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Renormalized quarkonium

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We extend our previous study of the vector charmonium states within a renormalization approach with boundary conditions to the full spectrum of charmonium and bottomonium. In light of the predicted spectrum we comply with assignments suggested in the literature. A comparison with the regularized quark model is also included.

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I. INTRODUCTION

The discovery of charmonium-like states by the *B* factories named *XYZ* mesons, which could not simply be described by the naive quark model, has increased the interest for new and more accurate charmonium potentials trying to accommodate these states.

The static charmonium potential can be parametrized phenomenologically as a Coulomb plus linear part together with spin-spin, spin-tensor, and spin-orbit terms as leading spin-dependent corrections [1,2]. From a more fundamental point of view, this potential can be derived from first principles of QCD in the lattice with relativistic corrections classified in powers of the inverse quark mass [3]. Recently these potentials have been improved either by new approaches in lattice QCD [4] or by matching the long-range part calculated by lattice simulations of the full QCD with results from perturbative QCD at short range [5]. Despite these improvements the short-range part of all these potentials still suffers from singularities which can be only handled by using *ad hoc* regulators. This triggers an unpleasant short-distance sensitivity.

In Ref. [6] we have developed an approach which reduces the effects of the *ad hoc* regulators, treating exactly the singular contributions of the potential following renormalization ideas. Instead of fitting the regulators to reproduce the ground state mass we use this mass as an input parameter of our calculation. The rest of the spectrum is *predicted* from the orthogonality with the ground state. The short-range uncertainties are encoded in this input and the rest of the spectrum only depends on the well established pieces of the potential.

We have shown in Ref. [6] that this procedure accurately reproduces the same numerical results as the standard constituent quark model (CQM) with extra regulators for the particular case of the $J^{PC} = 1^{--}$ states. In this paper we extend the calculation to the full charmonium and bottomonium spectrum showing that for all purposes the regulators only account for the ground states of $\bar{c}c$ and $\bar{b}b$ systems with J^{PC} quantum numbers. Therefore the approach constitutes a useful tool to complement more fundamental potentials. For completeness, we will also compare the results with those of the original model (including regulators), where generally an almost perfect agreement is found. Once these short-range uncertainties have been resolved we try to analyze which of the currently existing *XYZ* states could be identified as purely quarkonium states.

The plan of the paper is as follows. Section II explains briefly the features of our constituent quark model and describes the renormalization approach extended to all possible channels of quantum numbers. Sections III and IV are devoted to commenting on the spectrum of charmonium and bottomonium sectors within the renormalization approach. Section III includes a comparison with the original model. We finish by summarizing the work and giving some conclusions in Sec. V.

II. RENORMALIZED QUARKONIUM MODEL

The renormalization procedure we use exploits the local character of the potentials and uses a radial regulator in terms of boundary condition at a given cutoff radius r_c which is made smaller than any other length scale of the problem (typically $r_c \sim 0.01$ fm is enough). Within the present context the procedure has been explained in detail in Ref. [6] for which we refer for further aspects and motivation. Here, we will only present its generalization for any channel different from the $J^{PC} = 1^{--}$.

The interquark potential we use is based on the one developed by Vijande *et al.* [7] which is able to describe meson phenomenology from the light to the heavy quark sector. It also successfully describes hadron phenomenology and hadronic reactions [8–11]. The quark model includes a screened linear confinement potential based on unquenched lattice calculations together with spin-dependent terms determined by perturbative one gluon exchange as a Fermi-Breit type interaction. Although a

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complete description of the potential and the parameter values is given in Ref. [12], it is instructive to write down the explicit expressions of the (unregularized) one gluon exchange (OGE) contribution to display its singularities explicitly.

