Relativistic correction to color-octet J/ψ production at hadron colliders

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The relativistic corrections to the color-octet J/ψ hadroproduction at the Tevatron and LHC are calculated up to $\mathcal{O}(v^2)$ in nonrelativistic QCD factorization frame. The short distance coefficients are obtained by matching full QCD with NRQCD results for the partonic subprocess $g + g \rightarrow J/\psi({}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_J^{[8]}) + g$, $q + \bar{q} \rightarrow J/\psi({}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_J^{[8]}) + g$ and $g + q(\bar{q}) \rightarrow J/\psi({}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_J^{[8]}) + q(\bar{q})$. The short distance coefficient ratios of relativistic correction to leading order for color-octet states ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, and ${}^3P_J^{[8]}$ at large p_T are approximately $-5/6$, $-11/6$, and $-31/30$, respectively, for each subprocess, and it is $1/6$ for color-singlet state ${}^3S_1^{[1]}$. If the higher order long distance matrix elements are estimated through velocity scaling rule with adopting $v^2 = 0.23$ and the lower order long distance matrix elements are fixed, the leading order cross sections of color-octet states are reduced by about a factor of 20–40% at large p_T at both the Tevatron and the LHC. Comparing with QCD radiative corrections to color-octet states, relativistic correction is ignored along with p_T increasing. Using long distance matrix elements extracted from the fit to J/ψ production at the Tevatron, we can find the unpolarization cross sections of J/ψ production at the LHC taking into account both QCD and relativistic corrections are changed by about 20–50% of that considering only QCD corrections. These results indicate that relativistic corrections may play an important role in J/ψ production at the Tevatron and the LHC.

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I. INTRODUCTION

Heavy quarkonium is an excellent candidate to probe quantum chromodynamics (QCD) from the high energy to the low energy regimes. Nonrelativistic QCD (NRQCD) factorization formalism was established [1] to describe the production and decay of heavy quarkonium. In the NRQCD approach, the production and decay of heavy quarkonium is factored into short distance coefficients and long distance matrix elements (LDMEs). The short distance coefficients indicate that the creation or annihilation of a heavy quark pair can be calculated perturbatively with the expansions by the strong coupling constant α_s. However, the LDMEs, which represent the evolution of a free heavy quark pair into a bound state, can be scaled by the relative velocity v between the quark and antiquark and obtained by lattice QCD or extracted from the experiment. v^2 is about 0.2–0.3 for charmonium and about 0.08–0.1 for bottomonium. The color-octet mechanism (COM) was introduced here. The heavy quark pair should be a color-singlet (CS) bound state at long distances, but it may be in a color-octet (CO) state at short distances. NRQCD had achieved great success since it was proposed. The COM was applied to cancel the infrared divergences in the decay widths of P-wave [2,3] and D-wave [4,5] heavy quarkonium. However, difficulties were still encountered. The large discrepancy between the experimental data and the theoretical calculation of J/ψ and ψ' unpolarization and polarization production at Tevatron is an interesting phenomenon that can verify NRQCD when solved [6,7]. Theoretical prediction with COM contributions was introduced and was found to fit with the experimental data on J/ψ production at Tevatron [8]. However, the CO contributions from gluon fragmentation indicated that the J/ψ was transversely polarized at large p_T, which is inconsistent with the experimental data [6].

The next-to-leading order (NLO) QCD corrections and other possible solutions for J/ψ hadroproduction were calculated to resolve the J/ψ hadronic production and polarization puzzle [9,10]. The calculation enhanced the CS cross sections at large p_T by approximately an order of magnitude. However, the large discrepancy between the CS predictions and experimental data remains unsolved. The relativistic correction to CS J/ψ hadroproduction was insignificant [11]. The NLO QCD corrections of COM J/ψ hadroproduction were also calculated to formulate a possible solution to the long-standing J/ψ polarization puzzle [12–14]. The spin-flip interactions in the spin density matrix of the hardronization of a color-octet charm quark pair had been examined in Ref. [15]. A similar large discrepancy was found in double-charmonium production at B factories [16–18]. A great deal of work had been performed on this area, and these discrepancies can apparently be resolved by including NLO QCD corrections [19–22] and relativistic corrections [23–26]. The data from B factories highlight that the COM LDMEs of J/ψ

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production may be smaller than previously expected [25–29]. Relativistic corrections have also been studied in Ref. [30] for heavy quarkonium decay, in Ref. [31] for J/ψ photoproduction, in Ref. [32] for J/ψ production in b decay, and in Ref. [33] for gluon fragmentation into spin triplet S-wave quarkonium. More information about heavy quarkonium physics can be found in Refs. [34–36].

In this paper, the effect of relativistic corrections to the COM J/ψ hadroproduction at Tevatron and LHC were estimated based on NRQCD. The short distance coefficients were calculated up to $\mathcal{O}(v^2)$. Many free LDMEs were realized at $\mathcal{O}(v^2)$, which were estimated according to the velocity scaling rules of NRQCD with $v^2 = 0.23$ [37].

The paper is organized as follows. In Sec. II, the frame of calculation is introduced for the relativistic correction of both the S-and P-wave states in NRQCD frame. Section III provides the numerical result. Finally, a brief summary of this work is presented.

II. RELATIVISTIC CORRECTIONS OF CROSS SECTION IN NRQCD

We only consider J/ψ direct production at high energy hadron colliders, which contributes 70% to the prompt cross section. The differential cross section of direct production can be obtained by integrating the cross sections of parton level as the following expression:

$$d\sigma(p + p(\bar{p}) \to J/\psi + X) = \sum_{a,b,d} \int dx_1 dx_2 f_{a/p}(x_1) f_{b/p(\bar{p})}(x_2) \times d\hat{\sigma}(a + b \to J/\psi + d), \quad (1)$$

where $f_{a(b)/p(\bar{p})}(x_i)$ is the parton distribution function (PDF), and x_i is the parton momentum fraction denoted by the fraction parton carried from proton or antiproton. The sum is over all the partonic subprocesses including

$$g + g \to J/\psi + g, \qquad g + q(\bar{q}) \to J/\psi + q(\bar{q}), \qquad q + \bar{q} \to J/\psi + g.$$

As shown at the beginning of this paper, under the NRQCD frame, the computation to cross section of each subprocess can be divided into two parts: short distance coefficients and LDMEs:

$$d\hat{\sigma}(a(k_1) + b(k_2) \to J/\psi(P) + d(k_3)) = \sum_n \frac{F_n(ab)}{m_c^{d_n-4}} \langle 0|\mathcal{O}_n^{J/\psi}|0\rangle. \quad (2)$$

On the right-hand side of the equation, the cross section is expanded to sensible Fock states noted by the subscript n. F_n, i.e., short distance coefficients, which describe the process that produces intermediate $Q\bar{Q}$ in a short range before heavy quark and antiquark hadronization to the physical meson state. Here we use initial partons to mark the short distance coefficients for different subprocesses. $\langle 0|\mathcal{O}_n^{J/\psi}|0\rangle$ are the long distance matrix elements that represent the hadronization $Q\bar{Q}$ evolutes to the CS final state by emitting soft gluons. $\mathcal{O}_n^{J/\psi}$ are local four fermion operators. The factor of $m_c^{d_n-4}$ is introduced to make F_n dimensionless.

