''}{a}.\tag{3}
$$

This time varying mass accounts for the interaction between the scalar and the gravitational fields. The energy of the field  $\chi$  is not conserved (its action is explicitly time dependent), and, more important, its quantization leads to particle creation at the expense of the classical gravitational background [[18](#page-9-14)–[20](#page-9-15)].

On the other hand, in the framework of general relativity theory, the scale factor of a FRW-type universe dominated by radiation ( $a \propto t^{1/2}$ ) satisfies the following relation:  $[a\ddot{a} + \dot{a}^2 = 0]$ , or, in the conformal time,  $a'' = 0$ . Therefore, for massless fields ( $m = 0$ ), there is no particle production since Eq. [\(2\)](#page-1-1) reduces to the same of a massless field in Minkowski space-time, and, as such, its quantization becomes trivial. This is the basis of the Parker theorem concerning the absence of massless particle production in the early stages of the Universe. Note that Parker's result does not forbid the production of massless particles in a very early de Sitter stage  $(a^{\prime\prime} \neq 0)$ . Potentially, we also see that massive particles can always be produced by a time varying gravitational field. As we shall see, such features are incorporated in the scenario proposed here.

In principle, for applications in cosmology, the above semiclassical results have three basic difficulties, namely:

- (i) The scalar field was treated as a test field, and, therefore, the FRW background is not modified by the newly produced particles.
- (ii) The particle production is an irreversible process, and, as such, it should be constrained by the second law of thermodynamics.
- (iii) There is no a clear prescription of how an irreversible mechanism of quantum origin can be incorporated in the EFE.

Later on, a possible macroscopic solution for these problems was put forward by Prigogine and co-workers [\[7](#page-9-5)] using nonequilibrium thermodynamics for open systems, and by Calvão *et al.* [\[8](#page-9-17)] through a covariant relativistic treatment for imperfect fluids (see also Ref. [\[9](#page-9-6)]). The novelty of such an approach is that particle production, at the expense of the gravitational field, is an irreversible process constrained by the usual requirements of nonequilibrium thermodynamics. This irreversible process is described by a negative pressure term in the stress tensor whose form is constrained by the second law of thermodynamics.<sup>1</sup> This macroscopic approach has also microscopically been justified by Zimdahl and collaborators through a relativistic kinetic theoretical formulation [\[22](#page-9-18)]. In comparison to the standard equilibrium equations, the irreversible creation process is described by two new ingredients: a balance equation for the particle number density and a negative pressure term in the stress tensor. Such quantities are related to each other in a very definite way by the second law of thermodynamics. Since the middle of the nineties, several interesting features of cosmologies with the creation of cold dark matter and radiation have been investigated by many authors [[10](#page-9-7),[23–](#page-9-19)[26](#page-10-0)].

<span id="page-2-7"></span>In what follows, as theoretically predicted by inflation and observationally indicated by the angular power spectrum of the temperature fluctuations, we consider the EFE for a homogeneous, isotropic, spatially flat universe with gravitationally induced particle production:

$$
8\pi G\rho = 3\frac{\dot{a}^2}{a^2},\tag{4}
$$

$$
8\pi G(p + p_c) = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2},
$$
 (5)

<span id="page-2-5"></span>where an overdot means time derivative,  $\rho$  and  $p$  are the dominant energy density and pressure of the cosmic fluid, respectively, and  $p_c$  is a dynamic pressure which depends on the particle production rate. Special attention has been paid to the simpler process termed ''adiabatic'' particle production. It means that particles and entropy are produced in the space-time, but the specific entropy (per particle),  $\sigma = S/N$ , remains constant [[8\]](#page-9-17). In this case, the creation pressure reads  $[7-11]$  $[7-11]$  $[7-11]$  $[7-11]$ 

$$
p_c = -\frac{(\rho + p)\Gamma}{3H},\tag{6}
$$

<span id="page-2-6"></span>where  $\Gamma$  with dimensions of  $(\text{time})^{-1}$  is the particle production rate and  $H = \dot{a}/a$  is the Hubble parameter [\[27\]](#page-10-1). In principle, the quantity  $\Gamma$  should be determined from quantum field theory in curved space-times by taking into account that particle production is an irreversible process.

But how is the evolution of  $a(t)$  affected by  $\Gamma$ ? By assuming a dominant cosmic fluid satisfying the equation of state,  $p = \omega \rho$ , where  $\omega$  is a constant, the EFE imply that

$$
\dot{H} + \frac{3}{2}(1+\omega)H^2\left(1 - \frac{\Gamma}{3H}\right) = 0.
$$
 (7)

<span id="page-2-1"></span>The de Sitter solution ( $\dot{H} = 0$ ,  $\Gamma = 3H = \text{constant}$ ) is now possible regardless of the equation of state defining the cosmic fluid. Since the Universe is evolving, such a solution is unstable, and, as long as  $\Gamma \ll 3H$ , conventional solutions without particle production are recovered. From the above equation, one may conclude that the main effect of  $\Gamma$  is to provoke a dynamic instability in the space-time thereby allowing a transition from a de Sitter regime  $(\Gamma \sim 3H)$  to a conventional solution, and vice versa (see Secs. [II A](#page-2-0) and [II B](#page-3-0) below).

# A. From an early de Sitter stage to the standard radiation phase

<span id="page-2-0"></span>Let us first discuss the transition from an initial de Sitter stage to the standard radiation phase. The main theoretical constraints are

- (i) The model must not only solve the horizon problem but also provide a quasiclassical boundary condition to quantum cosmology (a hint on how to solve the initial singularity problem).
- (ii) Massless particles cannot be quantum mechanically produced in the conventional radiation phase (Parker's theorem).