The OGE potentials are

$$V^{C}_{\rm OGE}(\vec{r}) = -\frac{4\alpha_s}{3}\frac{1}{r}, \tag{1}$$

$$V^{T}_{\rm OGE}(\vec{r}) = \frac{1}{3}\frac{\alpha_s}{m^2}\frac{1}{r^3}S_{12}, \tag{2}$$

$$V^{SO}_{\rm OGE}(\vec{r}) = \frac{2\alpha_s}{m^2}\frac{1}{r^3}\vec{L}\cdot\vec{S}. \tag{3}$$

As we see, the tensor and spin-orbit potentials present $1/r^3$ singularities at short distances. Note that we have discarded the Dirac delta function (which has the same dimensions as $1/r^3$) which is traditionally included. While this may seem weird, these are distributions around the origin which are not seen by the compact support test functions implied by the boundary condition regularization below the radial cutoff radius [6]. This also applies to any derivatives of the Dirac delta function. The result was suggested [13] and explicitly checked by using a momentum space regularization with so-called counterterms [14].

Once the constituent quark model we use has been presented, we will apply the renormalization with boundary conditions to eliminate the regulators of the model and treat exactly the singular contribution of the potential. This scheme has been explained in detail in Ref. [6] and we will provide here only the most relevant additional aspects to deal with all J^{PC} quarkonium states.

In Ref. [6] it is shown that for unique and finite normalizable solutions the number of free independent parameters of the regularized theory depends on the behavior of the solution at the origin and can be established by a simple analysis of the potentials. The different cases that we can find are

(i) Uncoupled channels with a singular attractive potential. In this case, besides the normalization condition which eliminates one constant, one more parameter is needed to renormalize the solution. If the potential is repulsive at short range all the constants which determine the wave function are defined and the bound state is predicted as it usually happens for the standard nonsingular quantum mechanics problem.

(ii) In the case of coupled channels the number of free parameters depends of the values of the potential near the origin, and more specifically on the corresponding eigenvalues of the coupled channel potential matrix. For two attractive potential eigenvalues we need three observables to fix the wave function. If there is one attractive eigenvalue only one parameter is needed. Finally, in the case of two repulsive eigenvalues the wave function is completely determined without any additional input parameter.

Focusing on quarkonium systems, we will work within the nonrelativistic framework, so the dynamics of the system is given by the Schrödinger equation. For a tensor and spin-orbit interaction we find the following situations:

(i) Singlet channel ($s = 0$, $l = J$)

$$-\frac{1}{2\mu}u_n''(r) + \left[V^{0J}_{J,J}(r) + \frac{J(J+1)}{2\mu r^2}\right]u_n(r) = E_n u_n(r) \tag{4}$$

(ii) Triplet uncoupled channel ($s = 1$, $l = J$)

$$-\frac{1}{2\mu}u_n''(r) + \left[V^{1J}_{J,J}(r) + \frac{J(J+1)}{2\mu r^2}\right]u_n(r) = E_n u_n(r). \tag{5}$$

(iii) Triplet coupled channel ($s = 1$, $l = J \pm 1$)

$$\begin{aligned} -\frac{1}{2\mu}u_n''(r) + \left[V^{1J}_{J-1,J-1}(r) + \frac{(J-1)J}{2\mu r^2}\right]u_n(r) + V^{1J}_{J-1,J+1}(r)w_n(r) &= E_n u_n(r),\\ -\frac{1}{2\mu}w_n''(r) + V^{1J}_{J+1,J-1}(r)u_n(r) + \left[V^{1J}_{J+1,J+1}(r) + \frac{(J+1)(J+2)}{2\mu r^2}\right]w_n(r) &= E_n w_n(r), \end{aligned} \tag{6}$$

where the energy is defined with respect to the $Q-\bar{Q}$ threshold, $E_n = M_n - m_Q - m_{\bar{Q}} = M_n - 2m$.

As mentioned, in order to determine the number of independent constants we have to study the potential at short distances for different channels. In the $r \to 0$ limit the dominant contributions are the tensor and the spin-orbit terms of the OGE potential. We need to know their character for the different cases:

(1) Singlet channel ($s = 0$, $l = J$). We have $\langle S_{12}\rangle = 0$ and $\langle \vec{L}\cdot\vec{S}\rangle = 0$, so the potential is regular and there is no observable free parameter. For the $^1S_0$ channel we have to take into account that in this case the potential has an attractive δ function and so is singular; therefore we have to set an observable for a regularized solution.

