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# Branching ratios and *CP* asymmetries in charmless nonleptonic *B* decays to radially excited mesons

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Nonleptonic two-body $B$ decays including radially excited $\pi(1300)$ or $\rho(1450)$ mesons in the final state are studied using the framework of a generalized naive factorization approach. Branching ratios and $CP$ asymmetries of $B\to P\pi(1300)$, $B\to V\pi(1300)$, $B\to P\rho(1450)$ and $B\to V\rho(1450)$ decays are calculated, where $P$ and $V$ stand for pseudoscalar and vector charmless mesons. Form factors for $B\to\pi(1300)$ and $B\to\rho(1450)$ transitions are estimated in the improved version of the Isgur-Scora-Grinstein-Wise quark model. In some processes, $CP$ asymmetries of more than 10% and branching ratios of $10^{-5}$ order are found, which could be reached in experiments.

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## I. INTRODUCTION

Most of the research on nonleptonic two-body $B$ decays have concentrated in processes where the final mesons are ground states or angular orbital excitations [1]. Radial excitations can be produced in $B$ decays. Decays involving radially excited mesons could be an alternative to those more traditionally studied, given additional and complementary information.

The physics involved in nonleptonic two-body $B$ decays allows us to study the interplay of QCD and electroweak interactions, to search for $CP$ violation, and overconstrain Cabbibo-Kobayashi-Maskawa (CKM) parameters in a precision test of the standard model [1,2].

In the quark model, mesons are $q\bar{q}'$ bound states of quark $q$ and antiquark $\bar{q}'$. The $q\bar{q}'$ state has an orbital angular momentum $l$ and spin $J$. Pseudoscalar and vector mesons have orbital angular momentum $l=0$. The angular orbital excitations: scalar, axial vector and tensor mesons have $l=1$. Mesons can be classified in spectroscopy notation by $n^{2s+1}l_J$, where $s=0$ or 1 for parallel or antiparallel quarks $q$ and $\bar{q}'$, respectively. Radial excitations are denoted by the principal quantum number $n$.

The radial excitations $\pi(1300)$ and $\rho(1450)$, with principal quantum number $n=2$, $u$ and $d$ quark content, can be produced in nonleptonic two-body $B$ decays. In spectroscopy notation, $\pi(1300)$ is denoted by $2^1S_0$ and $\rho(1450)$ by $2^3S_1$. To simplify, we denote $\pi(1300)$ by $\pi'$ and $\rho(1450)$ by $\rho'$.

In Ref. [3], the authors interested in factorization-breaking effects in $B$ decays consider $B$ decays to final states with small decay constants, such as $\bar{B}^0\to D^+\pi'^-$ and $\bar{B}^0\to D^{*+}\pi'^-$ decays.

Production of charmless radially excited vector mesons in nonleptonic two-body $B$ decays is considered in Ref. [4]. The authors make a prediction for the ratio $\mathrm{Br}(B\to\rho'\pi)/\mathrm{Br}(B\to\rho\pi)$. This ratio is given in terms of the form factor $A_0$, which is calculated in a constituent quark model [4]. We compare our calculations with their result and experimental data available.

In this paper, we present a study on the exclusive modes $B\to P\pi'$, $B\to V\pi'$, $B\to P\rho'$ and $B\to V\rho'$, where $P$ and $V$ are the pseudoscalar and vector mesons, $\pi$, $\eta$, $\eta'$ and $K$ and $\rho$, $\omega$, $K^*$ and $\phi$, respectively. We compute branching ratios of these processes using the effective weak Hamiltonian, with tree and penguin contributions. Matrix elements are calculated in the generalized naive factorization approach [5,6]. The form factors for $B\to P$ and $B\to V$ transitions are calculated in the Bauer-Stech-Wirbel (WSB) model [7] and light-cone sum rule (LCSR) approach [8]. Form factors for $B\to\pi'$ and $B\to\rho'$ transitions are calculated in the improved version of the Isgur-Scora-Grinstein-Wise (ISGW) quark model, called ISGW2 model [9,10].

We also calculate $CP$-violating asymmetries in the framework of generalized naive factorization approach [11]. $CP$ asymmetries allow us to determine interior angles of the unitary triangle and test the unitarity of the CKM matrix. Specifically, in this work, we calculate direct $CP$ violation for charged $B^\pm$ decays and $CP$ asymmetries for neutral $B^0(\bar{B}^0)$ decays. For some channels, asymmetries of order 10% are found.

In general, we use the method and formulas developed in Refs. [5,6,11], to estimate branching ratios and $CP$-violating asymmetries. We make the respective changes in the processes studied in this work, i.e., masses, decay constants and form factors.

The decay constants $f_{\pi'}$ and $f_{\rho'}$ are not well-determined input parameters. The range of values for the $f_{\pi'}$ decay constant obtained from different methods and its impact in channels $B\to P\pi'$ and $B\to V\pi'$ is discussed in this work. Some branching ratios are sensitive to the decay constants $f_{\pi'}$ and $f_{\rho'}$. This fact will allow us to determine decay constants by experiment in cases where branching ratios are measured.

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This paper is organized as follows. In Sec. II, we present the framework used to calculate branching ratios of nonleptonic two-body $B$ decays, the effective Hamiltonian and the generalized naive factorization approach. Input parameters, mixing schemes, decay constants and form factors are discussed in Sec. III. In Sec. IV, we discuss the amplitudes involving radially excited mesons and calculate numerical results for branching ratios. $CP$-violating asymmetries for charged and neutral channels are presented in Sec. V. Our conclusions are given in Sec. VI. In the appendixes, we give the amplitudes for $B \to P\pi'$, $V\pi'$, $P\rho'$ and $V\rho'$ processes, which are taken from the appendixes in Refs. [5,6], without considering annihilation and interchange contributions, with the appropriate changes in the processes calculated in this work.

## II. FRAMEWORK

### A. Effective Hamiltonian

The framework to study $B$ decays is the effective weak Hamiltonian [12]. For $\Delta B = 1$ transitions, it is written as

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\Big[\sum_{j=u,c} V_{jb}V_{jq}^*(C_1(\mu)O_1^j(\mu) + C_2(\mu)O_2^j(\mu)) - V_{tb}V_{tq}^*\Big(\sum_{i=3}^{10} C_i(\mu)O_i(\mu)\Big)\Big] + \text{H.c.}, \quad (1)$$

where $G_F$ is the Fermi constant, $C_i(\mu)$ are Wilson coefficients at the renormalization scale $\mu$, $O_i(\mu)$ are local operators and $V_{ij}$ are the respective CKM matrix elements involved in the transitions. The local operators for $b \to q$ transitions are

$$\begin{aligned}
O_1^j &= \bar{q}_\alpha\gamma^\mu L u_\alpha \cdot \bar{u}_\beta\gamma_\mu L b_\beta \\
O_2^j &= \bar{q}_\alpha\gamma^\mu L u_\beta \cdot \bar{u}_\beta\gamma_\mu L b_\alpha \\
O_{3(5)} &= \bar{q}_\alpha\gamma^\mu L b_\alpha \cdot \sum_{q'}\bar{q}'_\beta\gamma_\mu L(R)q'_\beta \\
O_{4(6)} &= \bar{q}_\alpha\gamma^\mu L b_\beta \cdot \sum_{q'}\bar{q}'_\beta\gamma_\mu L(R)q'_\alpha \\
O_{7(9)} &= \frac{3}{2}\bar{q}_\alpha\gamma^\mu L b_\alpha \cdot \sum_{q'}e_{q'}\bar{q}'_\beta\gamma_\mu R(L)q'_\beta \\
O_{8(10)} &= \frac{3}{2}\bar{q}_\alpha\gamma^\mu L b_\beta \cdot \sum_{q'}e_{q'}\bar{q}'_\beta\gamma_\mu R(L)q'_\alpha,
\end{aligned} \quad (2)$$

where $q = d$ or $s$, $O_1^j$ and $O_2^j$ are the current-current operators ($j = u, c$), $O_3 - Q_6$ the QCD penguins operators and $Q_7$–$Q_{10}$ the electroweak penguins operators. The indexes $\alpha$ and $\beta$ mean $SU(3)$ color degrees, $L$ and $R$ are the left and right projector operators, respectively. The sum extends over active quarks $u$, $d$, $s$ and $c$ at the scale of $B$ meson $\mu = \mathcal{O}(m_b)$.

In order to calculate the branching ratios and $CP$ asymmetries in this work, we use the next to leading order Wilson coefficients for $\Delta B = 1$ transitions obtained in the naive dimensional regularization scheme (NDR) at the energy scale $\mu = m_b(m_b)$, $\Lambda^{(5)}_{\overline{MS}} = 225$ MeV and quark top mass $m_t = 170$ GeV. These coefficients are taken from Ref. [12], see Table 22. Those values are $c_1 = 1.082$, $c_2 = -0.185$, $c_3 = 0.014$, $c_4 = -0.035$, $c_5 = 0.009$, $c_6 = -0.041$, $c_7/\alpha = -0.002$, $c_8/\alpha = 0.054$, $c_9/\alpha = -1.292$ and $c_{10}/\alpha = 0.263$, where $\alpha = 1/137$ is the fine structure constant.

### B. Generalized naive factorization approach

The decay amplitude of a nonleptonic two-body $B$ decay can be calculated using the effective weak Hamiltonian by

$$\mathcal{M}(B \to M_1M_2) = \langle M_1M_2|H_{\rm eff}|B\rangle = \frac{G_F}{\sqrt{2}}\sum_{i=1}^{10} C_i(\mu)\langle O_i(\mu)\rangle, \quad (3)$$

where the hadronic matrix elements $\langle O_i(\mu)\rangle$ are defined by $\langle M_1M_2|O_i(\mu)|B\rangle$ and $M_i$ are final state mesons. In the naive factorization hypothesis, hadronic matrix elements $\langle O_i(\mu)\rangle$ are evaluated by the product of decay constants and form factors. These matrix elements are energy $\mu$ scale and renormalization scheme independent; consequently, there is no term to cancel the energy $\mu$ dependency in the Wilson coefficients, and the amplitudes for nonleptonic two-body $B$ decays are scale and renormalization scheme dependent.

The improved naive factorization approach [5,6] is formulated to solve the problem of energy scale dependency by including some perturbative QCD contributions in Wilson coefficients. This is considered in order to isolate the energy $\mu$ dependency from the matrix element $\langle O_i(\mu)\rangle$ and join it with the Wilson coefficients to produce effective Wilson coefficients $c_i^{\rm eff}$, which are scale $\mu$ independent. Schematically,

$$\sum_i C_i(\mu)\langle O_i(\mu)\rangle = \sum_i C_i(\mu)g_i(\mu)\langle O_i\rangle_{\rm tree} = \sum_i c_i^{\rm eff}\langle O_i\rangle_{\rm tree}, \quad (4)$$

where $g_i(\mu)$ are perturbative QCD corrections to Wilson coefficients and $\langle O_i\rangle_{\rm tree}$ are tree-level hadronic matrix elements. Explicit expressions for the effective Wilson coefficients $c_i^{\rm eff}$ are given in Refs. [6]. These coefficients are recalculated with the current CKM parameters [13]. Effective Wilson coefficients, for transitions $b \to d$ and $b \to s$, are shown in Table I. They are evaluated at the factorizable scale $\mu = m_b$, with an averaged momentum transfer of $k^2 = m_b^2/2$ and using the central values for CKM parameters from Ref. [13].

In the factorizable decay amplitude, the effective Wilson coefficients appear as linear combinations. Thus, to simplify decay amplitudes, $a_i$ coefficients are introduced

TABLE I. Effective Wilson coefficients $c_i^{\rm eff}$ for $b \to d$ and $b \to s$ transitions. Evaluated at $\mu_f = m_b$ and $k^2 = m_b^2/2$, where the central values for Wolfenstein parameters $\lambda = 0.2253$, $A = 0.808$, $\rho = 0.132$ and $\eta = 0.341$ are used, see Ref. [13].

| $c_i^{\rm eff}$ | $b \to d$ | $b \to s$ |
|---|---|---|
| $c_1^{\rm eff}$ | 1.1680 | 1.1680 |
| $c_2^{\rm eff}$ | −0.3652 | −0.3652 |
| $c_3^{\rm eff}$ | 0.0226 + i0.0041 | 0.0227 + i0.0044 |
| $c_4^{\rm eff}$ | −0.0461 − i0.0122 | −0.0464 − i0.0133 |
| $c_5^{\rm eff}$ | 0.0133 + i0.0041 | 0.0134 + i0.0044 |
| $c_6^{\rm eff}$ | −0.0483 − i0.0122 | −0.0486 − i0.0133 |
| $c_7^{\rm eff}/\alpha$ | −0.0264 − i0.0343 | −0.0270 − i0.0365 |
| $c_8^{\rm eff}/\alpha$ | 0.0551 | 0.0551 |
| $c_9^{\rm eff}/\alpha$ | −1.4229 − i0.0343 | −1.4235 − i0.0365 |
| $c_{10}^{\rm eff}/\alpha$ | 0.4804 | 0.4804 |

$$
\begin{aligned}
a_i &\equiv c_i^{\rm eff} + \frac{1}{N_c} c_{i+1}^{\rm eff}\ (i = {\rm odd}),\\
a_i &\equiv c_i^{\rm eff} + \frac{1}{N_c} c_{i-1}^{\rm eff}\ (i = {\rm even}),
\end{aligned}
\tag{5}
$$

where index $i$ runs over $1, \ldots, 10$, and $N_c = 3$ is the color number of QCD. Effective coefficients $a_i$ for $b \to d$ and $b \to s$ transitions are shown in Table II.

The improved naive factorization approach contains various sources of theoretical uncertainties, i.e., renormalization scheme dependence, $1/N_C$ value, and $k^2$ dependence, which we discuss in the following.

