Quasidegenerate neutrinos in type II seesaw models

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We present an analysis of normal and inverted hierarchical neutrino mass models within the framework of tri-bimaximal mixing. Considering the neutrinos to be quasidegenerate (QDN), we study two different neutrino mass models with mass eigenvalues $(m_1, -m_2, m_3)$ and (m_1, m_2, m_3) for both normal hierarchical and inverted hierarchical cases. Parameterizing the neutrino mass matrix using best-fit oscillation and cosmology data for a QDN scenario, we find the right-handed Majorana mass matrix using the type I seesaw formula for two types of Dirac neutrino mass matrices: charged lepton type and up quark type. Incorporating the presence of the type II seesaw term which arises naturally in generic left-right symmetric models along with the type I term, we compare the predictions for neutrino mass parameters with the experimental values. Within such a framework and incorporating both oscillation as well as cosmology data, we show that a QDN scenario of neutrino masses can still survive in nature with some minor exceptions. A viable extension of the standard model with an Abelian-gauged flavor symmetry is briefly discussed which can give rise to the desired structure of the Dirac and Majorana mass matrices.

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I. INTRODUCTION

Recent neutrino oscillation experiments have provided significant amount of evidence which confirms the existence of the nonzero yet tiny neutrino masses [1]. We know that the smallness of three standard model (SM) neutrino masses [1] can be naturally explained via the seesaw mechanism. In general, the seesaw mechanism can be of three types: type I [2], type II [3], and type III [4]. All these mechanisms involve the inclusion of additional fermionic or scalar fields to generate tiny neutrino masses at tree level. Although these seesaw models can naturally explain the smallness of neutrino mass compared to the electroweak scale, we are still far away from understanding the origin of neutrino mass hierarchies as suggested by experiments. Recent neutrino oscillation experiments T2K [5], Double ChooZ [6], Daya-Bay [7], and RENO [8] provide the values of various neutrino oscillation parameters as follows:

$$\begin{aligned}
\Delta m_{21}^2 &= (7.12\text{–}8.20)\times 10^{-5}, \\
|\Delta m_{31}^2| &= (2.21\text{–}2.64)\times 10^{-3}, \\
\sin^2\theta_{12} &= 0.27\text{–}0.37, \\
\sin^2\theta_{23} &= 0.37\text{–}0.67, \\
\sin^2\theta_{13} &= 0.017\text{–}0.033.
\end{aligned} \tag{1}$$

The above recent data have positive evidence for nonzero θ_{13} as well, which was earlier thought to be zero or negligibly small. The values of these mixing angles have nontrivial impact on the neutrino mass hierarchy as studied in a recent paper [9] where the author showed that the atmospheric angle θ_{23} is found to discriminate the possible hierarchies in the type I and type II seesaw frameworks using different texture zero-mass matrices. In this context, to know the actual hierarchy of the neutrino masses has become equally important like the issue of nonzero θ_{13} both from neutrino physics as well as from a cosmology point of view. The recent cosmological upper bound [10] on the sum of three absolute neutrino masses $\sum_i m_i \leq 0.28$ eV has ruled out quasidegenerate neutrino (QDN) mass models with $m_i \geq 0.1$ eV. This has made studying the survivability of QDN models as important as the issue of the normal and inverted hierarchical natures of neutrino masses.

Detailed analysis of normal versus inverted hierarchical neutrino masses using different approaches started just after the discovery of the neutrino oscillation phenomena. The inverted hierarchical neutrino was studied exclusively in Ref. [11] considering neutrino as a pseudo-Dirac particle with nonconservation of $L_e - L_\mu - L_\nu$, where L_l denotes lepton number corresponding to individual lepton (l) generation. Use of specific grand unified models explaining the seesaw mechanisms has also been done in the last few years to study the hierarchy of neutrino masses. An analysis done in Ref. [12] showed that every normal neutrino mass hierarchy solution of a grand unified model corresponds to an inverted hierarchy solution. It was also mentioned in their work that any future observation of inverted hierarchy would tend to disfavor the grand unified models based on the conventional type I seesaw mechanism. But models with type II and type III or models based on conserved $L_e - L_\mu - L_\nu$ symmetry may favor the inverted hierarchical nature of neutrino masses. Models based on seesaw mechanism with three right-handed neutrinos can also generate inverted hierarchical neutrino masses [13] within the framework of bimaximal mixing.

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As stressed earlier, along with the hierarchy of neutrino masses, the explanation of nonzero θ_{13} as well as *CP* violation is also an unsolved agenda in neutrino physics. From the supernova neutrinos point of view, it was shown [14] that one can discriminate the inverted hierarchy from the normal one if $\sin^2\theta_{13} \geq$ a few $\times 10^{-4}$. If a particular neutrino mass hierarchy is assumed this can bias cosmological parameter constraints [15] like the dark energy equation of state parameter as well as the sum of the neutrino masses.