(2) Triplet uncoupled channel ($s = 1$, $l = J$). We have $\langle S_{12}\rangle = +2$ and $\langle \vec{L}\cdot\vec{S}\rangle = -1$, so the potential is singular attractive and an observable must be fixed for a regularized solution.

(3) Triplet coupled channel ($s = 1$, $l = J \pm 1$). If we denote $l = J - 1$ and $l' = J + 1$, we will have

(i) $\langle {}^3l_J|S_{12}|{}^3l'_J\rangle = \frac{6\sqrt{J(J+1)}}{2J+1}$,
(ii) $\langle {}^3l_J|S_{12}|{}^3l_J\rangle = -\frac{2(J-1)}{2J+1}$,
(iii) $\langle {}^3l'_J|S_{12}|{}^3l'_J\rangle = -\frac{2(J+2)}{2J+1}$,
(iv) $\langle {}^3l_J|\vec{L}\cdot\vec{S}|{}^3l_J\rangle = J-1$,
(v) $\langle {}^3l'_J|\vec{L}\cdot\vec{S}|{}^3l'_J\rangle = -(J+2)$,

and diagonalizing the potential matrix the eigenvalues are

$$E(J) = -10 \pm 6\sqrt{1+J+J^2} \tag{7}$$

with $J \geq 1$, so we always have one negative eigenvalue and it requires one parameter.

In order to describe a bound state we seek normalizable solutions

$$\int_0^\infty [u(r)^2 + w(r)^2] = 1, \tag{8}$$

where $w(r) = 0$ for the uncoupled channels. This imposes conditions on the wave functions both at infinity as well as at the origin.

Moreover, the set of Eqs. (4)–(6), must be accompanied by asymptotic conditions at infinity. Once we have discarded the irregular function at long distances, the wave functions at infinity have the following behavior:

$$u(r) \to A_{J-1}e^{-\epsilon r}, \qquad w(r) \to A_{J+1}e^{-\epsilon r}, \tag{9}$$

where A_{J-1} is the normalization factor and the asymptotic $J+1/J-1$ ratio parameter is defined by $\eta = A_{J+1}/A_{J-1}$. Ideally, one would integrate the Schrödinger equation taking its solutions at infinity, Eq. (9), which depend on the binding energy and η. The singular structure of the problem at short distances requires a specific analysis of the coupled equations as has been done extensively elsewhere [15] and we adapt here for our particular situation. The result amounts to integrate from infinity for the physical value of M_0 and η (or M_0 in the case of singlet channels). Generally, the solutions diverge strongly at the origin, so that the normalization of the state is precluded. However, there is a particular value of η which guarantees that the wave function becomes normalizable.[1] The rest of the spectrum is then built by imposing orthogonality of states in coupled channels (see Ref. [6] for details in the J/ψ case).

[1]Thus, if one imposes the regularity condition at the origin one will determine η and therefore the wave function of the bound state. In practice, however, the converging solution is rather elusive since integrated-in solutions quickly run into the diverging solution due to the round-off errors and dominate over the converging solution.

III. PHENOMENOLOGY OF CHARMONIUM STATES

The charmonium spectrum consists of eight narrow states below the open-charm threshold (3.73 GeV) and several tens of states above the threshold, some of them wide because they decay into charmed mesons, some of them still narrow because their decay to open-charm is forbidden by some conservation rule. Below the threshold all states are well established. Above threshold, however, there are new charmonium-like states that are very difficult to accommodate theoretically.

In Tables 4 and 9 of Ref. [16] the updated new *conventional* and *unconventional* states in the $c\bar{c}$, $b\bar{c}$, and $b\bar{b}$ sectors are given. If we focus on the $c\bar{c}$ region, there are three new states which have been recognized as $q\bar{q}$ pairs in the PDG [17] during the last years. They are the h_c which is the ${}^1P_1$ state of charmonium, singlet partner of the long-known χ_{cJ} triplet ${}^3P_J$ states, the $\eta_c(2S)$ which is the first radial excitation of the pseudoscalar ground state $\eta_c(1S)$, and the $Z(3930)$ whose assignment as the 2^3P_2 state, $\chi_{c2}(2P)$, seems widely accepted. The rest of the resonances of this sector, namely, the $X(3872)$, $X(3915)$, $Y(3940)$, $X(3940)$ still lack for a clear assignment.

