New physics in ϵ' from chromomagnetic contributions and limits on left-right symmetry

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New physics in the chromomagnetic flavor changing transition $s \to dg^*$ can avoid the strong Glashow-Iliopoulos-Maiani suppression of the standard model and lead to large contributions to CP-violating observables, in particular to the ϵ' parameter, that we address here. We discuss the case of the left-right symmetric models, where this contribution implies bounds on the phases of the right-handed quark mixing matrix, or in generic models with large phases a strong bound on the left-right symmetry scale. To the leading order, a numeric formula for ϵ' as a function of the short-distance coefficients for a wide class of models of new physics is given.

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Flavor-changing processes still offer one of the best means for spotting signs of physics beyond the standard model (SM). The K decays into pions are among the best studied channels both experimentally and theoretically, and despite the hadronic uncertainties in the theoretical predictions, can serve as a great tool for probing new physics. The reason is that for a number of observables, in particular the CP violating ones, the SM contribution is extremely small, mainly due to the Glashow-Iliopoulos-Maiani (GIM) mechanism [1] and the fact that the CP violating phase in the SM is suppressed by the smallness of the quark mixing angles. In turn, in the SM the GIM mechanism is intimately related to the chiral nature of the weak interactions. As a result, probes of processes involving the GIM mechanism are well suited to test for nonchiral interactions.

This is paradigmatic in one popular extension of the SM, left-right (LR) symmetry [2], which altogether gives a framework for restoration of parity in fundamental interactions, nonzero neutrino masses, as well as violation of lepton number and flavor [3] both at the reach of the coming round of experiments, fitting especially well with the scenario of TeV scale LR symmetry [4–6]. The related direct searches at LHC, with important signatures through the new interactions and same-sign dileptons [7], can explore this possibility up to ~6 TeV [8] and are already beginning to probe this interesting region [9–11]. It is then important to assess the bounds on the model from existing phenomena.

In the LR models, based the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group, modifications of GIM are mainly due to the new right-handed gauge boson W_R and to its mixing with the standard weak gauge boson W_L. Bounds on the scale of the new right-handed gauge interaction were already addressed since the early days, a notorious example being the ΔM_K box diagram [12,13] where the GIM enhancement adds to a chiral enhancement of the matrix elements, and still leads today to the strongest bound on the scale of LR symmetry, $M_{W_R} \gtrsim 2.5$–3 TeV [4]. Similar effects hold for the CP-violation parameter ϵ [14]. The bottom line is that the interplay of nonchiral interactions with the hierarchy of quark masses and mixings can lead to dramatic effects in loop diagrams. This is especially true in the phenomenology of strange mesons, and in particular for the direct CP-violation parameter ϵ' and the chromomagnetic loop, that we address.

The gluonic penguin operators have been traditionally associated to ϵ'/ϵ, because they pointed immediately to a possibly sizable effect. However, in the SM a partial cancellation between the dominant gluonic and electroweak penguins translates into a large theoretical uncertainty, linked to hadronic matrix elements. For a review on the evaluation of ϵ'/ϵ and additional literature, we refer to Refs. [15–17]. In any case, ϵ' is naturally tiny in the SM, and can serve as an efficient tool for constraining new physics.

In this work, we address the contributions of nonchiral interactions in the chromomagnetic operator, and its effect on ϵ'. In the analysis we first give a parametrization of the effects of new physics in ϵ' which is applicable to a wide class of models with nonchiral interactions. In the context of left-right symmetry, as it is known current-current operators mediated through the left-right gauge boson mixing gives large contributions to the $K \to \pi\pi$ amplitude. This issue was studied in detail in Refs. [4,18–20], together with the other flavor constraints on the model. However, the effect of the chromomagnetic operator was not considered. We study its impact due to the effective $K \to \pi\pi$ transition whose hadronic matrix element computed within the chiral quark model (χQM) [21].

The χQM provides an interpolation between short-distance QCD and its effective description in terms of the octet of Goldstone mesons, below the scale of chiral symmetry breaking (for a recent discussion, see Ref. [22]). The chiral Lagrangian coefficients are determined order by order in the momentum expansion by integration of the constituent quarks and depend on three nonperturbative parameters: the constituent quark mass and the quark and gluon condensates. Via a fit of the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decays, the authors of Ref. [15] obtained a nontrivial phenomenological determination of these three parameters that allowed for a correlated calculation of ϵ'/ϵ and of the $\Delta S = 2$ bag parameter B_K within the χQM approach, at next-to-leading order (NLO) in the chiral expansion [23–25]. We will use these values of the parameters in our analysis.

For the LR model, we shall see that only the chromomagnetic operator plays a dominant role in ϵ', once other existing constraints from K and B physics are considered, and implies a bound on the free phases involved, in the hypothesis of TeV scale LR symmetry. For other new-physics models, constraints from ϵ' via the chromomagnetic operator were studied in Refs. [26–28].

The paper is organized as follows: In Sec. I, we describe the effective operators involved when nonchiral new physics is present, including the dipole ones. We also review the short distance coefficients in the case of the LR theory. In Sec. II, by running with the mixed anomalous dimensions, we compute the Wilson coefficients at 0.8 GeV, which is our chosen scale for matching with chiral perturbation theory. In Sec. III, we describe (and update) the bosonization of the chromomagnetic operator. This enables us to make contact with the $K \to \pi\pi$ amplitude and with ϵ' in Sec. IV in general and in Sec. V for the LR model. In Sec. VI, we draw our conclusions.

I. NEW PHYSICS

The effective Lagrangian for flavor changing can be written in the form $\mathrm{L}_{\Delta S=1} = -(G_F/\sqrt{2})\sum_i C_i Q_i + \mathrm{H.c.}$, where Q_i are the relevant operators and C_i the corresponding coefficients (and G_F the Fermi constant). In the standard model the $\Delta S = 1$ processes are usually described by a (over)complete set of operators [29,30]. They involve tree-level operators as well as QED and QCD penguins. In models with both chiralities such as the left-right model, the standard set of operators has to be extended. In the case of $\Delta S = 1$ discussed here the complete set at low energy involves 28 operators,