In view of the importance of understanding the hierarchies as stressed above, this paper presents an analysis of normal and inverted neutrino mass hierarchies incorporating the contributions from both type I and type II seesaw mechanisms. However, our analysis does not attempt to explain nonzero θ_{13} as has already been explained in the literature by considering deviations from the TBM mixing using different corrections. The present analysis is done within the framework of TBM mixing which theoretically, is a very close approximate description of neutrino mixing. In the present analysis we use the appropriate neutrino mass patterns in the framework of TBM mixing by considering neutrinos to be QDN. The mass matrices are parameterized for the QDN case with the help of present neutrino oscillation data and the cosmological upper bound on the sum of neutrino masses. Using these QDN mass matrices and considering two possible structures of the Dirac neutrino mass matrix m_{LR}: charged lepton (CL) type and up quark (UQ) type, we calculate the right-handed Majorana mass matrix using type I seesaw formula. We then take into account the contributions from type II seesaw term in a generic left-right symmetric theory [16]. In the presence of both type I and type II seesaw contributions, we perform our detailed analysis to calculate the predictions for neutrino parameters to show the survivability of the QDN scenario. In the end, we also outline a simple extension of the standard model by an Abelian-gauged flavor symmetry which can give rise to the specific structure of Dirac neutrino mass matrices used in the analysis.

This paper is organized as follows: in Sec. II, we discuss the methodology of the type II seesaw mechanism. In Sec. III, we discuss our numerical analysis and results. In Sec. IV, we outline a simple extension of standard model by an Abelian-gauged flavor symmetry which can naturally give rise to the desired structure of mass matrices. We conclude in Sec. V.

II. METHODOLOGY AND TYPE II SEESAW MECHANISM

The type I seesaw framework is the simplest mechanism for generating tiny neutrino masses and mixing. There is also another type of noncanonical seesaw formula (known as the type-II seesaw formula) [3] where a left-handed Higgs triplet Δ_L picks up a vacuum expectation value (vev). This is possible both in the minimal extension of the standard model by Δ_L or in other well-motivated extensions like left-right symmetric model (LRSM) [16]. The seesaw formula can be written as

$$m_{LL} = m_{LL}^{II} + m_{LL}^{I}, \tag{2}$$

where the usual type I seesaw formula is given by the expression,

$$m_{LL}^{I} = -m_{LR} M_{RR}^{-1} m_{LR}^{T}. \tag{3}$$

Here, m_{LR} is the Dirac neutrino mass matrix. The above seesaw formula with both type I and type II contributions can naturally arise in extension of standard model with three right-handed neutrinos and one copy of Δ_L. However, we will use this formula in the framework of LRSM where M_{RR} arises naturally as a result of parity breaking at high energy and both the type I and type II terms can be written in terms of M_{RR} as we will see below.

In this present analysis, we consider m_{LR} in a diagonal form and M_{RR} in general nondiagonal form. In LRSM with Higgs triplets, M_{RR} can be expressed as $M_{RR} = v_R f_R$ with v_R being the vev of the right-handed triplet Higgs field Δ_R imparting Majorana masses to the right-handed neutrinos and f_R is the corresponding Yukawa coupling. The first term m_{LL}^{II} in Eq. (2) is due to the vev of $SU(2)_L$ Higgs triplet. In the usual LRSM, m_{LL}^{II} and M_{RR} are proportional to the vev's of the electrically neutral components of scalar Higgs triplets Δ_L and Δ_R, respectively. Thus, $m_{LL}^{II} = f_L v_L$ and $M_{RR} = f_R v_R$, where $v_{L,R}$ denote the vev's and $f_{L,R}$ are symmetric 3×3 matrices. The left-right symmetry demands $f_R = f_L = f$. The induced vev for the left-handed triplet v_L can be shown for generic LRSM to be

$$v_L = \gamma \frac{M_W^2}{v_R}$$

with $M_W \simeq 80.4$ GeV being the weak boson mass such that

$$|v_L| \ll M_W \ll |v_R|.$$

In general, γ is a function of various couplings in the scalar potential of generic LRSM and without any fine-tuning γ is expected to be of the order unity ($\gamma \sim 1$). The type II seesaw formula in Eq. (2) can now be expressed as

$$m_{LL} = \gamma (M_W/v_R)^2 M_{RR} - m_{LR} M_{RR}^{-1} m_{LR}^{T}. \tag{4}$$

With the above seesaw formula (4), the neutrino mass matrices are constructed by considering contributions from both type I and type II terms. Here, M_{RR} is defined as $M_{RR} = v_R f_R$. If f_R is held fixed, both terms in Eq. (4) vary as $1/v_R$. Here, we hold M_{RR} fixed, so the first term is v_R dependent while second term is fixed. However, different choices of v_R for fixed M_{RR} would lead to different values of m_{LL}^{II} while keeping m_{LL}^{I} unchanged. This ambiguity is seen in the literature where different choices of v_R are made according to convenience [17–20]. However, in this paper, we will always take v_R as $v_R = \gamma \frac{M_W^2}{v_L} \simeq \gamma \times 10^{15}$ GeV