The Wilson coefficients depend on renormalization energy scale $\mu$, though improved naive factorization was built to resolve this problem. In addition, Wilson coefficients at next leading order also depend on the renormalization scheme; see Ref. [12] where an analysis is presented for naive dimensional regularization (NDR) and 't Hooft Veltman (HV) schemes. Thus, we have a dependency on the renormalization scheme used in the calculation of effective Wilson coefficients $c_i^{\rm eff}$. The hadronic matrix elements do no have terms to cancel this dependency in the physical amplitude. In this work, we use the next-to-leading order Wilson coefficients, obtained in the naive dimensional regularization scheme at the energy scale $\mu = m_b(m_b)$.

TABLE II. Effective coefficients $a_i$ for $b \to d$ and $b \to s$ transitions (in units of $10^{-4}$ for $a_3, \ldots, a_{10}$).

| $a_i$ | $b \to d$ | $b \to s$ |
|---|---|---|
| $a_1$ | 1.046 | 1.046 |
| $a_2$ | 0.024 | 0.024 |
| $a_3$ | 72 | 72 |
| $a_4$ | −386 − i108 | −388 − i118 |
| $a_5$ | −28 | −28 |
| $a_6$ | −438 − i108 | −441 − i118 |
| $a_7$ | −0.59 − i2.50 | −0.63 − i2.66 |
| $a_8$ | 3.38 − i0.83 | 3.36 − i0.89 |
| $a_9$ | −92.2 − i2.50 | −92.2 − i2.66 |
| $a_{10}$ | 0.44 − i0.83 | 0.43 − i0.89 |

In the naive generalized factorization approach, the parameter $1/N_C$ is considered as a phenomenological parameter, which includes nonfactorization effects and it is varied to model the nonfactorization contributions in matrix elements. Although $N^{\rm eff}$ is scale energy $\mu$ and renormalization scheme independent, it is a gauge and infrared regulator dependent quantity, see Ref. [14]. Thus, we consider $N_C$ the parameter defined and fixed to $N_C = 3$ in QCD.

The effective coefficients $c_i^{\rm eff}$ depend on an averaged momentum transfer $k^2$. In a specific model and from two-body decays, kinematics has been estimated to lie in the range $m_b^2/4 < k^2 < m_b^2/2$. It is found that the *CP* average branching ratios are not sensibly dependent on if $k^2$ is varied, see Refs. [5,6].

## III. INPUT PARAMETERS AND FORM FACTORS

### A. Input parameters

The CKM matrix is parametrized in terms of Wolfenstein parameters $\lambda$, $A$, $\bar\rho$ and $\bar\eta$ [15],

$$
\begin{pmatrix}
1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\bar\rho - i\bar\eta)\\
-\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2\\
A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2 & 1
\end{pmatrix},
\tag{6}
$$

with $\bar\rho = \rho(1-\lambda^2/2)$ and $\bar\eta = \eta(1-\lambda^2/2)$, including $\mathcal{O}(\lambda^5)$ corrections [16]. The Wolfenstein parameters are determined by unitarity constraint of three family of quarks and a global fit to experimental data. The central values $\lambda = 0.2253$, $A = 0.808$, $\bar\rho = 0.132$, and $\bar\eta = 0.341$ are used in calculations, see Ref. [13].

Running quark masses enter in loop calculation of effective Wilson coefficients. Furthermore, they are present in the equation of motion necessary to calculate the chiral factor, which multiplies the matrix elements of penguin terms $a_6$ and $a_8$ in the effective weak Hamiltonian. These contributions are only present in the processes involving pseudoscalar mesons $\pi$, $\eta$, $\eta'$, $K$ and $\pi'$ in final states.

Since the energy release in $B$ decay is of order $m_b$, the scale energy for evaluation of running quark masses should be $\mu \approx m_b$. The values $m_u(m_b) = 3.2$ MeV, $m_d(mb) = 6.4$ MeV, $m_s(m_b) = 127$ MeV, $m_c(m_b) = 0.95$ GeV and $m_b(m_b) = 4.34$ GeV are used in calculations, see Ref. [17].

The decay constants of pseudoscalar and vector mesons are determined using branching ratio of mesons and $\tau$ semileptonic decays, respectively. The central values, $f_\pi = 130$, $f_K = 160$, $f_\rho = 212$, $f_\omega = 195$, $f_{K^*} = 221$ and $f_\phi = 237$ MeV are extracted using experimental data, see Ref. [13].

In Ref. [18] is presented the argument that decay constants of radial excited pseudoscalar mesons are suppressed relative to the pion decay constant. Decay constant $f_{\pi'}$ computed in models has also been found to be small, see Ref. [19]. In unquenched lattice QCD [19], the ratio $f_{\pi'}/f_{\pi} = 0.078(93)$ is calculated. Using $f_{\pi} = 131$ MeV, the value of decay constant $f_{\pi'} = 16.3 \pm 6.1$ MeV is obtained. From the experimental bound on branching ratio $\mathrm{Br}(\tau \to \pi' \nu_{\tau})$ [20] is established a bound in the decay constant, $f_{\pi'} < 8.4$ MeV [3]. In Ref. [21], the authors obtain $f_{\pi'} = 26$ MeV and $f_{\rho'} = 128$ MeV, using a light cone quark model. Recently, in the large-$N_c$ limit and using QCD spectral sum rules in Ref. [22], the authors estimate the decay constant $f_{\rho'} = 182 \pm 5$ MeV.

In order to calculate branching ratios and $CP$ asymmetries for channels $B \to P\pi'$ and $B \to V\pi'$, two values of decay constant are used $f_{\pi'} = 0$ and 26 MeV, which are the minimum and maximum in the range of values obtained by different methods. For channels $B \to P\rho'$ and $B \to V\rho'$, the decay constant $f_{\rho'} = 128$ MeV is used to calculate branching ratios and $CP$ asymmetries.

The mixing of the $\eta - \eta'$, $\rho^0 - \omega$ and $\omega - \phi$ systems are considered by mixing in decay constants and form factors. Ideal mixing is considered for $\omega - \phi$, i.e., the mesons have quark content $\omega = 1/\sqrt{2}(u\bar{u} + d\bar{d})$ and $\phi = s\bar{s}$.

Two-mixing angle formalism is used to describe mixing in $\eta - \eta'$ system, see Refs. [23,24]. The physical states $\eta$ and $\eta'$ are defined in terms of flavor octet $\eta_8$ and singlet $\eta_0$. In Ref. [24], we find a complete fit of mixing parameters to experimental data, resulting for decay constants the following central values $f^u_{\eta} = 76.2$, $f^u_{\eta'} = 61.8$, $f^s_{\eta} = -110.5$ and $f^s_{\eta'} = 138$ MeV. Decay constants $f^c_{\eta} = -(2.4 \pm 0.2)$ and $f^c_{\eta'} = -(6.3 \pm 0.6)$ MeV are used to include the $\eta_c$ meson in the mixing scheme. When scalar and pseudoscalar densities in penguin terms $a_6$ and $a_8$ are evaluated, the correct chiral behavior must be ensured. Thus, these matrix elements are multiplied by the factor $r_{\eta^{(\prime)}}$. The numerical values $r_{\eta} = -0.689$ and $r_{\eta'} = 0.462$ are used, see Ref. [25].

For the meson $B$ lifetime, the values $\tau_{B^-} = (1.638 \pm 0.011) \times 10^{-6}$ s and $\tau_{\bar{B}^0} = (1.525 \pm 0.009) \times 10^{-6}$ s are used, and for $B$ mass $m_{B^-} = 5279.17 \pm 0.29$ and $m_{\bar{B}^0} = 5279.50 \pm 0.30$ MeV [13], which are required to calculate branching ratios.

### B. Form factors

The WSB model [7] and LCSR approach [8] are used to calculate form factors for $B \to P$ and $B \to V$ transitions. Since the WSB model and LCSR approach provide form factors only for the above transitions, form factors for $B \to \pi'$ and $B \to \rho'$ transitions are calculated using the ISGW2 quark model [10].

The transitions $B \to P$ and $B \to V$ can be written in terms of form factors by the following expressions

$$\langle P(p_P)|V_{\mu}|B(p_B)\rangle \equiv \left[(p_B + p_P)_{\mu} - \frac{m_B^2 - m_P^2}{q^2} q_{\mu}\right] F_1(q^2) + \left[\frac{m_B^2 - m_P^2}{q^2}\right] q_{\mu} F_0(q^2) \tag{7}$$

and

$$\langle V(p_V, \epsilon)|(V_{\mu} - A_{\mu})|B(p_B)\rangle \equiv -\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu *} p_B^{\alpha} p_V^{\beta} \frac{2V(q^2)}{(m_B + m_V)} - i\left[\left(\epsilon^*_{\mu} - \frac{\epsilon^* \cdot q}{q^2} q_{\mu}\right)(m_B + m_V) A_1(q^2) - \left((p_B + p_V)_{\mu} - \frac{(m_B^2 - m_V^2)}{q^2} q_{\mu}\right)(\epsilon^* \cdot q) \frac{A_2(q^2)}{(m_B + m_V)} + \frac{2m_V(\epsilon^* \cdot q)}{q^2} q_{\mu} A_0(q^2)\right], \tag{8}$$

where $q = (p_B - p_P)$ or $q = (p_B - p_V)$ and $\epsilon$ is the polarization of the vector meson $V$. The following restrictions are imposed over form factors in order to cancel poles at $q^2 = 0$

$$\begin{aligned} F_1(0) &= F_0(0), \\ 2m_V A_0(0) &= (m_B + m_V)A_1(0) - (m_B - m_V)A_2(0). \end{aligned} \tag{9}$$

In the case of the WSB model, a single pole dominance model is used for the $q^2$ momentum squared dependency

$$f(q^2) = \frac{f(0)}{(1 - q^2/m_*^2)}, \tag{10}$$

where $m_*^2$ is the pole mass given by the vector meson, and $f(0)$ is the form factor at zero momentum transfer.

The WSB model is a relativistic constituent quark model where the meson-meson matrix elements are evaluated from the average integral corresponding to meson functions, which are solutions of a relativistic harmonic oscillator potential [7]. The wave function depends on the parameter $\omega^2 = \langle \vec{p}_T^{\,2} \rangle$, which represents the average transverse quark momentum. The estimate of the form factors are sensible to the $\omega$ value used. The central values for form factors reported in Ref. [7] are calculated with the value $\omega = 0.40$. Additionally, we use the values $\omega = 0.35$ and $\omega = 0.50$ to estimate form factors in the WSB model and obtain the uncertainties associated to the form factors due to reasonable variations of the parameter $\omega$.

The LCSR approach uses the method of QCD sum rules on the light cone [8]. The second set of parameters are used in the calculations, see Ref. [8]. A fit parametrization is

TABLE III. Form factors at zero momentum transfer for $B \to P$ and $B \to V$ transitions, evaluated in the WSB quark model [7] and LCSR approach [8].

| Transition | $F_1 = F_0$ | $V$ | $A_1$ | $A_2$ | $A_0$ |
|---|---|---|---|---|---|
| $B \to \pi$ | $0.333 \pm 0.027$ | | | | |
| | $[0.258 \pm 0.031]$ | | | | |
| $B \to K$ | $0.379 \pm 0.020$ | | | | |
| | $[0.331 \pm 0.041]$ | | | | |
| $B \to \eta$ | $0.307 \pm 0.034$ | | | | |
| | $[0.275 \pm 0.036]$ | | | | |
| $B \to \eta'$ | $0.254 \pm 0.046$ | | | | |
| | $[-]$ | | | | |
| $B \to \rho$ | | $0.329 \pm 0.052$ | $0.283 \pm 0.045$ | $0.283 \pm 0.046$ | $0.281 \pm 0.041$ |
| | | $[0.323 \pm 0.029]$ | $[0.242 \pm 0.024]$ | $[0.221 \pm 0.023]$ | $[0.303 \pm 0.028]$ |
| $B \to \omega$ | | $0.328 \pm 0.052$ | $0.281 \pm 0.044$ | $0.281 \pm 0.045$ | $0.280 \pm 0.041$ |
| | | $[0.293 \pm 0.029]$ | $[0.219 \pm 0.025]$ | $[0.198 \pm 0.022]$ | $[0.281 \pm 0.030]$ |
| $B \to K^*$ | | $0.369 \pm 0.046$ | $0.328 \pm 0.040$ | $0.331 \pm 0.042$ | $0.321 \pm 0.036$ |
| | | $[0.411 \pm 0.033]$ | $[0.292 \pm 0.028]$ | $[0.259 \pm 0.027]$ | $[0.374 \pm 0.034]$ |

utilized for the $q^2$ dependency of the form factors. The authors in Ref. [8] make variations of parameters and determine the dependency for the form factors from the input parameters. These estimates can be considered the uncertainties associated with the prediction of the LCSR approach for the form factors.

In Table III, values for form factors involved in transitions $B \to P$ and $B \to V$ are shown, evaluated at zero momentum transfer, in the WSB quark model [7] and LCSR approach [8], with the corresponding errors from input parameter dependency. Form factor in $B \to \pi$ transition, evaluated in the LCSR approach, is small at one sigma error compared to the WSB model. This result is in accordance with current experimental data [13]. The pseudoscalar meson $\eta'$ is too heavy to be treated in the LCSR approach, its value is not reported. In general, the form factors calculated in LCSR approach are smaller than those calculated in the WSB model, for this reason the branching ratios are also smaller.

The improved version of the ISGW model [9], the so-called ISGW2 model [10], a nonrelativistic quark model is used in this work. Although, in the ISGW model is possible to calculate transitions to a radially excited pseudoscalar and vector mesons, the ISGW2 model is better because the constrains imposed by heavy quark symmetry, hyperfine distortions of wave functions, and form factor more realistic at high recoil momentum transfer. These additional features incorporated in the ISGW2 model allow us to make more reliable estimations.