In this section, our calculation on the differential cross section for this process in the NRQCD factorization formula is divided into three parts, namely, kinematics, long distance matrix elements, and short distance coefficients.

A. Kinematics

We denote the three relative momenta between heavy quark and antiquark as $2\vec{q}$, with $|\vec{q}| \sim m_c v$, in J/ψ rest frame, where m_c is the mass of charm quark and v is the three relative velocity of quark or antiquark in this frame. Thus, the momenta for the quark and antiquark are expressed as [24,38,39]

$$p_c = (E_q, \vec{q}), \qquad p_{\bar{c}} = (E_q, -\vec{q}), \quad (3)$$

where $E_q = \sqrt{m_c^2 + |\vec{q}|^2}$ is the rest energy of both the quark and antiquark, and $2E_q$ is the invariable mass of J/ψ. When boosting to an arbitrary frame,

$$p_c \to \tfrac{1}{2}P + q, \qquad p_{\bar{c}} \to \tfrac{1}{2}P - q, \quad (4)$$

where P is the four momenta of J/ψ, and q receives the boost from $(0, \vec{q})$.

The Lorentz invariant Mandelstam variables are defined as

$$\begin{aligned} s &= (k_1 + k_2)^2 = (P + k_3)^2, \\ t &= (k_1 - P)^2 = (k_2 - k_3)^2, \\ u &= (k_1 - k_3)^2 = (k_2 - P)^2. \end{aligned}$$

with the relationship $s + t + u = P^2 = 4E_q^2$. Here, s is $|\vec{q}|^2$ independence. To expand t, u in terms of $E_q(i.e.|\vec{q}|^2)$, we can first write down t, u in the center of initial partons mass frame:

$$\begin{aligned} t(|\vec{q}|) &= -(s - 4E_q^2)(1 - cos\theta)/2 = \frac{s - 4E_q^2}{s - 4m_c^2} t(0), \\ u(|\vec{q}|) &= -(s - 4E_q^2)(1 + cos\theta)/2 = \frac{s - 4E_q^2}{s - 4m_c^2} u(0), \end{aligned} \quad (5)$$

where $t(0)$, $u(0)$ are Lorentz invariants of $|\vec{q}|^2$ independence and satisfies $s + t(0) + u(0) = 4m_c^2$. These relations between $t(|\vec{q}|)(u(|\vec{q}|))$ and $t(0)(u(0))$ are also satisfied when boosting to arbitrary frame. In our subsequent calculation and result, we adopt $t(u)$ to represent $t(0)(u(0))$ directly for simplification.

The FEYNARTS [40] package was used to generate Feynman diagrams and amplitudes, and the FEYNCALC

[41] package was used to handle amplitudes. The numerical phase space was integrated with Fortran.

B. Long distance matrix elements

According to NRQCD factorization, the differential cross section of each partonic subprocess up to next order in v^2 to CS state ${}^3S_1^{[1]}$ and CO states ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_J^{[8]}$ can be expressed as

$$\begin{aligned} d\sigma = {} & d\sigma_{lo}[{}^3S_1^{[1]}] + d\sigma_{lo}[{}^1S_0^{[8]}] + d\sigma_{lo}[{}^3S_1^{[8]}] \\ & + d\sigma_{lo}[{}^3P_J^{[8]}] + d\sigma_{rc}[{}^3S_1^{[1]}] + d\sigma_{rc}[{}^1S_0^{[8]}] \\ & + d\sigma_{rc}[{}^3S_1^{[8]}] + d\sigma_{rc}[{}^3P_J^{[8]}]. \end{aligned} \tag{6}$$

In this expression, relativistic correction parts, denoted as "rc," can easily be distinguished from LO, denoted as "lo." Superscript [1] corresponds to CS, and Superscript [8] corresponds to CO. In addition, each differential cross section to different Fock states should be divided in short distance coefficient part and LDMEs. We can introduce $F({}^{2s+1}L_J^{[c]})$ to express the short distance coefficient of the LO cross section, corresponding to $G({}^{2s+1}L_J^{[c]})$ for relativistic correction. Many LDMEs are presented, all of which are denoted by $\langle 0|\mathcal{O}^{J/\psi}({}^{2s+1}L_J^{[c]})|0\rangle$ and $\langle 0|\mathcal{P}^{J/\psi}({}^{2s+1}L_J^{[c]})|0\rangle$ for the LO and relativistic correction term, respectively. The explicit expressions of the ten four-fermion operators are [1]

$$\begin{aligned}
\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|0\rangle &= \langle 0|\chi^\dagger\sigma^i\psi(a_\psi^\dagger a_\psi)\psi^\dagger\sigma^i\chi|0\rangle, \\
\langle 0|\mathcal{P}^{J/\psi}({}^3S_1^{[1]})|0\rangle &= \left\langle 0\left|\frac{1}{2}\left[\chi^\dagger\sigma^i\psi(a_\psi^\dagger a_\psi)\psi^\dagger\sigma^i\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi + \text{H.c.}\right]\right|0\right\rangle, \\
\langle 0|\mathcal{O}^{J/\psi}({}^1S_0^{[8]})|0\rangle &= \langle 0|\chi^\dagger T^a\psi(a_\psi^\dagger a_\psi)\psi^\dagger T^a\chi|0\rangle, \\
\langle 0|\mathcal{P}^{J/\psi}({}^1S_0^{[8]})|0\rangle &= \left\langle 0\left|\frac{1}{2}\left[\chi^\dagger T^a\psi(a_\psi^\dagger a_\psi)\psi^\dagger T^a\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi + \text{H.c.}\right]\right|0\right\rangle, \\
\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[8]})|0\rangle &= \langle 0|\chi^\dagger T^a\sigma^i\psi(a_\psi^\dagger a_\psi)\psi^\dagger T^a\sigma^i\chi|0\rangle, \\
\langle 0|\mathcal{P}^{J/\psi}({}^3S_1^{[8]})|0\rangle &= \left\langle 0\left|\frac{1}{2}\left[\chi^\dagger T^a\sigma^i\psi(a_\psi^\dagger a_\psi)\psi^\dagger T^a\sigma^i\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi + \text{H.c.}\right]\right|0\right\rangle, \\
\langle 0|\mathcal{O}^{J/\psi}({}^3P_0^{[8]})|0\rangle &= \frac{1}{3}\left\langle 0\left|\chi^\dagger T^a\left(-\frac{i}{2}\overleftrightarrow{D}\cdot\sigma\right)\psi(a_\psi^\dagger a_\psi)\psi^\dagger T^a\left(-\frac{i}{2}\overleftrightarrow{D}\cdot\sigma\right)\chi\right|0\right\rangle, \\
\langle 0|\mathcal{O}^{J/\psi}({}^3P_1^{[8]})|0\rangle &= \frac{1}{2}\left\langle 0\left|\chi^\dagger T^a\left(-\frac{i}{2}\overleftrightarrow{D}\times\sigma\right)\psi(a_\psi^\dagger a_\psi)\psi^\dagger T^a\left(-\frac{i}{2}\overleftrightarrow{D}\times\sigma\right)\chi\right|0\right\rangle, \\
\langle 0|\mathcal{O}^{J/\psi}({}^3P_2^{[8]})|0\rangle &= \left\langle 0\left|\chi^\dagger T^a\left(-\frac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}\right)\psi(a_\psi^\dagger a_\psi)\psi^\dagger T^a\left(-\frac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}\right)\chi\right|0\right\rangle, \\
\langle 0|\mathcal{P}^{J/\psi}({}^3P_J^{[8]})|0\rangle &= \left\langle 0\left|\frac{1}{2}\left[\chi^\dagger T^a\left(-\frac{i}{2}\overleftrightarrow{D}^i\sigma^j\right)\psi(a_\psi^\dagger a_\psi)\psi^\dagger T^a\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\left(-\frac{i}{2}\overleftrightarrow{D}^i\sigma^j\right)\chi + \text{H.c.}\right]\right|0\right\rangle,
\end{aligned} \tag{7}$$