To begin with, let us assume a radiation dominated universe ( $\omega = 1/3$ ,  $\Gamma = \Gamma_r$ ). The dynamics is determined by the ratio  $\Gamma_r/3H$  [see Eq. ([7\)](#page-2-1)]. The most natural choice would be a ratio which favors no epoch in the evolution of the Universe  $(\Gamma_r/3H = \text{constant})$ . However, the particle production must be strongly suppressed,  $\Gamma_r/3H \ll 1$ , when the Universe enters the radiation phase. The simplest formula satisfying such a criterion is linear, namely,

$$
\frac{\Gamma_r}{3H} = \frac{H}{H_I},\tag{8}
$$

<span id="page-2-2"></span>where  $H<sub>I</sub>$  is the inflationary expansion rate associated to the initial de Sitter ( $H \leq H_I$ ). It is worth to notice that such particle creation rates have been previously discussed by several authors (see Refs. [\[7](#page-9-5)[,9](#page-9-6),[25](#page-10-2),[29](#page-10-3)] and references therein). It is also worth noticing that for adiabatic photon creation the form of the blackbody spectrum is preserved in the course of the expansion [[30](#page-10-4)].

<span id="page-2-3"></span>Now, inserting Eq. ([8](#page-2-2)) into Eq. ([7](#page-2-1)) it becomes

$$
\dot{H} + 2H^2 \left( 1 - \frac{H}{H_I} \right) = 0. \tag{9}
$$

<span id="page-2-4"></span>The solution of the above equation can be written as

$$
H(a) = \frac{H_I}{1 + Da^2},
$$
 (10)

where  $D \ge 0$  is an integration constant. Note that  $H = H<sub>I</sub>$ is a special solution of Eq. ([9\)](#page-2-3) describing the exponentially

<sup>&</sup>lt;sup>1</sup>The quantum approach is unable to provide the entropy burst accompanying the particle production since it is adiabatic and reversible.

expanding de Sitter space-time. This solution is unstable with respect to the critical value  $D = 0$ . For  $D > 0$ , the Universe starts without a singularity and evolves continuously towards a radiation stage,  $a \sim t^{1/2}$ , when  $Da^2 \gg 1$ . By integrating Eq.  $(10)$ , we obtain the scale factor:

$$
H_{I}t = \ln \frac{a}{a_{*}} + \frac{\lambda^{2}}{2} \left(\frac{a}{a_{*}}\right)^{2},
$$
 (11)

<span id="page-3-1"></span>where  $\lambda^2 = D a_*^2$  is an integration constant and  $a_*$  defines the transition from the de Sitter stage to the beginning of the standard radiation epoch.<sup>2</sup> At early times  $(a \ll a_*)$ , when the logarithmic term dominates, one finds  $a \approx$  $a_*e^{H_I t}$ , while at late times,  $a \gg a_*$ ,  $H \ll H_I$ , Eq. [\(11\)](#page-3-1) reduces to  $a \approx a_*(\frac{2H_I}{\lambda^2}t)^{1/2}$ , and the standard radiation phase is reached.

It should be noticed that the time scale  $H_I^{-1}$  provides the greatest value of the energy density,  $\rho_I = 3H_I^2/8\pi G$ , characterizing the initial de Sitter stage which is supported by the maximal radiation production rate,  $\Gamma_r = 3H_l$ . From Eqs.  $(4)$  $(4)$  and  $(10)$  we obtain the radiation energy density:

$$
\rho_r = \rho_I \left[ 1 + \lambda^2 \left( \frac{a}{a_*} \right)^2 \right]^{-2} . \tag{12}
$$

As expected, we see again that the conventional radiation phase,  $\rho_r \sim a^{-4}$ , is attained when  $a \gg a_*$ .

Now we pose the following question: how does the cosmic temperature evolve at the very early stages? For adiabatic production of relativistic particles the energy density scales as  $\rho_r \sim T^4$  [\[8,](#page-9-17)[9](#page-9-6)], and the above equation implies that

$$
T = T_I \left[ 1 + \lambda^2 \left( \frac{a}{a_*} \right)^2 \right]^{-1/2}, \tag{13}
$$

where  $T_I$  is the temperature of the initial de Sitter phase which must be uniquely determined by the scale  $H<sub>I</sub>$ . We see that the expansion proceeds isothermally during the de Sitter phase ( $a \ll a_*$ ) which means that the supercooling and subsequent reheating that is taking place in several inflationary variants are avoided [[34](#page-10-5),[35](#page-10-6)]. In other words, there is no ''graceful exit'' problem.

After the de Sitter stage, the temperature decreases continuously in the course of the expansion. For  $a \gg a_*$  $(H \ll H_I)$ , we obtain  $T \sim a^{-1}$ . Accordingly, the comoving number of photons becomes constant since  $n \propto a^{-3}$ , as expected for the standard radiation stage.<sup>3</sup>

In this context, we also need to answer the following question: what about the initial temperature  $T_I$ ? Since the model starts as a de Sitter space-time, the most natural choice is to define  $T<sub>I</sub>$  as the Gibbons-Hawking temperature [\[36](#page-10-7)] of its event horizon,  $T_I = H_I/2\pi$ . Naively, one may expect  $T_I$ of the same order or smaller than the Planck temperature because of the classical description. From EFE we have  $\rho_I$  =  $3m_{\text{Pl}}^2 H_I^2 / 8\pi$  (where  $m_{\text{Pl}} \simeq 1.22 \times 10^{19}$  GeV), and since the energy density is  $\rho_I = N_*(T)T_I^4$ , one finds  $T_I \sim H_I \sim$  $10^{19}$  GeV [where  $N_*(T) = \pi^2 g_*(T)/30$  depends on the number of effectively massless particles].

Naturally, due to the initial de Sitter phase, the model is free of particle horizons. A light pulse beginning at  $t = -\infty$ will have traveled by the cosmic time  $t$  a physical distance,  $d_H(t) = a(t) \int_{-\infty}^{t} \frac{d\tilde{t}}{a(\tilde{t})}$ , which diverges, thereby implying the absence of particle horizons. The latter feature means that the local interactions may homogenize the whole Universe.