The results of the study of the full charmonium spectrum up to total spin $J = 2$ and for the first radial excitations within the renormalization with boundary conditions scheme are shown in Table I. In the following we will discuss our predictions for the different channels except the $J^{PC} = 1^{--}$ which has been extensively discussed in Ref. [6] already.

In columns four and five we compare the calculated masses within the renormalization approach with the experimental data. As we see, once the experimental value of the ground state mass is taken as input in the calculation, the rest of the spectrum is reproduced fairly well.

In order to size the relevance of form factors in the excited spectrum we also show in columns six and seven the comparison with the original model including form factors [7] and taking the predicted mass as a parameter of the renormalization approach. The observed tiny deviations are below a few MeV corresponding to a marginal influence of the form factors on the excited states. These results actually point to the idea that the gluonic regulators are fitted just to provide the ground state energies. Once we have eliminated possible bias due to the use of regulators, we are in a favorable position to discuss possible $q\bar{q}$ assignments for the old and new states.

A. η_c and η_{c2} states

An $\eta_c(1S)$ candidate was observed thirty years ago by CBAL [21] and MARK II [22] Collaborations with a mass measurement very close to the updated world average 2980.3 ± 1.2. In the renormalization scheme this state is taken as a parameter and in essence fixes the spin-spin contact term interaction.

TABLE I. Masses, in MeV, of charmonium states. nL labels the radial and angular momentum quantum numbers. When partial waves are coupled they refer to the dominant component. We compare with the well established states in Ref. [17] and assign possible XYZ mesons.

Particle	J^{PC}	nL	REN1 (MeV)	Experimental data (MeV)	REN2 (MeV)	CQM (MeV)
η_c	0^{-+}	$1S$	Input	2980.3 ± 1.2	Input	2991
		$2S$	3634	3637 ± 4	3640	3643
		$3S$	4046		4050	4054
χ_{c0}	0^{++}	$1P$	Input	3414.75 ± 0.31	Input	3452
		$2P$	3872	$3915 \pm 3 \pm 2$ [18]	3909	3910
		$3P$	4209		4243	4242
h_c	1^{+-}	$1P$	3516	3525.42 ± 0.29	3516	3515
		$2P$	3957		3957	3956
		$3P$	4279		4279	4278
ψ	1^{--}	$1S$	Input	3096.916 ± 0.011	Input	3096
		$2S$	3704	3686.093 ± 0.034	3703	3703
		$1D$	3796	3775.2 ± 1.7	3796	3796
		$3S$	4098	4039 ± 1	4097	4097
		$2D$	4152	4153 ± 3	4153	4153
		$4S$	4390	$4361 \pm 9 \pm 9$ [19]	4389	4389
		$3D$	4425	4421 ± 4	4426	4426
		$5S$	4615	4634^{+8+5}_{-7-8} [20]	4614	4614
		$4D$	4640	$4664 \pm 11 \pm 5$ [19]	4641	4641
χ_{c1}	1^{++}	$1P$	Input	3510.66 ± 0.07	Input	3504
		$2P$	3955		3947	3947
		$3P$	4278		4272	4272
η_{c2}	2^{-+}	$1D$	3812		3812	3812
		$2D$	4166	$4156^{+25}_{-20} \pm 15$	4166	4166
		$3D$	4437		4437	4437
χ_{c2}	2^{++}	$1P$	Input	3556.20 ± 0.09	Input	3531
		$2P$	3974	$3929 \pm 5 \pm 2$	3968	3969
		$1F$	4043		4043	4043
ψ_2	2^{--}	$1D$	Input		3810	3810
		$2D$	4164		4164	4164
		$3D$	4436		4436	4436

The search for a reproducible $\eta_c(2S)$ signal has a long history. Recently, Belle [23] found a signal in $B \to K\eta_c(2S)$ in the exclusive $\eta_c(2S) \to K^0_S K^- \pi^+$ decay mode [a favorite all-charged final state for $\eta_c(1S)$], at $3654 \pm 6 \pm 8$ MeV. Since then measurements of $\eta_c(2S)$ in that mass region have been reported by *BABAR* [24], CLEO [25], and Belle [26] in $\gamma\gamma$-fusion to $K\bar{K}\pi$ final states and by *BABAR* [27] and Belle [28] in double charmonium production.

