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Runaway, D term and *R*-symmetry breaking

Tatsuo Azeyanagi,[1] Tatsuo Kobayashi,[2] Atsushi Ogasahara,[2] and Koichi Yoshioka[3]

[1]*Center for the Fundamental Laws of Nature, Harvard University, Cambridge, Massachusetts 02138, USA*

[2]*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

[3]*Department of Physics, Keio University, Kanagawa 223-8522, Japan*

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We study the D-term effect on runaway directions of the F-term scalar potential. A minimal renormalizable model is presented where supersymmetry is broken without any pseudomoduli. The model is applied to the hidden sector of gauge mediation for spontaneously breaking *R* symmetry and generating nonvanishing gaugino masses at the one-loop order.

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I. INTRODUCTION

Supersymmetry is expected to be one of the key ingredients to describe physics beyond the Standard Model (SM). While tree-level supersymmetry breaking within the SM sector leads to light sfermions, the breaking sector is separated from the SM one and is mediated by some effective operators or quantum effects. Among various mechanisms for realizing this scenario, the gauge mediation, relevant to this paper, is one of the most promising candidates with a strong prophetic power (for a review, see Ref. [1]).

It is known that pseudomoduli directions are present in the supersymmetry-breaking vacuum of O'Raifeartaigh-like models with the canonical Kähler potential [2]. An important implication of this result is that, if such models are used as the hidden sector of gauge mediation, gaugino masses are generally suppressed or the vacuum is unstable somewhere along the pseudomoduli [3]. There have been various ways in the literature to avoid such a phenomenologically unfavorable situation, such as including nonminimal terms in the potential [4], quantum effects from specific scalar and/or vector multiplets [5], or accepting metastable vacua [6]. Another way, as we discussed before, is to introduce gauge multiplets and take non-negligible D term into account for making the vacuum stable without pseudomoduli.

In our previous paper, we classified supersymmetry-breaking models with nonvanishing F and D terms [7]. First, the models are divided into two categories based on whether the F-term potential has a supersymmetric minimum (at finite field configuration). We then add the D term by gauging flavor symmetry and analyze the vacuum of the full scalar potential in each category. For models that do not satisfy the F-flatness conditions, we found that the full potential generally shows runaway behavior. On the other hand, when the F-flatness conditions are satisfied, supersymmetry can be broken without pseudomoduli only in the presence of the Fayet-Iliopoulos (FI) term. By using the latter class of models, we constructed a model of gauge mediation, where gaugino masses are generated at the one-loop order.

In this paper, we discuss another possibility for the classification: the F-term potential is minimized at some infinite field configuration, i.e., it shows a runaway behavior. It is found that the runaway direction of the F-term potential can be uplifted by the D term, and a supersymmetry-breaking vacuum emerges at finite field configuration. There are several reasons to explore this class of models in detail. First of all, contrary to our previous result, there is no need to add the FI term for supersymmetry breaking.[1] Secondly, the vacuum automatically suppresses pseudomoduli directions associated with the F-term potential, since it has a runaway behavior and is stabilized by the D-term potential. We propose a minimal model with such properties and couple it with an appropriate messenger sector. In this model including the messenger sector, *R* symmetry is spontaneously broken at the tree level even though the model contains chiral superfields with $U(1)_R$ charge 0 or 2 only. We notice that *R* symmetry breaking does not occur for O'Raifeartaigh-like models with such a $U(1)_R$ charge assignment [9]. This class of supersymmetry breaking can therefore provide a realistic model for gauge mediation, where leading-order gaugino masses are obtained at the stable vacuum.

The outline of this paper is as follows. In Sec. II, we briefly review our classification of F- and D-term supersymmetry breaking. In Sec. III, we discuss the case in which the F-term potential shows runaway behaviors. After some general arguments, a minimal model is presented to realize the vacuum property listed above. Further, appropriate messenger sectors are discussed and shown to be viable for generating gaugino masses. Section IV is devoted to summarizing our results and discussions on future directions. In the Appendix, we show the potential analysis of the model given in Sec. III.

[1]The FI term also has some difficulty when incorporated into local supersymmetric theory [8].