Since photons are not produced in the radiation phase, the big bang nucleosynthesis may work in the conventional way [[37](#page-10-8)]. Subsequently, the Universe enters the cold dark matter [Einstein-de Sitter,  $a(t) \propto t^{2/3}$ ] dominated phase. Finally, we have also verified that a large class of  $\Gamma_r$  is capable to overcome the graceful exit and fine-tuning problem. Indeed we have found that the correct transition from an early de Sitter to the radiation phase is valid even for  $\Gamma_r \propto H^n$  with  $n \ge 2$  [\[38\]](#page-10-9). The details are of course different depending on the power  $n$ , but the qualitative fact of the transition is universal, and to our opinion this is very good news because it shows that it can be a clue for a general graceful exit mechanism.

### <span id="page-3-0"></span>B. From Einstein-de Sitter to a late time de Sitter stage

Due to the conservation of the baryon number the remaining question is the production rate of cold dark matter particles and the overall late time evolution. In other words, what is the form of  $\Gamma_{DM}$ ? For simplicity, we consider here only the dominant CDM component.

In principle,  $\Gamma_{DM}$  should be determined from quantum field theory in curved space-times. In the absence of a rigorous treatment, we consider (phenomenologically) the following fact [[39](#page-10-10)[–41](#page-10-11)]: All available observations are in accordance with the  $\Lambda$ CDM evolution both at the background and perturbative levels.

<span id="page-3-2"></span>Now, we recall that a flat  $\Lambda$ CDM model evolves like

$$
\dot{H} + \frac{3}{2}H^2 \bigg[ 1 - \left(\frac{H_f}{H}\right)^2 \bigg] = 0,\tag{14}
$$

<span id="page-3-3"></span>where  $H_f^2 = \Lambda/3$  sets the Hubble scale of the final de Sitter stage ( $H \geq H_f$ ). Such behavior should be compared to that predicted for a dust filled model ( $\omega = 0$ ,  $\Gamma = \Gamma_{DM}$ ) with particle production [see Eq. ([7\)](#page-2-1)]:

$$
\dot{H} + \frac{3}{2}H^2 \left( 1 - \frac{\Gamma_{\rm DM}}{3H} \right) = 0.
$$
 (15)

 $2$ This kind of evolution was first discussed by G.L. Murphy [\[31\]](#page-10-12) by studying possible effects of the second viscosity in the very early universe. Later on, it was also investigated in a more general framework involving cosmic strings by J. D. Barrow [\[32\]](#page-10-13) who coined the expression ''deflationary universes''. It has also been discussed in connection with decaying  $\Lambda(t)$  models [[33](#page-10-14)].

Since  $n_r \propto T^3$ , the average photon concentration reads  $n_r =$  $n_I[1 + \lambda^2(a/a_*)^2]^{-3/2}.$ 

By comparing Eqs.  $(14)$  and  $(15)$  $(15)$  $(15)$ , we see that the same background evolution requires that  $\Gamma_{DM}/3H = (H_f/H)^2$ . Thus, in the background solution, the particle creation rate does not depend on a given scale but rather it is considered homogeneous. The limiting value of the creation rate,  $\Gamma_{\text{DM}} = 3H_f$ , leads to a late time de Sitter phase ( $\dot{H} = 0$ ,  $H = H<sub>f</sub>$ ) thereby showing that the de Sitter solution now becomes an attractor at late times. With this proviso, the solution of Eq.  $(15)$  reads

<span id="page-4-1"></span>
$$
H^{2} = H_{0}^{2} E(z)^{2} = H_{0}^{2} [\tilde{\Omega}_{m} (1+z)^{3} + \tilde{\Omega}_{\Lambda}], \quad (16)
$$

where  $\tilde{\Omega}_\Lambda \equiv (H_f/H_0)^2 = 1 - \tilde{\Omega}_m$  is smaller than unity and  $1 + z = a^{-1}$ . Such a solution mimics the Hubble function  $H(z)$  of the traditional flat  $\Lambda$  cosmology, with  $\Omega_{\Lambda}$  playing the dynamical role of  $\Omega_{\Lambda}$  (dark energy appearing in the concordance model).4 The dark matter parameter  $(\Omega_{DM} = 1)$  is also replaced by an effective parameter,  $(\Omega_{DM})_{eff} = 1 - \tilde{\Omega}_{\Lambda}$ , which quantifies the amount of matter that is clustering. This explains why this model is in agreement with the dynamical determinations related to the amount of the cold dark matter at the cluster scale and, simultaneously, may also be compatible with the position of the first acoustic peak in the pattern of CMB anisotropies which requires  $\Omega_{\text{total}} = 1$ .

By integrating Eq.  $(16)$  $(16)$  $(16)$  we obtain

$$
a(t) = \left(\frac{\tilde{\Omega}_m}{\tilde{\Omega}_\Lambda}\right)^{1/3} \sinh^{\frac{2}{3}} \left(\frac{3H_0 \sqrt{\tilde{\Omega}_\Lambda}}{2} t\right).
$$
 (17)

Note that the late time dynamics is determined by a single parameter, namely,  $\tilde{\Omega}_{\Lambda} = 1 - \tilde{\Omega}_{m}$ , and is identical to that predicted by the flat  $\Lambda$ CDM model. Using the current, a joint statistical analysis, involving the latest observational data (SNIa [[42](#page-10-15)], BAO [[43](#page-10-16)] and CMB shift parameter [\[41\]](#page-10-11)) is implemented. We find that the overall likelihood function peaks at  $\tilde{\Omega}_m = 0.274 \pm 0.011$  with  $\chi^2_{\text{tot}}(\tilde{\Omega}_m) \simeq 543.18$  for 557 degrees of freedom. Since the current statistical results are in excellent agreement with those provided by WMAP7 [[41](#page-10-11)], for the rest of the paper we will restrict our present analysis to the choice  $(\Omega_m, \sigma_{8,0}) = (0.273, 0.811)$ , where  $\sigma_{8,0}$  is the rms mass fluctuations on scales of  $8h^{-1}$  Mpc at redshift  $z = 0$ .