where χ and ψ are the Pauli spinors describing anticharm quark creation and charm quark annihilation, respectively. T is the $SU(3)$ color matrix. σ is the Pauli matrices and $\mathbf{D}$ is the gauge-covariant derivative with $\overleftrightarrow{\mathbf{D}} = \overrightarrow{\mathbf{D}} - \overleftarrow{\mathbf{D}}$. $\overleftrightarrow{D}^{(i}\sigma^{j)}$ is used as the notation for the symmetric traceless component of a tensor: $\overleftrightarrow{D}^{(i}\sigma^{j)} = (\overleftrightarrow{D}^i\sigma^j + \overleftrightarrow{D}^i\sigma^j)/2 - \overleftrightarrow{D}^k\sigma^k\delta^{ij}/3$. Here we have

$$v^2 = \frac{\langle 0|\mathcal{P}^{J/\psi}({}^{2s+1}L_J^{[c]})|0\rangle}{m_c^2\langle 0|\mathcal{O}^{J/\psi}({}^{2s+1}L_J^{[c]})|0\rangle}. \tag{8}$$

It should be noted that

$$\begin{aligned} \langle 0|\mathcal{O}^{J/\psi}({}^3P_J^{[8]})|0\rangle &= (2J+1)(1+\mathcal{O}(v^2))\langle 0|\mathcal{O}^{J/\psi}({}^3P_0^{[8]})|0\rangle, \\ \langle 0|\mathcal{P}^{J/\psi}({}^3P_J^{[8]})|0\rangle &= (2J+1)(1+\mathcal{O}(v^2))\langle 0|\mathcal{P}^{J/\psi}({}^3P_0^{[8]})|0\rangle \\ &\sim \mathcal{O}(v^2)\langle 0|\mathcal{O}^{J/\psi}({}^3P_J^{[8]})|0\rangle. \end{aligned} \tag{9}$$

To NLO in v^2, we can ignore $\mathcal{O}(v^4)$ terms and set

$$\langle 0|\mathcal{P}^{J/\psi}({}^3P_J^{[8]})|0\rangle = (2J+1)\langle 0|\mathcal{P}^{J/\psi}({}^3P_0^{[8]})|0\rangle.$$

So there are four CO LDMEs for P-wave, four CO LDMEs for S-wave and two CS LDMEs at NLO in v^2. The LDMEs of heavy quarkonium decay may be determined by potential model [23,42], lattice calculations [43], or phenomenological extraction from experimental data [11,44]. But it is very difficult to determine the production of CO LDMEs. Recently, two groups fitted CO LDMEs $\langle 0|\mathcal{O}^{J/\psi}({}^{2s+1}L_J^{[8]})|0\rangle$ to NLO in α_s. It is

$$\begin{aligned} \langle 0|\mathcal{O}^{J/\psi}({}^1S_0^{[8]})|0\rangle &= (8.90\pm 0.98)\times 10^{-2}\,\text{GeV}^3, \\ \langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[8]})|0\rangle &= (0.3\pm 0.12)\times 10^{-3}\,\text{GeV}^3, \\ \langle 0|\mathcal{O}^{J/\psi}({}^3P_0^{[8]})|0\rangle/m_c^2 &= (0.56\pm 0.21)\times 10^{-2}\,\text{GeV}^3, \end{aligned} \tag{10}$$

with data of J/ψ production and polarization at $p_t >$ 7 GeV at Tevatron in Ref. [13] and

$$\langle 0|\mathcal{O}^{J/\psi}({}^1S_0^{[8]})|0\rangle = (4.50 \pm 0.72) \times 10^{-2}\ \mathrm{GeV}^3,$$
$$\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[8]})|0\rangle = (3.12 \pm 0.93) \times 10^{-3}\ \mathrm{GeV}^3,$$
$$\langle 0|\mathcal{O}^{J/\psi}({}^3P_0^{[8]})|0\rangle = (-1.21 \pm 0.35) \times 10^{-2}\ \mathrm{GeV}^5, \quad (11)$$

with data of J/ψ production at $p_t > 3$ GeV at Tevatron and $p_T > 2.5$ GeV at HERA in Ref. [14]. The two series CO LDMEs are not consistent with each other. For the three CO P-wave LDMEs $\langle 0|\mathcal{O}^{J/\psi}({}^3P_J^{[8]})|0\rangle$, it is hard to determine. To simplify the discussion of the numerical result, it is assumed that

$$\langle 0|\mathcal{O}^{J/\psi}({}^3P_J^{[8]})|0\rangle = (2J+1) < 0|\mathcal{O}^{J/\psi}({}^3P_0^{[8]})|0\rangle. \quad (12)$$

At the same time, we can estimate the relation between their order from the Gremm-Kapustin relation [45] in the weak-coupling regime

$$v^2 = v_1^2 = v_8^2 = \frac{M_{J/\psi} - 2m_c^{\mathrm{pole}}}{2m_c^{\mathrm{QCD}}}, \quad (13)$$

where m_c^{QCD} is the mass of charm quark that appears in the NRQCD actions and m_c^{pole} is the pole mass of charm quark. This equation was given only for CS in Ref. [45]. This is the same with Ref. [33], and we can get $v_1^2 = v_8^2$. If we select $M_{J/\psi} = 3.1$ GeV and $m_c^{\mathrm{QCD}} = m_c^{\mathrm{pole}} = 1.39$ GeV, we can get $v^2 \sim 0.23$.

After those presses, there are three CO LDMEs in the numerical calculation.