II. SUPERSYMMETRY BREAKING WITH F AND D TERMS

We first review our previous result of the classification of supersymmetry breaking with both F and (Abelian) D terms [7]. Throughout this paper, we assume the Kähler potential is canonical. The superpotential W has a polynomial form of chiral superfields ϕ_i with U(1) charges q_i (the latin indices label their species). The scalar potential V is then given by

$$V = V_F + V_D, \tag{2.1}$$

where V_F and V_D are the contributions from F and D terms:

$$V_F = \sum_i |F_i|^2, \qquad F_i = -\left(\frac{\partial W}{\partial \phi_i}\right), \tag{2.2}$$

$$V_D = \frac{g^2}{2} D^2, \qquad D = \sum_i q_i |\phi_i|^2 + \xi. \tag{2.3}$$

Here g is the U(1) gauge coupling constant and ξ is the coefficient of the possible Fayet-Iliopoulos term [10]. In the following, we abbreviate field derivatives of the superpotential as $W_{\phi_i}(= \partial W/\partial \phi_i)$.

We first divide models into two categories. The criterion for the classification is whether the F-flatness condition, $V_F = 0$, is satisfied or not at its minimum defined by $\partial V_F/\partial \phi_i = 0$. If the F-flatness condition is (not) satisfied, a model is called in the second (first) class. We then add the U(1) D term and analyze the tree-level behavior of the full scalar potential. For the first class of models, the scalar potential shows supersymmetric runaway behaviors when the D-term contribution is included.[2] For the second class of models, on the other hand, supersymmetry can be broken without any pseudomoduli. That is, however, realized only in the presence of the FI term. (See Table I for the classification.)

There is another possibility in the classification, which we did not consider previously: the F-term potential V_F has runaway behavior, i.e., the minimization $\partial V_F/\partial \phi_i = 0$ is not satisfied (with finite field configuration). In the following, we focus on this class of models, giving some general arguments and discussing a minimal model that shows that the runaway direction is uplifted by the D-term potential V_D. The model also has the property that pseudomoduli are absent in the vacuum. By coupling it with an appropriate messenger sector, we show that R symmetry is spontaneously broken and gaugino masses are generated at the one-loop order.

[2]A similar behavior was studied in an explicit example in Ref. [11].

TABLE I. Classification of supersymmetry breaking with F and D terms.

	At $\frac{\partial V_F}{\partial \phi_i} = 0$	FI term $(\xi) = 0$	FI term $(\xi) \neq 0$
First class	$V_F \neq 0$	runaway	runaway
Second class	$V_F = 0$	SUSY	SUSY breaking

III. RUNAWAY AND D-TERM UPLIFT

A. General arguments

A well-known runaway behavior of the scalar potential arises from a nonperturbative superpotential which has inverse powers of field variables. Supersymmetry breaking occurs along such a runaway direction if a suitable superpotential is added to lift it up [12]. It is, however, noted in this case that the D-term potential is vanishing.[3] On the other hand, there is another type of runaway behavior related to symmetries of theory. We are interested in how the runaway potential is affected by a nonvanishing D term, which is given by gauging non-R flavor symmetry. In this paper, we focus on the Abelian D term, but non-Abelian generalization is straightforward.

We first see how the runaway directions related to U(1) gauge symmetry occur. Consider a theory with U(1) (gauge) symmetry. The superpotential is then invariant under the complexified U(1) which contains a charge-dependent scale transformation as its real part. An important point is that the F-flatness conditions are satisfied with the variables obtained by the U(1) transformation from the original solution. Let us assume that there exists a field configuration $\phi_i = \phi_i^{(0)}$ that realizes the following situation: $F_i = 0$ for all fields with seminegative charges $(q_i \leq 0)$,[4] while $F_j \neq 0$ for at least one field with positive charge $(q_j > 0)$. Under the above scale transformation acting on $\phi_i^{(0)}$,

$$\phi_i^{(0)} \to e^{q_i \alpha} \phi_i^{(0)} \qquad (\alpha \in \mathbb{R}), \tag{3.1}$$

the F terms behave as

$$W_{\phi i} = 0 \qquad (q_i \leq 0), \tag{3.2}$$

$$W_{\phi i} \to e^{-q_i \alpha} W_{\phi i} \qquad (q_i > 0), \tag{3.3}$$

and the F-term potential approaches to zero as $\alpha \to \infty$. At the same time, some values of positively charged fields go to infinity. The F-term potential satisfying the above assumption, therefore, shows a runaway behavior along the direction related to the U(1) symmetry. Similarly, the

[3]Deviation from D-flat directions has been discussed for several models recently [13].