In the ISGW2 model, $B \to P'$ transition is written as

$$\langle P(p_P)|V_\mu|B(p_B)\rangle \equiv f'_+(q^2)(p_B + p_P)_\mu + f'_-(q^2)(p_B - p_P)_\mu, \quad (11)$$

and matrix elements of vector and axial vector currents for $B \to V'$ transition are written as

$$\langle V(p_V, \epsilon)|V_\mu|B(p_B)\rangle \equiv i g'(q^2)\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}(p_B + p_V)^\alpha \times (p_B - p_V)^\beta \quad (12)$$

and

$$\langle V(p_V, \epsilon)|A_\mu|B(p_B)\rangle \equiv f'(q^2)\epsilon^*_\mu + a'_+(q^2)(\epsilon^* \cdot p_B) \times (p_B + p_V)_\mu + a'_-(q^2) \times (\epsilon^* \cdot p_B)(p_B - p_V)_\mu, \quad (13)$$

where $q = (p_B - p_P)$ or $(p_B - p_V)$ in the respective case.

The transitions $B \to \pi'$ and $B \to \rho'$ can be considered at the quark level by $b \to q_1$, where the pseudoscalar $\pi'$ and the vector meson $\rho'$ have quark content $q_1\bar{q}_2$, and $q_2$ is the spectator quark. In the following we write down the basic expressions for form factors in $B \to \pi'$ and $B \to \rho'$ transitions in the ISGW2 model. In the formulas $X$ are $\pi'$ or $\rho'$, respectively.

The form factors $f'_+$ and $f'_-$, which parametrize the transition $B \to \pi'$, are determined in ISGW2 model by the expressions

$$\begin{aligned} f'_+ + f'_- &= \sqrt{\frac{3}{2}}\left[\left(1 - \frac{m_2}{m_1}\right)U - \frac{m_2}{m_1}V\right]F_3^{(f'_+ + f'_-)} \\ f'_+ - f'_- &= \sqrt{\frac{3}{2}\frac{\tilde{m}_B}{m_1}}\left[U + \frac{m_2}{\tilde{m}_X}V\right]F_3^{(f'_+ - f'_-)}, \end{aligned} \quad (14)$$

where

$$\begin{aligned} U &= \frac{\beta_B^2 - \beta_X^2}{2\beta_{BX}^2} + \frac{\beta_B^2\tau}{3\beta_{BX}^2}, \\ V &= \frac{\beta_B^2}{6\beta_{BX}^2}\left(1 + \frac{m_1}{m_b}\right)\left[7 - \frac{\beta_B^2}{\beta_{BX}^2}(5 + \tau)\right] \\ \tau &\equiv \frac{m_1^2\beta_X^2(\tilde{\omega} - 1)}{\beta_B^2\beta_{BX}^2}, \qquad \tilde{\omega} = \frac{t_m - t}{2\tilde{m}_B\tilde{m}_X} + 1. \end{aligned} \quad (15)$$

The parameters $\beta_B = 0.43$ and $\beta_X = 0.41$ are obtained from the model, where $\beta_{BX} = 1/2(\beta_B^2 + \beta_X^2)$. The exponents $n_B$ and $n_X$ in the term $F_3^{(i)}$ are $(-1/2, +1/2)$ and $(+1/2, -1/2)$ in the cases $(f'_+ + f'_-)$ and $(f'_+ - f'_-)$, respectively. The term $F_3^{(i)}$ is given by

$$F_3^{(i)} = \left(\frac{\bar{m}_B}{\tilde{m}_B}\right)^{n_B}\left(\frac{\bar{m}_X}{\tilde{m}_X}\right)^{n_X}\left(\frac{\tilde{m}_X}{\tilde{m}_B}\right)^{1/2}\left(\frac{\beta_X\beta_B}{\beta_{BX}^2}\right)^{3/2} \times\left[1 + \frac{1}{12}r_{XB}^2(t_m - t)\right]^{-2}, \tag{16}$$

where

$$r_{XB}^2 = \frac{3}{4m_b m_1} + \frac{3m_2^2}{2\bar{m}_B\bar{m}_X\beta_{BX}^2} + \frac{1}{\bar{m}_B\bar{m}_X}\left(\frac{16}{33 - 2n_f}\right) \times \ln\left[\frac{\alpha_s(\mu_{\mathrm{QCD}})}{\alpha_s(m_1)}\right]. \tag{17}$$

The parameters $m_1$ and $m_2$ are the masses of the quarks $q_1$ and $q_2$, $\bar{m}_1$ is the hyperfine averaged mass, $\tilde{m}$ is the sum of the masses of constituent quarks, $t_m = (m_B - m_X)^2$ is the maximum momentum transferred, $n_f$ is the number of active flavors at the $b$ scale and $\alpha_s(\mu)$ is the QCD coupling at the $\mu$ scale.

The transition $B \to \rho'$ is parametrized by the form factors $f'$, $g'$, $a'_+$ and $a'_-$ and determined in the ISGW2 model by the equations

$$f' = C_{f'}\sqrt{\frac{3}{2}}\tilde{m}_B(1 + \tilde{\omega})UF_3^{(f')}$$
$$g' = \frac{1}{2}\sqrt{\frac{3}{2}}\left[\left(\frac{1}{m_1} - \frac{m_2\beta_B^2}{2\mu_-\tilde{m}_X\beta_{BX}^2}\right)U + \frac{m_2\beta_B^2\beta_X^2}{3\mu_-\tilde{m}_X\beta_{BX}^4}\right]F_3^{(g')} \tag{18}$$

and

$$\begin{aligned} a'_+ + a'_- = {}& -\sqrt{\frac{2}{3}}\frac{\beta_B^2}{m_1 m_b\beta_{BX}^2}\left\{\frac{7m_2^2\beta_X^4}{8\tilde{m}_B\beta_{BX}^4}\left[1 + \frac{1}{7}\tau\right] - \frac{5m_2^2\beta_X^2}{4\beta_{BX}^4}\left[1 + \frac{1}{5}\tau\right] - \frac{3m_2^2\beta_X^4}{8\tilde{m}_B\beta_B^2\beta_{BX}^2} + \frac{3m_2\beta_X^2}{4\beta_B^2}\right\}F_3^{(a'_+ + a'_-)} \\ a'_+ - a'_- = {}& \sqrt{\frac{2}{3}}\frac{3\tilde{m}_B}{2m_b\tilde{m}_X}\left\{1 - \frac{\beta_B^2}{\beta_{BX}^2}\left[1 + \frac{1}{7}\tau\right] - \frac{m_2\beta_X^2}{2\tilde{m}_B\beta_{BX}^2}\left(1 - \frac{5\beta_B^2}{3\beta_{BX}^2}\left[1 + \frac{1}{5}\tau\right]\right) - \frac{7m_2^2\beta_B^2\beta_X^2}{12m_1\tilde{m}_B\beta_{BX}^4}\left(1 - \frac{\beta_X^2}{\beta_{BX}^2} + \frac{\beta_B^2\tau}{7\beta_{BX}^2}\right)\right\}F_3^{(a'_+ - a'_-)}, \end{aligned} \tag{19}$$

where

$$\mu_\pm = \left(\frac{1}{m_1} \pm \frac{1}{m_b}\right), \tag{20}$$

and $C_{f'}$ is a relativistic correction to the form factor $f'$. The exponents $n_B$ and $n_X$ in the factors $F_3^{(i)}$ for $i = f'$, $g'$, $(a'_+ - a'_-)$ and $(a'_+ - a'_-)$ are $(+1/2, +1/2)$, $(-1/2, 11/2)$, $(-3/2, +1/2)$ and $(-1/2, -1/2)$, respectively.

The relevant parameters which determine the numerical value of the above form factors, including masses of quarks and mesons involved in the exclusive channel, are the variational parameters $\beta_B$, $\beta_X$ and the $C_{f'}$ relativistic correction to the form factor $f'$. In the ISGW2 model, when we vary the parameters $\beta_B = 0.43 \pm 0.01$, $\beta_X = 0.41 \pm 0.01$ and $C_F = 0.776 \pm 0.019$ by 2.5%, we get variations between 10 and 20% in predictions of the form factors. These variations are considered the uncertainties of the model to form factors predictions.

Form factors in ISGW2 model are related to form factors in the WSB model by the following relations

$$\begin{aligned} &F_1(q^2) = f'_+(q^2), \qquad V(q^2) = (m_B + m_V)g'(q^2), \\ &A_1(q^2) = (m_B + m_V)^{-1}f'(q^2), \\ &A_2(q^2) = -(m_B + m_V)a'_+(q^2), \\ &A_0(q^2) = \frac{1}{2m_V}[f'(q^2) + (m_B^2 - m_V^2)a'_+(q^2), + q^2a'_-(q^2)]. \end{aligned} \tag{21}$$

The form factors for $B \to \pi'$ and $B \to \rho'$ transitions, calculated at momentum transfer $q^2 = m_\pi^2$, are presented in Table IV, including estimates of uncertainties. To estimate branching ratios, it is necessary to calculate form factors at different momentum transfers, namely at $q^2 = m_K^2, m_\eta^2, m_{\eta'}^2, m_\omega^2, m_{K^*}^2$ and $m_\phi^2$.

Mixing $\eta - \eta'$ effect is not included in the WSB model prediction. $SU(3)$ symmetry is used to consider mixing in $B \to \eta$ and $B \to \eta'$ transitions, which imply the relations $F^{B\pi}(0) = \sqrt{3}F^{B\eta_0}(0) = \sqrt{6}F^{B\eta_8}(0)$ and

$$\begin{aligned} F^{B\eta} &= F^{B\eta_8}\cos\theta - F^{B\eta_0}\sin\theta, \\ F^{B\eta'} &= F^{B\eta_8}\sin\theta + F^{B\eta_0}\cos\theta, \end{aligned} \tag{22}$$

where $\theta = -15.4°$ is the mixing angle [24]. Using $F^{B\pi}(0) = 0.333$ from the WSB model, form factors $F^{B\eta}(0) = 0.181$ and $F^{B\eta'}(0) = 0.148$ are obtained. In the LCSR approach, using the form factor $F^{B\pi}(t)$ and $SU(3)$ symmetry, the form factors $F^{B\eta}(t)$ and $F^{B\eta'}(t)$ are estimated.

The $\rho^0 - \omega$ mixing is introduced in hadronic matrix element $B \to \rho^0$. Nevertheless, the effect in $B \to \omega$ transitions is negligible and it is not included in branching ratios calculations. In the limit of isospin symmetry, physical states $\rho^0$ and $\omega$ are expressed in terms of isospin eigenstates $\rho^I$ and $\omega^I$ by a rotation matrix

TABLE IV. Form factors at momentum transfer $q^2 = m_\pi^2$ for $B \to \pi'$ and $B \to \rho'$ transitions, evaluated in the ISGW2 quark model [10].

| Transition | $F_1 = F_0$ | $V$ | $A_1$ | $A_2$ | $A_0$ |
|---|---|---|---|---|---|
| $B \to \pi'$ | $0.25 \pm 0.04$ | | | | |
| $B \to \rho'$ | | $0.455 \pm 0.030$ | $0.118 \pm 0.019$ | $-0.117 \pm 0.024$ | $0.429 \pm 0.075$ |

$$|\rho^0\rangle = |\rho^I\rangle + \epsilon|\omega^I\rangle \qquad |\omega\rangle = |\omega^I\rangle - \epsilon'|\rho^I\rangle, \tag{23}$$

where numerical values for mixing parameters are $(1+\epsilon) = (0.092 + 0.016i)$ and $(1-\epsilon') = (1.011 + 0.030i)$. Including isospin effects, hadronic matrix element for $B \to \rho^0$ transition is modified by the factor $(1+\epsilon)$, see Ref. [26].

## IV. AMPLITUDES AND BRANCHING RATIOS

The amplitudes for processes studied in this work are explicitly written in the Appendixes. These amplitudes are given in terms of decay constants and form factors and contain all the contributions of the effective weak Hamiltonian. Basically, the formulas for the decay amplitudes are taken from the appendixes in Refs. [5,6], with the appropriate changes for the processes studies in this article, i.e., masses, decay constants and form factors, without annihilation and exchange contributions.

Appendix A contains the amplitudes for $B \to P\pi'$ decays, where $P$ is the pseudoscalar meson $\pi$, $\eta$, $\eta'$ or $K$. The amplitude for the process $\bar{B}^0 \to \pi^-\pi'^-$ is not written, but it can be obtained directly from $\bar{B}^0 \to \pi^+\pi'^+$, interchanging $\pi$ by $\pi'$. Similarly, the amplitude for $B^- \to \pi^-\pi'^0$ can be obtained from $B^- \to \pi^0\pi'^-$.

To compare $B \to P\pi'$ with $B \to P\pi$ modes, besides obvious differences in decay constants and form factors, a point to remark is the following one. In the penguin sector the more important contributions come from terms $a_4$ and $a_6$. Particularly, the $a_6$ and $a_8$ coefficients are enhanced by a chiral factor, which is proportional to the squared mass of pseudoscalar $P$ or pseudoscalar radial excitation $\pi'$. In the case of radial excitation $\pi'$, this contribution can be two orders of magnitude bigger than contribution of the pseudoscalar meson $\pi$. This enhancement effect is shown in the branching ratios of channels $B \to \pi\pi'$, $B \to \eta\pi'$ and $B \to \eta'\pi'$.

In Appendix B, the amplitudes for $B \to V\pi'$ processes are shown, where $V$ is the vector meson $\rho$, $\omega$, $K^*$ or $\phi$. In these modes, the increased factor in the $a_6$ and $a_8$ penguin contributions occur like in $B \to P\pi'$ modes. This effect appears in the branching ratios of modes $B \to \rho\pi'$ and $B \to \omega\pi'$, with the exception of channel $\bar{B}^0 \to \rho^-\pi'^+$.

The amplitudes for $B \to P\rho'$ processes are given in Appendix C. The processes $B \to \pi\rho'$ are not written, but they can be obtained from amplitudes $B \to \rho\pi'$ in Appendix B, interchanging $\pi'$ by $\pi$ and $\rho$ by $\rho'$.