C. Short distance coefficients calculation

The short distance coefficients can be evaluated by matching the computations of perturbative QCD and NRQCD:

$$d\sigma|_{\mathrm{pert\ QCD}} = \sum_n \frac{F_n}{m_c^{d_n-4}} \langle 0|\mathcal{O}_n^{c\bar{c}}|0\rangle|_{\mathrm{pert\ NRQCD}}. \quad (14)$$

The covariant projection operator method should be adopted to compute the expression on the left-hand side of the equation. Using this method, spin-singlet and spin-triplet combinations of spinor bilinears in the amplitudes can be written in covariant form. For the spin-singlet case,

$$\sum_{s\bar{s}} v(s)\bar{u}(\bar{s})\left\langle \frac{1}{2}, s; \frac{1}{2}, \bar{s}|0,0\right\rangle = \frac{1}{2\sqrt{2}(E_q+m)}(-\not{p}_{\bar{c}}+m_c)\gamma_5\frac{\not{P}+2E_q}{2E_q}(\not{p}_c+m_c). \quad (15)$$

For spin-triplet case, the expression is defined as

$$\sum_{s\bar{s}} v(s)\bar{u}(\bar{s})\left\langle \frac{1}{2}, s; \frac{1}{2}, \bar{s}|1, S_z\right\rangle = \frac{1}{2\sqrt{2}(E_q+m)}(-\not{p}_{\bar{c}}+m_c)\not{\epsilon}\frac{\not{P}+2E_q}{2E_q}(\not{p}_c+m_c), \quad (16)$$

where ϵ denotes the polarization vector of the spin-triplet state. In our calculation, Dirac spinors are normalized as $\bar{u}u = -\bar{v}v = 2m_c$.

The differential cross section of each state then satisfies:

$$d\sigma({}^{(2s+1)}L_J^{[c]}) \sim \bar{\sum}|\mathcal{M}(a+b \to (c\bar{c})({}^{(2s+1)}L_J^{[c]}) + d)|^2 \times \langle 0|\mathcal{O}^{J/\psi}({}^{2s+1}L_J^{[c]})|0\rangle, \quad (17)$$

where $\bar{\sum}$ means sum over the final state color and polarization and average over initial states. According to this expression and Eq. (8), expanding the cross section to next leading order of v^2 is to expand the amplitude squared on the right side of the above expression to $\mathcal{O}(|\vec{q}|^2)$.

Next, we prepare to expand the short distance coefficients to the next order in v^2. First, we expand each Fock state amplitude, including the S-wave and P-wave states, in terms of the relative momentum $|\vec{q}|$:

$$\mathcal{M}(a+b \to (c\bar{c})({}^3S_1^{[1,8]}) + d) = \epsilon_\rho\left(\mathcal{M}_t^\rho|_{q=0} + \frac{1}{2}q^\alpha q^\beta \frac{\partial^2\left(\sqrt{\frac{m_c}{E_q}}\mathcal{M}_t^\rho\right)}{\partial q^\alpha \partial q^\beta}\Bigg|_{q=0}\right) + \mathcal{O}(q^4), \quad (18)$$

$$\mathcal{M}(a+b \to (c\bar{c})({}^1S_0^{[8]}) + d) = \mathcal{M}_s|_{q=0} + \frac{1}{2}q^\alpha q^\beta \frac{\partial^2\left(\sqrt{\frac{m_c}{E_q}}\mathcal{M}_s\right)}{\partial q^\alpha \partial q^\beta}\Bigg|_{q=0} + \mathcal{O}(q^4), \quad (19)$$

$$\mathcal{M}(a+b \to (c\bar{c})({}^3P_J^{[8]}) + d) = \epsilon_\rho(s_z)\epsilon_\sigma(L_z)\left(\frac{\partial \mathcal{M}_t^\rho}{\partial q^\sigma}\Bigg|_{q=0} + \frac{1}{6}q^\alpha q^\beta \frac{\partial^3\left(\sqrt{\frac{m_c}{E_q}}\mathcal{M}_t^\rho\right)}{\partial q^\alpha \partial q^\beta \partial q^\sigma}\Bigg|_{q=0}\right) + \mathcal{O}(q^4). \quad (20)$$

The factor $\sqrt{\frac{m_c}{E_q}}$ comes from the relativistic normalization of $c\bar{c}$ state. Odd power terms of four-momentum q vanish in either the S-wave or the P-wave amplitudes, where $\mathcal{M}_t$ and $\mathcal{M}_s$ are inclusive production amplitudes to triplet and singlet $c\bar{c}$, respectively:

$$\mathcal{M}_s = \sum_{s\bar{s}}\sum_{ij}\left\langle\frac{1}{2}, s; \frac{1}{2}, \bar{s}|0, 0\right\rangle\langle 3i; \bar{3}j|1, 8a\rangle \times \mathcal{A}(a + b \to c^i + \bar{c}^j + d),$$
$$\mathcal{M}_t = \sum_{s\bar{s}}\sum_{ij}\left\langle\frac{1}{2}, s; \frac{1}{2}, \bar{s}|1, S_z\right\rangle\langle 3i; \bar{3}j|1, 8a\rangle \times \mathcal{A}(a + b \to c^i + \bar{c}^j + d).$$

In evaluating the amplitudes in power series in $|\vec{q}|$, it needs to be integrated over the space angle to $\vec{q}$. We can obtain the following replacements to extract the contribution of fixed power of $|\vec{q}|$:

For the S-wave case,

$$q^\alpha q^\beta \to \frac{1}{3}|\vec{q}|^2\Pi^{\alpha\beta}. \tag{21}$$

For the P-wave case,

$$q^\alpha q^\beta q^\sigma \to \frac{1}{5}|\vec{q}|^3[\Pi^{\alpha\beta}\epsilon^\sigma(L_z) + \Pi^{\alpha\sigma}\epsilon^\beta(L_z) + \Pi^{\beta\sigma}\epsilon^\alpha(L_z)], \tag{22}$$

where $\Pi^{\mu\nu} = -g^{\mu\nu} + \frac{P^\mu P^\nu}{P^2}$ and $\epsilon(L_z)$ is the orbital polarization vector of P-wave states. Subsequently, by multiplying the complex conjugate of the amplitude, the amplitude squared up to the next order can be obtained:

$$\sum|\mathcal{M}(^3S_1^{[1,8]})|^2 = \mathcal{M}_t^\rho(0)\mathcal{M}_t^{\lambda *}(0)\sum_{s_z}\epsilon_\rho\epsilon_\lambda^* + \frac{1}{3}|\vec{q}|^2\left[\left(\Pi^{\alpha\beta}\frac{\partial^2\left(\sqrt{\frac{m_c}{E_q}}\mathcal{M}_t^\rho\right)}{\partial q^\alpha\partial q^\beta}\right)_{q=0}\mathcal{M}_t^{*\lambda}(0)\right]\left(\sum_{s_z}\epsilon_\rho\epsilon_\lambda^*\right)_{q=0} + \mathcal{O}(v^4), \tag{23}$$

$$\sum|\mathcal{M}(^1S_0^{[8]})|^2 = \mathcal{M}_s(0)\mathcal{M}_s^*(0) + \frac{1}{3}|\vec{q}|^2\left[\left(\Pi^{\alpha\beta}\frac{\partial^2\left(\sqrt{\frac{m_c}{E_q}}\mathcal{M}_s\right)}{\partial q^\alpha\partial q^\beta}\right)_{q=0}\mathcal{M}_s^*(0)\right] + \mathcal{O}(v^4), \tag{24}$$