[20]. It is worth mentioning that here the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry-breaking scale (as in generic LRSM) v_R is the same as the scale of parity breaking [17]. Using this form of v_R, the seesaw formula (4) becomes

$$m_{LL} = \gamma\left(\frac{M_W}{\gamma \times 10^{15}}\right)^2 M_{RR} - m_{LR} M_{RR}^{-1} m_{LR}^T. \tag{5}$$

Since a type II term is inversely proportional to γ, smaller values of this parameter (say, $\gamma \sim 0$) would give rise to a more dominating type II term whereas $\gamma \sim 1$ would correspond to the minimum possible contributions from a type II term.

After fixing the symmetry-breaking scales as above, we carry out a complete analysis of the normal and inverted hierarchical models of neutrino masses in the framework of TBM mixing. We vary the dimensionless parameter γ from 0.001 to 1.0 and check the survivability of neutrino mass models with contributions from type I and type II terms. We adopt a *natural selection* for the survival of neutrino mass models which have the least deviation of γ from unity. The nearer the value of γ to one the better the chance for the survival of the model in question. Thus, the value of γ is an important parameter for the proposed natural selection of the neutrino mass models in question.

III. NUMERICAL ANALYSIS AND RESULTS

For detail numerical analysis we use the specific μ-τ symmetric neutrino mass matrix [21] which gives rise to TBM type mixing pattern

$$m_{LL} = \begin{pmatrix} A & B & B \\ B & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ B & \frac{1}{2}(A+B-D) & \frac{1}{2}(A+B+D) \end{pmatrix}, \tag{6}$$

TABLE I. Input parameters and predictions for different parameters consistent with experiments using type I seesaw only.

Parameters	IH(+ − +)	IH(+ + +)	NH(+ − +)	NH(+ + +)
$\Delta m^2_{21}[10^{-5}\ \mathrm{eV}^2]$	7.65	7.65	7.65	7.65
$\lvert\Delta m^2_{13}\rvert[10^{-3}\ \mathrm{eV}^2]$	2.40	2.40	2.40	2.40
m_3(eV)	0.08	0.08	0.10	0.10
$\sin^2\theta_{23}$	0.50	0.50	0.50	0.50
$\sin^2\theta_{12}$	0.33	0.33	0.33	0.33
m_1(eV)	0.09340	0.09340	0.08674	0.08675
m_2(eV)	−0.09380	0.09380	−0.08717	0.08717
$\sum_i m_i$(eV)	0.267	0.267	0.274	0.274
A	0.031	0.09353	0.02877	0.08688
B	−0.0624	0.00013	−0.05797	0.00014
D	0.08	0.08	0.10	0.10

which has eigenvalues $m_1 = A - B$, $m_2 = A + 2B$, and $m_3 = D$. Then we parameterize the above matrix for the QDN case. From presently available cosmological constraints, the upper bound on the sum of neutrino masses has come down to the lowest value $\sum_i m_i \leq 0.28$ eV [10], which has ruled out QDN neutrino models with $m_i \geq 0.1$ eV. Parametrization of the matrix (6) is done with this upper bound and taking the largest allowed value $m_i \leq 0.1$ eV consistent with the latest cosmological data. A classification for threefold QDN neutrino masses [22] with a maximum Majorana *CP*-violating phase in their eigenvalues is used here. *CP* phase patterns in the mass eigenvalues for both NH and IH are taken as $(m_1, -m_2, m_3)$ (denoted as $+ - +$) and (m_1, m_2, m_3) (denoted as $+ + +$). Using the best-fit global values of neutrino oscillation observational data [23] on solar and atmospheric neutrino mass squared differences, and taking $m_i \leq 0.1$ eV, predictions for neutrino parameters are calculated within the cosmological upper bounds mentioned above. First, we calculate the neutrino mass parameters using the above

TABLE II. Right-handed Majorana neutrino masses in GeV.