In Fig. [1](#page-4-2), we show the overall evolution (radiation, matter, and dark energy dominated eras) of our complete cosmological scenario which coincides exactly with the one recently discussed in Refs. [\[12,](#page-9-12)[13](#page-9-8)] following a slightly different approach. Note also that by replacing the value of  $\Gamma_{DM}$  into the definition of the creation

<span id="page-4-2"></span>

FIG. 1 (color online). Overall evolution of the scale factor predicted by the matter creation model (solid line) and the traditional CDM cosmology (open points). In this plot we have adopted the best fit,  $\hat{\Omega}_m = 0.273$ , from WMAP7 [\[41\]](#page-10-11).

pressure [see Eq. [\(6\)](#page-2-6)] one obtains that it is negative and constant  $(p_c = -3H_f^2/8\pi G = -3\tilde{\Omega}_\Lambda H_0^2/8\pi G)$ .

# III. THE EVOLUTION OF THE LINEAR GROWTH FACTOR

<span id="page-4-0"></span>In this section, we briefly discuss the basic equation which governs the behavior of the linear matter perturbations  $\delta_m \equiv \delta \rho_m / \rho_m$  on subhorizon scales in the CCDM model, assuming that the particle creation rate remains homogeneous and only the corresponding effective dark matter forms structures. The reason for introducing the growth analysis here is to give the reader the opportunity to appreciate also at the perturbative level, the relative strength and similarities of the CCDM and  $\Lambda$ CDM models used to constrain the growth index. As discussed by Jesus et al. [[44](#page-10-17)] based on the neo-Newtonian approach [[45](#page-10-18)], the evolution equation of the matter fluctuations of a CCDM cosmology reads

<span id="page-4-3"></span>
$$
\frac{d^2\delta_m}{d\eta^2} + F(\eta)\frac{d\delta_m}{d\eta} + G(\eta)\delta_m = 0, \qquad (18)
$$

whose solution is  $\delta_m(\eta) \propto \mathcal{D}(\eta)$ , with  $\mathcal{D}(\eta)$  denoting the linear growing mode (usually scaled to unity at the present time). The functions appearing in  $(18)$  are defined by

$$
F(\eta) = \frac{\tilde{\Omega}_m(1 + 6c_p) + 2\tilde{\Omega}_\Lambda e^{3\eta}(8 + 3c_p)}{2(\tilde{\Omega}_m + \tilde{\Omega}_\Lambda e^{3\eta})}, \quad (19)
$$

 $\frac{4}{10}$ Observationally there is no reason to distinguish between  $\tilde{\Omega}_{\Lambda}$ which appears in Eq.  $(16)$  and that of the cosmological constant term parametrized by  $\Omega_{\Lambda}$ .

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$$
G(\eta) = \frac{9\tilde{\Omega}_m^2}{2(\tilde{\Omega}_m + \tilde{\Omega}_\Lambda e^{3\eta})^2} + \frac{15\tilde{\Omega}_\Lambda e^{3\eta}(1 + c_p) - 3\tilde{\Omega}_m(2 + c_p)}{\tilde{\Omega}_m + \tilde{\Omega}_\Lambda e^{3\eta}},
$$
 (20)

where  $\eta = \ln a(t)$  and  $\tilde{\Omega}_m = 1 - \tilde{\Omega}_\Lambda$ . The quantity  $c_p$  can be viewed as the ''effective adiabatic'' sound speed

$$
c_p \equiv c_{\text{eff ad}}^2 = \frac{\delta p_c}{\delta \rho_m},\tag{21}
$$

where  $\delta p_c$  is the perturbation of creation pressure. Obviously using the above equation and Eq. [\(6](#page-2-6)) we find that the corresponding effective adiabatic sound speed must be negative  $c_p = c_{\text{eff,ad}}^2 < 0$ . Note that the latter restriction is valid also for the interacting dark energy models (see Ref.  $[46]$  $[46]$  $[46]$ ).<sup>5</sup> We would like to stress that for simplicity we are using a constant  $c_p$  (for more discussions see Ref. [\[44\]](#page-10-17)). In this context, the functions  $F(\eta)$  and  $G(\eta)$ defined above can be written in terms of  $\Omega_m(\eta)$  as

$$
F(\Omega_m) = \frac{1 + c_p + 15(1 - \Omega_m)}{2},
$$
 (22)

<span id="page-5-3"></span>and

$$
G(\Omega_m) = \frac{9}{2}\Omega_m^2 + 15(1 + c_p)(1 - \Omega_m) - 3(2 + c_p)\Omega_m,
$$
\n(23)

<span id="page-5-4"></span>where the definition

$$
\Omega_m(\eta) = 1 - \Omega_\Lambda(\eta) = \frac{\tilde{\Omega}_m}{\tilde{\Omega}_m + \tilde{\Omega}_\Lambda e^{3\eta}},\qquad(24)
$$

has been adopted. At this point, we remind the reader that solving Eq. ([18](#page-4-3)) for the  $\Lambda$ CDM cosmology,<sup>6</sup> we derive the well-known perturbation growth factor (see Ref. [\[47\]](#page-10-20)):

$$
\mathcal{D}_{\Lambda}(z) = \frac{5\tilde{\Omega}_m E(z)}{2} \int_z^{+\infty} \frac{(1+u)du}{E^3(u)}.
$$
 (25)

Obviously, for  $E(z) \approx \tilde{\Omega}_m^{1/2} (1+z)^{3/2}$  it gives the standard result  $\mathcal{D}(z) \simeq a = e^{\eta} = (1+z)^{-1}$ , which corresponds to the matter dominated epoch, as expected.