[4]One can relax the condition such that $F_i \neq 0$ for some neutral fields. The following arguments of runaway behavior still hold, since neutral fields and their F terms are unchanged along the runaway direction. The only difference is the size of the F-term potential at infinity of the field space.

runaway also occurs if scalar fields realize the following situation: $F_i = 0$ for all fields with semipositive charges ($q_i \geq 0$), while $F_j \neq 0$ for at least one field with negative charge ($q_j < 0$). In this case, the runaway direction corresponds to $\alpha \to -\infty$.[5]

Then we include the U(1) D term and examine how the scalar potential is modified. If the charge-dependent scale transformation is applied and the parameter α is taken to infinity, the D term behaves as

$$D \to \sum_{q_i > 0} q_i |\phi_i|^2 \qquad (\alpha \to \infty). \tag{3.4}$$

Similarly, for $\alpha \to -\infty$, the D term is dominated by negatively charged fields. Along the U(1) runaway direction discussed above, the D term grows up as $|\alpha| \to \infty$ and uplifts the full scalar potential away from the origin. This result is irrelevant to whether a nonvanishing FI term exists or not. We here comment on an important point that since a runaway F-term potential does not satisfy its stationary conditions at finite field configuration, the theory is expected to have no pseudomoduli by formulation.

It is noted that while the U(1) runaway direction is stabilized, the full potential is not necessarily stabilized in the direction (3.1). For example, one may have other directions, such as $\mathrm{U}(1)_R$-related runaway, orthogonal to the stabilized direction. Moreover, the potential must be carefully analyzed so that it is indeed minimized, not a saddle point. A more general argument seems hard to complete, and we leave these issues to future work. In the following sections, we present a minimal tree-level model with F-term runaway lifted by the D term of gauged flavor symmetry.

B. A minimal model

In this section we study a renormalizable model satisfying the assumption we have made above: symmetry-related runaway directions of the F-term potential can be uplifted by the D term, and a supersymmetry-breaking vacuum emerges at finite field configuration. The model is explicitly shown to have a stable vacuum with no pseudomoduli, where supersymmetry is broken. By coupling it with an appropriate messenger sector, we show that R symmetry is spontaneously broken and gaugino masses are generated at the one-loop order.

1. Supersymmetry-breaking sector and runaway

The model consists of six chiral multiplets $X_\pm$, X_0, $\varphi_\pm$ and φ_0, where the subscripts denote their U(1) charges. The assignment of U(1) and $\mathrm{U}(1)_R$ charges is summarized below. The superpotential is

$$W = fX_0 + \lambda X_0 \varphi_+ \varphi_- + mX_+ \varphi_- + \lambda' X_- \varphi_+ \varphi_0. \tag{3.5}$$

This form can be the most generic, renormalizable one if φ_0 and X_- have the charges $+1$ and -1 under an additional Z_N symmetry ($N > 2$). We here comment on the role of each term in the superpotential (3.5): the first and second terms make the origin unstable along the meson $\varphi_+\varphi_-$. The third term lifts up the φ_- direction, and naively supersymmetry is broken with the F-term potential coming from the first three terms. However, the potential minimum is found to run away to infinity of the moduli space along the φ_+ direction (with a finite value of $\varphi_+\varphi_-$). It is further noticed that anomaly cancellation requires the existence of a negatively charged field (X_-). Without the last fourth term, the value of X_- is free and the direction $X_-\varphi_+$ becomes D flat, along which the potential minimum goes to infinity and supersymmetry is recovered. In this way, the superpotential (3.5) is regarded as minimal one for the present purpose.