$$
\begin{aligned}
Q_1^{\mathrm{LL}} &= (\bar{s}_\alpha u_\beta)_{\mathrm{L}}(\bar{u}_\beta d_\alpha)_{\mathrm{L}}, & Q_1^{\mathrm{RR}} &= (\bar{s}_\alpha u_\beta)_{\mathrm{R}}(\bar{u}_\beta d_\alpha)_{\mathrm{R}},\\
Q_2^{\mathrm{LL}} &= (\bar{s}u)_{\mathrm{L}}(\bar{u}d)_{\mathrm{L}}, & Q_2^{\mathrm{RR}} &= (\bar{s}u)_{\mathrm{R}}(\bar{u}d)_{\mathrm{R}},\\
Q_3 &= (\bar{s}d)_{\mathrm{L}}(\bar{q}q)_{\mathrm{L}}, & Q_3' &= (\bar{s}d)_{\mathrm{R}}(\bar{q}q)_{\mathrm{R}},\\
Q_4 &= (\bar{s}_\alpha d_\beta)_{\mathrm{L}}(\bar{q}_\beta q_\alpha)_{\mathrm{L}}, & Q_4' &= (\bar{s}_\alpha d_\beta)_{\mathrm{R}}(\bar{q}_\beta q_\alpha)_{\mathrm{R}},\\
Q_9 &= \frac{3}{2}(\bar{s}d)_{\mathrm{L}} e_q (\bar{q}q)_{\mathrm{L}}, & Q_9' &= \frac{3}{2}(\bar{s}d)_{\mathrm{R}} e_q (\bar{q}q)_{\mathrm{R}},\\
Q_{10} &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_{\mathrm{L}} e_q (\bar{q}_\beta q_\alpha)_{\mathrm{L}}, & Q_{10}' &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_{\mathrm{R}} e_q (\bar{q}_\beta q_\alpha)_{\mathrm{L}},\\
Q_1^{\mathrm{RL}} &= (\bar{s}_\alpha u_\beta)_{\mathrm{R}}(\bar{u}_\beta d_\alpha)_{\mathrm{L}}, & Q_1^{\mathrm{LR}} &= (\bar{s}_\alpha u_\beta)_{\mathrm{L}}(\bar{u}_\beta d_\alpha)_{\mathrm{R}},\\
Q_2^{\mathrm{RL}} &= (\bar{s}u)_{\mathrm{R}}(\bar{u}d)_{\mathrm{L}}, & Q_2^{\mathrm{LR}} &= (\bar{s}u)_{\mathrm{L}}(\bar{u}d)_{\mathrm{R}},\\
Q_5 &= (\bar{s}d)_{\mathrm{L}}(\bar{q}q)_{\mathrm{R}}, & Q_5' &= (\bar{s}d)_{\mathrm{R}}(\bar{q}q)_{\mathrm{L}},\\
Q_6 &= (\bar{s}_\alpha d_\beta)_{\mathrm{L}}(\bar{q}_\beta q_\alpha)_{\mathrm{R}}, & Q_6' &= (\bar{s}_\alpha d_\beta)_{\mathrm{R}}(\bar{q}_\beta q_\alpha)_{\mathrm{L}},\\
Q_7 &= \frac{3}{2}(\bar{s}d)_{\mathrm{L}} e_q (\bar{q}q)_{\mathrm{R}}, & Q_7' &= \frac{3}{2}(\bar{s}d)_{\mathrm{R}} e_q (\bar{q}q)_{\mathrm{L}},\\
Q_8 &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_{\mathrm{L}} e_q (\bar{q}_\beta q_\alpha)_{\mathrm{R}}, & Q_8' &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_{\mathrm{R}} e_q (\bar{q}_\beta q_\alpha)_{\mathrm{L}},\\
Q_g^{\mathrm{L}} &= \frac{g_s m_s}{8\pi^2}\bar{s}\sigma_{\mu\nu}t^a G_a^{\mu\nu}\mathrm{L}d, & Q_g^{\mathrm{R}} &= \frac{g_s m_s}{8\pi^2}\bar{s}\sigma_{\mu\nu}t^a G_a^{\mu\nu}\mathrm{R}d,\\
Q_\gamma^{\mathrm{L}} &= \frac{e m_s}{8\pi^2}\bar{s}\sigma_{\mu\nu}F_a^{\mu\nu}\mathrm{L}d, & Q_\gamma^{\mathrm{R}} &= \frac{e m_s}{8\pi^2}\bar{s}\sigma_{\mu\nu}F_a^{\mu\nu}\mathrm{R}d.
\end{aligned}
\tag{1}
$$

The notation is $(\bar{q}q)_{\mathrm{L,R}} = \bar{q}\gamma_\mu(1 \mp \gamma_5)q$, L, $\mathrm{R} = (1 \mp \gamma_5)/2$, and the summation on $q = u, d, s$ is implicit. $Q_{1,2}^{\mathrm{LL}}$ are the SM operators usually denoted as $Q_{1,2}$. The dipole operators $Q_{g,\gamma}$ are normalized with m_s, for an easy comparison with existing calculations, and for keeping unaltered the anomalous dimension. It is known that some of the operators above are accompanied by an enhancement due to the different chiral structure, either in the short distance coefficient, in the running, or in the matrix element. Such a situation occurs for the chromomagnetic operators $Q_g^{\mathrm{L,R}}$, as we describe below.

At leading order the operators generated by the SM and the LR short distance physics are Q_2^{AB}, Q_4, Q_4', Q_6, Q_6', Q_7, Q_7', Q_9, Q_9', Q_g^A, Q_γ^A, with A, $B = \mathrm{L}$, R. Their coefficients are the current-current ones:

$$
\begin{aligned}
C_2^{\mathrm{LL}} &= \lambda_u^{\mathrm{LL}}, & C_2^{\mathrm{LR}} &= \zeta^* \lambda_u^{\mathrm{LR}},\\
C_2^{\mathrm{RR}} &= \beta\lambda_u^{\mathrm{RR}}, & C_2^{\mathrm{RL}} &= \zeta\lambda_u^{\mathrm{RL}};
\end{aligned}
\tag{2}
$$

the penguin ones are

$$
\begin{aligned}
C_4 = C_6 &= \frac{\alpha_s}{4\pi}\Sigma_i \lambda_i^{\mathrm{LL}} F_1^{\mathrm{LL}}(x_i),\\
C_4' = C_6' &= \frac{\alpha_s}{4\pi}\beta\Sigma_i \lambda_i^{\mathrm{RR}} F_1^{\mathrm{RR}}(\beta x_i),\\
C_7 = C_9 &= \frac{\alpha e_u}{4\pi}\Sigma_i \lambda_i^{\mathrm{LL}} E_1^{\mathrm{LL}}(x_i),\\
C_7' = C_9' &= \frac{\alpha e_u}{4\pi}\beta\Sigma_i \lambda_i^{\mathrm{RR}} E_1^{\mathrm{RR}}(\beta x_i);
\end{aligned}
\tag{3}
$$

and the dipole ones are

$$
\begin{aligned}
m_s C_g^{\mathrm{L}} &= \Sigma_i[m_s\lambda_i^{\mathrm{LL}}F_2^{\mathrm{LL}} + \zeta m_i \lambda_i^{\mathrm{RL}}F_2^{\mathrm{LR}} + m_d\beta\lambda_i^{\mathrm{RR}}F_2^{\mathrm{RR}}],\\
m_s C_g^{\mathrm{R}} &= \Sigma_i[m_d\lambda_i^{\mathrm{LL}}F_2^{\mathrm{LL}} + \zeta^* m_i \lambda_i^{\mathrm{LR}}F_2^{\mathrm{LR}} + m_s\beta\lambda_i^{\mathrm{RR}}F_2^{\mathrm{RR}}],\\
m_s C_\gamma^{\mathrm{L}} &= \Sigma_i[m_s\lambda_i^{\mathrm{LL}}E_2^{\mathrm{LL}} + \zeta m_i \lambda_i^{\mathrm{RL}}E_2^{\mathrm{LR}} + m_d\beta\lambda_i^{\mathrm{RR}}E_2^{\mathrm{RR}}],\\
m_s C_\gamma^{\mathrm{R}} &= \Sigma_i[m_d\lambda_i^{\mathrm{LL}}E_2^{\mathrm{LL}} + \zeta^* m_i \lambda_i^{\mathrm{LR}}E_2^{\mathrm{LR}} + m_s\beta\lambda_i^{\mathrm{RR}}E_2^{\mathrm{RR}}].
\end{aligned}
\tag{4}
$$

In the above, $e_u = 2/3$ is the u-quark charge, $x_i = m_i^2/m_{W_{\mathrm{L}}}^2$ with $i = u, c, t$, and $F_{(1,2)}^{AB}$ and E_2^{AB} are the loop functions, given in Appendix A. Then, $\beta = M_{W_{\mathrm{L}}}^2/M_{W_{\mathrm{R}}}^2$ is the ratio of the electroweak to the LR scale and ζ is the

W_L-W_R mixing. Note that in the hypothesis of LR symmetry at TeV scale, $\beta \sim 10^{-3}$. Also ζ is of order β or less; for instance, in the minimal LR models it is $\zeta \simeq -\beta e^{i\alpha} \frac{2x}{1+x^2}$, with $x < 1$ the (modulus of the) ratio of the two vacuum expectation values of the Higgs bidoublet and α its phase. We will consider below the specific case of minimal LR models, referring to Refs. [4,31] for definitions and details. Finally, $\lambda_i^{AB} = V_{is}^{*A} V_{id}^{B}$, where V_L and V_R are the Cabibbo-Kobayashi-Maskawa matrix and its right-handed analogue. A crucial new ingredient in V_R is the presence of (five) additional phases, besides the Dirac one. These can be parametrized as ($U = u, c, t$, $D = d, s, b$)

$$V_{UD}^{R} = e^{i\theta_U} \hat{V}_{UD}^{R} e^{i\theta_D}, \tag{5}$$

with $\hat{V}_R$ the mixing matrix in standard Cabibbo-Kobayashi-Maskawa form.

The terms in the expressions (2)–(4) for the coefficients should be understood as generated at the decoupling of the relevant heavy states, and thus at different scales, namely: M_{W_L} or m_t for the current-current and top-dominated loops, m_c for the charm dominated loops etc, and m_{W_R} for the right-right (RR) current-current.