In Appendix D, the decay amplitudes for $B \to V\rho'$ processes are given by the factorized term

$$\begin{aligned} X^{B\rho',V} &= \langle V|(\bar{q}_3 q_2)_{V-A}|0\rangle\langle\rho'|(\bar{q}_1 b)_{V-A}|B\rangle \\ &= -if_V m_V\Big[(\epsilon_V^* \cdot \epsilon_{\rho'}^*)(m_B + m_{\rho'})A_1^{B\rho'}(m_V^2) \\ &\quad - (\epsilon_V^* \cdot p_B)(\epsilon_{\rho'}^* \cdot p_B)\frac{2A_2^{B\rho'}(m_V^2)}{(m_B + m_{\rho'})} \\ &\quad + i\epsilon_{\mu\nu\alpha\beta}\epsilon_V^\mu \epsilon_{\rho'}^\nu p_B^\alpha p_{\rho'}^\beta \frac{2V^{B\rho'}(m_V^2)}{(m_B + m_{\rho'})}\Big], \end{aligned} \tag{24}$$

where $\epsilon_V$ and $\epsilon_{\rho'}$ are the polarization vectors of the vectors mesons $V$ and $\rho'$, respectively. This notation is introduced to simplify expressions in Appendix D.

The amplitudes for processes which contain the meson $\pi^0$, $\pi'^0$, $\rho^0$ or $\rho'^0$ in the final state, are multiplied by the factor $1/\sqrt{2}$ due to the wave function of these neutral mesons, i.e., $1/\sqrt{2}(\bar{u}u - \bar{d}d)$.

From decay amplitude and input parameters, branching ratios are straightforward calculated. The decay rate for processes $B \to P\pi'$ is given by

$$\Gamma(B \to P\pi') = \frac{\lambda^{1/2}(m_B^2, m_P^2, m_{\pi'}^2)}{16\pi m_B^3}|\mathcal{M}(B \to P\pi')|^2, \tag{25}$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$. The decay rate for processes $B \to V\pi'$ and $B \to P\rho'$ are calculated using Eq. (25). In this case, the squared amplitude is proportional to $|\epsilon_V \cdot p_{\pi'}|^2$ and $|\epsilon_{\rho'} \cdot p_P|^2$, respectively. In processes $B \to V\rho'$, the squared amplitude is involved due to interfering terms proportional to $X^{B\rho',V}$ and $X^{BV,\rho'}$ contributions.

The branching ratios listed in Tables V and VI are $CP$-averaged conjugate modes. Charged and neutral channels are calculated by

$$\frac{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)}{2}, \qquad \frac{\Gamma(B^0 \to f) + \Gamma(\bar{B}^0 \to \bar{f})}{2}, \tag{26}$$

where $f$ is the two-body meson final state, i.e., $f = P\pi'$, $V\pi'$, $P\rho'$ or $V\rho'$.

Form factors in transitions $B \to P$ and $B \to V$ are calculated using two representative methods, the WSB quark model [7] and LCSR approach [8]. The branching ratios calculated in LCSR approach are listed between squared brackets in Tables V and VI. In transitions $B \to \pi'$ and $B \to \rho'$, the improved version of the ISGW nonrelativistic quark model [10] is used.

TABLE V. Branching ratios (in units of $10^{-6}$) averaged over $CP$ conjugate modes for $B \to P\pi'$ and $B \to V\pi'$ decays, using the WSB [7] model and LCSR approach [8], decay constants $f_{\pi'} = 26$ and $f_{\pi'} = 0.0$ MeV. (−) means same value.

| Mode | $\mathcal{B}$ |
|---|---|
| $\bar{B}^0 \to \pi^+\pi'^-$ | 5.4 ± 1.1[5.5 ± 1.1]5.5 ± 1.1 |
| $\bar{B}^0 \to \pi^-\pi'^+$ | 38.3 ± 6.2[24.6 ± 5.6]0 |
| $\bar{B}^0 \to \pi^0\pi'^0$ | 6.2 ± 1.0[4.2 ± 1.0]0.03 ± 0.01 |
| $B^- \to \pi^-\pi'^0$ | 2.8 ± 0.5[2.8 ± 0.5]2.9 ± 0.6 |
| $B^- \to \pi^0\pi'^-$ | 0.22 ± 0.04[0.14 ± 0.04]0.004 |
| $\bar{B}^0 \to \eta\pi'^0$ | 4.0 ± 0.7[2.6 ± 0.7]0.04 ± 0.01 |
| $B^- \to \eta\pi'^-$ | 14.1 ± 2.3[9.0 ± 2.3]0.05 ± 0.01 |
| $\bar{B}^0 \to \eta'\pi'^0$ | 2.2 ± 0.3[1.4 ± 0.4]0.007 ± 0.001 |
| $B^- \to \eta'\pi'^-$ | 8.1 ± 1.3[5.0 ± 1.3]0.01 ± 0.001 |
| $\bar{B}^0 \to K^-\pi'^+$ | 6.0 ± 1.1[−]− |
| $\bar{B}^0 \to K^0\pi'^0$ | 3.7 ± 0.7[3.7 ± 0.7]3.4 ± 0.7 |
| $B^- \to K^-\pi'^0$ | 3.6 ± 0.7[3.5 ± 0.7]3.2 ± 0.6 |
| $B^- \to \bar{K}^0\pi'^-$ | 7.3 ± 1.4[7.4 ± 1.4]7.3 ± 1.4 |
| $\bar{B}^0 \to \rho^-\pi'^+$ | 13.7 ± 2.6[−]− |
| $\bar{B}^0 \to \rho^+\pi'^-$ | 21.7 ± 6.2[27.8 ± 5.1]0 |
| $\bar{B}^0 \to \rho^0\pi'^0$ | 3.3 ± 1.1[4.3 ± 0.9]0.02 ± 0.01 |
| $B^- \to \rho^0\pi'^-$ | 15.3 ± 4.3[19.4 ± 3.6]0.04 ± 0.01 |
| $B^- \to \rho^-\pi'^0$ | 13.1 ± 3.1[14.9 ± 2.8]7.4 ± 1.4 |
| $\bar{B}^0 \to \omega\pi'^0$ | 6.7 ± 1.9[7.3 ± 1.6]0.05 ± 0.005 |
| $B^- \to \omega\pi'^-$ | 9.9 ± 3.1[10.7 ± 2.6]0.1 ± 0.05 |
| $\bar{B}^0 \to K^{*-}\pi'^+$ | 2.8 ± 0.5[−]− |
| $\bar{B}^0 \to K^{*0}\pi'^0$ | 0.8 ± 0.2[0.7 ± 0.1]0.8 ± 0.2 |
| $B^- \to K^{*-}\pi'^0$ | 1.7 ± 0.4[1.7 ± 0.3]1.5 ± 0.3 |
| $B^- \to \bar{K}^{*0}\pi'^-$ | 3.4 ± 0.7[−]− |
| $\bar{B}^0 \to \phi\pi'^0$ | 0.004[−]− |
| $B^- \to \phi\pi'^-$ | 0.009 ± 0.002[−]− |

In Table V, branching ratios for processes $B \to P\pi'$ and $B \to V\pi'$ are shown, where $P$ is the pseudoscalar meson $\pi$, $\eta$, $\eta'$ or $K$ and $V$ is the vector meson $\rho$, $\omega$, $K^*$ or $\phi$. Branching ratios are calculated using two different values in decay constant $f_{\pi'} = 26$ MeV (first and second column) and $f_{\pi'} = 0$ MeV (third column).

In Table V, channels with $K$, $K^*$ or $\phi$ in the final state, have equal branching ratio calculated in the WSB model or in the LCSR approach. The branching ratio in processes $\bar{B} \to \pi^+\pi'^-$ and $\bar{B} \to \rho^-\pi'^+$ have independent value of the decay constant $f_{\pi'}$, since decay amplitude has only one contribution proportional to $f_\pi$ or $f_\rho$, respectively.

Using the value $f_{\pi'} = 26$ MeV in calculations, channels $\bar{B}^0 \to \pi^-\pi'^+$, $B^- \to \eta\pi'^-$, $\bar{B}^0 \to \rho^-\pi'^+$, $\bar{B}^0 \to \rho^+\pi'^-$, $B^- \to \rho^0\pi'^-$ and $B^- \to \rho^-\pi'^0$ have branching ratios of the order of $10^{-5}$. The channels with branching ratios of order below $10^{-6}$ are $B^- \to \pi^0\pi'^-$, $\bar{B}^0 \to K^{*0}\pi'^0$, $\bar{B}^0 \to \phi\pi'^0$ and $B^- \to \phi\pi'^-$.

Numerical values for branching ratios of processes $B \to P\rho'$ and $B \to V\rho'$ are listed in Table VI. The branching ratios are calculated using the decay constant $f_{\rho'} = 128$ MeV. Branching ratio prediction for the decays $\bar{B}^0 \to \pi^-\rho'^+$, $\bar{B}^0 \to \rho^-\rho'^+$ $\bar{B}^0 \to \rho^+\rho'^-$ and $B^- \to \rho^-\rho'^0$ are of order $10^{-5}$. The branching ratio of the channels $\bar{B}^0 \to \pi^0\rho'^0$, $\bar{B}^0 \to \eta^{(\prime)}\rho'^0$, $\bar{B}^0 \to K^0\rho'^0$, $B^- \to K^-\rho'^0$, $B^- \to \bar{K}^0\rho'^-$, $\bar{B}^0 \to \rho^0\rho'^0$, $\bar{B}^0 \to \omega\rho'^0$, $\bar{B}^0 \to \phi\rho'^0$ and $B^- \to \phi\rho'^-$, are suppressed of order below $10^{-6}$. The modes $\bar{B}^0 \to \pi^+\rho'^-$, $\bar{B}^0 \to \pi^0\rho'^0$, $B^- \to \pi^-\rho'^0$, $B^- \to \eta\rho'^-$, $\bar{B}^0 \to \rho^+\rho'^-$, $\bar{B}^0 \to \omega\rho'^0$ and $B^- \to \omega\rho'^-$ have different branching ratios when form factors are calculated using the WSB model or the LCSR approach.

The authors in Ref. [4] can calculate the ratios

$$R_{\rho^+} = \frac{\mathrm{Br}(\bar{B}^0 \to \pi^-\rho'^+)}{\mathrm{Br}(\bar{B}^0 \to \pi^-\rho^+)}, \quad R_{\rho^0} = \frac{\mathrm{Br}(B^- \to \pi^-\rho'^0)}{\mathrm{Br}(B^- \to \pi^-\rho^0)} \tag{27}$$

and obtain approximately $R_{\rho^+} \approx R_{\rho^0} \approx 2$. Using the world-averaged experimental data from Refs. [13,27], $\mathrm{Br}(\bar{B}^0 \to \pi^\mp\rho^\pm) = 23.0 \pm 2.3 \times 10^{-6}$ and $\mathrm{Br}(B^- \to \pi^-\rho^0) = 8.3 \pm 1.2 \times 10^{-6}$, it is possible to predict the branching ratios $\mathrm{Br}(\bar{B}^0 \to \pi^-\rho'^+) = 46.0 \times 10^{-6}$ and $\mathrm{Br}(B^- \to \pi^-\rho'^0) = 16.6 \times 10^{-6}$. These numbers can be compared with our $CP$-averaged branching ratios calculated in Table VI, $\mathrm{Br}(\bar{B}^0 \to \pi^-\rho'^+) = 14.6 \pm 4.9[14.6 \pm 4.9] \times 10^{-6}$ and

TABLE VI. Branching ratios (in units of $10^{-6}$) averaged over $CP$ conjugate modes for $B \to P\rho'$ and $B \to V\rho'$ decays, using the WSB [7] model and LCSR approach [8], and decay constant $f_{\rho'} = 128$ MeV. (−) means same value.

| Mode | $\mathcal{B}$ | Mode | $\mathcal{B}$ |
|---|---|---|---|
| $\bar{B}^0 \to \pi^+\rho'^-$ | 9.8 ± 1.6[6.5 ± 1.6] | $\bar{B}^0 \to \rho^-\rho'^+$ | 37.8 ± 11.9[−] |
| $\bar{B}^0 \to \pi^-\rho'^+$ | 14.6 ± 4.9[−] | $\bar{B}^0 \to \rho^+\rho'^-$ | 28.3 ± 8.6[21.2 ± 4.1] |
| $\bar{B}^0 \to \pi^0\rho'^0$ | 0.02 ± 0.01[0.009 ± 0.001] | $\bar{B}^0 \to \rho^0\rho'^0$ | 0.14 ± 0.05[0.13 ± 0.04] |
| $B^- \to \pi^0\rho'^-$ | 8.3 ± 2.8[−] | $B^- \to \rho^0\rho'^-$ | 7.5 ± 2.2[6.0 ± 1.2] |
| $B^- \to \pi^-\rho'^0$ | 5.8 ± 0.9[3.8 ± 1.0] | $B^- \to \rho^-\rho'^0$ | 20.7 ± 6.5[−] |
| $\bar{B}^0 \to \eta\rho'^0$ | 0.008 ± 0.002[0.006 ± 0.002] | $\bar{B}^0 \to \omega\rho'^0$ | 0.15 ± 0.05[0.04 ± 0.02] |
| $B^- \to \eta\rho'^-$ | 3.3 ± 0.6[2.0 ± 0.5] | $B^- \to \omega\rho'^-$ | 10.2 ± 8.0[5.7 ± 1.4] |
| $\bar{B}^0 \to \eta'\rho'^0$ | 0.04 ± 0.01[−] | $\bar{B}^0 \to K^{*-}\rho'^+$ | 7.7 ± 2.5[−] |
| $B^- \to \eta'\rho'^-$ | 2.0 ± 0.4[1.2 ± 0.3] | $\bar{B}^0 \to K^{*0}\rho'^0$ | 6.7 ± 2.0[−] |
| $\bar{B}^0 \to K^-\rho'^+$ | 1.1 ± 0.4[−] | $B^- \to K^{*-}\rho'^0$ | 6.3 ± 1.9[6.3 ± 1.8] |
| $\bar{B}^0 \to K^0\rho'^0$ | 0.13 ± 0.02[0.11 ± 0.3] | $B^- \to \bar{K}^{*0}\rho'^-$ | 9.1 ± 2.9[−] |
| $B^- \to K^-\rho'^0$ | 0.6 ± 0.2[−] | $\bar{B}^0 \to \phi\rho'^0$ | 0.009 ± 0.002[−] |
| $B^- \to \bar{K}^0\rho'^-$ | 0.01 ± 0.007[−] | $B^- \to \phi\rho'^-$ | 0.03 ± 0.01[−] |

$\mathrm{Br}(B^- \to \pi^- \rho'^0) = 5.8 \pm 0.9[3.8 \pm 1.0] \times 10^{-6}$, which could be indicating an overestimate of the ratios in Eq. (27), by a factor of 2.