Now, for any type of DE, an efficient parametrization of the matter perturbations is based on the growth rate of clustering originally introduced by Peebles [\[47\]](#page-10-20). This is

$$
f(\eta) = \frac{d \ln \delta_m}{d \eta} \simeq \Omega_m^{\gamma}(\eta),\tag{26}
$$

<span id="page-5-6"></span><span id="page-5-0"></span>which implies

$$
\mathcal{D}(a) = \exp\biggl[\int_1^a \frac{\Omega_m^{\gamma}(x)}{x} dx\biggr],\tag{27}
$$

where  $\gamma$  is the so called growth index (see Refs. [[48–](#page-10-21)[53](#page-10-22)]) which plays a key role in cosmological studies, especially in the light of recent large redshift surveys (like the WiggleZ and SDSS (DR9); see Refs. [[54](#page-10-23)[–56](#page-10-24)] and references therein). As an example, it was theoretically shown that for DE models which adhere to general relativity the growth index  $\gamma$  is well approximated in terms of the equation of state parameter  $\gamma_{\text{DE}} \simeq \frac{3(w-1)}{6w-5}$  $\frac{6w-1}{6w-5}$  (see Refs. [[48](#page-10-21)[,49,](#page-10-25)[52](#page-10-26)[,53](#page-10-22)]), which boils down to  $\gamma_{\Lambda} \approx 6/11$  for the ACDM cosmology  $w(z) = -1$ . Notice, that in the case of the braneworld model of Dvali *et al.* [[57](#page-10-27)] we have  $\gamma_{\text{DGP}} \approx 11/16$  (see also Refs. [\[52](#page-10-26),[58](#page-10-28)]), while for the  $f(R)$  gravity models we have  $\gamma_R \simeq 0.41 - 0.43$  [\[59\]](#page-10-29) at the present time.

<span id="page-5-1"></span>Differentiating Eq. [\(26\)](#page-5-0) with respect to  $\eta$  we have

$$
\frac{df}{d\eta} + f^2 = \frac{1}{\delta_m} \frac{d^2 \delta_m}{d\eta^2}.
$$
 (28)

<span id="page-5-2"></span>Using Eqs.  $(18)$  $(18)$  $(18)$ ,  $(26)$ , and  $(28)$  $(28)$  $(28)$ , we find after some algebra

$$
\frac{df}{d\Omega_m}\frac{d\Omega_m}{d\eta} + f^2 + F(\eta)f + G(\eta) = 0, \qquad (29)
$$

where

$$
\frac{d\Omega_m}{d\eta} = -3\Omega_m(\eta)[1 - \Omega_m(\eta)].
$$
 (30)

Inserting the ansatz  $f \approx \Omega_m^{\gamma(\Omega_m)}$  into Eq. ([29](#page-5-2)), using simultaneously Eqs. ([22](#page-5-3))–[\(24\)](#page-5-4) and performing a first order Taylor expansion around  $\Omega_m = 1$  (for a similar analysis see Refs. [\[52](#page-10-26)[,53\]](#page-10-22)) we find that the asymptotic value of the growth index to the lowest order is

<span id="page-5-5"></span>
$$
\gamma \simeq \frac{3(13 + 12c_p)}{11 + 6c_p} = \frac{\gamma_{\Lambda}(13 + 12c_p)}{2(1 + \gamma_{\Lambda}c_p)}.
$$
 (31)

Inverting the above equation we have

$$
c_p \simeq \frac{13\gamma_\Lambda - 2\gamma}{2\gamma_\Lambda(\gamma - 6)}.\tag{32}
$$

We have checked for various values of  $c_p$  and  $\Omega_m$ , that using Eq.  $(31)$  $(31)$  $(31)$  in Eq.  $(27)$  $(27)$  the latter provides an excellent approximation to the numerically obtained form of  $D(\eta)$ that appears in Eq. [\(18](#page-4-3)). Indeed the difference between the two approaches is less than 0.1%–0.2%. Finally, from the above analysis it becomes clear that the possible difference between the CCDM and  $\Lambda$ CDM predictions is quantified only at the perturbative level, via the value of  $\gamma$ , because the two cosmological models share the same Hubble parameter as well as the same number of free parameters,

 ${}^{5}$ Based on a scalar field description (in our case see Sec. [V\)](#page-7-0) one can show that the sound speed  $c_{s\phi}^2 = \delta p_{\phi}/\delta \rho_{\phi}$  is exactly unity. Indeed using similar arguments to those of Ref. [\[46](#page-10-19)] we also prove that  $\delta \rho_{\phi} = \delta(\frac{\dot{\phi}^2}{2} + V) = \dot{\phi} \delta \dot{\phi} = \delta(\frac{\dot{\phi}^2}{2} - V) = \delta p_{\phi}$ which implies  $c_{s\phi}^2 = 1$ . Valiviita *et al.* [[46](#page-10-19)] proposed that in order to avoid instabilities in the dark energy we have to have  $c_{s\phi}^2 = 1$  and  $c_{\text{eff,ad}}^2 < 0$ . Obviously both conditions are fulfilled

here.<br>
<sup>6</sup>For the usual  $\Lambda$ CDM cosmological model we have  $\frac{df}{d\Omega_m} \times \frac{d\Omega_m}{d\eta} + f^2 + X(\eta)f + G(\eta) = 0$  where  $X(\eta) = \frac{1}{2} + \frac{3}{2} \times$  $[1 - \Omega_m(\eta)]$  and  $\mathcal{G}(\eta) = -\frac{3}{2}\Omega_m(\eta)$ .

namely, the dimensionless matter density at the present epoch  $\tilde{\Omega}_m$  and the growth index. In the case of  $\gamma = \gamma_{\Lambda} \simeq$  $6/11$  we find  $c_p^* \approx -1.008$ . Hence expanding Eq. [\(31\)](#page-5-5) around  $c_p^*$  we can write

$$
\gamma \approx \frac{6}{11} + \gamma_{c_p^*}(c_p - c_p^*),\tag{33}
$$

where

$$
\gamma_{c_p^*} = \frac{d\gamma}{dc_p}(c_p^*) = \frac{162}{(11 + 6c_p^*)^2}.
$$
 (34)

# <span id="page-6-0"></span>IV. FITTING THE CCDM GROWTH INDEX TO THE DATA

In the following we briefly present some details of the statistical method and on the observational sample that we adopt in order to constrain either the growth index or the effective adiabatic sound speed, presented in the previous section.

