

Damping of tensor modes in inflation

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We discuss the damping of tensor modes due to anisotropic stress in inflation. The effect is negligible in standard inflation and may be significantly large in inflation models which involve drastic production of free-streaming particles.

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I. INTRODUCTION

The spatial flatness and homogeneity of the present Universe strongly suggest that a period of de Sitter expansion or inflation had occurred in the early Universe [1]. During inflation, quantum fluctuations of the inflaton field may give rise to energy density perturbations (scalar modes) [2], which can serve as the seeds for the formation of large-scale structures of the Universe. In addition, a spectrum of gravitational waves (tensor modes) is produced from the de Sitter vacuum [3].

Gravitational waves are very weakly coupled to matter, so once produced, they remain as a stochastic background until today, and thus provide a potentially important probe of the inflationary epoch. Detection of these primordial waves by using terrestrial wave detectors or the timing of millisecond pulsars [4] would indeed require an experimental sensitivity of several orders of magnitude beyond the current reach. However, like scalar perturbation, horizon-sized tensor perturbation induces large-scale temperature anisotropy of the cosmic microwave background (CMB) via the Sachs-Wolfe effect [5]. The recent seven-year WMAP anisotropy data has placed an upper limit on the contribution of tensor modes to the CMB anisotropy, in terms of the tensor-to-scalar ratio, which is $r < 0.36$ [6]. More stringent limits, $r < 0.17$ – 0.19 , have been made by combining several other cosmological measurements [7]. In addition, the tensor modes uniquely induce CMB B-mode polarization which is the primary aim of ongoing and future CMB experiments [8].

As is well known, gravitational waves propagate freely in the expanding Universe [9]. This is under the assumption that the Universe is a perfect fluid. In the presence of non-vanishing anisotropic stress, an additional source term to the gravitational wave equation should be included [10]. The effect of anisotropic stress on cosmological gravitational waves due to free-streaming neutrinos after the neutrino-matter decoupling in the early Universe was first numerically calculated in Ref. [11], and incorporated in an integrodifferential equation for the wave propagation [12]. In fact, this equation can be also applied for any unknown free-streaming relativistic particles [13]. It was found that the anisotropic

stress reduces the wave amplitude, thus lowering the tensor-mode induced CMB anisotropy and polarization [11–14].

In this paper, we will discuss the effect of anisotropic stress on tensor modes in inflation. Here, the anisotropic stress is due to free-streaming relativistic particles produced during inflation. The generating source of these relativistic particles could be de Sitter quantum fluctuations of the inflaton itself in standard slow-roll inflation [2], a thermal component in warm inflation [15], isolated bursts of instantaneous massless particle production [16], particle production in trapped inflation in which the inflaton rolls slowly down a steep potential by dumping its kinetic energy into light particles at the trapping points along the inflaton trajectory [17,18], or electromagnetic dissipation in natural inflation [19,20].

II. PARTICLE PRODUCTION IN INFLATION

Here, we assume a flat Friedmann-Robertson-Walker metric,

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2), \quad (1)$$

where $a(\eta)$ and $d\eta = dt/a(t)$ are the scale factor and conformal time respectively. For simplicity, we treat the inflaton energy density ρ_ϕ as approximately constant, and then we have

$$a = -\frac{1}{H\eta}, \quad H^2 = \frac{8\pi G}{3}\rho_\phi. \quad (2)$$

We denote the energy density of the free-streaming relativistic particles χ produced during inflation by ρ_χ and define a ratio,

$$f \equiv \frac{\rho_\chi}{\rho_\phi}. \quad (3)$$

In the standard slow-roll inflation, a candidate for the χ particle is any weakly interacting field whose quanta are gravitationally produced during inflation. These de Sitter quantum fluctuations have a characteristic energy density roughly equal to $\rho_\chi \sim H^4$ [2], so we have $f \sim GH^2$. The WMAP results [6] have set an upper limit on the inflation scale which means that $f < 10^{-10}$. We will see below that this small f implies anisotropic stress too weak to affect the tensor modes in the standard slow-roll inflation.