Up to now, the only measured branching ratios reported are $\mathrm{Br}(B^+ \to \pi^+ \rho(1450)^0) = 1.4^{+0.6}_{-0.9} \times 10^{-6}$ [28], and upper limit at 90% confidence level for the decays $\mathrm{Br}(B^+ \to \rho(1450)^0 K^+) < 11.7 \times 10^{-6}$ [29] and $\mathrm{Br}(B^0 \to \rho(1450)^- K^+) < 2.1 \times 10^{-6}$ [30], see Ref. [27].

Our estimate of this $CP$-averaged branching ratios are $\mathrm{Br}(B^- \to \pi^- \rho'^0) = 5.8 \pm 0.9[3.8 \pm 1.0] \times 10^{-6}$, and $\mathrm{Br}(B^+ \to K^+ \rho'^0) = 0.6 \pm 0.2[0.6 \pm 0.2] \times 10^{-6}$, $\mathrm{Br}(B^0 \to K^+ \rho'^- = 1.1 \pm 0.4[1.1 \pm 0.4] \times 10^{-6})$, see Table VI. These results could be indicating that the decay constant $f_{\rho'}$ and the form factors in the transitions $B \to \rho'$ are overestimated.

## V. $CP$ ASYMMETRIES

Direct $CP$-violation asymmetry in charged $B^{\pm}$ decays is defined by [1,2]

$$A_{CP} = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)}. \tag{28}$$

For $b \to q$ transitions (where $q = d$, $s$), the decay amplitudes can be written generically by

$$\mathcal{M} = V_{ub} V_{uq}^* T - V_{tb} V_{tq}^* P, \tag{29}$$

where $T$ is current-current contributions, $P$ is penguin QCD and electroweak contributions, see the decay amplitudes in the Appendixes.

In the standard model, CKM matrix elements contain weak phases due to the weak dynamics. In the generalized naive factorization approach, the effective Wilson coefficients $c_i^{\mathrm{eff}}(\mu)$ are complex numbers, which contain a strong phase due to QCD interactions [11]. Thus, phase contributions from the terms $T$ and $P$ in the decay amplitude can be factorized in terms of weak and strong phases. This results in the contributions $T'$ and $P'$. The decay amplitude $\mathcal{M}$ and its $CP$-conjugate amplitude $\bar{\mathcal{M}}$ can be written as

$$\begin{aligned} \mathcal{M} &= e^{i\phi_1} e^{i\delta_1} T' + e^{i\phi_2} e^{i\delta_2} P', \\ \bar{\mathcal{M}} &= e^{-i\phi_1} e^{i\delta_1} T' + e^{-i\phi_2} e^{i\delta_2} P', \end{aligned} \tag{30}$$

where $\phi_i$ weak-decay phases change sign in a $CP$ conjugate transformation and $\delta_i$ strong phases are conserved. Using this factorization of phases, direct $CP$-violation asymmetry can be calculated by

$$A_{CP} = \frac{|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\bar{\mathcal{M}}|^2} = \frac{2\sin(\Delta\phi)\sin(\Delta\delta) r}{1 + r^2 + 2r\cos(\Delta\phi)\cos(\Delta\delta)}, \tag{31}$$

where $r$ is the ratio $P'/T'$, $\Delta\phi = \phi_1 - \phi_2$ and $\Delta\delta = \delta_1 - \delta_2$ are the difference in the weak- and the strong-phase contributions to the terms $T'$ and $P'$. Three conditions must be fulfilled for the existence of direct $CP$ violation. A $CP$-violation weak phase $\Delta\phi$, final state interactions which induce a strong phase $\Delta\delta$, and two different contributions to the amplitude of comparable size $T' \approx P'$, see Refs. [1,2].

In the framework of generalized naive factorization, the effective Wilson coefficients $c_i^{\mathrm{eff}}$ are complex numbers. The imaginary part of these coefficients are due to calculable QCD perturbative contributions [11]. This effect induces a strong phase in the amplitudes, which is required to have direct $CP$ violation.

Direct $CP$-violating asymmetries have been calculated using the form factors based on both the WSB [7] model and LCSR approach [8]. The $CP$ asymmetries depend weakly on the form factors. However, it is not the case for the weak decay constant $f_{\pi'}$. In the modes $B \to P\pi'$ and $B \to V\pi'$, dependency in the decay constant $f_{\pi'}$ will be discussed.

For the charged modes $B^{\pm} \to P\pi'$, $B^{\pm} \to V\pi'$, direct $CP$-violating asymmetries are listed in Table VII. Table VII shows the dependency of direct $CP$ asymmetries in the decay constant $f_{\pi'}$. Results using $f_{\pi'} = 26$ MeV are shown in the first and second columns, where the calculations are done in the WSB model and LCSR approach, respectively. In the third column are results using $f_{\pi'} = 0$ MeV. In this case, numerical results are independent of the model calculation of the form factors.

The processes $B^{\pm} \to K^{\pm}\pi'^0$, $B^{\pm} \to \rho^{\pm}\pi'^0$, $B^{\pm} \to K^{*\pm}\pi'^0$ have direct $CP$-violating asymmetries of order 10%. The channels $B^{\pm} \to \bar{K}^0\pi'^{\pm}$, $B^{\pm} \to \bar{K}^{*0}\pi'^{\pm}$ and $B^{\pm} \to \phi\pi'^{\pm}$ have only one contribution to its decay amplitude, in consequence direct $CP$ asymmetry is equal to zero. Direct $CP$-violating asymmetry in channel $B^{\pm} \to \pi^{\pm}\pi'^0$ depends on the use of WSB model or the LCSR approach.

In the channels $B^{\pm} \to \eta\pi'^{\pm}$, $B^{\pm} \to K^{\pm}\pi'^0$ and $B^{\pm} \to K^{*\pm}\pi'^0$, direct $CP$ violation asymmetries are not sensible to the value of decay constant $f_{\pi'}$. When the value $f_{\pi'} = 0$ MeV, direct $CP$ violation asymmetry in channel $B^{\pm} \to \pi^{\pm}\pi'^0$ is equal to zero. In the same case, modes $B^{\pm} \to \pi^0\pi'^{\pm}$, $B^{\pm} \to \eta'\pi'^{\pm}$, $B^{\pm} \to \rho^0\pi'^{\pm}$ and $B^{\pm} \to \omega\pi'^{\pm}$ have an increase in the direct $CP$-violation asymmetry. On the contrary, channel $B^{\pm} \to \rho^{\pm}\pi'^0$ has a decrease.

In Table VIII, direct $CP$-violating asymmetries for the channels $B^{\pm} \to P\rho'$, $B^{\pm} \to V\rho'$ are shown, using the WSB model and LCSR approach, and the decay constant $f_{\rho'} = 128$ MeV.

Direct $CP$-violating asymmetry corresponding to the channels $B^{\pm} \to K^{\pm}\rho'^0$, $B^{\pm} \to \omega\rho'^{\pm}$ and $B^{\pm} \to K^{*\pm}\rho'^0$ are bigger than 10%, which make them good candidates to be observed experimentally. In channels $B^{\pm} \to \bar{K}^0\rho'^{\pm}$, $B^{\pm} \to \bar{K}^{*0}\rho'^{\pm}$ and $B^{\pm} \to \phi\rho'^{\pm}$, the decay amplitude has only one contribution, thus the direct $CP$-violating asymmetry is automatically equal to zero. The other channels have direct $CP$-violating asymmetries of less than 10% order.

TABLE VII. Direct $CP$-violating asymmetries in percent for $B^{\pm} \to P\pi'$ and $B^{\pm} \to V\pi'$ decays, using the WSB [7] model and LCSR approach [8], decay constants $f_{\pi'} = 26$ and $f_{\pi'} = 0.0$ MeV.

| Final state | $A_{CP}$ |
|---|---|
| $\pi^{\pm}\pi'^{0}$ | $-1.6 \pm 0.3[-1.1 \pm 0.1]0.0$ |
| $\pi^{0}\pi'^{\pm}$ | $-0.4 \pm 0.1[-0.5 \pm 0.1]1.7 \pm 0.1$ |
| $\eta\pi'^{\pm}$ | $5.4 \pm 0.1[5.5 \pm 0.1]5.2 \pm 0.1$ |
| $\eta'\pi'^{\pm}$ | $5.9 \pm 0.1[6.2 \pm 0.1]23.5 \pm 0.5$ |
| $K^{\pm}\pi'^{0}$ | $-14.0 \pm 1.0[-14.0 \pm 1.0] - 15.0 \pm 1.0$ |
| $\bar{K}^{0}\pi'^{\pm}$ | 0[0]0 |
| $\rho^{0}\pi'^{\pm}$ | $-6.0[-6.0 \pm 1.0] - 23.4 \pm 1.0$ |
| $\rho^{\pm}\pi'^{0}$ | $-22.0 \pm 1.0[-23.0 \pm 1.0]5.3 \pm 1.0$ |
| $\omega\pi'^{\pm}$ | $-7.0 \pm 1.0[-7.0 \pm 1.0]14.9 \pm 1.0$ |
| $K^{*\pm}\pi'^{0}$ | $-27.0 \pm 1.0[-27.0 \pm 1.0] - 30.0 \pm 1.0$ |
| $\bar{K}^{*0}\pi'^{\pm}$ | 0[0]0 |
| $\phi\pi'^{\pm}$ | 0[0]0 |

The channels $B^{\pm} \to \pi^{0}\rho'^{\pm}$, $B^{\pm} \to \pi^{\pm}\rho'^{0}$ and $B^{\pm} \to \omega\rho'^{\pm}$ have direct $CP$-violating asymmetries which depend on the use of the WSB model or the LCSR approach in evaluating the form factors in transitions $B \to \pi$ and $B \to \omega$.

In neutral $B^0$ decays, because of the $B^0 - \bar{B}^0$ mixing, it is required to include time-dependent measurements in $CP$- violation asymmetries. The $CP$-violation time-dependent asymmetry is defined as

$$\begin{aligned} A_f(t) &= \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to \bar{f})} \\ &= C_f \cos(\Delta m t) + S_f \sin(\Delta m t), \end{aligned} \tag{32}$$

where $f$ is a two-body final state. The coefficients $C_f$ and $S_f$ are defined by

TABLE VIII. Direct $CP$-violating asymmetries in percent for $B^{\pm} \to P\rho'$ and $B^{\pm} \to V\rho'$ decays, using the WSB [7] model and LCSR approach [8], and decay constant $f_{\rho'} = 128$ MeV.

| Final state | $A_{CP}$ |
|---|---|
| $\pi^{0}\rho'^{\pm}$ | $-5.1 \pm 2.4[-7.3 \pm 3.2]$ |
| $\pi^{\pm}\rho'^{0}$ | $5.9 \pm 1.1[6.6 \pm 1.0]$ |
| $\eta\rho'^{\pm}$ | $5.7 \pm 0.5[5.8 \pm 0.5]$ |
| $\eta'\rho'^{\pm}$ | $5.4 \pm 0.5[5.5 \pm 0.5]$ |
| $K^{\pm}\rho'^{0}$ | $-20.5 \pm 1.0[-20.0 \pm 1.0]$ |
| $\bar{K}^{0}\rho'^{\pm}$ | 0[0] |
| $\rho^{0}\rho'^{\pm}$ | 0[0] |
| $\rho^{\pm}\rho'^{0}$ | 0[0] |
| $\omega\rho'^{\pm}$ | $15.0 \pm 5.0[16.8 \pm 2.6]$ |
| $K^{*\pm}\rho'^{0}$ | $-20.7 \pm 1.8[-20.7 \pm 1.7]$ |
| $\bar{K}^{*0}\rho'^{\pm}$ | 0[0] |
| $\phi\rho'^{\pm}$ | 0[0] |

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \qquad S_f = \frac{-2\,\mathrm{Im}(\lambda_f)}{1 + |\lambda_f|^2}, \tag{33}$$

given in terms of the ratio $\lambda_f$, which is defined by

$$\lambda_f = \frac{V_{tb}^{*} V_{td} \langle f | H_{\mathrm{eff}} | \bar{B}^0 \rangle}{V_{tb} V_{td}^{*} \langle \bar{f} | H_{\mathrm{eff}} | B^0 \rangle}. \tag{34}$$

The coefficients $C_f$ and $S_f$ are functions of $\lambda_f$. The quantity $\lambda_f$ is independent of phase conventions and physically meaningful, in consequence the coefficients $C_f$ and $S_f$ are observables. $CP$ violation in the interference of decays with and without mixing is encoded in the coefficient $S_f \neq 0$. $CP$ violation in decays means $C_f \neq 0$.