### A. The growth data

The growth data that we will use in this work based on 2dF, VVDS, SDSS and WiggleZ galaxy surveys, for which their combination parameter of the growth rate of structure,  $f(z)$ , and the redshift-dependent rms fluctuations of the linear density field,  $\sigma_8(z)$ , is available as a function of the redshift,  $f(z)\sigma_8(z)$ . The  $f\sigma_8 \equiv A$  estimator is almost a model-independent way of expressing the observed growth history of the Universe [[60](#page-10-30)]. In particular the data used are based on

- (i) The 2dF (Percival et al. [[61](#page-10-31)]), SDSS-luminous red galaxies (Tegmark et al.  $[62]$ ) and VVDS (Guzzo et al. [\[63\]](#page-10-33)) based growth results as collected by Song and Percival [\[60\]](#page-10-30). This sample contains 3 entries.
- (ii) The SDSS (DR7) results (2 entries) of Samushia *et al.* [\[54\]](#page-10-23) based on spectroscopic data of  $\sim$ 106000 luminous red galaxies in the redshift bin  $0.16 < z < 0.44$ .
- (iii) The WiggleZ results of Blake *et al.* [\[55\]](#page-10-34) based on spectroscopic data of  $\sim$ 152000 galaxies in the

<span id="page-6-1"></span>TABLE I. Summary of the observed growth rate and references.

Index	Z.			Growth rate $(A_{obs})$ References Symbols (in Fig. 3)
	0.17	$0.510 \pm 0.060$	[60, 61]	Open circles
$\mathcal{D}_{\mathcal{L}}$	0.35	$0.440 \pm 0.050$	[60, 62]	Open circles
3	0.77	$0.490 \pm 0.180$	[60, 63]	Open circles
4	0.25	$0.351 \pm 0.058$	[54]	Open triangles
5	0.37	$0.460 \pm 0.038$	$\left[54\right]$	Open triangles
6	0.22.	$0.420 \pm 0.070$	[55]	Solid circles
7	0.41	$0.450 \pm 0.040$	$\left[55\right]$	Solid circles
8	0.60	$0.430 \pm 0.040$	$\left[55\right]$	Solid circles
9	0.78	$0.380 \pm 0.040$	$\left[55\right]$	Solid circles
10	0.57	$0.427 \pm 0.066$	[56]	Solid square

redshift bin  $0.1 < z < 0.9$ . This data set contains 4 entries.

(iv) The SDSS (DR9) results of Reid *et al.* [[56](#page-10-24)] based on spectroscopic data of  $\sim$ 264000 galaxies in the redshift bin  $0.43 < z < 0.70$ . This data set includes 1 entry.

In Table [I](#page-6-1) we list the precise numerical values of the data points with the corresponding errors bars.

#### B. Observational constraints

In order to constrain the CCDM growth index (or  $c_p$ ) we perform a standard  $\chi^2$  minimization procedure between the growth data measurements (see previous section),  $A_{obs}$  =  $f_{obs}(z)\sigma_{8,obs}(z)$ , with the growth values predicted by the CCDM model at the corresponding redshifts,  $A(\mathbf{p}, z) =$  $f(\mathbf{p}, z)\sigma_8(\mathbf{p}, z)$  with  $\sigma_8(\mathbf{p}, z)=\sigma_{8,0}D(\mathbf{p}, z)$ . The vector **p** contains the free parameters of the cosmological model. In particular, the essential free parameters entering in the theoretical expectation are  $\mathbf{p} \equiv (\gamma, \tilde{\Omega}_m)$ . The  $\chi^2$  function is defined as

$$
\chi^2(z_i|\mathbf{p}) = \sum_{i=1}^N \left[ \frac{A_{\text{obs}}(z_i) - A(\mathbf{p}, z_i)}{\sigma_i} \right]^2, \tag{35}
$$

where  $\sigma_i$  is the observed growth rate uncertainty. Note that we sample  $\gamma \in [0.1, 1.3]$  in steps of 0.001.

In the left panel of Fig. [2](#page-6-2) we show the variation of  $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$  around the best  $\gamma$  fit value for the CCDM cosmology. We find that the likelihood function peaks at  $\gamma = 0.60 \pm 0.072$  (or  $c_p = -1 \pm 0.011$ ; see the right panel of Fig. [2\)](#page-6-2) with  $\chi^2_{\text{min}} \approx 7.75$  for 8 degrees of freedom. Notice, that for the physically acceptance range  $0.22 \leq \tilde{\Omega}_m \leq 0.32$ , we obtain either  $0.50 \leq \gamma \leq 0.70$ or  $-1.016 \le c_p \le -0.983$  with  $\chi^2_{\text{min}}/8 \in [0.90, 1.03]$ . Hence the effective sound speed varies very little in function of  $\Omega_m$ . Alternatively, considering the  $\Lambda$ CDM theoretical value of  $\gamma$ (= 6/11) and minimizing with respect to

<span id="page-6-2"></span>

FIG. 2. Left panel: The variance  $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$  around the best fit  $\gamma$  value for the CCDM cosmology. Note that the cross corresponds to  $(\gamma_{\Lambda}, \Delta \chi^2_{1\sigma}) = (\frac{6}{11}, 1)$ . Right panel: The  $\Delta \chi^2$ versus  $c_p$ . The corresponding cross is  $(c_p, \Delta \chi^2_{1\sigma}) = (-1.008, 1)$ .

<span id="page-7-1"></span>

FIG. 3. Comparison of the observed and theoretical evolution (see solid line) of the growth rate of clustering  $A(z) = f(z)\sigma_8(z)$ . The thin-line error bars correspond to  $1\sigma \gamma$  uncertainties. The different growth data sets are represented by different symbols (see Table [I](#page-6-1) or definitions).