A similar set of operators $Q_{1,2c}^{AB}$ with the c-quark replacing u, is also generated by the short distance physics, and also by renormalization at scales larger than m_c. On the other hand, the further operators involving the t quark are not explicitly required: for the LL and RR operators this is due to the GIM cancelation above m_t (also in the running); for the LR ones, they are only generated at electroweak scale through the LR-mixing ζ.[1] Last, there are also penguin operators built through the LR-mixing, which are chirally suppressed and give subleading (negligible) contributions.

From (4), it can be seen that the coefficients $C_g^{L,R}$ receive a large contribution in the LR model, due to the different GIM mechanism. In fact, the mass insertion on the external fermion legs in the SM (m_s) is replaced in the LR model by a mass insertion inside the loop (m_i). The loop is then dominated by m_c leading to an enhancement of $m_c/m_s \sim 100$. In addition, the factor $\lambda_c^{AB}/\lambda_t^{LL}$ gives a further large enhancement of 10^3, which compensates the LR-scale suppression ζ. Both Q_g^L and Q_g^R are present, and the LR contribution ends up being a factor $\sim$200 larger than the SM one, at short distance:

$$\frac{|H_g^{(LR)}|}{|H_g^{(SM)}|} \simeq \frac{2m_c F_2^{LR}(x_c)|V_{cd}^* V_{cs}\zeta|}{m_s F_2^{LL}(x_t)|V_{td}^* V_{ts}|} \simeq 2 \times 10^5 \zeta \simeq 200. \tag{6}$$

[1] Clearly, if $Q_{1,2t}^{LR}$ were generated at high scale, they should be taken into account, because due to the mixed chirality a GIM cancelation is not effective in the running (see also comments in Appendix B). Also, some more operators of the form $(\bar{s}d)_L \times (\bar{d}d + \bar{s}s)_{L,R}$ mixing with the penguins are present in general, see Ref. [32], but they are not generated in the LR model and it is also difficult to generate them in models where new physics sets in at scales higher than the electroweak scale.

Here, the factor 2 accounts for the contributions LR + RL, and we considered $\hat{V}_R \simeq V_L$, which is a general prediction of minimal LR models [4].

The new phases (5) contained in V_R, together with the enhancement above, can directly induce a sizeable CP violation. It is therefore important to address the effect of this operator on ϵ', which we study along the lines of Refs. [33,34]. In order to deal with this low energy phenomenon, two steps are necessary: the first is to renormalize the coefficients at low energy, in the range of chiral perturbation theory; the second is to use the matrix elements $\langle 2\pi|Q_i|K_0\rangle$, or equivalently to match with chiral perturbation theory. In the following section, we renormalize the full set of coefficients down to the scale of 0.8 GeV, and in the next we match with the chiral Lagrangian in the context of the chiral quark model.

The need to evaluate the Wilson coefficients at such a low QCD scale is dictated by the requirement to use the matrix elements calculated in the context of chiral quark model in chiral perturbation theory, whose cutoff is the chiral symmetry breaking scale. In order to assess quantitatively the scale dependence of the result, we remark that by varying the matching scale between 0.8 and 1 GeV, the chromomagnetic $C_g^{L,R}$ and the current-current ones $C_{1,2}^{LR,RL}$ vary at most by 5% and 10%, respectively. These uncertainties are well below those of the matrix elements.

II. RUNNING TO LOW SCALE

The mixing of operators (1) is described in detail in Appendix B. At leading order, the operators can be split into two sets, of opposite chiralities, corresponding to the two columns in (1). The low energy coefficients together with the matrix elements of all the operators are also sufficient to give an estimate of the impact on ϵ' for quite a large class of models of new physics. This will be presented in Sec. IV.

In the particular case of the LR model, the low energy coefficients are shown in Table I. The running takes into account the whole set of operators including the SM penguins, but we show the result only for the operators containing the LR scale β or ζ which have an impact on ϵ', and the magnetic operator $Q_\gamma^{L,R}$ which is also enhanced. The results are normalized to λ_u, to compare with existing calculations. The coefficients $C_g^{L,R}$, compared with the complex part of the SM result, $C_g^L(\text{SM}) \simeq 0.34\lambda_t$ [33], confirm the important role of Q_g from new physics.

In the detail of the running it is worth noting that, despite the reduction of $\sim$0.5 due to their own anomalous dimension, $C_g^{L,R}$ receive contributions from $C_{1,2}^{LL,LR,RL,RR}$. The largest additional contribution is due to $C_{1,2c}^{RL,LR}$ at scales above m_c, while the contributions from $C_{1,2u}^{RL,LR}$ are

TABLE I. Coefficients for the dominant new operators in the minimal LR model, evaluated at $\mu = 0.8$ GeV.

$C_1^{\mathrm{RL,LR}}$	$\lambda_u(1.07)\lvert\zeta\rvert e^{\pm i(\alpha-\theta_{s,d}-\theta_u)}$
$C_2^{\mathrm{RL,LR}}$	$\lambda_u(0.80)\lvert\zeta\rvert e^{\pm i(\alpha-\theta_{s,d}-\theta_u)}$
C_1^{RR}	$\lambda_u(-0.54)\beta e^{-i(\theta_s-\theta_d)}$
C_2^{RR}	$\lambda_u(1.24)\beta e^{-i(\theta_s-\theta_d)}$
$C_g^{\mathrm{L,R}}$	$\lambda_u(-10.7)\lvert\zeta\rvert e^{\pm i(\alpha-\theta_{d,s}-\theta_c)}$
$C_\gamma^{\mathrm{L,R}}$	$\lambda_u(-3.31)\lvert\zeta\rvert e^{\pm i(\alpha-\theta_{d,s}-\theta_c)}$

suppressed by the u quark mass. This is due to the internal mass insertion in the (two) loop graphs responsible for the operator mixing in the anomalous dimension matrix, and is another consequence of the nonchiral nature of these operators. As a side result, this additional contribution preserves the same combination of phases appearing in the original short-distance $C_g^{\mathrm{L,R}}$.

From Table I, we can calculate the contributions to ϵ' of these operators. This requires the evaluation of matrix elements which we review now for $Q_g^{\mathrm{L,R}}$.

III. BOSONIZATION OF Q_g

The bosonization of Q_g was addressed in Ref. [33] in the context of the chiral quark model. Here we review the computation, which leads a minor numerical correction.

Under chiral $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}}$ rotations, the Q_g operators transform as $(\mathbf{3}_{\mathrm{L}}, \bar{\mathbf{3}}_{\mathrm{R}})$, and thus they give rise to particular terms in the chiral Lagrangian. While by symmetry arguments there are diverse possibilities (see e.g., Ref. [35] in naive dimensional analysis) in the context of the chiral quark model, only one form arises [33]. This is true in the SM as with the separate operators Q_g^{L}, Q_g^{R}. One has

$$\mathcal{L}_{Q_g} = \mathrm{Tr}[(\Sigma^\dagger X\lambda_- + \lambda_- X^\dagger\Sigma)D_\mu\Sigma^\dagger D^\mu\Sigma], \qquad (7)$$

where $\lambda_- = (\lambda_6 - i\lambda_7)/2$, and where the matrix of two coefficients $X = \mathrm{diag}(0, G_8^{\mathrm{R}}, G_8^{\mathrm{L}})$ replaces the single coefficient $G_8^{(4)}$ and the running quark mass matrix $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$ of the analogous calculation in the SM. In fact, the coefficients $C_g^{\mathrm{L,R}}$ of the $\Delta S = 1$ transition induce a tiny breaking of the chiral symmetry which plays the same role as the s and d quark masses in the SM. We also observe that from the point of view of chiral perturbation theory the above Lagrangian is $O(p^4)$ in the LL and RR terms proportional to light quark masses, but $O(p^2)$ in the LR and RL terms, proportional to m_c. In any case, the coefficients $G_8^{\mathrm{L,R}}$ will, respectively, be proportional to $C_g^{\mathrm{L,R}}$. To determine them, it is convenient to evaluate some amplitude both through the above chiral Lagrangian and in the chiral quark model, and compare the results [33]. We do this in the "unrotated" picture [17]. The simplest process is the off-shell $K^- \to \pi^-$ transition, which at one loop is given by the three diagrams shown in Fig. 1. The two external gluon lines are attached to the gluons in the external thermal QCD vacuum and lead to a coefficient proportional to the gluon condensate. To deal with the thermal and color average, it is best to adopt Fock-Schwinger fixed-point gauge ($x^\mu A_\mu = 0$) [36]. Due to this gauge choice, translations are broken and two fixed "sink" points for the gluon momentum are defined, chosen here to be $x = 0$ and x at the K and π vertices. Then the three diagrams in Fig. 1 are