If the final state $f$ is a $CP$ eigenstate, i.e., $CP|f\rangle = \pm|f\rangle$, and decay amplitudes are dominated by only one weak phase term contribution, then $\langle f|H_{\mathrm{eff}}|\bar{B}^0\rangle = \langle \bar{f}|H_{\mathrm{eff}}|B^0\rangle$, $C_f = 0$ and $S_f = \eta_f \sin(2\phi)$, where $\eta_f$ is the $CP$ eigenvalue of $f$ and $2\phi$ is the difference in weak phase between the $B^0 \to f$ and $B^0 \to \bar{B}^0 \to f$ decay path. A contribution of another term to the decay amplitude with a different weak phase make the value of $S_f$ depends on the strong phase. In this situation is also possible that $C_f \neq 0$.

$CP$-violating asymmetry coefficients $C_f$ and $S_f$ for neutral $B^0(\bar{B}^0)$ decays with radial excited mesons $\pi'$ and $\rho'$ in final state are shown in Tables IX and X, respectively. For the processes $B^0(\bar{B}^0) \to \phi\pi'$ and $B^0(\bar{B}^0) \to \phi\rho'$ the coefficients $C_f$ and $S_f$ are equal to zero, since there is only one contribution to the decay amplitude in the respective channels.

The calculation results for the coefficients $C_f$ and $S_f$ are practically equal when the form factors are estimated using the WSB model or the LCSR approach. Nevertheless, for the modes $B \to P\pi'$ and $B \to V\pi'$, there is a dependency with respect to using the values for the decay constant $f_{\pi'} = 26$ or $f_{\pi'} = 0$ MeV. The channels with a strange meson in the final state have the same value of the coefficients using the two different values in decay constant $f_{\pi'}$.

The channels with $C_f \approx 0$ and $S_f \neq 0$ are $B^0 \to K^0\pi'^0$, $B^0 \to K^{*0}\pi'^0$, $B^0 \to K^0\rho'^0$ and $B^0 \to K^{*0}\rho'^0$. In these channels, where there is only present $CP$ violation in the interference of the decay and in the mixing, it is possible to relate the coefficient $S_f$ to fundamental parameters in the standard model, i.e., interior angles of the unitary triangle.

$CP$ violation in neutral $B^0(\bar{B}^0)$ mesons is involved when a final state $f$ and its $CP$ conjugate transformation state $\bar{f}$ are both common final states of $B^0$ and $\bar{B}^0$ mesons. The final states $f$ and $\bar{f}$ are not $CP$ eigenstates, i.e., $CP|f\rangle \neq |\bar{f}\rangle$. For this case, time evolutions of the four decays $B^0(t) \to f$, $B^0(t) \to \bar{f}$, $\bar{B}^0(t) \to \bar{f}$, $B^0(t) \to \bar{f}$ and $\bar{B}^0(t) \to f$ are studied in terms of four basic matrix elements

TABLE IX. *CP*-violating asymmetry parameters $C_f$ and $S_f$ in percent for neutral $B^0(\bar{B}^0) \to P\pi'$ and $B^0(\bar{B}^0) \to V\pi'$ decays, using the WSB [7] model and LCSR approach [8], decay constants $f_{\pi'} = 26$ and $f_{\pi'} = 0$ MeV.

| Final state | $C_f$ | $S_f$ |
|---|---|---|
| $\pi^0\pi'^0$ | $-0.7 \pm -0.1[0.8 \pm 0.1] - 10.1 \pm 0.1$ | $-2.7 \pm 0.3[-3.4 \pm 0.2] - 30.6 \pm 0.1$ |
| $\eta\pi'^0$ | $0.4 \pm 0.1[0.6 \pm 0.2]5.2 \pm 0.1$ | $1.7 \pm 0.4[2.3 \pm 0.6]21.5 \pm 0.5$ |
| $\eta'\pi'^0$ | $0.5 \pm 0.1[0.7 \pm 0.2]23.5 \pm 0.5$ | $2.0 \pm 0.5[2.7 \pm 0.7]40.9 \pm 0.5$ |
| $K^{\mp}\pi'^{\pm}$ | $-15.0 \pm 1.0[-15.0 \pm 1.0] - 15.0 \pm 1.0$ | $28.0 \pm 1.0[28.0 \pm 1.0]28.0 \pm 1.0$ |
| $K^0\pi'^0$ | 0[0]0 | $69.0 \pm 1.0[69.0 \pm 1.0]70.0 \pm 1.0$ |
| $\rho^0\pi'^0$ | $1.0 \pm 0.5[1.2 \pm 0.5] - 23.4 \pm 1.0$ | $5.0 \pm 1.0[4.8 \pm 0.8] - 58.0 \pm 1.0$ |
| $\omega\pi'^0$ | $1.0 \pm 0.5[1.0 \pm 0.5]14.9 \pm 1.0$ | $3.0 \pm 1.0[3.6 \pm 0.8]46.5 \pm 1.0$ |
| $K^{*\mp}\pi'^{\pm}$ | $-30.0 \pm 1.0[-30.0 \pm 1.0] - 30.0 \pm 1.0$ | $-20.0 \pm 1.0[-20.0 \pm 1.0] - 20.0 \pm 1.0$ |
| $K^{*0}\pi'^0$ | 0[0]0 | $70.0 \pm 1.0[70.0 \pm 1.0]70.0 \pm 1.0$ |
| $\phi\pi'^0$ | 0[0]0 | 0[0]0 |

$$g = \langle f|H_{\rm eff}|B^0\rangle, \qquad h = \langle f|H_{\rm eff}|\bar{B}^0\rangle,$$
$$\bar{g} = \langle \bar{f}|H_{\rm eff}|B^0\rangle, \qquad \bar{h} = \langle \bar{f}|H_{\rm eff}|\bar{B}^0\rangle. \tag{35}$$

The following two *CP*-violating asymmetries are introduced

$$\bar{A}_f(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)} = \bar{C}_f \cos(\Delta m t) + \bar{S}_f \sin(\Delta m t) \tag{36}$$

and

$$\bar{A}_{\bar{f}}(t) = \frac{\Gamma(B^0(t) \to \bar{f}) - \Gamma(\bar{B}^0(t) \to \bar{f})}{\Gamma(B^0(t) \to \bar{f}) + \Gamma(\bar{B}^0(t) \to \bar{f})} = \bar{C}_{\bar{f}} \cos(\Delta m t) + \bar{S}_{\bar{f}} \sin(\Delta m t), \tag{37}$$

where the coefficients of $\cos(\Delta m t)$ and $\sin(\Delta m t)$ are defined by

$$\bar{C}_f = \frac{|g|^2 - |h|^2}{|g|^2 + |h|^2}, \qquad \bar{S}_f = \frac{-2\,{\rm Im}(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\frac{h}{g})}{1 + |h/g|^2} \tag{38}$$

TABLE X. *CP*-violating asymmetries parameters $C_f$ and $S_f$ in percent for neutral $B^0(\bar{B}_0) \to P\rho'$ and $B^0(\bar{B}^0) \to V\rho'$ decays, using the WSB [7] model and LCSR approach [8], and decay constant $f_{\rho'} = 128$ MeV.

| Final state | $C_f$ | $S_f$ |
|---|---|---|
| $\pi^0\rho'^0$ | $-57.3 \pm 6.7[-62.6 \pm 7.7]$ | $-78.6 \pm 1.7[-75.1 \pm 5.6]$ |
| $\eta\rho'^0$ | $-44.3 \pm 1.4[-44.6 \pm 1.2]$ | $-87.3 \pm 2.1[-80.8 \pm 8.6]$ |
| $\eta'\rho'^0$ | $-5.9 \pm 1.3[-4.6 \pm 1.3]$ | $-48.3 \pm 1.2[-47.2 \pm 1.3]$ |
| $K^{\mp}\rho'^{\pm}$ | $-14.5 \pm 1.0[-14.5 \pm 1.0]$ | $-12.9 \pm 1.0[-12.9 \pm 1.0]$ |
| $K^0\rho'^0$ | 0[0] | $64.5 \pm 1.0[64.5 \pm 1.0]$ |
| $\rho^0\rho'^0$ | $-23.0 \pm 1.0[-23.0 \pm 1.0]$ | $-58.0 \pm 1.0[-58.0 \pm 1.0]$ |
| $\omega\rho'^0$ | $17.5 \pm 4.5[17.5 \pm 4.5]$ | $46.5 \pm 1.0[46.5 \pm 1.0]$ |
| $K^{*\mp}\rho'^{\pm}$ | $-30.0 \pm 1.0[-30.0 \pm 1.0]$ | $-20.0 \pm 1.0[-20.0 \pm 1.0]$ |
| $K^{*0}\rho'^0$ | 0[0] | $65.0 \pm 1.0[65.0 \pm 1.0]$ |
| $\phi\rho'^0$ | 0[0] | 0[0] |

and

$$\bar{C}_{\bar{f}} = \frac{|\bar{h}|^2 - |\bar{g}|^2}{|\bar{h}|^2 + |\bar{g}|^2}, \qquad \bar{S}_{\bar{f}} = \frac{-2\,{\rm Im}(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\frac{\bar{g}}{\bar{h}})}{1 + |\bar{g}/\bar{h}|^2}. \tag{39}$$

The condition for *CP* violation is that width decays $\Gamma(B^0(t) \to f) \neq \Gamma(\bar{B}^0 \to f)$ and $\Gamma(B^0(t) \to \bar{f}) \neq \Gamma(\bar{B}^0 \to \bar{f})$, which means, in terms of *CP*-violating asymmetry coefficients, $\bar{C}_f \neq -\bar{C}_{\bar{f}}$ and (or) $\bar{S}_f \neq -\bar{S}_{\bar{f}}$.

Numerical values in percent for the *CP*-violating asymmetry parameters $\bar{C}_f$, $\bar{S}_f$, $\bar{C}_{\bar{f}}$ and $\bar{S}_{\bar{f}}$ in the $B^0(\bar{B}^0) \to \pi\pi'$, $B^0(\bar{B}^0) \to \rho\pi'$, $B^0(\bar{B}^0) \to \pi\rho'$ and $B^0(\bar{B}^0) \to \rho\rho'$ decays are listed in Table XI. The form factors for the transitions $B \to \pi$ and $B \to \rho$ are calculated using the WSB model and LCSR approach.

The *CP*-violating asymmetry parameters for the final states $\rho^+\rho'^-$, $\rho^-\rho'^+$ have the same value if they are calculated using the WSB model or LCSR approach. The parameters for the final states with a $\pi'$ mesons are only calculated using the values in decay constant $f_\pi = 26$ MeV. When the value $f_{\pi'} = 0$ MeV is used, results are not reported, since there are zero contributions, $g = 0$ and $cg = 0$, with the consequence that $\bar{C}_f = -\bar{C}_{\bar{f}} = 100\%$, $\bar{S}_f = \infty$ and $\bar{S}_{\bar{f}} = 0$.

No significant direct *CP* asymmetry for the mode $B^{\pm} \to \rho^0(1450)\pi^{\pm}$ is observed, see Ref. [28], reporting $A_{CP}(\%) = -6.0^{+36.0}_{-42.0}$. For this channel we have obtained $A_{CP}(\%) = 5.9 \pm 1.1[6.6 \pm 1.0]$; note that we have defined direct *CP* asymmetry differently by a sign. One of the reasons for this work is to estimate these asymmetries and to motivate the experimental measurement of them.

## VI. CONCLUSIONS

In the framework of generalized naive factorization we calculate branching ratios and *CP*-violating asymmetries of exclusive nonleptonic two-body $B$ decays including the radial excited $\pi(1300)$ or $\rho(1450)$ meson in the final state. Branching ratios and *CP*-violating asymmetries for the exclusive channels $B \to P\pi'$, $B \to V\pi'$, $B \to P\rho'$ and

TABLE XI. $CP$-violating asymmetry parameters $\bar{C}_f$, $\bar{S}_f$, $\bar{C}_{\bar{f}}$ and $\bar{S}_{\bar{f}}$ in percent for $B^0(\bar{B}^0) \to \pi\pi'$, $B^0(\bar{B}^0) \to \rho\pi'$, $B^0(\bar{B}^0) \to \pi\rho'$ and $B^0(\bar{B}^0) \to \rho\rho'$ decays, using the WSB [7] model and LCSR approach [8].

| Final states | $\bar{C}_f$, $\bar{C}_{\bar{f}}$ | $\bar{S}_f$, $\bar{S}_{\bar{f}}$ |
|---|---|---|
| $\pi^+\pi'^-$, $\pi^-\pi'^+$ | $77.5 \pm 0.7[61.8 \pm 6.9]$ | $-70.7 \pm 0.9[-52.1 \pm 8.8]$ |
| | $61.5 \pm 0.9[76.7 \pm 5.8]$ | $66.5 \pm 0.8[80.9 \pm 5.1]$ |
| $\rho^+\pi'^-$, $\rho^-\pi'^+$ | $30.0 \pm 13.3[33.0 \pm 16.7]$ | $-30.1 \pm 13.3[-33.1 \pm 16.6]$ |
| | $-94.3 \pm 4.2[-92.6 \pm 5.9]$ | $-88.0 \pm 1.0[-84.8 \pm 4.9]$ |
| $\pi^+\rho'^-$, $\pi^-\rho'^+$ | $24.1 \pm 19.1[37.1 \pm 24.6]$ | $-22.1 \pm 16.5[-32.4 \pm 25.5]$ |
| | $5.8 \pm 1.0[5.4 \pm 1.0]$ | $5.5 \pm 1.0[5.1 \pm 1.0]$ |
| $\rho^+\rho'^-$, $\rho^-\rho'^+$ | $16.0 \pm 1.0[16.0 \pm 1.0]$ | $-5.4 \pm 1.0[-5.4 \pm 1.0]$ |
| | $16.0 \pm 1.0[16.0 \pm 1.0]$ | $16.0 \pm 1.0[16.0 \pm 1.0]$ |

$B \to V\rho'$ (where, $P$ and $V$ denote a pseudoscalar and vector meson, respectively) have been estimated using all the contributions coming from the effective weak Hamiltonian $H_{\rm eff}$.