 $\Omega_{m0}$  we find  $\Omega_m = 0.243 \pm 0.034$  (see also Ref. [[53](#page-10-22)]) with  $\chi^{2}_{\rm min}/\rm{dof} \simeq 7.37/8.$ 

Our best-fit  $\gamma$  value is in agreement within  $1\sigma (\Delta \chi^2_{1\sigma} \simeq 1)$ uncertainty with the theoretically predicted value of  $\gamma_{\Lambda} \simeq$  $6/11$  (see the cross in Fig. [2\)](#page-6-2). Finally, in Fig. [3,](#page-7-1) we plot the measured  $A_{obs}(z)$  with the estimated growth rate function,  $A(z) = f(z)\sigma_8(z)$  (see solid line).

We would like to finish this section with the following observation: if the particles are created proportional to the DM density (see, for instance Refs. [\[9,](#page-9-6)[25](#page-10-2),[29](#page-10-3)]) then at each point the growth of matter density perturbations will speed up with respect to an homogeneous particle creation rate, changing the growth factor and thus it could potentially affect the observational constraints. However, in our case we do not face such a problem because the corresponding particle creation rate term  $[\Gamma_{DM}3H = (H_f/H)^2]$  in the matter dominated era becomes  $\Gamma_{DM}/3H \propto 1/\rho$ . Notice that in order to obtain the latter relation we use Eq.  $(4)$ .

#### V. SCALAR FIELD DESCRIPTION

<span id="page-7-0"></span>Matter creation models constitute a possible way to explain the cosmic acceleration without the introduction of a dark energy component. However, it is sometimes desirable to represent the cosmic evolution in a field theoretical language, i.e., in terms of the dynamics of an ordinary scalar field  $(\phi)$ . In a point of fact, all the dynamical stages discussed here can be described through a simple scalar field model (for a similar analysis see Ref. [[25](#page-10-2)]).

To begin with, let us replace  $\rho$  and  $p_{\text{tot}} = p + p_c$  in Eqs. ([4\)](#page-2-5) and [\(5](#page-2-7)) by corresponding scalar field expressions

$$
\rho \to \rho_{\phi} = \frac{\dot{\phi}^2}{2} + V(\phi), \qquad p_{\text{tot}} \to p_{\phi} = \frac{\dot{\phi}^2}{2} - V(\phi).
$$
\n(36)

<span id="page-7-4"></span>Inserting the latter into the Friedmann's equations we can separate the scalar field contributions and express them in terms of H and  $\dot{H}$ , i.e.,

$$
\dot{\phi}^2 = -\frac{1}{4\pi G}\dot{H},\qquad(37)
$$

$$
V = \frac{3H^2}{8\pi G} \left( 1 + \frac{\dot{H}}{3H^2} \right) = \frac{3H^2}{8\pi G} \left( 1 + \frac{aH'}{3H} \right),\tag{38}
$$

<span id="page-7-2"></span>where  $\dot{H} = aHH'$  and prime here denote the derivative with respect to the scale factor. Now, considering that  $dt = da/aH$ , Eq. ([37](#page-7-2)) can be integrated to give

<span id="page-7-3"></span>
$$
\phi = \int \left( -\frac{\dot{H}}{4\pi G} \right)^{1/2} dt = \frac{1}{\sqrt{4\pi G}} \int \left( -\frac{H'}{aH} \right)^{1/2} da. \tag{39}
$$

### A. Early de Sitter radiation: Deflationary stage

In this case the Hubble function is given by Eq. ([10\)](#page-2-4). Obviously we can integrate Eq.  $(39)$ , in the interval [0, a], to obtain

$$
\phi(a) = \frac{1}{\sqrt{2\pi G}} \sinh^{-1}(\sqrt{D}a),
$$

$$
= \frac{1}{\sqrt{2\pi G}} \ln(\sqrt{D}a + \sqrt{D}a^2 + 1).
$$
(40)

Note that at the time of inflation ( $a = a_*$ ) the corresponding scalar field is

$$
\phi_* = \frac{1}{\sqrt{2\pi G}} \ln(\lambda + \sqrt{\lambda^2 + 1}).\tag{41}
$$

Also after some simple algebra, the potential energy becomes

$$
V(a) = \frac{H_I^2}{8\pi G} \frac{3 + Da^2}{(1 + Da^2)^2},
$$
 (42)

or

$$
V(\phi) = \frac{H_I^2}{8\pi G} \frac{3 + \sinh^2(\sqrt{2\pi G}\phi)}{[1 + \sinh^2(\sqrt{2\pi G}\phi)]^2}.
$$
 (43)

In the context of slow-roll approximation, one can prove that the density fluctuations are of the form  $\delta_H \sim$  $H^2/\dot{\phi}^2 \sim 10^{-5}$  (see Ref. [\[64\]](#page-10-35)). Using Eqs. [\(10\)](#page-2-4) and [\(37\)](#page-7-2) the function  $H^2/\dot{\phi}^2$  becomes

$$
\frac{H^2}{\dot{\phi}^2} = -4\pi G \frac{H^2}{\dot{H}} = -4\pi G \frac{H}{aH'} = 2\pi G \frac{(1+Da^2)}{Da^2}.
$$
 (44)

At the epoch of inflation ( $a = a_*$ ) we get

$$
\frac{H^2}{\dot{\phi}^2}(a_*) = 2\pi G \frac{(1+\lambda^2)}{\lambda^2}.
$$
 (45)

Inserting the latter into the density fluctuation constrain we obtain

$$
\frac{1}{\lambda^2} \sim \frac{\delta_H}{2\pi G} - 1 \sim \frac{\delta_H}{2\pi l_{\rm pl}^2},\tag{46}
$$

or

$$
\lambda \sim \sqrt{\frac{2\pi}{\delta_H}} l_{\rm pl},\tag{47}
$$

where  $l_{\text{pl}} \simeq \sqrt{G}$  is the Planck length in units  $\hbar = c \equiv 1$ .