Our predicted mass for the $\eta_c(2S)$ is 3634 MeV, in very good agreement with the updated world average reported in Ref. [17]. Nothing is known of the next excitation, $\eta_c(3S)$. Our prediction is around 4.05 GeV.

The potential at short distances is regular for the η_{c2} states. Therefore the mass of the ground state for the η_{c2} meson is not a parameter and we predict 3812, 4166, and 4437 MeV for the ground state and the first two radial excitations.

We can clearly identify a second state $\eta_{c2}(2S)$ with mass $M = 4166$ MeV and width $\Gamma = 122.9$ MeV with the resonance recently reported by Belle at $M = 4156^{+25}_{-20} \pm 15$ MeV with a width $\Gamma = 139^{+111}_{-61} \pm 21$ MeV [29] in the $e^+e^- \to D^*\bar{D}^*J/\psi$. The decay of η_{c2} to $D\bar{D}$ is forbidden being the $X(4160) \to D^*\bar{D}^*$, the most favored decay channel as shown by the data.

B. h_c and χ_{cJ} states

Two experiments reported the observation of the $h_c(1P)$ in 2005. CLEO [30,31] obtained a 6σ statistical significance in the isospin-forbidden decay chain $e^+e^- \to \psi(2S) \to \pi^0 h_c$, $h_c \to \gamma\eta_c(1S)$. E835 [32] found 3σ evidence in $p\bar{p} \to h_c$, $h_c \to \gamma\eta_c(1S)$, $\eta_c(1S) \to \gamma\gamma$.

The precision measurement of its mass was reported by CLEO in 2008 [33], $3525.28 \pm 0.19 \pm 0.12$ MeV. Later BESIII [34] has confirmed this with a mass of $3525.40 \pm 0.13 \pm 0.18$ MeV. It was important to measure the mass of this state because the spin-averaged centroid of the triplet states,

TABLE II. The theoretical masses, in MeV, of the first two radial excitations of h_c compared with the spin-averaged centroid, in MeV, of the triplet states. We compare with the experimental data [17].

	CQM		REN		
nL	$M(h_c)$	$C_{\rm the}$	$M(h_c)$	$C_{\rm the}$	$C_{\rm exp}$
$1P$	3515	3513	3516	3513	3525.30 ± 0.20
$2P$	3956	3955	3957	3955	

$$\langle m(1^3P_J)\rangle \equiv \frac{m_{\chi_{c0}} + 3m_{\chi_{c1}} + 5m_{\chi_{c2}}}{9}, \tag{10}$$

is expected to be near the $h_c(1P)$ mass. The lattice data show a vanishing long-range component of the spin-spin potential. Thus, the potential appears to be entirely dominated by its short-range, deltalike, part, suggesting that the ${}^1P_1$ should be close to the center-of-gravity of the ${}^3P_J$ system. So it makes the hyperfine mass splitting, $\Delta m_{hf}[h_c(1P)] = \langle m(1^3P_J)\rangle - m[h_c(1P)]$ an important measurement of the spin-spin interaction.

The centroid of the 1^3P_J states ($\chi_{0,1,2}$) is known to be [17] 3525.30 ± 0.04 MeV and then the hyperfine splittings are $+0.02 \pm 0.23$ MeV, from CLEO and -0.10 ± 0.22 MeV from BESIII.

Table I shows the masses for three radial excitations of the singlet ${}^1P_1$ and the triplet ${}^3P_J$ mesons. In Table II we show the comparison between the centroid of ${}^3P_J$ states and the corresponding h_c mass for the ground state and the first radial excitation, showing that our spin-spin interaction is negligible for these channels and it is in perfect agreement with the lattice expectations and the experimental measurements for the ground state.