The form factors in $B \to P$ and $B \to V$ transitions are estimated using the WSB model [7] and LCSR approach [8]. In order to obtain form factors in $B \to \pi'$ and $B \to \rho'$ transitions, we use the improved version of the nonrelativistic ISGW quark model [9], called ISGW2 model [10]. The factorized decay amplitudes for these decays are listed in the Appendixes.

We obtain branching ratios for 52 exclusive channels. Some of these decays can be reached in experiments. In fact, decays $\bar{B}^0 \to \pi^-\pi'^+$, $B^- \to \eta\pi'^-$, $\bar{B}^0 \to \rho^-\pi'^+$, $\bar{B}^0 \to \rho^+\pi'^-$, $B^- \to \rho^0\pi'^-$, $B^- \to \rho^-\pi'^0$, $\bar{B}^0 \to \pi^-\rho'^+$, $\bar{B}^0 \to \rho^-\rho'^+$, $\bar{B}^0 \to \rho^+\rho'^-$, and $B^- \to \rho^-\rho'^0$ have branching ratios of the order of $10^{-5}$.

We also studied the dependency of branching ratios in channels $B \to P\pi'$ and $B \to V\pi'$ with respect to the decay constant $f_{\pi'}$. The more sensible modes to the value in decay constant $f_{\pi'}$ are $\bar{B}^0 \to \pi^-\pi'^+$, $\bar{B}^0 \to \pi^0\pi'^0$, $B^- \to \eta\pi'^-$, $B^- \to \eta'\pi'^-$, $\bar{B}^0 \to \rho^+\pi'^-$, $B^- \to \rho^0\pi'^-$, $\bar{B}^0 \to \omega\pi'^0$, and $B^- \to \omega\pi'^-$. These channels could be the best scenario to determine the decay constant $f_{\pi'}$ in nonleptonic two-body $B$ decays.

In general, we can explain the large branching ratios in decays $\bar{B}^0 \to \pi^-\pi'^+$, $\bar{B}^0 \to \rho^-\pi'^+$, $\bar{B}^0 \to \rho^+\pi'^-$, and $B^- \to \rho^-\pi'^0$ by the effect of the enhancement of the chiral factor that multiply the penguin contributions $a_6$ and $a_8$ in the effective weak Hamiltonian $H_{\rm eff}$.

Direct $CP$-violating asymmetry in channels $B^\pm \to K^\pm\pi'^0$, $B^\pm \to \rho^\pm\pi'^0$, $B^\pm \to K^{*\pm}\pi'^0$, $B^\pm \to K^\pm\rho'^0$, $B^\pm \to \omega\rho'^\pm$, and $B^\pm \to K^{*\pm}\rho'^0$ are more than 10% order. In the modes $B^\pm \to \eta'\pi^\pm$, $B^\pm \to \rho^0\pi'^\pm$ and $B^\pm \to \omega\pi'^\pm$, estimation of direct $CP$-violating asymmetry using the value of the decay constant $f_{\pi'} = 0$ MeV, give an increase with respect to the calculations using the value $f_{\pi'} = 26$ MeV. On the contrary the channel $B^\pm \to \rho^\pm\pi'^0$ has a decrease in its estimation. When the value in the decay constant $f_{\pi'} = 0$ MeV is used, estimation of direct $CP$-violating asymmetry in modes $B^\pm \to \eta'\pi'^\pm$, $B^\pm \to \rho^0\pi'^\pm$ and $B^\pm \to \omega\pi'^\pm$ are more than 10% order.

In the neutral modes $B^0 \to K^0\pi'^0$, $B^0 \to K^{*0}\pi'^0$, $B^0 \to K^0\rho'^0$ and $B^0 \to K^{*0}\rho'^0$, we have estimated the $CP$- violating asymmetry coefficients $C_f \approx 0$ and $S_f$ at more than 60%.

For the channels $\bar{B}^0 \to \pi^-\rho'^+$ and $B^- \to \pi^-\rho'^0$, our predictions are lower like the ones obtained by Ref. [4], although our value is the same order of magnitude that the only experimental branching ratio measured $B^- \to \pi^-\rho'^0$ [28]. The estimations of the bracing ratios for the channels $B^+ \to K^+\rho'^0$ and $B^0 \to K^+\rho'^-$ are consistent with the upper limits measured by *BABAR* Collaboration [29,30].

Finally, we want to mention that, even the direct $CP$-violation asymmetry $A_{CP}(\%) = -6.0^{+36.0}_{-42.0}$ for the mode $B^\pm \to \rho^0(1450)\pi^\pm$ is consistent with zero, and our predictions for that asymmetry are compatible with the central value.

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## APPENDIX A: MATRIX ELEMENTS FOR $B \to P\pi'$ DECAYS

$$\mathcal{M}(\bar{B}^0 \to \pi^-\pi'^+) = -i\frac{G_F}{\sqrt{2}} f_{\pi'} F_0^{B\to\pi}(m_{\pi'}^2)(m_B^2 - m_\pi^2) \times \left\{ V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^* \times \left[ a_4 + a_{10} + 2(a_6 + a_8) \times \frac{m_{\pi\prime}^2}{(m_b - m_u)(m_d + m_u)} \right] \right\}, \quad \text{(A1)}$$

$$\mathcal{M}(\bar{B}^0 \to \pi^0 \pi'^0) = i\frac{G_F}{2\sqrt{2}} f_\pi F_0^{B\to\pi'}(m_\pi^2)(m_B^2 - m_{\pi'}^2)\Big\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\Big[-a_4 + \frac{1}{2}a_{10} - \frac{3}{2}(a_7 - a_9) - (2a_6 - a_8)\frac{m_\pi^2}{(m_b - m_d)(m_d + m_d)}\Big]\Big\} + i\frac{G_F}{\sqrt{2}} f_{\pi'} F_0^{B\to\pi}(m_{\pi'}^2)(m_B^2 - m_\pi^2)\Big\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\Big[-a_4 + \frac{1}{2}a_{10} - \frac{3}{2}(a_7 - a_9) - (2a_6 - a_8)\frac{m_{\pi'}^2}{(m_b - m_d)(m_d + m_d)}\Big]\Big\}, \tag{A2}$$

$$\mathcal{M}(B^- \to \pi^- \pi'^0) = -i\frac{G_F}{2} f_\pi F_0^{B\to\pi'}(m_\pi^2)(m_B^2 - m_{\pi'}^2)\{V_{ub}V_{ud}^* a_1\} - i\frac{G_F}{\sqrt{2}} f_{\pi'} F_0^{B\to\pi}(m_{\pi'}^2)(m_B^2 - m_\pi^2)\Big\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\frac{3}{2}\Big[a_9 + a_{10} - a_7 + 2a_8\frac{m_{\pi'}^2}{(m_b - m_u)(m_d + m_u)}\Big]\Big\}, \tag{A3}$$

$$\mathcal{M}(\bar{B}^0 \to \eta^{(\prime)} \pi'^0) = -i\frac{G_F}{2} f_{\pi'} F_0^{B\to\eta^{(\prime)}}(m_{\pi'}^2)(m_B^2 - m_{\eta^{(\prime)}}^2)\Big\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\Big[-a_4 + \frac{1}{2}a_{10} - (2a_6 - a_8) \times \frac{m_{\pi'}^2}{(m_b - m_d)(m_d + m_d)} + \frac{3}{2}(a_9 - a_7)\Big]\Big\} + i\frac{G_F}{2} f^u_{\eta^{(\prime)}} F_0^{B\to\pi'}(m_{\eta^{(\prime)}}^2)(m_B^2 - m_{\pi'}^2)\Big\{V_{ub}V_{ud}^* a_2 + V_{cb}V_{cd}^* a_2\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} - V_{tb}V_{td}^*\Big[a_4 + 2(a_3 - a_5) + \frac{1}{2}(a_9 - a_7 - a_{10}) + (2a_6 - a_8)\frac{m_{\eta^{(\prime)}}^2}{(m_b - m_d)(m_s + m_s)} \times \Big(\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} - 1\Big) r_{\eta^{(\prime)}} + (a_3 - a_5 + a_9 - a_7)\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} + \Big(a_3 - a_5 + \frac{1}{2}(a_7 - a_9)\Big)\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}\Big]\Big\}, \tag{A4}$$

$$\mathcal{M}(B^- \to \eta^{(\prime)} \pi'^-) = i\frac{G_F}{\sqrt{2}} f_{\pi'} F_0^{B\to\eta^{(\prime)}}(m_{\pi'}^2)(m_B^2 - m_{\eta^{(\prime)}}^2)\Big\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*\Big[a_4 + a_{10} + 2(a_6 + a_8)\frac{m_{\pi'}^2}{(m_b - m_u)(m_d + m_u)}\Big]\Big\} + i\frac{G_F}{\sqrt{2}} f^u_{\eta^{(\prime)}} F_0^{B\to\pi'}(m_{\eta^{(\prime)}}^2)(m_B^2 - m_{\pi'}^2)\Big\{V_{ub}V_{ud}^* a_2 + V_{cb}V_{cd}^* a_2\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} - V_{tb}V_{td}^*\Big[a_4 + 2(a_3 - a_5) + \frac{1}{2}(a_9 - a_7 - a_{10}) + (2a_6 - a_8)\frac{m_{\eta^{(\prime)}}^2}{(m_b - m_d)(m_s + m_s)}\Big(\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} - 1\Big) r_{\eta^{(\prime)}} + (a_3 - a_5 + a_7 - a_9)\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} + \Big(a_3 - a_5 + \frac{1}{2}(a_7 - a_9)\Big)\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}\Big]\Big\}, \tag{A5}$$

$$\mathcal{M}(\bar{B}^0 \to K^- \pi'^+) = -i\frac{G_F}{\sqrt{2}} f_K F_0^{B\to\pi'}(m_K^2)(m_B^2 - m_{\pi'}^2)\Big\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*\Big[a_4 + a_{10} + 2(a_6 + a_8)\frac{m_{K^-}^2}{(m_b - m_u)(m_u + m_s)}\Big]\Big\}, \tag{A6}$$

$$\mathcal{M}(\bar{B}^0 \to \bar{K}^0 \pi'^0) = -i\frac{G_F}{2} f_K F_0^{B\to\pi'}(m_K^2)(m_B^2 - m_{\pi'}^2) V_{tb}V_{ts}^*\Big\{a_4 - \frac{1}{2}a_{10} + (2a_6 - a_8)\frac{m_{K^0}^2}{(m_b - m_d)(m_d + m_s)}\Big\} - i\frac{G_F}{2} f_{\pi'} F_0^{B\to K}(m_{\pi'}^2)(m_B^2 - m_K^2)\Big\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\frac{3}{2}(a_9 - a_7)\Big\}, \tag{A7}$$

$$\mathcal{M}(B^- \to K^- \pi'^0) = -i\frac{G_F}{2} f_K F_0^{B\to\pi'}(m_K^2)(m_B^2 - m_{\pi'}^2)\bigg\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*\bigg[a_4 + a_{10} + 2(a_6 + a_8)\frac{m_{K^-}^2}{(m_b - m_u)(m_u + m_s)}\bigg]\bigg\} - i\frac{G_F}{2} f_{\pi'} F_0^{B\to K}(m_{\pi'}^2)(m_B^2 - m_K^2)\bigg\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\frac{3}{2}(a_9 - a_7)\bigg\}, \tag{A8}$$

$$\mathcal{M}(B^- \to \bar{K}^0 \pi'^-) = -i\frac{G_F}{\sqrt{2}} f_K F_0^{B\to\pi'}(m_K^2)(m_B^2 - m_{\pi'}^2) V_{tb}V_{ts}^*\bigg\{a_4 - \frac{1}{2}a_{10} + (2a_6 - a_8)\frac{m_{K^0}^2}{(m_b - m_d)(m_d + m_s)}\bigg\}. \tag{A9}$$