The scalar potential $V = V_F + V_D$ is explicitly given by

$$\begin{aligned} V_F = &|f + \lambda\varphi_+\varphi_-|^2 + |m\varphi_-|^2 + |\lambda'\varphi_+\varphi_0|^2 \\ &+ |\lambda' X_- \varphi_+|^2 + |\lambda X_0 \varphi_- + \lambda' X_- \varphi_0|^2 \\ &+ |\lambda X_0 \varphi_+ + mX_+|^2, \end{aligned} \tag{3.6}$$

$$V_D = \frac{g^2}{2} D^2 = \frac{g^2}{2}(|X_+|^2 + |\varphi_+|^2 - |X_-|^2 - |\varphi_-|^2)^2. \tag{3.7}$$

The F-term potential V_F has the U(1) runaway directions discussed in the previous section. We find along the following direction

$$\begin{aligned} &X_+ = X_- = X_0 = \varphi_0 = 0, \\ &\varphi_+ = \sqrt{\frac{f}{\lambda}} e^{\alpha}, \qquad \varphi_- = -\sqrt{\frac{f}{\lambda}} e^{-\alpha}, \end{aligned} \tag{3.8}$$

and all the F terms vanish except for the positively charged field F_{X_+},

$$\begin{aligned} &F_{X_-} = F_{X_0} = F_{\varphi_+} = F_{\varphi_-} = F_{\varphi_0} = 0, \\ &F^*_{X_+} = m\sqrt{\frac{f}{\lambda}} e^{-\alpha}. \end{aligned} \tag{3.9}$$

The runaway direction is parametrized by α. As $\alpha \to \infty$, the positively charged field φ_+ goes to infinity and the F-term potential (F_{X_+}) approaches to zero. Furthermore, no $\mathrm{U}(1)_R$ runaway is expected since all the scalar fields in the present model have R charges 0 or 2 [14].

It is easily seen that the F-term runaway direction (3.8) is stabilized by the D-term contribution because, along this direction, the D term increases as

$$D \to |\varphi_+|^2 \qquad (\alpha \to \infty). \tag{3.10}$$

[5]The runaway behavior related to $\mathrm{U}(1)_R$ symmetry is understood in a similar way [14]. A difference from non-R symmetry discussed in this section is that the superpotential carries a nonzero charge, and one can specify R-charge assignment for runaway to occur.

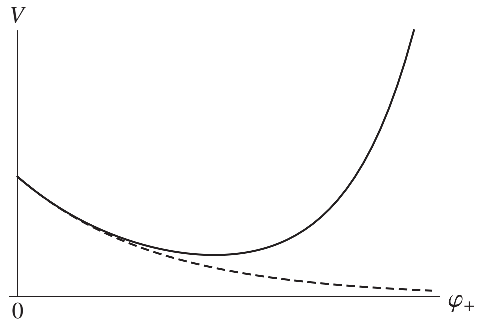

FIG. 1. A typical behavior of V_F (dashed line) and V (solid line) along the runaway direction (3.8). The runaway direction of V_F is uplifted by the D-term contribution V_D.

(See also Fig. 1.) We again note that even when the runaway is lifted by the D term, it does not necessarily mean that the vacuum of the scalar potential is in the direction (3.8).

2. Supersymmetry-breaking vacuum

We then analyze the scalar potential in detail to confirm that a stable supersymmetry-breaking vacuum is obtained in some parameter region. The parameters appearing in the superpotential are assumed to be real and positive without loss of generality.

The vacuum is identified by solving the stationary conditions for the full scalar potential V. A trivial configuration satisfying the stationary conditions is the origin at which all the scalar fields vanish. This point however is unstable and we do not consider it in the following. By some calculation (the detail is summarized in the Appendix), we can show that the vacuum satisfies

$$X_+ = X_0 = \varphi_0 = 0, \tag{3.11}$$

where several F terms vanish;

$$F_{X_-} = F_{\varphi_+} = F_{\varphi_-} = 0. \tag{3.12}$$

Then the stationary conditions for X_+, X_0, and φ_0 automatically hold, and the scalar potential simplifies to

$$V = |f + \lambda\varphi_+\varphi_-|^2 + |m\varphi_-|^2 + |\lambda' X_-\varphi_+|^2 + \frac{g^2}{2}D^2, \tag{3.13}$$

with $D = |\varphi_+|^2 - |\varphi_-|^2 - |X_-|^2$. With this reduced potential, the stationary condition for X_- reads

$$\frac{\partial V}{\partial X_-^*} = X_-(|\lambda'\varphi_+|^2 - g^2 D) = 0, \tag{3.14}$$

indicating $X_- = 0$ or $D = |\lambda'\varphi_+/g|^2$. We discuss these two cases separately below, solving the remaining stationary conditions for $\varphi_\pm$.