(m, n)	(6, 2)	(8, 4)
IH(+ + +)	$\begin{pmatrix} 3.8\times10^5 & -2.26\times10^5 & -4.67\times10^6 \\ -2.26\times10^5 & 7.23\times10^{10} & -1.18\times10^{11} \\ -4.67\times10^6 & -1.18\times10^{11} & 3.09\times10^{13} \end{pmatrix}$	$\begin{pmatrix} -2.19\times10^3 & -1.34\times10^3 & -5.75\times10^5 \\ -1.34\times10^3 & 4.32\times10^8 & -1.45\times10^{10} \\ -5.75\times10^5 & -1.45\times10^{10} & 7.87\times10^{13} \end{pmatrix}$
IH(+ − +)	$\begin{pmatrix} 123061 & -1.043\times10^8 & -2.16\times10^9 \\ -1.04\times10^8 & 2.80\times10^{10} & -1.04\times10^{12} \\ -2.16\times10^9 & -1.04\times10^{12} & 1.20\times10^{13} \end{pmatrix}$	$\begin{pmatrix} 733.40 & -6.22\times10^5 & -2.66\times10^8 \\ -6.22\times10^5 & 1.67\times10^8 & -1.28\times10^{11} \\ -2.66\times10^8 & -1.28\times10^{11} & 3.04\times10^{13} \end{pmatrix}$
NH(+ + +)	$\begin{pmatrix} 3.95\times10^5 & -2.91\times10^5. & -6.01\times10^6 \\ -2.91\times10^5 & 6.72\times10^{10} & 9.64\times10^{10} \\ -6.01\times10^6 & 9.64\times10^{10} & 2.87\times10^{13} \end{pmatrix}$	$\begin{pmatrix} 2.36\times10^3 & -1.73\times10^3 & -7.40\times10^5 \\ -1.73\times10^3 & 4.01\times10^8 & 1.19\times10^{10} \\ -7.40\times10^5 & 1.19\times10^{10} & 7.3\times10^{13} \end{pmatrix}$
NH(+ − +)	$\begin{pmatrix} 1.3\times10^5 & -1.12\times10^8 & -2.32\times10^9 \\ -1.12\times10^8 & 1.93\times10^{10} & -8.92\times10^{11} \\ -2.32\times10^9 & -8.92\times10^{11} & 8.27\times10^{12} \end{pmatrix}$	$\begin{pmatrix} 790.241 & -6.70\times10^5 & -2.86\times10^8 \\ -6.70\times10^5 & 1.16\times10^8 & -1.10\times10^{11} \\ -2.86\times10^8 & -1.10\times10^{11} & 2.10\times10^{13} \end{pmatrix}$

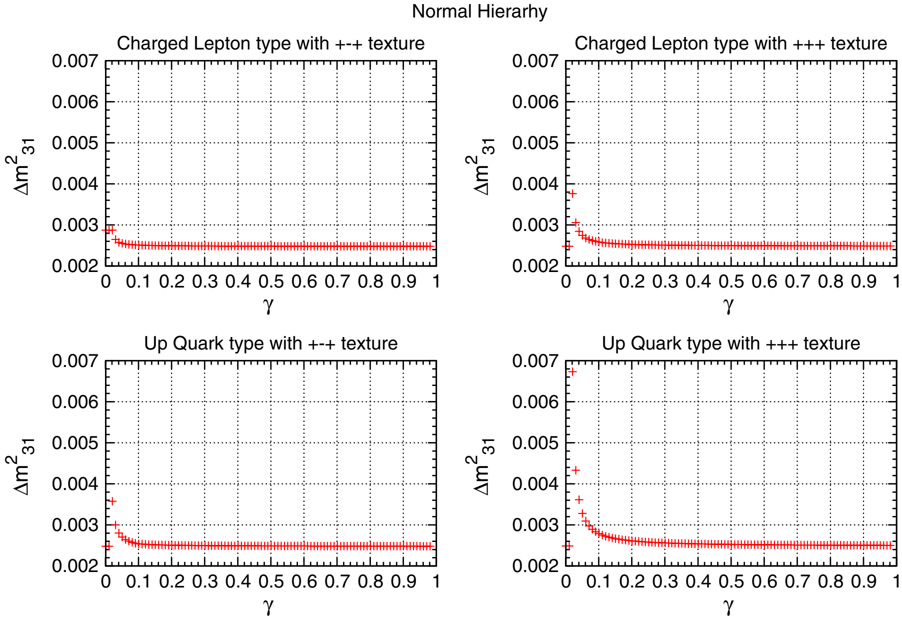

FIG. 1 (color online). Variation of the predicted values of Δm^2_{31} as a function of γ in NH case for both charged lepton and up quark type m_{LR} as well as both types of maximal Majorana CP phases.