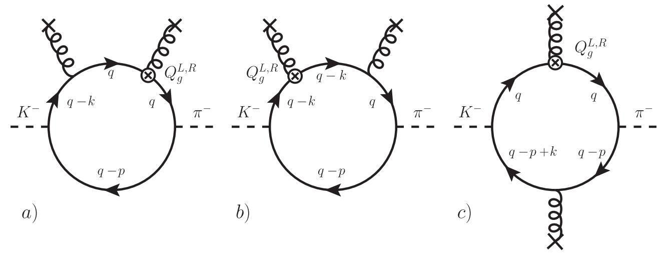

FIG. 1. Diagrams for the bosonization of Q_g, in the fixed point gauge. Note the flow of gluon momentum k.

$$A(K^- \to \pi^-) = \frac{2m^2}{f^2}\int\frac{\mathrm{d}^4q}{16\pi^4}(a + b + c),$$

$$\begin{aligned} a &= \mathrm{tr}[(\not{q} - \not{k} + m)\not{A}(\not{q} + m)q_g(\not{q} + m)(\not{q} - \not{p} + m)]\Delta[q,m]^2\Delta[q-k,m]\Delta[q-p,m],\\ b &= \mathrm{tr}[(\not{q} - \not{k} + m)q_g(\not{q} - \not{k} + m)\not{A}(\not{q} + m)(\not{q} - \not{p} + m)]\Delta[q,m]\Delta[q-k,m]^2\Delta[q-p,m],\\ c &= \mathrm{tr}[(\not{q} + m)q_g(\not{q} + m)(\not{q} - \not{p} + m)\not{A}(\not{q} + \not{k} - \not{p} + m)]\Delta[q,m]^2\Delta[q-p,m]\Delta[q-p+k,m], \end{aligned} \qquad (8)$$

where p is the K and π momentum; k is the incoming gluon momentum; $\Delta[p,m] = 1/(p^2 - m^2)$; $q_g = -iG_F g_s m_s (C_g^{\mathrm{L}}\mathrm{L} + C_g^{\mathrm{R}}\mathrm{R})G^{\alpha\beta}\sigma_{\alpha\beta}/\sqrt{2}$ and m is the constituent quark mass. Also (see Ref. [36]) $\not{A} = (g_s/2)\gamma_\mu G_{\mu\nu}\partial/\partial k_\nu$ where a derivative with respect to the gluon momentum has to be taken, after which k is set to zero.[2] By the same prescriptions, one shows that no external gluon momentum flows through the chromomagnetic operator. Finally, the two gluon field-strengths are averaged in the gluon condensate, $G_{\alpha,\beta}G_{\gamma,\delta} \to (\pi^2/6g_s^2)\langle\frac{\alpha}{\pi}GG\rangle(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma})$.

[2] Due to the absence of translational invariance, the use of the one-external-gluon effective quark propagator in the fixed point gauge [37] is not correct in diagram (b), where the gluon momentum flows to the origin (K) passing through the operator insertion. This leads to a mismatch between diagrams a and b and to a numerical correction of the result in Ref. [33].

The loop integration produces a term at order zero in p which is canceled in the leading chiral Lagrangian in agreement with the Feinberg-Kabir-Weinberg theorem [38]. The second order term gives the desired result

$$A(K^- \to \pi^-) = \frac{iG_F}{\sqrt{2}} m_s \frac{C_g^{\rm L} + C_g^{\rm R}}{16\pi^2} \left\langle \frac{\alpha}{\pi} GG \right\rangle p^2 \frac{(7+1+8)}{48 f^2 m}, \tag{9}$$

the three numbers being relative to diagrams a, b, c. Comparing this amplitude with the one calculated from the chiral Lagrangian (7), one finally finds the coefficients

$$G_8^{\rm L,R} = 2\frac{1}{12m}\left\langle \frac{\alpha}{\pi} GG \right\rangle \frac{G_F}{\sqrt{2}} \frac{m_s C_g^{\rm L,R}}{16\pi^2}. \tag{10}$$

The factor of 2 corrects 11/4 appearing in Ref. [33] and leads to a 30% reduction of the matrix element. This result is also confirmed by a similar calculation within the "rotated" picture.

Before using this result for the calculation of ϵ', let us note that the additional contribution from the *off-shell* chromomagnetic sdg vertex, shown in Ref. [39] to be of the same order as Q_g in the SM, is strongly suppressed in the case of nonchiral interactions (again because the mass insertion happens inside the W loop).

Finally, a double insertion of Q_g, leads directly to (long-distance) $\Delta S = 2$ processes and can be calculated similarly. However, the process is doubly loop and G_F suppressed and the result is negligible for both ΔM_K and ϵ (see Appendix D).

IV. RESULT FOR ϵ'

The direct CP parameter ϵ' is defined by

$$\epsilon' = \frac{i}{\sqrt{2}}\omega\left(\frac{{\rm Im}A_2}{{\rm Re}A_2} - \frac{{\rm Im}A_0}{{\rm Re}A_0}\right)\frac{q}{p} e^{i(\delta_2 - \delta_0)}, \tag{11}$$

where p, q are the K^0, $\bar{K}^0$ mixing parameters and $\omega \equiv A_2/A_0 \simeq 1/22.2$. The ratio $p/q \simeq 1$ with an excellent approximation. The isospin amplitudes A_I ($I = 0, 2$) are defined from the $\Delta S = 1$ effective Hamiltonian as $\langle (2\pi)_I | (-i) H_{\Delta S=1} | K^0 \rangle = A_I e^{i\delta_I}$, where δ_I are the strong phases of $\pi\pi$ scattering. We calculate the imaginary part of the amplitudes, while for the real part we take the experimental value: ${\rm Re}A_0 = 3.33 \times 10^{-7}$ GeV and ${\rm Re}A_2 = 1.49 \times 10^{-8}$ GeV.

The amplitudes A_0 and A_2 for the standard operators are collected in Appendix C. The ones of $Q_g^{\rm L,R}$ are easily calculated from the chiral Lagrangian (7). One has the isospin decomposition

$$A_0^{Q_g^{\rm L+R}} = \sqrt{\frac{2}{3}} A_\pm^{Q_g^{\rm L+R}}, \qquad A_2^{Q_g^{\rm L+R}} = \sqrt{\frac{1}{3}} A_\pm^{Q_g^{\rm L+R}}, \tag{12}$$

where the amplitude for $K^0 \to \pi^+\pi^-$ is

$$\begin{aligned} A_\pm^{Q_g^{\rm L+R}} &= \frac{\sqrt{2}}{f^3} m_\pi^2 (G_8^{\rm L} - G_8^{\rm R}) \\ &= \frac{G_F m_\pi^2}{6 m f^3} \left\langle \frac{\alpha}{\pi} GG \right\rangle \frac{m_s (C_g^{\rm L} - C_g^{\rm R})}{16\pi^2}, \end{aligned} \tag{13}$$

and the one for $K^0 \to \pi^0\pi^0$ vanishes. In the following we use $f = 93$ MeV, and for the gluon condensate and constituent quark mass we adopt the central values $\langle \frac{\alpha}{\pi} GG \rangle = (334~{\rm MeV})^4$, and $m = 200$ MeV [15], obtained by consistently fitting in the model the $\Delta I = 1/2$ selection rule.

Using the running and the matrix elements, one can generically describe the contributions to ϵ' of the different operators, as a weighted sum of the coefficients contributing from the desired scales μ_n:

$$|\epsilon'| = \left| \sum_n \sum_i w_i(\mu_n) {\rm Im} C_i(\mu_n) \right|, \tag{14}$$

where, at each scale μ_n, the $C_i(\mu_n)$ are the coefficients and $w_i(\mu_n)$ their weights. The scales μ_n can be either taken as the ones where the short distance coefficients are generated, i.e., m_W, m_c, etc., in which case the w_i account for the running and the matrix elements, or some low energy scale if one includes the running in the C_i. In Table II, we collect the numeric values of w_i computed by taking into account the complete running from a choice of different scales down to 0.8 GeV, together with the required matrix elements (see Appendix C).