(1) $X_- = 0$: In this case, for a large mass ($m^2 \gg \lambda f$, $\lambda^2 f/g$) or a small one ($m^2 \ll \lambda f$, $g^2 f/\lambda$), we can approximately write down the analytic solution to the stationary conditions for $\varphi_\pm$. For the large mass regime, the solution is given by

$$\varphi_+ = \pm\frac{\lambda f}{gm}, \qquad \varphi_- = \mp\frac{\lambda^2 f^2}{gm^3}, \qquad (\text{large } m), \tag{3.15}$$

where the F and D components other than (3.12) become

$$F_{X_+} = \pm\frac{\lambda^2 f^2}{gm^2}, \qquad F_{X_0} = -f + \frac{\lambda^4 f^3}{g^2 m^4}, \qquad F_{\varphi_0} = 0, \qquad D = \frac{\lambda^2 f^2}{g^2 m^2}, \tag{3.16}$$

at the leading order. Therefore, the scalar potential is dominated by F_{X_0}, i.e., $V \simeq f^2$. On the other hand, for the small mass regime, we have

$$\varphi_+ = \pm\sqrt{\frac{f}{\lambda}}\left(1 - \frac{m^2}{4\lambda f} + \frac{\lambda m^2}{8g^2 f}\right), \tag{3.17}$$

$$\varphi_- = \mp\sqrt{\frac{f}{\lambda}}\left(1 - \frac{m^2}{4\lambda f} - \frac{\lambda m^2}{8g^2 f}\right), \qquad (\text{small } m), \tag{3.18}$$

where the F and D components are

$$F_{X_+} = \mp m\sqrt{\frac{f}{\lambda}}, \qquad F_{X_0} = -\frac{m^2}{2\lambda}, \qquad F_{\varphi_0} = 0, \qquad D = \frac{m^2}{2g^2}, \tag{3.19}$$

and the scalar potential is found to be dominated by F_{X_+}, i.e., $V \simeq fm^2/\lambda$. Notice that, in both regimes of m, the R symmetry is unbroken as $F_{\varphi_0} = 0$. The stability of these vacua is read off from the eigenvalues of the squared mass matrix for the scalar fields. We find that all eigenvalues except for X_- are positive semidefinite. Along the X_- direction, the eigenvalue is given by

$$M_{X_-}^2 = (\lambda'^2 - g^2)|\varphi_+|^2 + g^2|\varphi_-|^2. \tag{3.20}$$

This eigenvalue is positive when $\lambda' \gtrsim g$ for the large mass regime and $\lambda' \gtrsim (\lambda m^2/f)^{1/2}$ for the small mass regime. Therefore the supersymmetry-breaking vacuum realized is stable for these parameters. Otherwise, $X_- = 0$ is a saddle point and the true vacuum is given by the following second case:

(2) $D = |\lambda'\varphi_+/g|^2$: In this case, the stationary conditions are complicated and we only present a typical numerical solution. For example, when the model parameters are set as $(m^2/f, \lambda, \lambda', g) = (2, 0.7, 0.1, 0.5)$, which do not satisfy the above

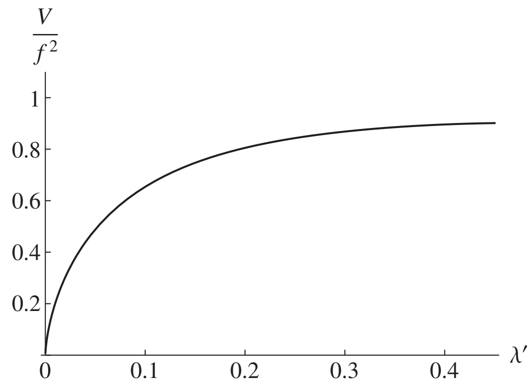

FIG. 2. The potential value at the vacuum as a function of λ' (for the $D = |\lambda' \varphi_+ / g|^2$ case). The other parameters are fixed to $(m^2/f, \lambda, g) = (2, 0.7, 0.5)$. In the limit $\lambda' \to 0$, supersymmetry is recovered, but some expectation values run away to infinity.