form of the matrix (6) taking into account only type I seesaw contributions. These predictions along with the input parameters for IH and NH cases in the presence of only type I seesaw are presented in Table I. Then we take into account contributions from the type II seesaw term given in Eq. (5) to study the survivability of the neutrino mass models. Using the inverse type I seesaw formula

$$M_{RR} = m^T_{LR} m^{-1}_{LL} m_{LR} \tag{7}$$

first we calculate the M_{RR} for each case using Dirac neutrino mass (m_{LR}) in the diagonal form. In this analysis, m_{LR} is being taken as either the charged lepton mass matrix or the up quark mass matrix. The general form of the Dirac neutrino mass is

$$m_{LR} = \begin{pmatrix} \lambda^m & 0 & 0 \\ 0 & \lambda^n & 0 \\ 0 & 0 & 1 \end{pmatrix} m_f, \tag{8}$$

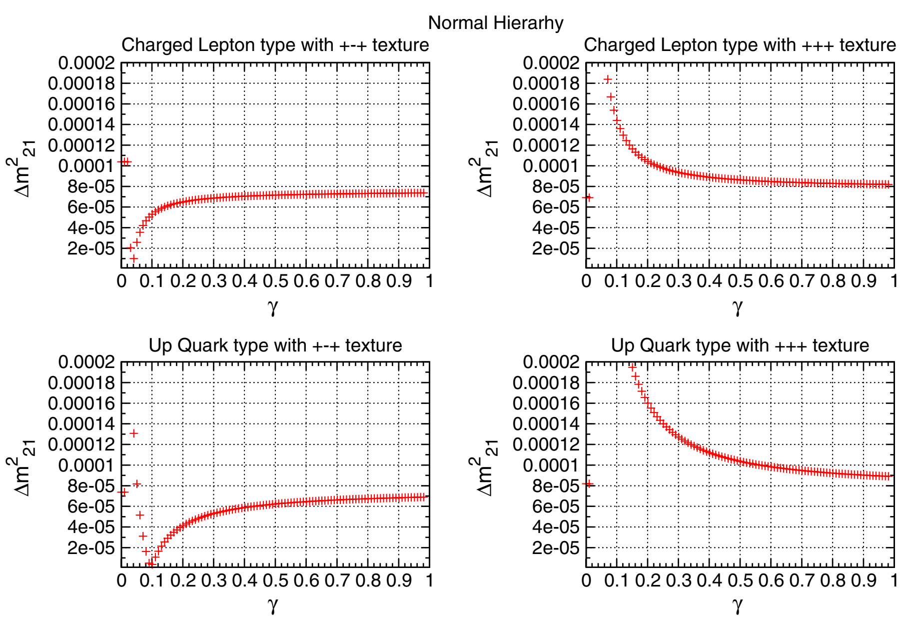

FIG. 2 (color online). Variation of the predicted values of Δm^2_{21} as a function of γ in NH case for both charged lepton and up quark type m_{LR} as well as both types of maximal Majorana CP phases.

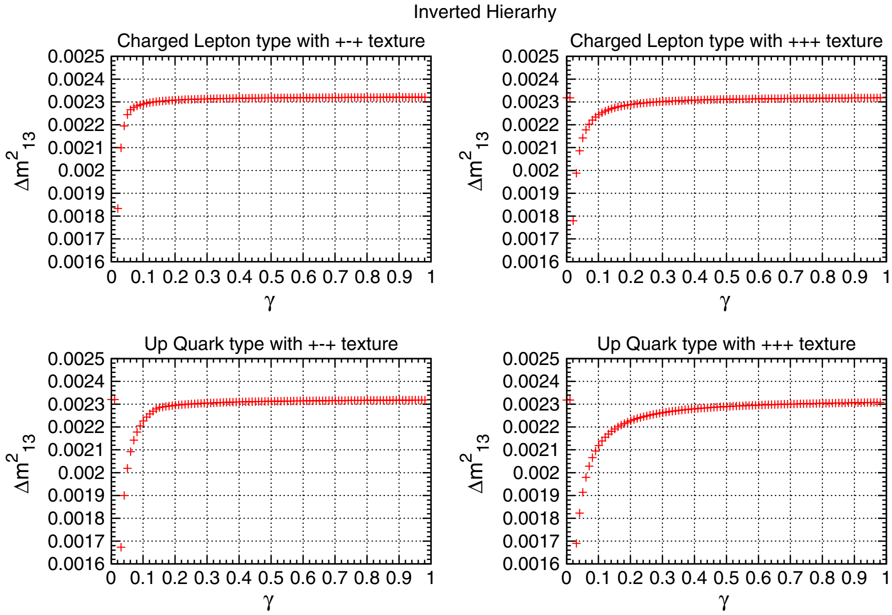

FIG. 3 (color online). Variation of the predicted values of Δm_{13}^2 as a function of γ in IH case for both charged lepton and up quark type m_{LR} as well as both types of maximal Majorana CP phases.

where m_f corresponds to $m_\tau \tan\beta$ for $(m, n) = (6, 2)$, $\tan\beta = 40$ in case of a charged lepton, and m_t for $(m, n) = (8, 4)$ in the case of up quarks [24,25]. $\lambda = 0.22$ is the standard Wolfenstein parameter. M_{RR} for both the cases are presented in the Table II.