$$\sum|\mathcal{M}(^3P_J^{[8]})|^2 = |\vec{q}|^2\frac{\partial\mathcal{M}_t^\rho}{\partial q^\alpha}\bigg|_{q=0}\frac{\partial\mathcal{M}_t^{*\lambda}}{\partial q^\beta}\bigg|_{q=0}\sum_{L_z}\epsilon_\alpha\epsilon_\beta^*\sum_{s_z}\epsilon_\rho\epsilon_\lambda^* + \frac{1}{15}|\vec{q}|^4\bigg[\bigg(\Pi^{\sigma\tau}\bigg(\frac{\partial^3}{\partial q^\alpha\partial q^\sigma\partial q^\tau} + \frac{\partial^3}{\partial q^\sigma\partial q^\alpha\partial q^\tau} + \frac{\partial^3}{\partial q^\tau\partial q^\sigma\partial q^\alpha}\bigg)\bigg(\sqrt{\frac{m_c}{E_q}}\mathcal{M}_t^\rho\bigg)\bigg) \times \frac{\partial\mathcal{M}_t^{*\lambda}}{\partial q^\beta}\bigg(\sum_{L_z}\epsilon_\alpha\epsilon_\beta^*\bigg)\bigg(\sum_{s_z}\epsilon_\rho\epsilon_\lambda^*\bigg)\bigg]_{q=0} + \mathcal{O}(v^6). \tag{25}$$

Any term, which is in the order of $|\vec{q}|^2$, must not be missed to obtain the correction up to the order of v^2. In the three expressions above, the first term on the right side of each equation can be expressed in terms of kinematics variables s, $t(|\vec{q}|)$, $u(|\vec{q}|)$. Here $t(|\vec{q}|)$, $u(|\vec{q}|)$ is $|\vec{q}|$ dependence and should be expanded by Eq. (5). The sum of terms in the order of $|\vec{q}|^2$ in the first term as well as all the second term is the contribution of the next leading order. Orbit polarization sum $\sum_{L_z}$ and spin-triplet polarization sum $\sum_{s_z}$ are equal to $\Pi^{\rho\lambda}(\Pi^{\alpha\beta})$. According to the expression of Π mentioned above, the q dependence of Π only appears in the denominator P^2 which equals to $4E_q^2$ and only contains even powers of four momentum q. So in the computation of unpolarized cross section to next order of v^2 as in Eqs. (23) and (25), expanding the polarization vector in order of v^2 is to handle the sum expression Π.

Therefore, the differential cross section in Eq. (6) takes the following form:

$$d\hat{\sigma}(a + b \to J/\psi + d) = \bigg(\frac{F(^3S_1^{[1]})}{m_c^2}\langle 0|\mathcal{O}^{J/\psi}(^3S_1^{[1]})|0\rangle + \frac{G(^3S_1^{[1]})}{m_c^4}\langle 0|\mathcal{P}^{J/\psi}(^3S_1^{[1]})|0\rangle + \frac{F(^1S_0^{[8]})}{m_c^2}\langle 0|\mathcal{O}^{J/\psi}(^1S_0^{[8]})|0\rangle + \frac{G(^1S_0^{[8]})}{m_c^4}\langle 0|\mathcal{P}^{J/\psi}(^1S_0^{[8]})|0\rangle + \frac{F(^3S_1^{[8]})}{m_c^2}\langle 0|\mathcal{O}^{J/\psi}(^3S_1^{[8]})|0\rangle + \frac{G(^3S_1^{[8]})}{m_c^4}\langle 0|\mathcal{P}^{J/\psi}(^3S_1^{[8]})|0\rangle + \frac{F(^3P_0^{[8]})}{m_c^2}\langle 0|\mathcal{O}^{J/\psi}(^3P_0^{[8]})|0\rangle + \frac{G(^3P_0^{[8]})}{m_c^4}\langle 0|\mathcal{P}^{J/\psi}(^3P_0^{[8]})|0\rangle\bigg) \times (1 + \mathcal{O}(v^4)). \tag{26}$$

The explicit expressions of the short distance coefficients to the relativistic correction of CO states $^1S_0^{[8]}$ and $^3S_1^{[8]}$, $^3P_J^{[8]}$ for partonic processes $gg \to J/\psi g$, $gq(\bar{q}) \to J/\psi q(\bar{q})$ and $q\bar{q} \to J/\psi g$ are relegated to the Appendix. The result of our relativistic correction of $^3S_1^{[1]}$ is consistent with that of Ref. [11] and was not given in this paper.

III. NUMERICAL RESULT AND DISCUSSION

We adopt the gluon distribution function CTEQ6 PDFs [46]. And the charm quark is set as $m_c = 1.5$ GeV. The ratios of the short distance coefficient between LO F and its relativistic correction G at the Tevatron with $\sqrt{s} = 1.96$ TeV and at the LHC with $\sqrt{s} = 7$ TeV or $\sqrt{s} = 14$ TeV are presented in Fig. 1. The ratios of $R[n] = G[n]/F[n]$ at the Tevatron and at the LHC are very close at large p_T. In the large p_T limit,

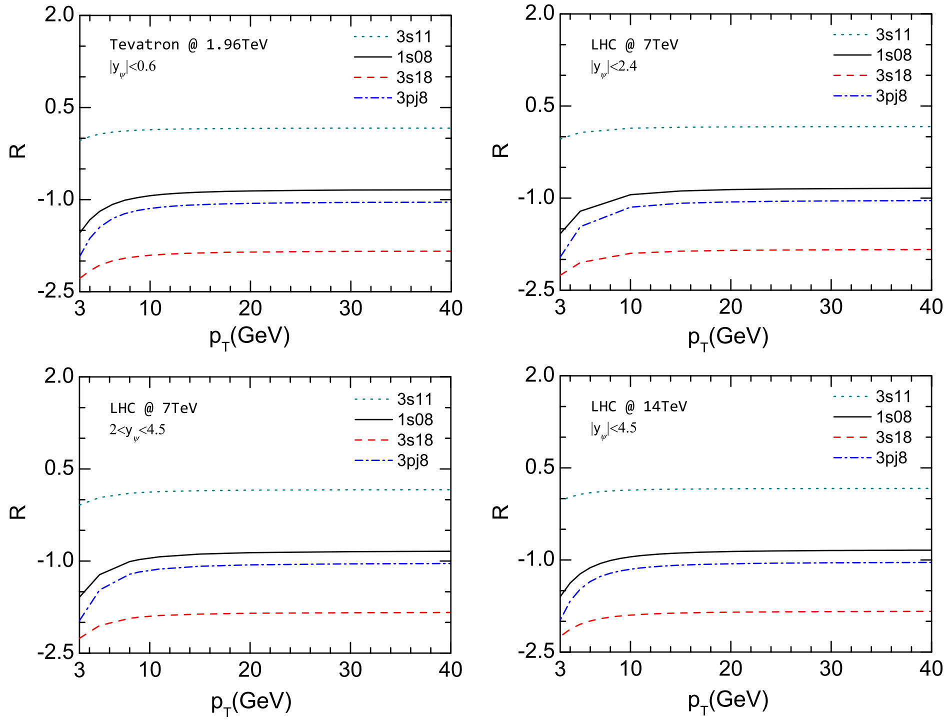

FIG. 1 (color online). The ratios of the short distance coefficient between LO F and its relativistic correction G at the Tevatron with $\sqrt{s} = 1.96$ TeV and at the LHC with $\sqrt{s} = 7$ TeV or $\sqrt{s} = 14$ TeV.