The mean $2P$ multiplet mass is predicted to be near 3.95 GeV. Although no $2P$ $c\bar{c}$ states have been clearly seen experimentally, there are a lot of states reported from the different collaborations which claim enhancements in that energy region. Concerning the rest of the new states, none of them appears in our calculation as a $c\bar{c}$ state except the $Z(3930)$. This fact agrees with the results of a recent calculation which shows that these states can be interpreted as molecular resonances [35].

The $Z(3930)$ was reported by Belle in $\gamma\gamma \to D\bar{D}$ with a mass and width $M = 3929 \pm 5 \pm 2$ MeV and $\Gamma = 29.9 \pm 10 \pm 2$ MeV [18]. The two photon width is measured to be $\Gamma_{\gamma\gamma}B(Z(3930) \to D\bar{D}) = 0.18 \pm 0.05 \pm 0.03$ keV. Moreover the $D\bar{D}$ angular distribution is consistent with $J = 2$. The $\chi_{c2}(2P)$ state is a good candidate for the $Z(3930)$. We obtained a mass of 3974 MeV and the total width $\Gamma = 49.1$ MeV is comparable with the experimental data. Finally the experimental two photon width compares nicely with our result $\Gamma_{\gamma\gamma}B(Z(3930) \to D\bar{D}) = 0.15$ keV.

IV. BOTTOMONIUM

In this section we extend the renormalization with boundary conditions scheme to the study of the bottomonium states. Although the B factories are not usually considered as ideal facilities for the study of the bottomonium spectrum since their energy is tuned to the peak of the $\Upsilon(4S)$ resonance, which decays in almost 100% of cases to a $B\bar{B}$ pair, *BABAR* and Belle have reported data samples at various energies in the bottomonium region that made possible discoveries like the η_b [36], $h_b(1P)$, and $h_b(2P)$ [37]. Nevertheless the number of states is lower than in the charmonium spectrum.

As in the previous section, column four of Table III shows the calculated masses within the renormalization

TABLE III. Masses in MeV of bottomonium states. The notation for the quantum numbers is the same as in Table I. We compare with the well established states in Ref. [17].

Particle	J^{PC}	nL	REN1 (MeV)	Experimental data (MeV)
η_b	0^{-+}	$1S$	Input	9390.9 ± 2.8
		$2S$	9957	$9999.0 \pm 3.5^{+2.8}_{-1.9}$ [38]
		$3S$	10306	
χ_{b0}	0^{++}	$1P$	Input	$9859.44 \pm 0.42 \pm 0.31$
		$2P$	10226	$10232.5 \pm 0.4 \pm 0.5$
		$3P$	10505	
h_b	1^{+-}	$1P$	9879	$9898.25 \pm 1.06^{+1.03}_{-1.07}$
		$2P$	10241	$10259.76 \pm 0.64^{+1.43}_{-1.03}$
		$3P$	10516	
Υ	1^{--}	$1S$	Input	9460.30 ± 0.26
		$2S$	9992	10023.26 ± 0.31
		$1D$	10117	
		$3S$	10331	10355.2 ± 0.5
		$2D$	10414	
		$4S$	10592	10579.4 ± 1.2
		$3D$	10653	
		$5S$	10805	10865 ± 8
		$4D$	10853	
		$6S$	10984	11019 ± 8
		$5D$	11023	
χ_{b1}	1^{++}	$1P$	Input	$9892.78 \pm 0.26 \pm 0.31$
		$2P$	10254	$10255.46 \pm 0.22 \pm 0.50$
		$3P$	10527	
η_{b2}	2^{-+}	$1D$	10123	
		$2D$	10419	
		$3D$	10658	
χ_{b2}	2^{++}	$1P$	Input	$9912.21 \pm 0.26 \pm 0.31$
		$2P$	10248	$10268.65 \pm 0.22 \pm 0.50$
		$1F$	10315	
Υ_2	2^{--}	$1D$	Input	10163.7 ± 1.4
		$2D$	10418	
		$3D$	10657	

TABLE IV. The theoretical masses, in MeV, of the first two radial excitations of h_b compared with the spin-averaged centroid, in MeV, of the triplet states. We compare with the experimental data [17].