condition for the stability of $X_- = 0$, the vacuum is located at

$$(\varphi_+, \varphi_-, X_-) \simeq (1.34, -0.252, 1.29) \times m, \tag{3.21}$$

where nonvanishing F and D components are

$$(F_{X_+}, F_{X_0}, F_{\varphi_0}, D) \simeq (-0.504, -0.527, 0.346, 0.144) \times f. \tag{3.22}$$

All the F and D terms become comparable to each other and contribute to the scalar potential $V \sim \mathcal{O}(f^2)$. The stability of this vacuum is confirmed numerically. We have also checked that, for a wider parameter region, the scalar potential has a similar behavior. In Fig. 2, we show the normalized potential V/f^2 characterizing supersymmetry breaking at the vacuum as a function of λ' for the large mass regime ($m^2 \gg \lambda f$, $\lambda^2 f/g$). Without the λ' term, supersymmetry recovers at infinity of the moduli space, as we mentioned before. As λ' becomes larger, X_- is stabilized for $\lambda' \gtrsim g$ and the vacuum is shifted up to (3.16), where $V/f^2 \simeq 1$.

3. Messenger sector and gaugino masses

We then discuss the gauge-mediation scenario by employing the above model as a supersymmetry breaking sector. Appropriate messenger fields and their superpotential are identified for generating one-loop-order gaugino masses for the two cases, $X_- = 0$ and $D = |\lambda' \varphi_+ / g|^2$, separately.

(1) $X_- = 0$: As we have shown, the F and D terms have different behaviors depending of whether the mass parameter m is large or small. For the large m case, (3.16), the supersymmetry-breaking scale is governed by F_{X_0} from which sfermions are expected to receive soft masses. For generating a similar size of gaugino masses, a simple way is to introduce the messenger fields M and $\tilde{M}$ which are vectorlike under the SM gauge symmetry and have the superpotential,

$$W = X_0 M\tilde{M} + m_M M\tilde{M}. \tag{3.23}$$

For $f \ll m_M^2$, the standard one-loop diagram of messenger fields generates a gaugino mass M_g;

$$M_g = \frac{T_R g_{\rm SM}^2}{8\pi^2} \frac{f}{m_M}, \tag{3.24}$$

where $g_{\rm SM}$ is the SM gauge coupling, and T_R is the Dynkin index of the SM gauge symmetry for M and $\tilde{M}$. The gaugino mass (3.24) is given at the vacuum discussed in the previous section. It is, however, noticed that the R symmetry is softly broken by the parameter m_M that makes $X_0 = 0$ a local minimum. Another way of obtaining gaugino masses would be to consider the direct gauge mediation, i.e., to generalize U(1) to non-Abelian symmetry containing the SM one. By adding a small supersymmetric mass for $\varphi_\pm$, they behave as messengers and would induce the SM gaugino masses without introducing additional multiplets.
For the small mass case, (3.19), the charged F term F_{X_+} dominates the supersymmetry-breaking scale. To split messenger masses with F_{X_+}, we must introduce two pairs of vectorlike messengers with U(1) charges $\pm q$ and $\pm(q-1)$ and couple them with X_+ in the superpotential. For further details of the messenger sector, the stability of the vacuum, and gaugino mass generation, see Ref. [7].

(2) $D = |\lambda' \varphi_+ / g|^2$: In the vacuum, the U$(1)_R$ symmetry given in Table II is broken by the F term of φ_0 which is neutral under both U(1) and U$(1)_R$. We consider the following messenger superpotential,

$$W = \varphi_0 M\tilde{M} + m_M M\tilde{M}, \tag{3.25}$$

where M and $\tilde{M}$ are vectorlike multiplets, charged under the SM gauge symmetry. It is noticed that the Z_N symmetry is softly broken in this messenger sector. Note that the superpotential (3.5) has an anomalous U(1)' symmetry, under which φ_0 and X_- are charged, but it is explicitly broken in (3.25). The R symmetry is spontaneously broken in the full potential with both (3.5) and (3.25), if the vacuum in the previous section is stable. The vacuum stability is ensured by taking the parameter m_M

TABLE II. The assignment of U(1) and U$(1)_R$ charges.