$$-\frac{M^2}{u} \sim -\frac{M^2}{t} < \frac{M^2}{p_T^2} \sim 0, \tag{27}$$

where M is the J/ψ mass. Then we can expand the short distance coefficients with M. The ratios of first order in the expansion are

$$\begin{aligned} R(^3S_1^{[1]})|_{p_T\gg M} &= \frac{G(^3S_1^{[1]})}{F(^3S_1^{[1]})}\Bigg|_{p_T\gg M} \sim \frac{1}{6}, \\ R(^1S_0^{[8]})|_{p_T\gg M} &= \frac{G(^1S_0^{[8]})}{F(^1S_0^{[8]})}\Bigg|_{p_T\gg M} \sim -\frac{5}{6}, \\ R(^3S_1^{[8]})|_{p_T\gg M} &= \frac{G(^3S_1^{[8]})}{F(^3S_1^{[8]})}\Bigg|_{p_T\gg M} \sim -\frac{11}{6}, \\ R(^3P_0^{[8]})|_{p_T\gg M} &= \frac{G(^3P_0^{[8]})}{F(^3P_0^{[8]})}\Bigg|_{p_T\gg M} \sim -\frac{31}{30}. \end{aligned} \tag{28}$$

These asymptotic behaviors of the ratios to each state are same for all the partonic subprocesses of gg, $gq(\bar{q})$, and qq. It is consistent with the curves in Fig. 1. The ratio $R(^3S_1^{[1]})$ is consistent with Ref. [11], and the ratio $R(^3S_1^{[8]})$ is consistent with Ref. [33].

As discussed in Sec. II, the LDMEs of relativistic correction are depressed by approximately 0.23 to LO. If we fix LDMEs $\langle 0|\mathcal{O}|0\rangle$ and estimate $\langle 0|\mathcal{P}|0\rangle$ through the velocity scaling rule with adopting $v^2 = 0.23$, then the LO cross sections of CO subprocesses are reduced by about a factor of $20\% \sim 40\%$ at large p_T at both Tevatron and LHC. In the CS case, the LO cross sections are enhanced by approximately 4% by the NLO relativistic corrections.[1]

The QCD corrections of both CO and CS states had been calculated in Ref. [12–14]. Ratios of NLO $\mathcal{O}(v^2)$, $\mathcal{O}(\alpha_s)$, and $\mathcal{O}(\alpha_s, v^2)$ to LO cross sections of J/ψ production at Tevatron are presented in Fig. 2. Here $v^2 = 0.23$, and QCD corrections are taken from Refs. [12–14]. The K factor of NLO QCD corrections is very large for $^3P_0^{[8]}$ and $^3S_1^{[1]}$ at large p_T, and it is about 1.3 for $^3S_1^{[8]}$ and 1.5 for $^1S_0^{[8]}$.

The ratio of $^3S_1^{[8]}$ is approximately $-11/6$. In the large p_T limit, the dominate contribution of this subprocess is $g^* \to c\bar{c}(^3S_1^{[8]})$. The propagator of virtual gluon g^* is proportional to $1/E_q^2$. This term offers a factor of -2 to the ratio $R(^3S_1^{[8]})$. And the factor of -2 at large p_T is same for the polarization of $^3S_1^{[8]}$ states. At the same time, the $^1S_0^{[8]}$

[1]In Ref. [11], the ratio of the CS cross sections enhanced by NLO relativistic corrections is about 1%. The difference comes from adopting the different LDMEs:

$$\begin{aligned} \langle 0|\mathcal{O}^{J/\psi}(^3S_1^{[1]})|0\rangle &= 1.64\ \text{GeV}^3, \\ \langle 0|\mathcal{P}^{J/\psi}(^3S_1^{[1]})|0\rangle &= 0.320\ \text{GeV}^5. \end{aligned} \tag{29}$$

Then

$$\langle 0|\mathcal{P}^{J/\psi}(^3S_1^{[1]})|0\rangle / \langle 0|\mathcal{O}^{J/\psi}(^3S_1^{[1]})|0\rangle / m_c^2 = 0.087, \tag{30}$$

which is much smaller than $v^2 \approx 0.23$.

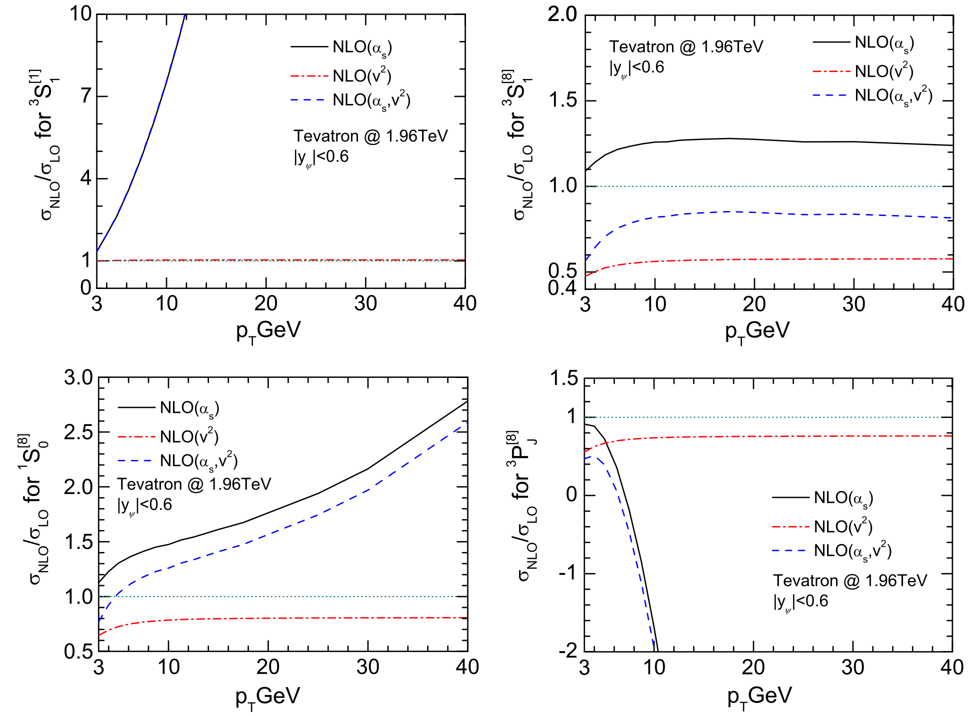

FIG. 2 (color online). Ratios of NLO$\mathcal{O}(v^2)$, $\mathcal{O}(\alpha_s)$, and $\mathcal{O}(\alpha_s, v^2)$ to LO cross sections of J/ψ production at Tevatron. Here $v^2 = 0.23$, and QCD corrections are taken form Refs. [12,13].

state is a scalar state and contributes to unpolarized production of J/ψ, and the K factor of NLO QCD corrections is much larger than relativistic corrections for $^3P_0^{[8]}$ and $^3S_1^{[1]}$ at large p_T. So the J/ψ polarization at large p_T is insensitive to the relativistic corrections.