However, some inflation models which involve particle production via interactions between ϕ and χ may allow a relatively large value for f . For example, in the trapped inflation model with an interaction of the type $g^2(\phi - \phi_i)^2 \chi_i^2$ [17], when ϕ rolls slowly down a steep potential by dumping its kinetic energy into χ_i particles at each trapping point at ϕ_i along the inflaton trajectory, a roughly constant energy density of all χ_i 's, $\rho_\chi = \sum_i \rho_{\chi_i}$, is maintained. This energy density can be estimated as $\rho_\chi \sim k_*^4$ with k_* being a characteristic energy scale given by $k_* \sim \sqrt{g(d\phi/dt)}$, where $d\phi/dt$ is the inflaton rolling speed. To have a successful trapped inflation, it is required that $f < 0.1$. Also, it is shown that the scattering rate of χ_i particles, which is given by $\Gamma \sim k_*^4/E^3$ with E being the energy of χ_i , is sufficiently slower than the expansion rate, $\Gamma \leq H$ [21]. Thus, in this model, the de Sitter vacuum may be populated with free-streaming χ_i particles which generate significantly large anisotropic stress to damp the tensor modes.

Another example which also provides us with a constant ρ_χ during inflation involves an interaction, $\phi/MF^{\mu\nu}\tilde{F}_{\mu\nu}$, where χ is a massless $U(1)$ gauge field, $F^{\mu\nu}$ is its field strength, and M is a mass scale [19,20]. The growth solution for the Fourier mode of the vector potential with + circular polarization is found as

$$A_+(\eta, k) \simeq \frac{1}{\sqrt{2k}} \left(\frac{k}{2\xi aH} \right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}} \quad (4)$$

in the interval $(8\xi)^{-1} \leq k/(aH) \leq 2\xi$, where $\xi \equiv 2(d\phi/dt)/(MH)$ is treated as constant. Hence, the energy density of the produced gauge quanta is given by

$$\begin{aligned} \rho_\chi &= \frac{1}{4\pi^2 a^4} \int dk k^2 (|dA_+/d\eta|^2 + k^2 |A_+|^2) \\ &\simeq \frac{6!}{2^{19}\pi^2} \frac{H^4}{\xi^3} e^{2\pi\xi}. \end{aligned} \quad (5)$$

These gauge quanta, in turn, source inflaton fluctuations which are highly non-Gaussian. The WMAP bound on non-Gaussianity implies that $\xi \lesssim 3$ [20]. When $\xi = 3$, $f \simeq 6.6 \times 10^3 \text{ GH}^2 \ll 1$. However, the value of ξ gets to increase toward the end of inflation. If $\xi = 5.16$ near the end of inflation and $\text{GH}^2 = 10^{-10}$, then we will have $f \simeq 0.1$. The gauge quanta, once produced, scatter with the inflaton fluctuations with a rate given by

$$\Gamma \sim \sigma n_{\delta\phi} \simeq \frac{H^2}{M^4} \times \frac{\rho_\chi}{H} \sim 10^{-4} \frac{e^{2\pi\xi}}{\xi^3} \left(\frac{H}{M} \right)^4 H, \quad (6)$$

where the energy of the gauge particle is of order H as shown in Eq. (4) and the number density of inflaton fluctuations is approximated as $n_{\delta\phi} \lesssim n_\chi$. As long as $\xi < 5.16$ and $H < 10^{-2}M$, we reach the condition $\Gamma < H$, under which the gauge quanta freeze out and decouple from the background.

III. GRAVITATIONAL WAVE EQUATION

In the weak field approximation, small metric fluctuations are ripples on the background metric:

$$g_{\mu\nu} = a^2(\eta)(\eta_{\mu\nu} + h_{\mu\nu}), \quad h_{\mu\nu} \ll 1, \quad (7)$$

where $\eta_{\mu\nu}$ is the Minkowski metric and Greek indices run from 0 to 3. In synchronous gauge, $h_{00} = h_{0i} = 0$, where i runs from 1 to 3. The remaining h_{ij} contain a transverse, traceless (TT) tensor which corresponds to a gravitational wave or tensor mode. Henceforth, we will work in the TT gauge, i.e., $h_k^k = \partial_i h^{ij} = 0$ and denote the two independent polarization states of the wave as $+$, \times . The propagation of gravitational waves in an expanding space-time is well studied. In the presence of anisotropic stress, the Fourier mode equation is given by [10]

$$\frac{d^2 \tilde{h}_{ij}}{d\eta^2} + \frac{2}{a} \frac{da}{d\eta} \frac{d\tilde{h}_{ij}}{d\eta} + k^2 \tilde{h}_{ij} = 16\pi G a^2 \pi_{ij}, \quad (8)$$

where π_{ij} is the Fourier mode of the TT part of the anisotropic stress tensor.