	X_+	X_-	X_0	φ_+	φ_-	φ_0
U(1)	+1	−1	0	+1	−1	0
U$(1)_R$	2	2	2	0	0	0

sufficiently large. The gaugino mass is evaluated for $f \ll m_M^2$ as

$$M_g = \frac{T_R g_{SM}^2}{8\pi^2} \frac{F_{\varphi_0}}{m_M}, \tag{3.26}$$

which comes from a one-loop diagram, where M, $\tilde{M}$ circulate in the loop.

From these observations, we conclude that this class of supersymmetry-breaking models is useful to build a simple, realistic hidden sector of gauge mediation.

IV. SUMMARY AND DISCUSSIONS

We have studied supersymmetry-breaking models with both F and D terms being nonvanishing. In particular, we have focused on the case that the F-term potential shows runaway behaviors that originate from symmetries of theory considered. The runaway directions are uplifted by the D term and a supersymmetry-breaking vacuum is realized at finite field configuration. An interesting property of this approach is that (phenomenologically disfavored) pseudomoduli are absent in the vacuum since they are related to the minimization of the F-term potential. Moreover, there is no need to add the FI term for supersymmetry breaking. Along this line, a minimal renormalizable model has been presented where supersymmetry is broken. For an application to gauge mediation, we have introduced appropriate messenger sectors and confirmed that R symmetry is spontaneously broken, and gaugino masses in the visible sector are generated at the comparable order of sfermion masses. This class of models may open up a new way to build realistic models of gauge mediation, circumventing the lemma proved by Komargodski and Shih [3].

As remarked, the D-term lifted runaway might be destabilized along other orthogonal directions such as the $U(1)_R$ runaway. We do not have any criteria to ensure that the D-term uplifting of runaway directions can lead to the stable and global minimum of the scalar potential. It would be interesting if one could carry out a general argument on this issue.

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APPENDIX: POTENTIAL ANALYSIS

In this Appendix, we give some details of the potential analysis for the model given in Sec. III B. In particular, we explain the derivation of the vacuum expectation values (3.11).

The field derivatives of the superpotential (3.5) are given by

$$\begin{aligned} W_{X_+} &= m\varphi_-, & W_{X_-} &= \lambda'\varphi_+\varphi_0, \\ W_{X_0} &= f + \lambda\varphi + \varphi_-, & W_{\varphi_+} &= \lambda X_0\varphi_- + \lambda' X_-\varphi_0, \\ W_{\varphi_-} &= \lambda X_0\varphi_+ + mX_+, & W_{\varphi_0} &= \lambda' X_-\varphi_+. \end{aligned} \tag{A1}$$

The scalar potential V is the sum of the contributions from F and D terms, V_F and V_D, and these explicit forms are written down in (3.6) and (3.7). The stationary conditions for V are then given by

$$\frac{\partial V}{\partial X_+^*} = m^*(\lambda X_0\varphi_+ + mX_+) + g^2 X_+ D = 0, \tag{A2}$$

$$\frac{\partial V}{\partial X_-^*} = |\lambda'\varphi_+|^2 X_- + \lambda'^*\varphi_0^*(\lambda X_0\varphi_- + \lambda' X_-\varphi_0) - g^2 X_- D = 0, \tag{A3}$$

$$\frac{\partial V}{\partial X_0^*} = \lambda^*\varphi_+^*(\lambda X_0\varphi_+ + mX_+) + \lambda^*\varphi_-^*(\lambda X_0\varphi_- + \lambda' X_-\varphi_0) = 0, \tag{A4}$$

$$\frac{\partial V}{\partial \varphi_+^*} = \lambda^*\varphi_-^*(f + \lambda\varphi_+\varphi_-) + |\lambda' X_-|^2\varphi_+ + \lambda^* X_0^*(\lambda X_0\varphi_+ + mX_+) + |\lambda'\varphi_0|^2\varphi_+ + g^2\varphi_+ D = 0, \tag{A5}$$