As discussed in Appendix C, the hadronic uncertainties present in the $\mathcal{B}$ factors of departure from vacuum saturation approximation can be sizable and may vary from 10 to 50% for the better known operators, to a factor of order one for Q_g and $Q_{1,2}^{\rm LR}$. For the SM operators, the values adopted in Table II are taken from the chiral quark model calculation [15]. For the LR operators $Q_{1,2}^{\rm LR}$, a determination is missing but an guess can be given by noting their similarity with operators $Q_{7,8}$.

For $Q_g^{\rm L,R}$, while the leading order chiral bosonization (7) results in a m_π^2/m_K^2 suppression [see (13)], this may cease to be true in higher orders and may result in a further enhancement. Together with chiral loops, this is likely to lead to order one correction coefficients $\mathcal{B}_{0,2}^g$ to be added to Eq. (12). While we stress the need for a dedicated assessment of these corrections in the view of new physics, in the present analysis we conservatively assume $\mathcal{B}_{0,2}^g = 1$, keeping in mind that a possible enhancement would make our bounds below stronger.

Clearly, taking the central values of the hadronic matrix elements is sufficient for the scope of assessing the relevance of contributions beyond the SM, and, in particular, in view of the leading role of the chromomagnetic operators shown by the present analysis. Also, the renormalization evolution is performed at leading order, as is the determination of the starting conditions in Eqs. (2)–(4). Nevertheless, since the penguins can be considered as

TABLE II. Weigths w_i (times 10) of the coefficients C_i in the determination of ϵ', applicable to coefficients from different scales. The weights relative to the opposite chirality operators have opposite sign.

$w_i \times 10$	$w_{1c}^{\rm RL}$	$w_{2c}^{\rm RL}$	$w_1^{\rm RL}$	$w_2^{\rm RL}$	$w_{1c}^{\rm LL}$	$w_{2c}^{\rm LL}$	$w_1^{\rm LL}$	$w_2^{\rm LL}$	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	$w_g^{\rm L}$
$\mu = m_W$	0.52	−0.068	160.	52.	0.086	−0.55	2.0	−0.24	−0.14	−3.4	8.1	22.	110.	340.	3.2	0.5	0.020
$\mu = m_c$	0.	0.	51.	15.	0.	0.	1.9	1.0	0.16	−0.74	2.7	8.0	32.	110.	2.8	1.9	0.038
$\mu = 0.8$ GeV	0.	0.	42.	11.	0.	0.	2.0	1.2	0.22	−0.55	2.2	6.7	25.	88.	2.8	2.1	0.043

NLO contributions, a NLO correction to the current-current starting conditions was also inserted (see Ref. [30]). In this respect let us again remark that while a NLO analysis is necessary for the SM, it is not crucial for assessing the constraint from ϵ' on new physics. We believe Table II with formula (14) to be useful in analyzing the impact of ϵ' for quite a wide class of new physics models.

V. CONSEQUENCES FOR THE LR MODEL

We can finally estimate the numerical impact on ϵ' of the new physics operators in the case of LR symmetry, using the values of the low energy coefficients summarized in Table I and the last line of Table II (or equivalently the appropriate short distance coefficients with the first lines). We find

$$\begin{aligned}|\epsilon'_{\rm LR}| \simeq ||\zeta| 1.25[\sin(\alpha-\theta_u-\theta_d)+\sin(\alpha-\theta_u-\theta_s)] \\ +|\zeta| 0.010[\sin(\alpha-\theta_c-\theta_d)+\sin(\alpha-\theta_c-\theta_s)] \\ +\beta 0.013 \sin(\theta_d-\theta_s)|, \qquad (15)\end{aligned}$$

where the first line is due to the dominant $Q_{1,2}^{\rm LR,RL}$, the second to $Q_g^{\rm L,R}$, and the last to $Q_{1,2}^{\rm RR}$.[3] The penguin contributions are only responsible for minor corrections in the above numeric result.

For TeV LR-scale ($\beta \sim 10^{-3}$), we see that the above contributions can give overdominant contributions to ϵ', even having assumed similar left and right quark mixing angles, as in the minimal LR models. This is true both for $Q_g^{\rm L,R}$ and for the other operators. Their impact may be different and it depends, in addition to the LR scale β, on the CP phase α and on the extra phases in $V_{\rm R}$. The actual implications for a given LR model depend thus on the available freedom in choosing these phases. Let us describe the bounds in different scenarios, assuming that the new physics can contribute as much as 100% to ϵ'.

Assuming generic O(1) free phases, the LR scale is constrained to lie above a large limit $M_{W_{\rm R}} \gtrsim 25$ TeV. In fact this amounts to the highest limit on the right-handed scale. It is also worth recalling that for order one phases another large bound of about $M_{W_{\rm R}} \gtrsim 15$ TeV results from ϵ, while limits from B mass difference and CP violation are less stringent [4,19]. The argument can however be turned around and (15) can be used to put constraints on the phases, in the scenario of TeV LR symmetry.

In the minimal LR models, phases are either strictly predicted or are free, depending on the choice of LR symmetry, which can be generalized parity ($\mathcal{P}$, exchanging fermions $\psi_{\rm L} \leftrightarrow \psi_{\rm R}$) or generalized charge conjugation ($\mathcal{C}$, exchanging fermions $\psi_{\rm L} \leftrightarrow \psi_{\rm R}^c$), following the analysis of Ref. [4].

An important common constraint resulting in both cases from ϵ is that $\theta_d - \theta_s$ is close to 0 (or marginally to π) at least as 10^{-2}. This implies that the contribution of $Q_{1,2}^{\rm RR}$ to ϵ' can be neglected [the last line in (15)]. This can already be seen from the low energy coefficients in Table I. From the same Table I, it is also important to note an other consequence of the ϵ constraint, namely, that thanks to $\theta_s \simeq \theta_d$ the coefficients are practically complex conjugated under L $\leftrightarrow$ R exchange. As a result, the contributions to ϵ', which are proportional to L − R combinations [as $C_g^{\rm L} - C_g^{\rm R}$, see (13)] are purely imaginary and thus with maximal imaginary part. On the contrary, only the combinations L + R would enter in other CP-violating observables involving an even number of mesons, making the imaginary part suppressed by $\theta_s - \theta_d \lesssim 10^{-2}$. This is the case for instance for the contribution of the magnetic operators $O_\gamma^{\rm L,R}$ to the CP asymmetry in $K \to \pi e^+ e^-$, which is thus suppressed, despite the enhancement of the Wilson coefficient. This situation can be contrasted with the one occurring, e.g., in supersymmetric models, where a correlation between ϵ' and $K \to \pi \ell^+ \ell^-$ can be inferred [26]. Nevertheless, the enhancement of $C_\gamma^{\rm L,R}$ would survive in CP-asymmetries with an odd number of mesons, like $K \to \pi\pi\ell^+\ell^-$ (whose analysis brings in the leptonic sector of the LR models and is beyond our study).

Let us then describe the impact of the first two lines in (15) to ϵ', for the two possible choices of LR symmetry.

In the case of $\mathcal{P}$, due to the hermiticity of the Yukawa couplings, the phases in $V_{\rm R}$ are all predicted in terms of the phase α, and they are all close to 0 or π. The neutron electric dipole moment (EDM) then poses a strong constraint which together with ϵ and ϵ' leads to the strong limit $M_{W_{\rm R}} > 8$–10 TeV [19]. As discussed in Ref. [4], a TeV-scale LR symmetry is still allowed by resorting to an unappealing fine-tuning with the QCD strong phase $\bar\theta$. In this case, the ϵ' gives alone a bound, because from ϵ one must have $x \sin\alpha \simeq 10^{-3}$ [4,19], so that the limit $x \to 0$ i.e., $\zeta \to 0$ that would suppress ϵ' in (15) is not permitted. Then, by using the values for the predicted phases (see

[3] Also $\lambda_c^{\rm LL}$ is complex, but its phase is $O(1/1000)$ and is subleading in this expression.