## APPENDIX B: MATRIX ELEMENTS FOR $B \to V\pi'$ DECAYS

$$\mathcal{M}(\bar{B}^0 \to \rho^- \pi'^+) = \sqrt{2} G_F f_\rho F_1^{B\to\pi'}(m_\rho^2) m_\rho(\epsilon \cdot p_{\pi'})\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*[a_4 + a_{10}]\}, \tag{B1}$$

$$\mathcal{M}(\bar{B}^0 \to \rho^+ \pi'^-) = \sqrt{2} G_F f_{\pi'} A_0^{B\to\rho}(m_{\pi'}^2) m_\rho(\epsilon \cdot p_{\pi'})\bigg\{V_{ub}V_{UK}^* a_1 - V_{tb}V_{td}^*\bigg[a_4 + a_{10} - 2(a_6 + a_8)\frac{m_{\pi'}^2}{(m_b + m_u)(m_u + m_d)}\bigg]\bigg\}, \tag{B2}$$

$$\mathcal{M}(\bar{B}^0 \to \rho^0 \pi'^0) = -\frac{G_F}{\sqrt{2}} m_\rho(\epsilon \cdot p_{\pi'})\bigg(f_\rho F_1^{B\to\pi'}(m_\rho^2)\bigg\{V_{ub}V_{ud}^* a_2 + V_{tb}V_{td}^*\bigg[a_4 - \frac{1}{2}a_{10} - \frac{3}{2}(a_7 + a_9)\bigg]\bigg\} + f_{\pi'} A_0^{B\to\rho}(m_{\pi'}^2)\bigg\{V_{ub}V_{ud}^* a_2 + V_{tb}V_{td}^*\bigg[a_4 - \frac{1}{2}a_{10} - (2a_6 - a_8)\frac{m_{\pi'}^2}{(m_b + m_d)(m_d + m_d)} + \frac{3}{2}(a_7 - a_9)\bigg]\bigg\}\bigg), \tag{B3}$$

$$\mathcal{M}(B^- \to \rho^0 \pi'^-) = G_F m_\rho(\epsilon \cdot p_{\pi'})\bigg(f_{\pi'} A_0^{B\to\rho}(m_{\pi'}^2)\bigg\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*\bigg[a_4 + a_{10} - 2(a_6 + a_8)\frac{m_{\pi'}^2}{(m_b + m_u)(m_u + m_d)}\bigg]\bigg\} + f_\rho F_1^{B\to\pi'}(m_\rho^2)\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[-a_4 + \frac{1}{2}a_{10} + \frac{3}{2}(a_7 + a_9)\bigg]\bigg\}\bigg), \tag{B4}$$

$$\mathcal{M}(B^- \to \rho^- \pi'^0) = G_F m_\rho(\epsilon \cdot p_{\pi'})\bigg(f_\rho F_1^{B\to\pi'}(m_\rho^2)\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*(a_4 + a_{10})\} + f_{\pi'} A_0^{B\to\rho}(m_{\pi'}^2)\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[-a_4 + \frac{1}{2}a_{10} - (2a_6 - a_8)\frac{m_{\pi'}^2}{(m_b + m_d)(m_d + m_d)} + \frac{3}{2}(a_9 - a_7)\bigg]\bigg\}\bigg), \tag{B5}$$

$$\mathcal{M}(\bar{B}^0 \to \omega \pi'^0) = \frac{G_F}{\sqrt{2}} m_\omega(\epsilon \cdot p_{\pi'})\bigg(-f_\omega F_1^{B\to\pi'}(m_\omega^2)\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[a_4 + 2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9 - a_{10})\bigg]\bigg\} + f_{\pi'} A_0^{B\to\omega}(m_{\pi'}^2)\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[-a_4 + \frac{1}{2}a_{10} - (2a_6 - a_8)\frac{m_{\pi'}^2}{(m_b + m_u)(m_u + m_d)} + \frac{3}{2}(a_9 - a_7)\bigg]\bigg\}\bigg), \tag{B6}$$

$$\mathcal{M}(B^- \to \omega \pi'^-) = G_F m_\omega(\epsilon \cdot p_{\pi'})\bigg(f_{\pi'} A_0^{B\to\omega}(m_{\pi'}^2)\bigg\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*\bigg[a_4 + a_{10} - 2(a_6 + a_8)\frac{m_{\pi'}^2}{(m_b + m_u)(m_u + m_d)}\bigg]\bigg\} + f_\omega F_1^{B\to\pi'}(m_\omega^2)\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[a_4 + 2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9 - a_{10})\bigg]\bigg\}\bigg), \tag{B7}$$

$$\mathcal{M}(\bar{B}^0 \to K^{*-}\pi'^{+}) = \sqrt{2}G_F f_{K^*} F_1^{B\to\pi'}(m_{K^*}^2) m_{K^*}(\epsilon\cdot p_{\pi'})\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*[a_4 + a_{10}]\}, \tag{B8}$$

$$\begin{aligned}\mathcal{M}(\bar{B}^0 \to \bar{K}^{*0}\pi'^{0}) = G_F m_{K^{*0}}(\epsilon\cdot p_{\pi'})\bigg(& f_{\pi'} A_0^{B\to K^*}(m_{\pi'}^2)\bigg\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \frac{3}{2}(a_9 - a_7)\bigg\} \\ &+ f_{K^*} F_1^{B\to\pi'}(m_{K^{*0}}^2) V_{tb}V_{ts}^*\bigg\{a_4 - \frac{1}{2}a_{10}\bigg\}\bigg),\end{aligned} \tag{B9}$$

$$\begin{aligned}\mathcal{M}(B^- \to K^{*-}\pi'^{0}) = G_F m_{K^*}(\epsilon\cdot p_{\pi'})\bigg(& f_{\pi'} A_0^{B\to K^*}(m_{\pi'}^2)\bigg\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \frac{3}{2}(a_9 - a_7)\bigg\} \\ &+ f_{K^*} F_1^{B\to\pi'}(m_{K^*}^2)\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*(a_4 + a_{10})\}\bigg),\end{aligned} \tag{B10}$$

$$\mathcal{M}(B^- \to \bar{K}^{*0}\pi'^{-}) = -\sqrt{2}G_F f_{K^*} F_1^{B\to\pi'}(m_{K^*}^2) m_{K^*}(\epsilon\cdot p_{\pi'}) V_{tb}V_{ts}^*\bigg\{a_4 - \frac{1}{2}a_{10}\bigg\}, \tag{B11}$$

$$\mathcal{M}(\bar{B}^0 \to \phi\pi'^{0}) = -G_F f_\phi F_1^{B\to\pi'}(m_\phi^2) m_\phi(\epsilon\cdot p_{\pi'}) V_{tb}V_{td}^*\bigg\{a_3 + a_5 - \frac{1}{2}(a_7 + a_9)\bigg\}, \tag{B12}$$

$$\mathcal{M}(B^- \to \phi\pi'^{-}) = -\sqrt{2}\mathcal{M}(\bar{B}^0 \to \phi\pi'^{0}). \tag{B13}$$

## APPENDIX C: MATRIX ELEMENTS FOR $B \to P\rho'$ DECAYS

$$\begin{aligned}\mathcal{M}(\bar{B}^0 \to \eta^{(\prime)}\rho'^{0}) = G_F m_{\rho'}(\epsilon\cdot p_{\eta^{(\prime)}})\bigg(& f_{\rho'} F_1^{B\to\eta^{(\prime)}}(m_{\rho'}^2)\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[-a_4 + \frac{1}{2}a_{10} + \frac{3}{2}(a_7 + a_9)\bigg]\bigg\} \\ &+ f_{\eta^{(\prime)}}^u A_0^{B\to\rho'}(m_{\eta^{(\prime)}}^2)\bigg\{V_{ub}V_{ud}^* a_2 + V_{cb}V_{cd}^* a_2 \frac{f_{\eta^{(\prime)}}^c}{f_{\eta^{(\prime)}}^u} - V_{tb}V_{td}^*\bigg[a_4 + 2(a_3 - a_5) \\ &+ \frac{1}{2}(a_9 - a_7 - a_{10}) - (2a_6 - a_8)\frac{m_{\eta^{(\prime)}}^2}{(m_b + m_d)(m_s + m_s)}\bigg(\frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u} - 1\bigg) r_{\eta^{(\prime)}} \\ &+ (a_3 - a_5 - a_7 + a_9)\frac{f_{\eta^{(\prime)}}^c}{f_{\eta^{(\prime)}}^u} + \bigg(a_3 - a_5 + \frac{1}{2}(a_7 - a_9)\bigg)\frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u}\bigg]\bigg\}\bigg),\end{aligned} \tag{C1}$$

$$\begin{aligned}\mathcal{M}(B^- \to \eta^{(\prime)}\rho'^{-}) = \sqrt{2}G_F m_{\rho'}(\epsilon\cdot p_{\eta^{(\prime)}})\bigg(& f_{\rho'} F_1^{B\to\eta^{(\prime)}}(m_{\rho'}^2)\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*[a_4 + a_{10}]\} + f_{\eta^{(\prime)}}^u A_0^{B\to\rho'}(m_{\eta^{(\prime)}}^2)\bigg\{V_{ub}V_{ud}^* a_2 \\ &+ V_{cb}V_{cd}^* a_2 \frac{f_{\eta^{(\prime)}}^c}{f_{\eta^{(\prime)}}^u} - V_{tb}V_{td}^*\bigg[a_4 + 2(a_3 - a_5) + \frac{1}{2}(a_9 - a_7 - a_{10}) - (2a_6 - a_8)\frac{m_{\eta^{(\prime)}}^2}{(m_b + m_d)(m_s + m_s)} \\ &\times\bigg(\frac{f_{\eta^{(\prime)}}^u}{f_{\eta^{(\prime)}}^s} - 1\bigg) r_{\eta^{(\prime)}} + (a_3 - a_5 - a_7 + a_9)\frac{f_{\eta^{(\prime)}}^c}{f_{\eta^{(\prime)}}^u} + \bigg(a_3 - a_5 - \frac{1}{2}(a_9 - a_7)\bigg)\frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u}\bigg]\bigg\}\bigg),\end{aligned} \tag{C2}$$

$$\mathcal{M}(\bar{B}^0 \to K^- \rho'^+) = \sqrt{2} G_F f_K A_0^{B\to\rho'}(m_K^2) m_{\rho'} (\epsilon \cdot p_K) \bigg\{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* \bigg[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{K^-}^2}{(m_b + m_u)(m_u + m_s)} \bigg] \bigg\}, \tag{C3}$$

$$\mathcal{M}(\bar{B}^0 \to \bar{K}^0 \rho'^0) = G_F m_{\rho'} (\epsilon \cdot p_K) \bigg( f_K A_0^{B\to\rho'}(m_{K^0}^2) V_{tb} V_{ts}^* \bigg[ a_4 - \frac{1}{2} a_{10} - (2a_6 - a_8) \frac{m_{K^0}^2}{(m_b + m_d)(m_d + m_s)} \bigg] + f_{\rho'} F_1^{B\to K}(m_{\rho'}^2) \bigg\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \frac{3}{2}(a_7 + a_9) \bigg\} \bigg), \tag{C4}$$

$$\mathcal{M}(B^- \to K^- \rho'^0) = G_F m_{\rho'} (\epsilon \cdot p_K) \bigg( f_K A_0^{B\to\rho'}(m_K^2) \bigg\{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* \bigg[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{K^-}^2}{(m_b + m_u)(m_u + m_s)} \bigg] \bigg\} + f_{\rho'} F_1^{B\to K}(m_{\rho'}^2) \bigg\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \frac{3}{2}(a_7 + a_9) \bigg\} \bigg), \tag{C5}$$

$$\mathcal{M}(B^- \to \bar{K}^0 \rho'^-) = -\sqrt{2} G_F f_K A_0^{B\to\rho'}(m_{K^0}^2) m_{\rho'} (\epsilon \cdot p_K) V_{tb} V_{ts}^* \bigg\{ a_4 - \frac{1}{2} a_{10} - (2a_6 - a_8) \frac{m_{K^0}^2}{(m_b + m_d)(m_d + m_s)} \bigg\}. \tag{C6}$$

## APPENDIX D: MATRIX ELEMENTS FOR $B \to V\rho'$ DECAYS

$$\mathcal{M}(\bar{B}^0 \to \rho^- \rho'^+) = X^{(\bar{B}^0 \rho'^+, \rho^-)} \{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* [a_4 + a_{10}] \}, \tag{D1}$$

$$\mathcal{M}(\bar{B}^0 \to \rho^0 \rho'^0) = [X^{(\bar{B}^0 \rho^0, \rho'^0)} + X^{(\bar{B}^0 \rho'^0, \rho^0)}] \bigg\{ V_{ub} V_{ud}^* a_2 + V_{tb} V_{td}^* \bigg[ a_4 - \frac{1}{2} a_{10} - \frac{3}{2}(a_7 + a_9) \bigg] \bigg\}, \tag{D2}$$

$$\mathcal{M}(B^- \to \rho^- \rho'^0) = X^{(B^- \rho^-, \rho'^0)} \{ V_{ub} V_{ud}^* a_2 \} + X^{(B^- \rho'^0, \rho^-)} \bigg\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \frac{3}{2}(a_7 + a_9 + a_{10}) \bigg\}, \tag{D3}$$

$$\mathcal{M}(\bar{B}^0 \to \omega \rho'^0) = X^{(\bar{B}^0 \rho'^0, \omega)} \bigg\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \bigg[ a_4 + 2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9 - a_{10}) \bigg] \bigg\} + X^{(\bar{B}^0 \omega, \rho'^0)} \bigg\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \bigg[ -a_4 + \frac{1}{2} a_{10} + \frac{3}{2}(a_7 + a_9) \bigg] \bigg\}, \tag{D4}$$

$$\mathcal{M}(B^- \to \omega \rho'^-) = X^{(B^- \omega, \rho'^-)} \{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* (a_4 + a_{10}) \} + X^{(B^- \rho'^-, \omega)} \bigg\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \bigg[ a_4 + 2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9 - a_{10}) \bigg] \bigg\}, \tag{D5}$$

$$\mathcal{M}(\bar{B}^0 \to K^{*-} \rho'^+) = X^{(\bar{B}^0 \rho'^+, K^{*-})} \{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10}) \}, \tag{D6}$$

$$\mathcal{M}(\bar{B}^0 \to \bar{K}^{*0} \rho'^0) = X^{(\bar{B}^0 \bar{K}^{*0}, \rho'^0)} \bigg\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \frac{3}{2}(a_7 + a_9) \bigg\} + X^{(\bar{B}^0 \rho'^0, \bar{K}^{*0})} \bigg\{ -V_{tb} V_{ts}^* \bigg[ a_4 - \frac{1}{2} a_{10} \bigg] \bigg\}, \tag{D7}$$

$$\mathcal{M}(B^- \to K^{*-}\rho'^0) = X^{(B^- K^{*-},\rho'^0)}\left\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\frac{3}{2}(a_7 + a_9)\right\} + X^{(B^- \rho'^0, K^{*-})}\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*(a_4 + a_{10})\}, \tag{D8}$$

$$\mathcal{M}(B^- \to \bar{K}^{*0}\rho'^-) = -X^{(B^- \rho'^-, \bar{K}^{*0})}V_{tb}V_{ts}^*\left\{a_4 - \frac{1}{2}a_{10}\right\}, \tag{D9}$$

$$\mathcal{M}(\bar{B}^0 \to \phi\rho'^0) = X^{(\bar{B}^0 \rho'^0, \phi)}\left\{V_{tb}V_{td}^*\left[a_3 + a_5 - \frac{1}{2}(a_7 + a_9)\right]\right\}, \tag{D10}$$

$$\mathcal{M}(B^- \to \phi\rho'^-) = -\sqrt{2}\mathcal{M}(\bar{B}^0 \to \phi\rho'^0). \tag{D11}$$


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