IV. ANISOTROPIC STRESS TENSOR

In this section, we will derive the evolution of the anisotropic stress tensor of the free-streaming relativistic particles χ . We will follow the methodology in Ref. [10], taking into account the particle production in inflation and assuming that the produced χ particles are decoupled from the background. Let $N_\chi(\mathbf{x}, \mathbf{p}, t)$ be the phase space density of χ particles; then, the physical energy density of χ particles is given by

$$\rho_\chi(t) = a^{-4}(t) \frac{1}{V} \int_V d^3\mathbf{x} d^3\mathbf{p} |\mathbf{p}| N_\chi(\mathbf{x}, \mathbf{p}, t). \quad (9)$$

In light of the results in Sec. II, in the following, we will assume that the physical energy density is constant during inflation: $d\rho_\chi(t)/dt = 0$. This suggests that we should deal with a rescaled phase space density instead, defined by

$$n_\chi(\mathbf{x}, \mathbf{p}, t) \equiv a^{-4}(t) N_\chi(\mathbf{x}, \mathbf{p}, t). \quad (10)$$

In the absence of collisions, the rescaled phase space density satisfies a Boltzmann equation in a metric $g_{ij}(\mathbf{x}, t)$,

$$\frac{\partial n_\chi}{\partial t} + \frac{\partial n_\chi}{\partial x^i} \frac{p^i}{p^0} + \frac{\partial n_\chi}{\partial p_i} \frac{p^j p^k}{2p^0} \frac{\partial g_{jk}}{\partial x^i} = 0, \quad (11)$$

where $p^i = g^{ij} p_j$ and $p^0 = \sqrt{g^{ij} p_i p_j}$. We now consider small perturbation,

$$g_{ij} = a^2 \delta_{ij} + \delta g_{ij}, \quad n_\chi = \bar{n}_\chi(a p^0) + \delta n_\chi, \quad (12)$$

where $\bar{n}_\chi(p)$ with $p = |\mathbf{p}|$ is just the rescaled local phase space density, which has a well defined energy spectrum as exemplified in Eq. (5). To first order in metric and density perturbation, Eq. (11) reads

$$\frac{\partial \bar{n}_\chi(p)}{\partial t} = 0, \quad (13)$$

$$\frac{\partial \delta n_\chi}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \delta n_\chi}{\partial x^i} = \frac{p}{2} \frac{\partial \bar{n}_\chi(p)}{\partial p} \hat{p}_i \hat{p}_j \frac{\partial}{\partial t} \left(\frac{\delta g_{ij}}{a^2} \right), \quad (14)$$

where $\hat{p}_i = p_i/p$.

Using Eq. (7) for the metric perturbation, we write down the tensor component of Eq. (14) as

$$\frac{\partial \delta n_\chi}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \delta n_\chi}{\partial x^i} = \frac{p}{2} \frac{\partial \bar{n}_\chi(p)}{\partial p} \hat{p}_i \hat{p}_j \frac{\partial h_{ij}}{\partial t}. \quad (15)$$

Let us introduce a dimensionless rescaled intensity perturbation defined by

$$\bar{\rho}_\chi J(\mathbf{x}, \hat{p}, t) \equiv \int dp 4\pi p^3 \delta n_\chi(\mathbf{x}, \mathbf{p}, t), \quad (16)$$

$$\bar{\rho}_\chi \equiv \int dp 4\pi p^3 \bar{n}_\chi(p). \quad (17)$$

Then, Eq. (15) becomes

$$\frac{\partial J}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial J}{\partial x^i} = -2\hat{p}_i \hat{p}_j \frac{\partial h_{ij}}{\partial t}, \quad (18)$$

where we have used integration by part and assumed that $\bar{n}_\chi(0) = \bar{n}_\chi(\infty) = 0$. We can then construct the spatial component of the stress tensor perturbation as

$$\delta T_{\chi j}^i = \int d^3\mathbf{p} \delta n_\chi(\mathbf{x}, \mathbf{p}, t) p \hat{p}_i \hat{p}_j, \quad (19)$$