Entering the values of M_{RR} in Eq. (5), we compute the type I + II neutrino mass matrix m_{LL} and find the mass eigenvalues and eigenvectors to compute the $|\Delta m_{31}^2|$ and Δm_{21}^2 and corresponding mixing angles for various values of γ. Deviations of these predicted Δm^2's from the data central values are then plotted in Figs. 1–4 against the parameter γ. It is observed from the figures that all the mass models survive at $\gamma \sim 1$ except the fact that for UQ type m_{LR}, the predictions for Δm_{21}^2 has deviated slightly from the 3σ range of experimental data. On the other hand, predictions for the mixing angles, i.e., θ_{12}, θ_{23}, show that all the models survive provided γ is close to 1 (or in other words, type II term has minimal contribution). The calculated values of neutrino parameters for $\gamma = 0.25, 0.50, 0.75, 1.0$ are given in Tables III,

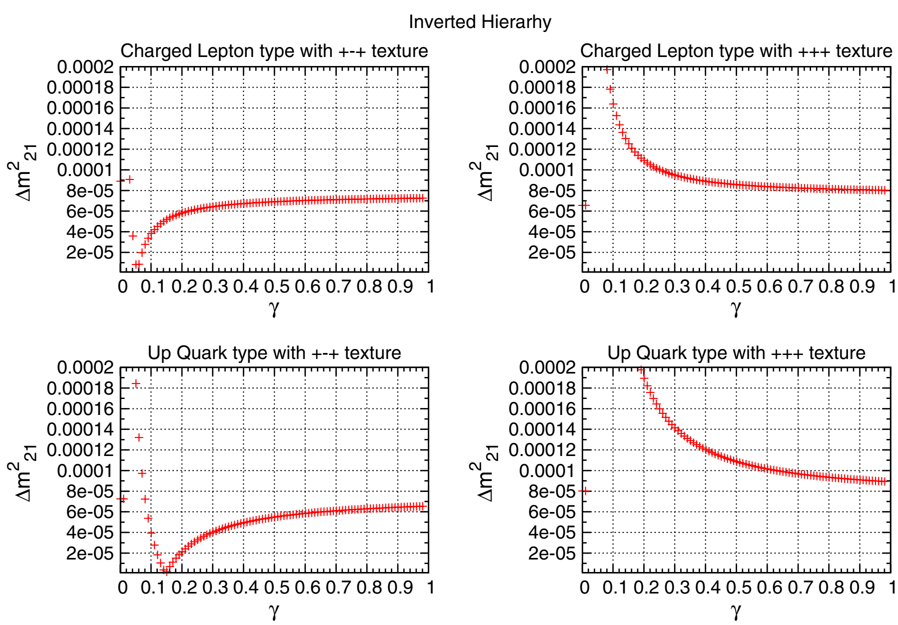

FIG. 4 (color online). Variation of the predicted values of Δm_{21}^2 as a function of γ in IH case for both charged lepton and up quark type m_{LR} as well as both types of maximal Majorana CP phases.

TABLE III. Predictions for neutrino parameters using type I + II seesaw for CL type m_{LR} with inverted hierarchy.

Parameters	IH(+ − +), $\gamma = 0.25$	IH(+ − +), $\gamma = 0.50$	IH(+ − +), $\gamma = 0.75$	IH(+ − +), $\gamma = 1.00$	IH(+ + +), $\gamma = 0.25$	IH(+ + +), $\gamma = 0.50$	IH(+ + +), $\gamma = 0.75$	IH(+ + +), $\gamma = 1.00$
$\Delta m^2_{21}[10^{-5}\ \mathrm{eV}^2]$	6.19	6.91	7.14	7.25	10.01	8.57	8.18	8.02
$\lvert\Delta m^2_{31}\rvert[10^{-3}\ \mathrm{eV}^2]$	2.31	2.32	2.32	2.32	2.29	2.31	2.31	2.31
$\sin^2\theta_{23}$	0.50	0.50	0.50	0.50	0.51	0.50	0.50	0.50
$\sin^2\theta_{12}$	0.33	0.33	0.33	0.33	0.15	0.22	0.26	0.33

IV, V, and VI. From this analysis we observe that QDN neutrino mass models can survive in nature within the framework of TBM mixing (with a few exceptions), with contributions from both type I and type II seesaw mechanisms.