	CQM		REN		
nL	$M(h_b)$	$C_{\rm the}$	$M(h_b)$	$C_{\rm the}$	$C_{\rm exp}$
$1P$	9879	9879	9879	9879	9899.87 ± 0.27
$2P$	10240	10240	10241	10240	10260.24 ± 0.36

approach up to total spin $J = 2$ and for the first radial excitations. As in the case of the charmonium spectrum, when applicable, we indicate the fact that the ground state mass is used as an input parameter.

In the $J^{\rm PC} = 1^{--}$ channel there are fewer pieces of data for the bottomonium spectrum than for the charmonium. The S and D wave states are almost degenerate for the highest excited states. In the charmonium these doublets have been measured and we can assign them to S and D waves. In the bottomonium the splittings are smaller and experimentally they have not been resolved. We follow the assignment of the Particle Data Group up to $4S$ states and assumed that the others are S wave. This is the reason why we assigned the experimentally measured state at $M = 11019$ MeV to our state $6S$ and not to our $5D$.

As a general trend the experimental data are well reproduced by the calculation and a number of excited states are given which can be tested at LHCb and other B factories. The $\eta_b(2S)$ has been very recently measured by the Belle Collaboration [38] with a mass $M = 9999.0 \pm 3.5^{+2.8}_{-1.9}$ which is in reasonable agreement with our $M = 9957$ MeV result.

The hyperfine mass splitting of singlet-triplet states, $\Delta m_{hf}[\eta_b(1S)] = m(\Upsilon(1S)) - m(\eta_b(1S))$, probes the spin dependence of bound-state energy levels, and, once measured, imposes constraints on theoretical descriptions. It is given experimentally by

$$\Delta m_{hf}[\eta_b(1S)] = 69.6 \pm 2.9 \text{ MeV}. \qquad (11)$$

In the renormalization scheme this splitting is fixed and we can only predict the splittings for radial excitations which have not yet been measured.

In the case of the centroid of the $\chi_{bJ}(nP)$ states with $n = 1, 2$ the masses are known to be [17] 9899.87 ± 0.27 MeV and 10260.24 ± 0.36 MeV, respectively. The hyperfine splittings measured by the Belle Collaboration [37] are $\Delta m_{hf}[h_b(1P)] = +1.6 \pm 1.5$ MeV and $\Delta m_{hf}[h_b(2P)] = +0.5^{+1.6}_{-1.2}$ MeV which are compatible with zero.

Table III shows the masses for three radial excitations of the singlet h_b and the triplet χ_{bJ} mesons. They are in reasonable agreement with the experimental data. In Table IV we show the comparison between the centroid of χ_{bJ} states and the corresponding h_b mass for the ground state and the first radial excitation, showing that our spin-spin interaction is negligible.

$\chi_{bJ}(1P)$ and $\chi_{bJ}(2P)$ with $J = 2, 1, 0$ were discovered earlier in 1982 [39,40] and 1983 [41,42], respectively. Their masses have not changed much since then and our theoretical prediction through both schemes are very close to the experimental values.

V. CONCLUSIONS

Based on an earlier work, we have reanalyzed the calculation of the charmonium spectrum in a constituent quark model using a renormalization scheme with boundary conditions. This approach avoids explicitly the introduction of phenomenological form factors taking as a parameter the mass of the ground state. Thus, the only relevant physical information on the form factors is to tune the value of the ground state energy. Once this fact has been established we have applied the renormalization framework to provide some basis to "bare" $q\bar{q}$ assignments of mesonic states.

We obtain a spectrum in reasonable agreement with the experimentally well established data. For instance, we obtain $\Delta m_{hf}[h_c(nP)]$ compatible with zero due to the fact that the hyperfine contact term has been included in the renormalization conditions. We also assign certain XYZ mesons according to our model.

For the phenomenologically successful model of Ref. [7] where *ad hoc* form factors are introduced as regulators, we find an almost perfect agreement with the renormalization approach. This result provides confidence on the way the original model took into account the unknown short-distance dynamics. In addition, we have extended this study to the bottomonium sector obtaining similar conclusions as in the charmonium sector.

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