which contributes to the anisotropic stress tensor π_{ij} . Following the same steps in Ref. [10], the free-streaming solution for the anisotropic stress tensor of χ particles in the presence of gravitational waves is found as

$$\pi_{ij}^\chi = -4\bar{\rho}_\chi \int_{\eta_i}^{\eta} K(k\eta - k\eta') \frac{d\tilde{h}_{ij}(\eta')}{d\eta'} d\eta', \quad (20)$$

where η_i is some initial time and the kernel is given by

$$K(u) = \frac{j_2(u)}{u^2} = -\frac{\sin u}{u^3} - \frac{3\cos u}{u^4} + \frac{3\sin u}{u^5}. \quad (21)$$

The integrodifferential equation (8) with π_{ij} given by Eq. (20) has been solved for the case in which χ particles are free-streaming neutrinos in the early Universe [12–14]. One would anticipate that the free-streaming solution of the anisotropic stress tensor (20) is a backreaction to the wave equation and thus reduces the wave amplitude.

In trapped inflation [17] or axionic inflation with $10^5 U(1)$ gauge fields [19], the time scale of particle production is much shorter than the expansion time, H^{-1} . For instance, trapped inflation produces particles in a time scale of order $k_*^{-1} \ll H^{-1}$. Therefore, inflation begins very shortly after particles are copiously produced. Let η_i be the moment when inflation begins. The initial condition, $\pi_{ij}^\chi(\eta_i) = 0$, which we have assumed in Eq. (20) is then justified.

Note that generically anisotropic stress should exist before inflation, i.e., $\pi_{ij}(\eta_i) \neq 0$, due to the fact that we do not really know the initial condition for inflation since here we do not have a physical model before inflation. However, soon after inflation starts, this preexisting anisotropic stress has decayed and become vanishingly small. Since we are mainly interested in χ particles produced during inflation, we have assumed that the generic anisotropic stress is absent at the beginning of inflation, namely $\pi_{ij}(\eta_i) = 0$. Otherwise, we will need to consider the effect of this generic anisotropic stress on gravitational waves in a brief period after the start of inflation.

V. DAMPING IN INFLATION

Let us decompose

$$\tilde{h}_{ij}(\eta, \mathbf{k}) = (8\pi G)^{\frac{1}{2}} H k^{-\frac{3}{2}} h(\eta, \mathbf{k}) \epsilon_{ij}(\mathbf{k}; \lambda), \quad (22)$$

where $\epsilon_{ij}(\mathbf{k}; \lambda)$ is the polarization tensor with $\lambda = +, \times$. Here, we have introduced a dimensionless wave amplitude h which is assumed to be the same for both polarizations. Then, Eq. (8) becomes

$$\frac{d^2 h}{du^2} - \frac{2}{u} \frac{dh}{du} + h = -\frac{24f}{u^2} \int_{u_i}^u K(u - u') \frac{dh}{du'} du', \quad (23)$$

where $f = \bar{\rho}_\chi/\rho_\phi$ is a constant. The homogeneous solution of Eq. (23) is known. Selecting the Bunch-Davis vacuum [22], it is given by

$$h_0 = -(1 + iu)e^{-iu}. \quad (24)$$

At the end of inflation (i.e., $u \rightarrow 0$), $|h_0| \rightarrow 1$. This reproduces the scale-invariant power spectrum predicted in standard slow-roll inflation.

Since the damping effect is expected to be secondary, we can make the Born approximation to replace dh/du' in the damping term in Eq. (23) by dh_0/du' . Then, we use the retarded Green's function method to find the particular solution,

$$h_p = -24f \int_{u_i}^u \frac{du'}{u'^2} G(u - u') \int_{u_i}^{u'} du'' K(u' - u'') \frac{dh_0}{du''}, \quad (25)$$

where the Green's function is constructed from the homogeneous solution, given by

$$G(u - u') = \frac{u}{u'} \left[\left(1 + \frac{1}{uu'} \right) \sin(u - u') - \frac{u - u'}{uu'} \cos(u - u') \right]. \quad (26)$$

Now, we set $a = 1$ at the time when inflation starts. This fixes $u_i = -k/H$ with $k/H = 1$ corresponding to the length scale which crosses the horizon at the onset of inflation. We then numerically evaluate the integral (25) by letting $u \rightarrow 0$ to obtain h_p for a range of values of k/H .