IV. A VIABLE MODEL WITH A GAUGED ABELIAN FLAVOR SYMMETRY

The standard model (SM) of particle physics, if extended by the inclusion of three right-handed neutrinos which are singlet under the SM gauge group, can give rise to tiny neutrino mass by the type I seesaw mechanism [2]. Alternatively, if the SM is extended by a scalar triplet, a tiny neutrino mass can arise from the type II seesaw mechanism [3] after the neutral component of the scalar triplet acquires a tiny vacuum expectation value. Being singlets under the gauge group, the mass matrix of the right-handed neutrinos can have off-diagonal terms as well. Similarly, the gauge structure of the SM does not prevent off-diagonal Dirac Yukawa couplings. In other words, both the Dirac and right-handed Majorana mass matrices can be nondiagonal in general. However, throughout our analysis in the previous sections, we have restricted the Dirac neutrino mass matrix to its diagonal form only. This can be achieved by incorporating additional symmetries (global or local) with family nonuniversal gauge charges so that off-diagonal mass terms are not allowed. In this paper, we take up one such highly motivated extended symmetry: the Abelian gauge extension of the SM. It should be noted that although our analysis in the previous sections considered the type II seesaw formula (4) for a general left-right symmetric model, here we outline a simpler extension of the standard model by an Abelian-gauged flavor symmetry to explain the desired form of the mass matrices.

The Abelian gauge extension of the standard model is one of the best motivating examples of beyond standard model physics. For a review see Ref. [26]. Such a model is also motivated within the framework of grand unified theory models, for example, E_6. The supersymmetric version of such models have an additional advantage in the sense that they provide a solution to the minimal supersymmetric standard model μ problem. Such an Abelian gauge extension of SM was studied recently in Ref. [27] in the context of neutrino mass and cosmology.

Here we consider an extension of the standard model gauge group with one Abelian $U(1)_X$ gauge symmetry. Thus, the model we propose here is an $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge theory with three chiral generations of SM and three additional right-handed neutrinos. We will consider family nonuniversal $U(1)_X$ couplings.

The fermion content of our model is

$$
\begin{aligned}
&Q_i = \begin{pmatrix} u \\ d \end{pmatrix} \sim \left(3, 2, \frac{1}{6}, n_{qi}\right), \qquad L_i = \begin{pmatrix} \nu \\ e \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}, n_{li}\right),\\
&u_i^c \sim \left(3^*, 1, \frac{2}{3}, n_{ui}\right), \qquad d_i^c \sim \left(3^*, 1, -\frac{1}{3}, n_{di}\right),\\
&e_i^c \sim (1, 1, -1, n_{ei}), \qquad \nu_i^c \sim (1, 1, 0, n_{ri}),
\end{aligned}
$$

where $i = 1, 2, 3$ goes over the three generations of the standard model and the numbers in the brackets correspond to the quantum number under the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$. The $U(1)_X$ gauge quantum numbers should be such that they do not give rise to anomalies. We consider the following solution of the anomaly matching conditions:

$$
\begin{aligned}
&n_{qi} = n_{ui} = n_{di} = 0, \qquad n_{li} = n_{ei} = n_{ri} = n_i\\
&\sum n_{li} = \sum n_{ei} = \sum n_{ri} = 0,\\
&\sum n_{li}^3 = \sum n_{ei}^3 = \sum n_{ri}^3 = 0.
\end{aligned}
$$

TABLE IV. Predictions for neutrino parameters using type I + II seesaw for CL type m_{LR} with normal hierarchy.

Parameters	NH(+ − +), $\gamma = 0.25$	NH(+ − +), $\gamma = 0.50$	NH(+ − +), $\gamma = 0.75$	NH(+ − +), $\gamma = 1.00$	NH(+ + +), $\gamma = 0.25$	NH(+ + +), $\gamma = 0.50$	NH(+ + +), $\gamma = 0.75$	NH(+ + +), $\gamma = 1.00$
$\Delta m^2_{21}[10^{-5}\mathrm{eV}^2]$	6.72	7.16	7.30	7.36	9.76	8.63	8.32	8.19
$\lvert\Delta m^2_{31}\rvert[10^{-3}\ \mathrm{eV}^2]$	2.48	2.48	2.48	2.47	2.51	2.49	2.49	2.48
$\sin^2\theta_{23}$	0.49	0.49	0.50	0.50	0.48	0.49	0.49	0.50
$\sin^2\theta_{12}$	0.33	0.33	0.33	0.33	0.20	0.25	0.27	0.33

TABLE V. Predictions for neutrino parameters using type I + II seesaw for UQ type m_{LR} with inverted hierarchy.