### B. Accelerating dark matter stage

In this case the late evolution of the Hubble function is given by Eq.  $(16)$ . Now, inserting Eq.  $(16)$  $(16)$  $(16)$  as well as its derivative  $(H')$  into Eq. [\(39\)](#page-7-3) and integrating we obtain

<span id="page-8-1"></span>
$$
\phi(a) = A \ln \left[ \frac{\sqrt{1 - \tilde{\Omega}_{\Lambda} + \tilde{\Omega}_{\Lambda} a^3} - \sqrt{1 - \tilde{\Omega}_{\Lambda}}}{\sqrt{1 - \tilde{\Omega}_{\Lambda} + \tilde{\Omega}_{\Lambda} a^3} + \sqrt{1 - \tilde{\Omega}_{\Lambda}}} \right], \quad (48)
$$

<span id="page-8-2"></span>where  $A = 1/\sqrt{24\pi G(1 - \tilde{\Omega}_\Lambda)}$ . From Eqs. [\(16\)](#page-4-1) and [\(38\)](#page-7-4) it is possible to show that

$$
V(a) = \frac{H_0^2}{8\pi G} \left[ \tilde{\Omega}_{\Lambda} - \frac{1 + 2\tilde{\Omega}_{\Lambda}}{2} a^{-3} \right].
$$
 (49)

Finally, combining Eqs. [\(48\)](#page-8-1) and ([49](#page-8-2)) we find

$$
V(\phi) = B + C \cosh(\omega \phi), \tag{50}
$$

where  $B = H_0^2 (5 \tilde{\Omega}_\Lambda + \tilde{\Omega}_\Lambda^2)/32 \pi G (1 - \tilde{\Omega}_\Lambda), C = -H_0^2 (1 +$  $2\tilde{\Omega}_{\Lambda}$ ) $\tilde{\Omega}_{\Lambda}/32\pi G(1-\tilde{\Omega}_{\Lambda})$  and  $\omega = \sqrt{24\pi G(1-\tilde{\Omega}_{\Lambda})}$ .

# VI. FINAL REMARKS

<span id="page-8-0"></span>A new cosmology based on the production of massless particles (in the early de Sitter phase) and CDM particles (in the transition to a late time de Sitter stage) has been discussed. The same mechanism avoids the initial singularity, particle horizon and the late time coincidence problem of the  $\Lambda$ CDM model has been phenomenologically eliminated ( $\Lambda \equiv 0$ ) because there is no dark energy in our accelerating scenario. In particular, this means that the dark energy component may be only a ''mirage'' (an effective description), since it can be mimicked (globally and locally) by the gravitationally induced particle production mechanism acting in the evolving Universe.

In this scenario, the standard cosmic phases—a radiation era followed by an Einstein-de Sitter evolution driven by nonrelativistic matter until redshifts of the order of a few are not modified. However, the model has two extreme accelerating phases (very early and late time de Sitter phases) powered by the same mechanism (particle creation). Therefore, it sheds some light on a possible connection among the different accelerating stages of the Universe. In particular our model, since  $H_f^2 = \tilde{\Omega}_{\Lambda} H_0^2$ , where  $\tilde{\Omega}_{\Lambda} \sim$ 0.7 and  $H_0 \approx 1.5 \times 10^{-42}$  GeV, it sets the ratio of the primeval and late time de Sitter scales to be  $\rho_I/\rho_f$  =  $(H_I/H_f)^2 \sim 10^{122}$ . This large number is ultimately obtained through a combination of thermodynamics and quantum field theory in curved space-times in virtue to the association of the Hawking-Gibbons temperature ( $T = H/2\pi$ ) to the early de Sitter phase. Due tothe maximal radiation production rate,  $\Gamma_{r,\text{max}} = 3H_I$ , the model deflates from an unstable de Sitter with initial expansion rate  $(H = H_I)$ . It is exactly this Gibbons-Hawking connection (and the present value of  $H_0$ ) that lead for the given number under the proviso that the Universe starts with the Planck temperature. In principle, such a result in the present context has no correlation with the so-called cosmological constant problem (in this connection see Ref. [\[33\]](#page-10-14)). In a forthcoming communication, it will be shown that the above ratio is preserved even if the initial de Sitter phase is powered by a production rate term proportional to  $H^n$ ,  $n \geq 2$ .

As it appears, the cosmic history discussed here is semiclassically complete. However, there is no guarantee that the initial de Sitter configuration is not only the boundary condition of a true quantum gravitational effect. In other words, the very early de Sitter phase may be the result of a quantum fluctuation which is further semiclassically supported by the creation of massless particles (in this connection see Ref. [[65](#page-10-36)] and references therein).

Naturally, the existence of an early isothermal de Sitter phase suggests that thermal fluctuations (within the de Sitter event horizon) may be the causal origin of the primeval seeds that will form the galaxies. Such a possibility and its consequences for the structure formation problem deserves a closer investigation and is clearly out of the scope of the present paper.

We stress that our model provides a natural solution to the horizon problem and finally it connects smoothly the radiation and the matter dominated eras, respectively. At late times it also mimics perfectly the cosmic expansion history of the concordance  $\Lambda$ CDM model. In this context, we discuss the behavior of the linear matter perturbations  $\delta_m$  on subhorizon scales for the CCDM model and the main results are shown in Figs. [2](#page-6-2) and [3.](#page-7-1)

For completeness, we have also represented the evolution of our model in terms of the dynamics of an ordinary scalar field  $(\phi)$  and derived analytically the scalar field potential for two regimes: (i) when the Universe evolves from an early de Sitter to a Radiation phase and (ii) when it goes to the CDM phase to a late time de Sitter stage.

At present, we also know that a more complete version of the late time evolution must be filled with CDM ( $\sim$  96%) and baryons ( $\sim$  4%), and, unlike  $\Lambda$  cosmology, the baryon to dark matter ratio is a redshift function [\[12](#page-9-12)[,13\]](#page-9-8). In particular, this means that studies involving the gas mass fraction may provide a crucial test of our scenario, potentially, modifying our present view of the dark sector. Some investigations along the above discussed lines are in progress and will be published elsewhere.

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