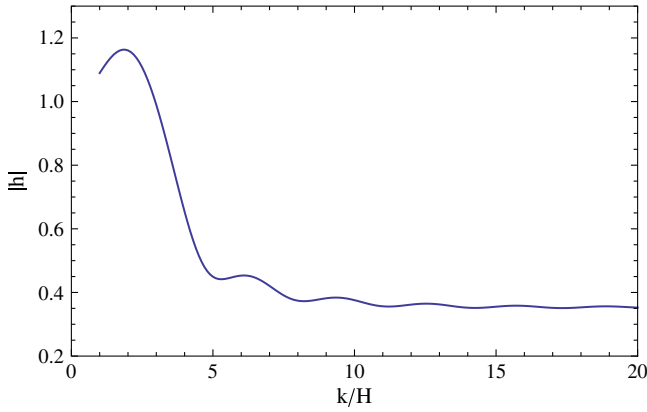


FIG. 1 (color online). The wave amplitude $|h|$ versus the wave number k/H at the end of inflation by the use of the Born approximation in Eq. (25), where $24f = 1$ is assumed and $k/H = 1$ corresponds to the length scale which leaves the horizon at the onset of inflation.

In Fig. 1, assuming that $24f = 1$, we plot the total wave amplitude, $|h| = |h_0 + h_p|$, at the end of inflation ($u \rightarrow 0$) against the wave number, k/H . The figure shows that the wave amplitude is reduced and becomes asymptotically flat at large k .

Apparently, the wave amplitude is enhanced for $k/H < 3$. However, we do not expect that the present work would give precise results for these low k modes. The enhancement should be an artifact due to the use of the approximation, $dh/du' \simeq dh_0/du'$, and the choice of the Bunch-Davis vacuum near the initial time, u_i . A quick way to resolve this problem, albeit artificial, is to integrate the damping effect on each k mode for the time interval from the start of inflation to the horizon crossing time, which means by setting $u = -1$ in Eq. (25). As such, $|h| = 1$ for $k/H = 1$ by default. Some improvements can also be made such as using an advanced solving method for the integrodifferential equation (8) and considering a smooth transit to the inflationary phase.

VI. CONCLUSION

We have discussed the damping effect of anisotropic stress on tensor modes due to free-streaming relativistic particles produced during inflation. The damping increases with the ratio of the particle energy density and the de Sitter vacuum energy, which ranges from 10^{-10} in standard

inflation to about 0.1 in inflation models which involve drastic particle production such as the trapped inflation or axionic inflation. In these inflation models, the particle production may significantly reduce the amplitude of the tensor modes.

Recently, new sources of gravitational waves during inflation have been proposed. They are anisotropic stress induced by quantum energy stress of conformal fields [23] and their associated fluctuations [24], by the Bremsstrahlung from the particle production events [21], and by the produced gauge field quanta which couple to inflaton [25,26]. All of these produced gravitational waves may experience the damping effect considered in the present work if copious free-streaming relativistic particles are also produced during inflation. As such, to properly take into account the damping, one needs to consider the full integrodifferential equation,

$$\frac{d^2 \tilde{h}_{ij}}{d\eta^2} + \frac{2}{a} \frac{da}{d\eta} \frac{d\tilde{h}_{ij}}{d\eta} + k^2 \tilde{h}_{ij} + 64\pi G a^2 \bar{\rho}_\chi \times \int_{\eta_i}^{\eta} K(k\eta - k\eta') \frac{d\tilde{h}_{ij}(\eta')}{d\eta'} d\eta' = 16\pi G a^2 \pi_{ij}^{\text{new}}, \quad (27)$$

where the damping term is taken from Eq. (20) and π_{ij}^{new} denotes a new source term for the anisotropic stress. When $f \ll 1$, the damping term can be neglected, and the equation reduces to that considered in Refs. [21,23–26]. Otherwise, Eq. (27) should be solved self-consistently to obtain the damped tensor power spectrum. At last, we note that in Ref. [26], the authors have considered the production of gravitational waves at the interferometric scales during the final stage of inflation when $\xi \sim 5$ –6. This large value of ξ may imply that $f \sim 1$, thus resulting in a large damping on the tensor modes.

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