Eq. (29) in Ref. [19]) and exploiting conservatively the free signs, in particular u and c opposite, one finds the numeric result[4]

$$|\epsilon'_{\rm LR}| \simeq 5.7 \times 10^{-6}(\beta/10^{-3}). \tag{16}$$

This result holds in the natural regime $x < 0.1$, where analytic expressions for the phases are available. Comparing $\epsilon'_{\rm LR}$ with 100% (50%) of $|\epsilon'|_{\rm exp} \simeq 3.92\times 10^{-6}$, we obtain the constraint $M_{W_{\rm R}} \gtrsim 3(4)$ TeV. Here the main contribution comes from $Q_{1,2}^{\rm LR,RL}$, while $Q_g^{\rm L,R}$ contribute subdominantly, with the effect of softening the limit with respect to the conclusions of Ref. [4].[5]

In the more interesting case of $\mathcal{C}$ as LR symmetry, the phases (5) in $V_{\rm R}$ are free. This time, the bound from ϵ together with the $B_{d,s}$ systems put the stronger constraint $\theta_d - \theta_s < 10^{-3}$ [4]. As a result, from (15) one has

$$|\epsilon'_{\rm LR}| \simeq |\zeta||2.50\sin(\alpha - \theta_u - \theta_d) + 0.020\sin(\alpha - \theta_c - \theta_d)|. \tag{17}$$

The first line due to $Q_{1,2}^{\rm LR,RL}$ is dominant, but one can note that the phase combination appearing there is independently constrained by the neutron EDM: as studied in Refs. [4,19,40] one has $|\zeta|\sin(\alpha - \theta_u - \theta_d) < 10^{-7}$. This implies that the current-current contribution can be neglected here, and thus we are left with the dominance of the second line, due to $Q_g^{\rm L,R}$. For $\epsilon'_{\rm LR}$ to at most saturate the experimental value, one has the constraint

$$|\zeta||\sin(\alpha - \theta_c - \theta_d)| < 2.0 \times 10^{-4} \tag{18}$$

(or a correspondingly more stringent one for a subdominant $|\epsilon'|_{\rm LR}$). This represents a correlated bound between the phases and the LR symmetry scale/mixing. For unconstrained phases $\theta_c + \theta_d$ one would require $M_{W_{\rm R}} \gtrsim 8$ TeV, or vanishing LR mixing, while for TeV-scale LR symmetry and large LR mixing, one puts a constraint of $\sim 10^{-1}$ on the involved combination of phases.

This constraint is relatively stronger than the one reported in Ref. [41], where a different evaluation of the Q_g matrix element was adopted, and where only the "charm" couplings were considered. As discussed above, the uncertainty (possible enhancement) in the matrix element of Q_g could make this bound even stronger by an order one factor.

The bound (18) is also stronger than the analogous limit inferred from the $s \to d\gamma$ decays [41,42], which through $Q_\gamma^{\rm L,R}$ involve the same enhancement and the same phases as the chromomagnetic operator.

VI. SUMMARY AND CONCLUSIONS

In this work we addressed the effect on ϵ' of new physics in the chromomagnetic dipole operators, which can have a huge enhancement with respect to the SM, especially in the presence of nonchiral interactions. The paradigmatic example for this effect appears in minimal left-right symmetric theories, where the $W_{\rm L}$-$W_{\rm R}$ gauge-boson mixing leads to an enhancement of 10^5 in the short distance loop coefficient, so that even with a scale of new physics in the TeV region, an enhancement of two orders of magnitude results. Together with the presence of new phases in the right quark mixing matrix, this can lead to a dramatic impact in ϵ'.

To evaluate quantitatively the effect, we considered the dipole operators together with the full set of four quark operators which can give rise to CP violation in $K \to \pi\pi$ decays. We considered their renormalization and mixing (at leading order) from short distance to the low scale of matching with chiral perturbation theory, where the matrix elements can be estimated. For the chromomagnetic dipole operators, we reevaluated the corresponding matrix element in the context of the chiral quark model (correcting the previous existing calculation). For the SM operators, we adopted the estimates consistently determined in previous analysis of ϵ'/ϵ [15]. These were also used for an estimate for the LR current-current operators $Q_{1,2}^{\rm LR}$ (see discussion in Sec. IV and Appendix C). The set of high energy operators considered is fairly complete, and can serve also for estimates of the impact of other new physics models on ϵ'.

We applied the results to the case of the minimal left-right model, showing that ϵ' receives contributions from the chromomagnetic operator as well as from the current-current ones $Q_{1,2}^{\rm LR,RL}$. These are in general large, but we noted that they are severely constrained by the nEDM, with the result that the chromomagnetic operators turn out to be dominant. One can expect this to be a fairly generic situation, because new CP phases contributing to $Q_{1,2}^{\rm LR,RL}$ are usually contributing as well to the nEDM.

In the LR model, focusing on the case of generalized charge-conjugation $\mathcal{C}$ taken as LR symmetry, where new phases are free, this allows us to derive a correlated constraint between the LR-gauge boson mixing (or LR-symmetry scale) and the relevant phases. The bound amounts to $\zeta \lesssim 10^{-4}$ (or $M_{W_{\rm R}} > 8$ TeV) for arbitrary phases or, in the scenario of TeV-scale LR symmetry, a constraint of $10^{-1,2}$ on the involved phases.

[4]This corresponds to the case $|\theta_d - \theta_s| \sim 10^{-2}$. There is also a second possibility, with $|\theta_d - \theta_s| \sim \pi + O(10^{-2})$ and $x\sin\alpha \approx 10^{-2}$, but it is disfavored by the $B_{d,s}$ mass differences and CP-violation. Moreover, in this case there are cancelations in each line of (15) and the situation is more ambiguous, since it depends on the precision to which the equality $V_{\rm L} \sim V_{\rm R}$ of mixing angles holds. We recall that with a global numerical study of this model [4], the angles could be deviated as much as 20%, which would spoil these cancelations in ϵ'. Therefore, also in this case one can expect a dominant contribution as in (16).

[5]We also do not agree with the strong bound derived in Ref. [20], using only the isospin-2 amplitude of the operator $Q_1^{\rm LR}$.

The dominance of the Q_g contribution can then lead to constraints on processes involving the same combination of phases. This is true for instance for the magnetic operators $O_\gamma^{\mathrm{L,R}}$ which are also GIM-enhanced, and as we argued will enter in $K \to \pi\pi e^+ e^-$ CP-violating asymmetries. Also, $(\alpha - \theta_c - \theta_d)$ enters in the charmed mesons physics, the analysis of which is beyond the scope of this work. Nevertheless, let us point out that it enters the decays of the D meson via $c \to u\gamma$, whose short distance contribution is overwhelmed by the long distance ones [43], but also it enters the CP-violation in the $D \to KK$, $\pi\pi$ channels, for which anomalous signals have been reported by the LHCb collaboration [44]. The interesting analysis of the related charm physics in the LR models will be the subject of a separate work.

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APPENDIX A: LOOP FUNCTIONS

The loop functions relevant for the SM and the LR model are [14,45–47]

$$F_1^{\mathrm{LL}} = \frac{x_j(-18 + 11x_j + x_j^2)}{12(x_j - 1)^3} - \frac{(4 - 16x_j + 9x_j^2)\ln x_j}{6(x_j - 1)^4},$$

$$F_2^{\mathrm{LL}} = \frac{x_j(2 + 3x_j - 6x_j^2 + x_j^3 + 6x_j \ln x_j)}{4(x_j - 1)^4},$$

$$F_2^{\mathrm{LR}} = \frac{-4 + 3x_j + x_j^3 - 6x_j \ln x_j}{2(x_j - 1)^3},$$

$$E_1^{\mathrm{LL}} = -\frac{x_j^2(5x_j^2 - 2x_j - 6)}{18(x_j - 1)^4}\ln x_j + \frac{19x_j^3 - 25x_j^2}{36(x_j - 1)^3} + \frac{4}{9}\ln x_j,$$

$$E_2^{\mathrm{LL}} = \frac{x_j(8x_j^2 + 5x_j - 7)}{12(x_j - 1)^3} + \frac{x_j^2(2 - 3x_j)}{2(x_j - 1)^4}\ln x_j,$$

$$E_2^{\mathrm{LR}} = \frac{5x_j^2 - 31x_j + 20}{6(x_j - 1)^2} - \frac{x_j(2 - 3x_j)}{(x_j - 1)^3}\ln x_j, \tag{A1}$$

where $x_j = (\frac{m_j}{M_W})^2$, $j = u, c, t$. In addition to these, one has $F_{1,2}^{\mathrm{RR}} = F_{1,2}^{\mathrm{LL}}(\beta x_i)$ and similarly for E_2^{RR}.