$$\frac{\partial V}{\partial \varphi_-^*} = \lambda^*\varphi_+^*(f + \lambda\varphi_+\varphi_-) + |m|^2\varphi_- + \lambda^* X_0^*(\lambda X_0\varphi_- + \lambda' X_-\varphi_0) - g^2\varphi_- D = 0, \tag{A6}$$

$$\frac{\partial V}{\partial \varphi_0^*} = \lambda'^* X_-^*(\lambda X_0\varphi_- + \lambda' X_-\varphi_0) + |\lambda'\varphi_+|^2\varphi_0 = 0. \tag{A7}$$

First, by using (A4) and (A7), we express X_0 and φ_0 in terms of the other fields,

$$X_0 = \frac{-m}{\lambda} \frac{X_+(|X_-|^2 + |\varphi_+|^2)}{\varphi_+(|X_-|^2 + |\varphi_+|^2 + |\varphi_-|^2)}, \tag{A8}$$

$$\varphi_0 = \frac{m}{\lambda'} \frac{X_+ X_-^* \varphi_-}{\varphi_+(|X_-|^2 + |\varphi_+|^2 + |\varphi_-|^2)}. \tag{A9}$$

If $X_+ \neq 0$, we find from (A2) and (A8),

$$g^2 D = \frac{-|m\varphi_-|^2}{|X_-|^2 + |\varphi_+|^2 + |\varphi_-|^2}. \tag{A10}$$

This implies that D is negative at the stationary point (if $X_+ \neq 0$). On the other hand, from (A3) and (A7), we have

$$|\lambda' X_- \varphi_+|^2 = |\lambda' \varphi_+ \varphi_0|^2 + g^2 |X_-|^2 D, \tag{A11}$$

while the equation $\varphi_+^*(\partial V/\partial \varphi_+^*) - \varphi_-^*(\partial V/\partial \varphi_-^*) = 0$ gives

$$|\lambda' X_- \varphi_+|^2 + \varphi_+ |\lambda' \varphi_+ \varphi_0|^2 - |m\varphi_-|^2 + g^2(|\varphi_+|^2 + |\varphi_-|^2) D - \varphi_- \lambda X_0 (\lambda X_0 \varphi_- + \lambda' X_- \varphi_0)^* + \lambda X_0 (\lambda X_0 \varphi_+ + m X_+)^* = 0. \tag{A12}$$

By using the relations,

$$\lambda X_0 (\lambda X_0 \varphi_+ + m X_+)^* = \frac{-|m X_+ \varphi_-|^2}{\varphi_+} \frac{|X_-|^2 + |\varphi_+|^2}{(|X_-|^2 + |\varphi_+|^2 + |\varphi_-|^2)^2}, \tag{A13}$$

$$\lambda X_0 (\lambda X_0 \varphi_- + \lambda' X_- \varphi_0)^* = \frac{|m X_+ \varphi_-|^2}{\varphi_-} \frac{|X_-|^2 + |\varphi_+|^2}{(|X_-|^2 + |\varphi_+|^2 + |\varphi_-|^2)^2}, \tag{A14}$$

following from (A8) and (A9), we find another expression for D from (A11) and (A12),

$$g^2 D = \frac{1}{|X_-|^2 + |\varphi_+|^2 + |\varphi_-|^2} \times \left[|m\varphi_-|^2 + \frac{2|m X_+ \varphi_+ \varphi_-|^2}{(|X_-|^2 + |\varphi_+|^2 + |\varphi_-|^2)^2} \right]. \tag{A15}$$

That implies D is positive at the stationary point. For both conditions (A10) and (A15) to be true, $D = 0$ is the only solution, but it cannot be satisfied because the origin of meson direction $\varphi_+ \varphi_-$ is destabilized by the F-term potential. Therefore, $X_+ \neq 0$, which is the assumption for (A10), should not be realized. In the end, from (A8) and (A9), we find the following vacuum expectation values (3.11):

$$X_+ = X_0 = \varphi_0 = 0.$$

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