Parameters	IH$(+-+)$, $\gamma = 0.25$	IH$(+-+)$, $\gamma = 0.50$	IH$(+-+)$, $\gamma = 0.75$	IH$(+-+)$, $\gamma = 1.00$	IH$(+++)$, $\gamma = 0.25$	IH$(+++)$, $\gamma = 0.50$	IH$(+++)$, $\gamma = 0.75$	IH$(+++)$, $\gamma = 1.00$
$\Delta m_{21}^2[10^{-5}\ \mathrm{eV}^2]$	3.28	5.49	6.20	6.53	15.98	10.86	9.49	8.94
$\lvert\Delta m_{31}^2\rvert[10^{-3}\ \mathrm{eV}^2]$	2.30	2.31	2.31	2.31	2.24	2.28	2.30	2.30
$\sin^2\theta_{23}$	0.50	0.50	0.50	0.50	0.54	0.52	0.51	0.50
$\sin^2\theta_{12}$	0.33	0.33	0.33	0.33	0.12	0.22	0.27	0.33

TABLE VI. Predictions for neutrino parameters using type I + II seesaw for UQ type m_{LR} with normal hierarchy.

Parameters	NH$(+-+)$, $\gamma = 0.25$	NH$(+-+)$, $\gamma = 0.50$	NH$(+-+)$, $\gamma = 0.75$	NH$(+-+)$, $\gamma = 1.00$	NH$(+++)$, $\gamma = 0.25$	NH$(+++)$, $\gamma = 0.50$	NH$(+++)$, $\gamma = 0.75$	NH$(+++)$, $\gamma = 1.00$
$\Delta m_{21}^2[10^{-5}\ \mathrm{eV}^2]$	4.83	6.23	6.68	6.89	14.00	10.36	9.33	8.90
$\lvert\Delta m_{31}^2\rvert[10^{-3}\ \mathrm{eV}^2]$	2.49	2.48	2.48	2.48	2.58	2.52	2.50	2.50
$\sin^2\theta_{23}$	0.49	0.50	0.50	0.50	0.46	0.48	0.49	0.50
$\sin^2\theta_{12}$	0.33	0.33	0.33	0.33	0.15	0.20	0.26	0.33

In particular, if we choose $n_1 = 0$, $n_2 = n$, $n_3 = -n$, only the following types of Dirac Yukawa terms will be present in the Lagrangian:

$$\mathcal{L}_Y \supset Y_\nu^{ii}\bar{L}_i H \nu_i^c + Y_e^{ii}\bar{L}_i H^\dagger e_i^c,$$

where H is the Higgs field responsible for breaking electroweak symmetry and has the quantum numbers $(1, 2, -\frac{1}{2}, 0)$ with respect to the gauge group. For the chosen Abelian charges, two singlet Higgs fields must exist $S_1(1, 1, 0, 0)$, $S_2(1, 1, 0, 2n)$ to give rise to a general structure of the right-handed Neutrino mass matrix. One of these singlet fields $S_2(1, 1, 0, 2n)$ (after acquiring a nonzero vev) also breaks the gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ to that of the standard model. Also, since the quarks have zero charges under the additional Abelian symmetry, they continue to have the usual CKM (Cabibbo-Kobayashi-Maskawa) structure of the mixing matrix. Such a model can have rich phenomenology from a collider as well as a cosmology point of view. However, for the purpose of our current work, we outline this model just to explain one possible origin of the specific structure (diagonal) of Dirac neutrino mass matrix m_{LR} used in the analysis. A more detailed investigation of such a model is left for future studies.

V. DISCUSSION

Analysis of the effect of Majorana CP phases in case of quasidegenerate neutrinos is done with contributions from both type I and type II seesaw formulas within the framework of TBM mixing. Fitting the neutrino mass matrix with best-fit oscillation and cosmology data, the right-handed neutrino mass matrix is calculated using type I seesaw formula only for both CL-type and UQ-type Dirac neutrino mass matrices. Adding a type II seesaw term (which arises in generic left-right symmetric models) to the type I term, the predictions for neutrino parameters are calculated. It is observed that for the minimal possible contribution of a type II seesaw term (which corresponds to the value of the dimensionless parameter γ in a type II seesaw term of order 1) to the neutrino mass matrix, all the neutrino mass models can survive in nature except the fact that for UQ type m_{LR} the predictions for Δm_{21}^2 have deviated slightly from the 3σ range of experimental data. Apart from this exception, all other predicted values of the neutrino parameters are consistent with neutrino oscillation data. Apart from neutrino oscillation data, these predictions are also within the limit of the cosmological upper bound $\sum_i m_i \leq 0.28$ eV. In view of above, the scenario of quasidegenerate neutrinos can survive in nature within the framework of type I and type II seesaw mechanisms and hence cannot be ruled out yet. However, here we stick to the TBM mixing framework and have not made any attempt to explain the nonzero θ_{13} as confirmed recently by several neutrino oscillation experiments. As an extension of this work, one can incorporate various corrections to m_{LL} to explain nonzero θ_{13}, which we have left for future studies.

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