APPENDIX B: RUNNING OF ALL $\Delta S = 1$ OPERATORS

For our purposes, the relevant operators are the Q_i appearing in (1) plus the eight $Q_{1,2c}^{AB}$ where u quark is replaced by c. At leading order (LO) the LR operators mix only with $Q_{g,\gamma}^{\mathrm{L}}$ (in addition to themselves). Similarly, the RL ones mix only with $Q_{g,\gamma}^{\mathrm{R}}$. The operators can thus be divided in two decoupled sets of 18 operators each, related by the exchange L ↔ R:

$$\{Q_{1c}^{\mathrm{RL}}, Q_{2c}^{\mathrm{RL}}, Q_1^{\mathrm{RL}}, Q_2^{\mathrm{RL}}, Q_{1c}^{\mathrm{LL}}, Q_{2c}^{\mathrm{LL}}, Q_1^{\mathrm{LL}}, Q_2^{\mathrm{LL}}, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, Q_9, Q_{10}, Q_g^{\mathrm{L}}, Q_\gamma^{\mathrm{L}}\}, \tag{B1}$$

$$\{Q_{1c}^{\mathrm{LR}}, Q_{2c}^{\mathrm{LR}}, Q_1^{\mathrm{LR}}, Q_2^{\mathrm{LR}}, Q_{1c}^{\mathrm{RR}}, Q_{2c}^{\mathrm{RR}}, Q_1^{\mathrm{RR}}, Q_2^{\mathrm{RR}}, Q_3', Q_4', Q_5', Q_6', Q_7', Q_8', Q_9', Q_{10}', Q_g^{\mathrm{R}}, Q_\gamma^{\mathrm{R}}\}, \tag{B2}$$

as in the two columns of Eq. (1). The corresponding vectors of coefficients in the two sets, $\vec{C}_{\mathrm{L,R}}(\mu)$, evolve separately according to the renormalization group equation

$$\left(\frac{\partial}{\partial \ln\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_{m_i}\frac{\partial}{\partial \ln m_i}\right)\vec{C}_{\mathrm{L,R}}(\mu) = \gamma^T(\mu)\vec{C}_{\mathrm{L,R}}(\mu), \tag{B3}$$

with $i = u, s, c$. The 18×18 anomalous dimension matrix γ is the same in the L and R sectors and reads [47,48]

$$
\frac{\alpha_s}{4\pi}\left(\begin{array}{cccccccccccccccccc}
-16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\frac{m_c}{m_s} & \frac{320}{3}\frac{m_c}{m_s} \\
-6 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3}\frac{m_c}{m_s} & \frac{128}{9}\frac{m_c}{m_s} \\
0 & 0 & -16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\frac{m_u}{m_s} & \frac{320}{3}\frac{m_u}{m_s} \\
0 & 0 & -6 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3}\frac{m_u}{m_s} & \frac{128}{9}\frac{m_u}{m_s} \\
0 & 0 & 0 & 0 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \\
0 & 0 & 0 & 0 & 6 & -2 & 0 & 0 & -2 & \frac{2}{3} & -2 & \frac{2}{3} & 0 & 0 & 0 & 0 & \frac{140}{27} & \frac{832}{81} \\
0 & 0 & 0 & 0 & 0 & 0 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 6 & -2 & -2 & \frac{2}{3} & -2 & \frac{2}{3} & 0 & 0 & 0 & 0 & \frac{140}{27} & \frac{832}{81} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{22}{9} & \frac{22}{3} & -4 & \frac{4}{3} & 0 & 0 & 0 & 0 & \left(\frac{280}{27}+6n_f\right) & -\frac{928}{81} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6-2n_f & -2+\frac{2n_f}{3} & -2n_f & \frac{2n_f}{3} & 0 & 0 & 0 & 0 & \left(12+\frac{140n_f}{27}\right) & -\frac{4}{9}\left(\frac{8n_f}{9}+12(n_f-3n_u)\right) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -6 & 0 & 0 & 0 & 0 & \left(-\frac{28}{3}-6n_f\right) & \frac{64}{9} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2n_f}{9} & \frac{2n_f}{3} & -\frac{2n_f}{9} & -16+\frac{2n_f}{3} & 0 & 0 & 0 & 0 & \left(-8-\frac{238n_f}{27}\right) & -\frac{4}{9}\left(\frac{8n_f}{9}-12(n_f-3n_u)\right) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{9}(n_f-3n_u) & \frac{1}{3}(3n_u-n_f) & \frac{1}{9}(n_f-3n_u) & \frac{1}{3}(3n_u-n_f) & 0 & -16 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{9} & -\frac{2}{3} & \frac{2}{9} & -\frac{2}{3} & 0 & 0 & -2 & 6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{9}(n_f-3n_u) & \frac{1}{3}(3n_u-n_f) & \frac{1}{9}(n_f-3n_u) & \frac{1}{3}(3n_u-n_f) & 0 & 0 & 6 & -2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{28}{3} & -\frac{32}{9} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{32}{3}
\end{array}\right) \tag{B4}
$$

where n_f and n_u are, respectively, the number of active quarks, and of active up-type quarks. The mixing of the $Q_{1,\ldots,10}$ operators with the (chromo)magnetic operators appears at NLO [48], and we adopt the anomalous dimensions in the 't Hooft-Veltman scheme. Note, the off-diagonal terms in the last two columns carry explicitly the ratio of mass insertions responsible of the operator mixing. In fact, while for the mixing of the LL and penguin operators with the magnetic ones the mass insertion on the external legs is m_s and coincides with the normalization of $Q_{g,\gamma}$, for the mixing $Q_{1,2}^{\mathrm{LR,RL}} \to Q_{g,\gamma}$ the mass insertion is that of the internal quark (m_u or m_c), breaking the usual GIM cancelations. It also follows that the traditional description in terms of the y_i and z_i variables [30] is no longer appropriate, and we need to perform the running of the whole vector of coefficients $\vec{C}_{\mathrm{L}}$ (and $\vec{C}_{\mathrm{R}}$). Clearly, for the SM part the results coincide, as the GIM cancelation is effective in the mixing with the dipole operators. Finally, the operators involving the charm quark are integrated out at their threshold, and accordingly the anomalous dimension matrix is projected, below this scale, on the remaining set of low energy operators $Q_{1,2}^{AB}$, Q_{3-10}, Q_g^{L}, Q_γ^{L}.

We perform the running choosing $\alpha_s(M_Z) = 0.1176$, and with starting coefficients introduced separately at the relative scales of decoupling. The running is performed down to 0.8 GeV where the matrix elements are evaluated.