If we fit the differential cross section of prompt J/ψ production at $p_t > 7$ GeV at the Tevatron [7] to NLO in α_s and v^2 [13], we can get CO LDMEs but with large errors for $^3S_1^{[8]}$ and $^3P_J^{[8]}$ states. In Ref. [13], they considered two combined LDMEs to fit the data:

$$M_{0,r_0}^{J/\psi} = <0|\mathcal{O}^{J/\psi}(^1S_0^{[8]})|0> + \frac{r_0}{m_c^2} < 0|\mathcal{O}^{J/\psi}(^3P_0^{[8]})|0>,$$
$$M_{1,r_1}^{J/\psi} = <0|\mathcal{O}^{J/\psi}(^3S_1^{[8]})|0> + \frac{r_1}{m_c^2} < 0|\mathcal{O}^{J/\psi}(^3P_0^{[8]})|0> . \quad (31)$$

Here r_0, r_1 determined from the short distance coefficient decomposition holding within a small error

$$d\hat{\sigma}[^3P_J^{[8]}] = r_0 d\hat{\sigma}[^1S_0^{[8]}] + r_1\hat{\sigma}[^3S_1^{[8]}]. \quad (32)$$

In Ref. [13], they found $r_0 = 3.9$ and $r_1 = -0.56$ using the NLO(α_s) results. When considering relativistic corrections as well as NLO(α_s) data we find $r_0 = 3.64$ and $r_1 = -0.84$. Then we can fit CDF J/ψ prompt production data to determine these two LDMEs as Fig. 3 shows (here, we do not consider the effect of the feed-down cross section form χ_{cJ} and $\psi\prime$ to the fit):

$$M_{0,3.64}^{J/\psi} = (11.0 \pm 0.3) \times 10^{-2}\ \text{GeV}^3,$$
$$M_{1,-0.84}^{J/\psi} = (0.16 \pm 0.02) \times 10^{-2}\ \text{GeV}^3, \quad (33)$$

comparing with fitting results only considering NLO(α_s) data

$$M_{0,3.9}^{J/\psi} = (9.0 \pm 0.3) \times 10^{-2}\ \text{GeV}^3,$$
$$M_{1,-0.56}^{J/\psi} = (0.13 \pm 0.02) \times 10^{-2}\ \text{GeV}^3. \quad (34)$$

About 20% difference is shown for either LDMEs between the two sets. Complete NLO(α_s) calculations show that

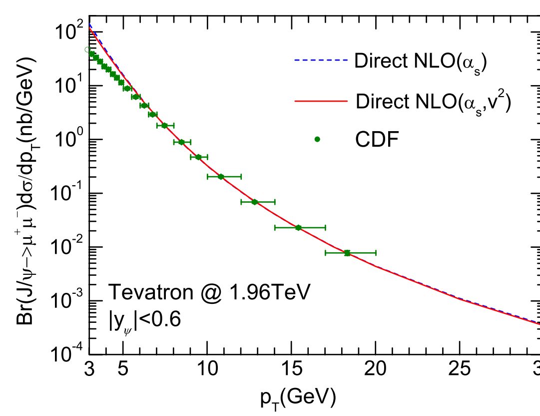

FIG. 3 (color online). Transverse momentum distribution of prompt J/ψ production at Tevatron. By fitting the CDF experimental data, we obtained the two sets of combined LDMEs $M_{0,r0}^{J/\psi}$ and $M_{1,r1}^{J/\psi}$ using the results of NLO(α_s) and NLO(α_s, v^2) short distance coefficients, respectively.

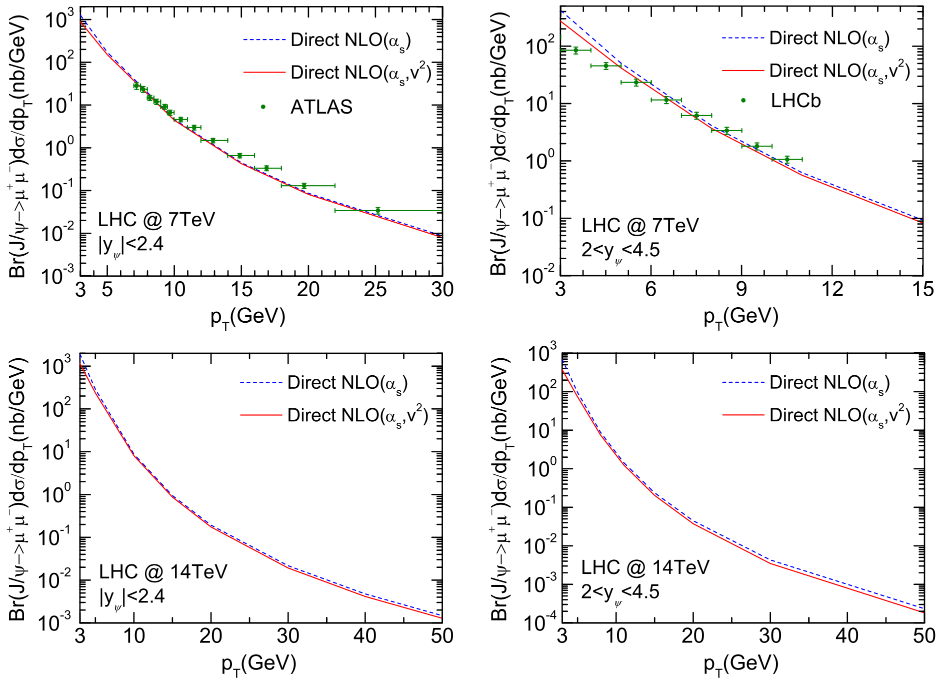

FIG. 4 (color online). Transverse momentum distribution of NLO(α_s) and NLO(α_s, v^2) to J/ψ direct production. The LHC experimental data can be found in Refs. [48,49].

LDMEs fitting the Tevatron data agree with all the LHC data. However, it does not agree well at the small p_T region [13]. The K factor curves in Fig. 2 imply that relativistic corrections suppress the tend of the K factors of NLO(α_s) mainly at the small p_T region. To investigate the effect of new fitting LDMEs to the total cross section at hadron colliders, especially at the small p_T region, we compare the cross sections of NLO(α_s) and NLO(α_s, v^2) at the LHC using the corresponding set of LDMEs above, and the results are shown in Fig. 4. NLO(α_s, v^2) results suppressed by about 50% ~ 20% along with p_T increasing comparing with NLO(α_s) results. But the calculations of relativistic correction of direct production fail to explain the trend of experimental data at the small p_T region, and it is still an open problem. It is expected to solve the problem by two ways. First, contribution from the feed-down of high excited charmonia production process as $p + p(\bar{p}) \longrightarrow \chi_{cJ} + X$ and $p + p(\bar{p}) \longrightarrow \psi\prime + X$ may account for 30% to prompt J/ψ production. In this case, the calculations of relativistic correction to feed-down parts are necessary. Second, recently, the calculation method of resummation of relativistic correction had been presented by Bodwin, Lee, and Yu and applied to calculate the resummation of relativistic correction to exclusive production $e^+e^- \longrightarrow J/\psi\eta_c$ at e^+e^- colliders that payed an important contribution to total cross section [47]. Whether contributions of resummation of relativistic correction may play an important role, further calculations are needed.