APPENDIX C: AMPLITUDES

For all the operators, the amplitudes $A_{0,2}$ are expressed in terms of their $K \to (\pi\pi)_{I=0,2}$ matrix elements $\langle Q_i \rangle_{0,2}$:

$$A_0 = \sum_i C_i \langle Q_i \rangle_0, \qquad A_2 = \sum_i C_i \langle Q_i \rangle_2. \tag{C1}$$

We report here the matrix elements for the relevant $Q_{1,2}^{\mathrm{LL,LR,RL,RR}}$ [14,15]:

$$\begin{aligned}
\langle Q_1^{\mathrm{LL}} \rangle_0 &= -\langle Q_1^{\mathrm{RR}} \rangle_0 = -\frac{1}{3\sqrt{6}} X \mathcal{B}_0^1,\\
\langle Q_1^{\mathrm{LL}} \rangle_2 &= -\langle Q_1^{\mathrm{RR}} \rangle_2 = \frac{4}{3\sqrt{3}} X \mathcal{B}_2^1,\\
\langle Q_2^{\mathrm{LL}} \rangle_0 &= -\langle Q_2^{\mathrm{RR}} \rangle_2 = \frac{5}{3\sqrt{6}} X \mathcal{B}_0^2,\\
\langle Q_2^{\mathrm{LL}} \rangle_2 &= -\langle Q_2^{\mathrm{RR}} \rangle_2 = \frac{4}{3\sqrt{3}} X \mathcal{B}_2^2,
\end{aligned} \tag{C2}$$

$$\begin{aligned}
\langle Q_1^{\mathrm{LR}} \rangle_0 &= -\langle Q_1^{\mathrm{RL}} \rangle_0 = \frac{\sqrt{2}(X + 9Y + 3Z)}{3\sqrt{3}} \mathcal{B}_0^{1,\mathrm{LR}},\\
\langle Q_1^{\mathrm{LR}} \rangle_2 &= -\langle Q_1^{\mathrm{RL}} \rangle_2 = \frac{1}{3}\sqrt{\frac{1}{3}}(X - 6Z) \mathcal{B}_2^{1,\mathrm{LR}},\\
\langle Q_2^{\mathrm{LR}} \rangle_0 &= -\langle Q_2^{\mathrm{RL}} \rangle_0 = \frac{\sqrt{2}(3X + 3Y + Z)}{3\sqrt{3}} \mathcal{B}_0^{2,\mathrm{LR}},\\
\langle Q_2^{\mathrm{LR}} \rangle_2 &= -\langle Q_2^{\mathrm{RL}} \rangle_2 = \frac{1}{3}\sqrt{\frac{1}{3}}(3X - 2Z) \mathcal{B}_2^{2,\mathrm{LR}},
\end{aligned} \tag{C3}$$

with

$$\begin{aligned}
X &\equiv -\langle \pi^- | \bar{d}\gamma_\mu \gamma_5 u | 0 \rangle \langle \pi^+ | \bar{u}\gamma^\mu s | \bar{K}^0 \rangle\\
&= i\sqrt{2} f_\pi (m_K^2 - m_\pi^2) \simeq 0.03i \text{ GeV}^3,\\
Y &\equiv -\langle \pi^+ \pi^- | \bar{u} u | 0 \rangle \langle 0 | \bar{d}\gamma_5 s | \bar{K}^0 \rangle\\
&= i\sqrt{2} f_K A^2 \simeq 0.22i \text{ GeV}^3,\\
Z &\equiv -\langle \pi^- | \bar{d}\gamma_5 u | 0 \rangle \langle \pi^+ | \bar{u} s | \bar{K}^0 \rangle\\
&= i\sqrt{2} f_\pi A^2 \simeq 0.18i \text{ GeV}^3,
\end{aligned} \tag{C4}$$

where $A \equiv m_K^2/(m_s + m_d)$, and $f_{\pi,K}$ the π and K decay constants, and the quark masses are evaluated at $\mu = 0.8$ GeV (i.e., $m_s \simeq 200$ MeV). Since we use the matrix elements at $\mu = 0.8$ GeV, also the $\mathcal{B}_i$ coefficients of departure from vacuum saturation have to be evaluated at this scale.

The SM ones, determined in the chiral quark model via a phenomenological approach based on the fit of the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decays, can be taken from Ref. [15] (see Table VI), where one can also find the "correlated" matrix elements for the operators $Q_{3,\ldots,10}$. For the above current-current operators, one finds the central values $\mathcal{B}_0^1 \simeq 9.5$, $\mathcal{B}_0^2 \simeq 2.9$, $\mathcal{B}_2^{1,2} \simeq 0.41$. For the gluonic and electromagnetic penguins relevant to ϵ'/ϵ, it is found $B_0^6 \simeq 1.6$ and $B_2^8 \simeq 0.92$.

Concerning the $\mathcal{B}^{1,2,\mathrm{LR}}$, their evaluation is still lacking both in the chiral quark model as on the lattice, and to our knowledge also in $1/N_c$ expansion. Some hints can be derived from the observation that the electromagnetic penguins $Q_{7,8}$ transform as $(\mathbf{8}_{\mathrm{L}}, \mathbf{8}_{\mathrm{R}})$ as do the $Q_{1,2}^{\mathrm{LR}}$. Then, their leading bosonization and chiral loops coincide (see Ref. [23]) so that one can expect the $\mathcal{B}$ parameters of $Q_{1,2}^{\mathrm{LR}}$ to be very similar to those of $Q_{8,7}$. For the isospin-2 amplitudes, this correspondence has even been argued to be exact [20] so that using the results reported in Ref. [15], we can set $\mathcal{B}_2^{1,2,\mathrm{LR}} = \mathcal{B}_2^{7,8} \simeq 0.92$. For isospin-0 amplitudes, the larger $\mathcal{B}_0^{7,8} \simeq 2.5$ hint for $\mathcal{B}_0^{1,2,\mathrm{LR}}$ also larger than one. In general, this is in accordance with the strong phases from pion rescattering in final state interactions which point to a correction factor of ~ 1.4 [49–51], and also more simply with the correction factor traditionally applied to Y in vacuum saturation to account for the renormalization to the K scale in the pion matrix element, $(1 + m_K^2/\Lambda_{\chi PT}^2) \sim 1.5$, which enhances the isospin-0 amplitudes in Eq. (C3). Therefore, for the present analysis, we adopt the conservative choice of central values $B_0^{1,2,\mathrm{LR}} \simeq 2$ with $O(1)$ uncertainty. For the left-right model, the impact of the isospin-0 amplitudes of $Q_{1,2}^{\mathrm{LR}}$ is fortunately limited. The result in the first line of (15) changes by 20% within a 1–3 range of $B_0^{1,2,\mathrm{LR}}$.

APPENDIX D: DOUBLE INSERTION OF Q_g LEADING TO $\Delta S = 2$

The gluonic dipole operators can give effects also on the $\Delta S = 2$ processes, via its double insertion or via its insertion together with other $\Delta S = 1$ operators, most notably the SM current-current operators $Q_{1,2,}^{\rm LL}$ which have large (real) coefficients. These combinations constitute true long-distance contributions. A first estimate was given in Ref. [52], and the resulting constraint is not relevant for ΔM_K and is marginal for ϵ [41]. In the presence of the whole set of new physics operators also chiral loops with multiple possible insertions should be evaluated, and we leave that for a future analysis. However, a simple double insertion of Q_g can be readily estimated. It leads to K-$\bar{K}$ mixing through the diagram in Fig. 2. The two external gluons are averaged in the vacuum gluon condensate, and the total mixing Hamiltionian is

$$H_{K\bar{K}} = -\frac{8}{3}\frac{M_K^2}{f^2}\frac{G_F^2}{2}m_s^2\frac{(C_g^{\rm L})^2 + (C_g^{\rm R})^2}{16\pi^2}\left\langle\frac{\alpha}{\pi}GG\right\rangle K\bar{K}. \quad \text{(D1)}$$

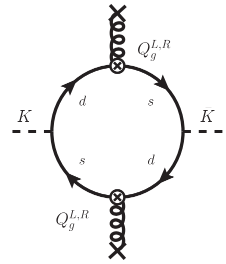

FIG. 2. Double insertion of Q_g for $\bar{K}^0 - K^0$ mixing.

Considering from Table I the low energy values $C_g^{\rm L,R} \simeq 2.7|\zeta|e^{\pm i(\alpha-\theta_c-\theta_{d,s})}$, we find the impact on the $K\bar{K}$ mixing to be negligibly small, despite the huge enhancement of the dipole loop: from $\Delta M_K = {\rm Re}(H_{K\bar{K}})/2M_K$, we have

$$\Delta M_K^{Q_g^{\rm L+R}} \lesssim 10^{-21}\ {\rm GeV}, \qquad (\text{for } \zeta \lesssim 10^{-3}), \quad \text{(D2)}$$

which is six orders of magnitude less than the experimental value. Similarly, also the effect on ϵ is negligible: we have $\epsilon \sim 0.3|\zeta|^2\cos(2\alpha)(\theta_d - \theta_s)$. Since $\zeta < 10^{-3}$ and $\theta_d - \theta_s$ is at most 10^{-2}, this gives no constraint.

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