IV. SUMMARY

In summary, we calculate the relativistic correction terms to CO states for J/ψ production at the Tevatron and at the LHC. The short distance coefficient ratios of relativistic correction to LO for CO states ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, and ${}^3P_J^{[8]}$ at large p_T are approximately $-5/6$, $-11/6$, and $-31/30$, respectively, and it is $1/6$ for the color-singlet state ${}^3S_1^{[1]}$. If NLO long distance matrix elements are estimated through the velocity scaling rule with adopting $v^2 = 0.23$, the cross sections are reduced by about a factor of 20–40% at large p_T to LO results of CO states at both the Tevatron and the LHC. Compared with the relativistic corrections to the CS state, the LO cross sections are enhanced by a factor of 4%. Thus, the result may affect the production of J/ψ at hadronic colliders. Because of the large results of QCD corrections at large p_T especially to ${}^3P_J^{[8]}$ states, relativistic corrections are small, even ignored, along with p_T increasing. But relativistic corrections can also affect the total cross section with a considerable contribution. We computed the unpolarized cross sections at the LHC with CO LDMEs extracted from the fit to J/ψ direct production at the Tevatron, and the results of NLO(α_s, v^2) suppress that of NLO(α_s) by about 20–50% at different p_T regions. These results indicate that relativistic corrections may play an important role in J/ψ production at the Tevatron and LHC.

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APPENDIX: SHORT DISTANCE COEFFICIENTS

The short distance coefficients of ${}^1S_0^{[8]}$ for $gg \to J/\psi g$ subprocess were

$$\begin{aligned}\frac{F_{gg}({}^1S_0^{[8]})}{m_c^2} &= \frac{1}{16\pi s^2}\frac{1}{64}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\times 640[M^{12}(t^2+tu+u^2) - M^{10}(4t^3+7t^2u+7tu^2+4u^3)\\ &+ M^8(8t^4+21t^3u+27t^2u^2+21tu^3+8u^4) - M^6(10t^5+35t^4u+57t^3u^2+57t^2u^3+35tu^4+10u^5)\\ &+ M^4(8t^6+33t^5u+66t^4u^2+81t^3u^3+66t^2u^4+33tu^5+8u^6) - M^2(t^2+tu+u^2)^2(4t^3+9t^2u+9tu^2+4u^3)\\ &+ (t^2+tu+u^2)^4]/[M(M^2-t)^2t(M^2-u)^2(M^2-t-u)u(t+u)^2], \end{aligned}\tag{A1}$$

$$\begin{aligned}\frac{G_{gg}({}^1S_0^{[8]})}{m_c^4} &= \frac{1}{16\pi s^2}\frac{1}{64}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}1280[5tu(t+u)(t^2+tu+u^2)^4 + 12M^{18}(t^2+tu+u^2)\\ &- 5M^{16}(11t^3+20t^2u+20tu^2+11u^3) + M^{14}(95t^4+280t^3u+358t^2u^2+280tu^3+95u^4)\\ &- 3M^{12}(16t^5+95t^4u+175t^3u^2+175t^2u^3+95tu^4+16u^5) - 2M^{10}(45t^6+72t^5u+21t^4u^2-22t^3u^3+21t^2u^4\\ &+ 72tu^5+45u^6) + M^8(198t^7+678t^6u+1141t^5u^2+1345t^4u^3+1345t^3u^4+1141t^2u^5+678tu^6+198u^7)\\ &- M^6(180t^8+756t^7u+1583t^6u^2+2224t^5u^3+2446t^4u^4+2224t^3u^5+1583t^2u^6+756tu^7+180u^8)\\ &+ M^4(85t^9+408t^8u+1000t^7u^2+1637t^6u^3+2028t^5u^4+2028t^4u^5+1637t^3u^6+1000t^2u^7+408tu^8+85u^9)\\ &- M^2(t^3+2t^2u+2tu^2+u^3)^2(17t^4+30t^3u+30t^2u^2+30tu^3+17u^4)]/[3M^3(M^2-t)^3t(M^2-u)^3u(t+u)^3\\ &\times(-M^2+t+u)]. \end{aligned}\tag{A2}$$

The short distance coefficients of ${}^3S_1^{[8]}$ for $gg \to J/\psi g$ subprocess were

$$\begin{aligned}\frac{F_{gg}({}^3S_1^{[8]})}{m_c^2} &= \frac{1}{16\pi s^2}\frac{1}{64}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{1}{3}256[27(t^2+tu+u^2)^3 + 19M^8(t^2+tu+u^2) - M^6(65t^3+111t^2u+111tu^2+65u^3)\\ &+ M^4(100t^4+227t^3u+300t^2u^2+227tu^3+100u^4) - 27M^2(3t^5+8t^4u+13t^3u^2+13t^2u^3\\ &+ 8tu^4+3u^5)]/[3M^3(M^2-t)^2(M^2-u)^2(t+u)^2], \end{aligned}\tag{A3}$$

$$\begin{aligned}\frac{G_{gg}({}^3S_1^{[8]})}{m_c^4} &= \frac{1}{16\pi s^2}\frac{1}{64}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{1}{3}(-512)[M^{14}(87t^2+22tu+87u^2) + M^{12}(-14t^3+335t^2u+335tu^2-14u^3)\\ &- 2M^{10}(399t^4+1612t^3u+2020t^2u^2+1612tu^3+399u^4) + M^8(2100t^5+8976t^4u+14497t^3u^2+14497t^2u^3\\ &+ 8976tu^4+2100u^5) - M^6(2590t^6+12096t^5u+23855t^4u^2+29314t^3u^3+23855t^2u^4+12096tu^5\\ &+ 2590u^6) + M^4(1620t^7+8498t^6u+19905t^5u^2+29152t^4u^3+29152t^3u^4+19905t^2u^5+8498tu^6\\ &+ 1620u^7) - 27M^2(15t^8+104t^7u+295t^6u^2+510t^5u^3+612t^4u^4+510t^3u^5+295t^2u^6+104tu^7+15u^8)\\ &+ 297tu(t+u)(t^2+tu+u^2)^3]/[9M^5(M^2-t)^3(M^2-u)^3(t+u)^3]. \end{aligned}\tag{A4}$$

The short distance coefficients of ${}^3P_J^{[8]}$ for $gg \to J/\psi g$ subprocess were

$$\frac{F_{gg}(^3P_J^{[8]})}{m_c^4} = \frac{1}{16\pi s^2}\frac{1}{64}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\times 2560[7M^{16}(t^3+2t^2u+2tu^2+u^3) - M^{14}(35t^4+99t^3u+120t^2u^2+99tu^3+35u^4) + M^{12}(84t^5+296t^4u+450t^3u^2+450t^2u^3+296tu^4+84u^5) - 3M^{10}(42t^6+171t^5u+304t^4u^2+362t^3u^3 + 304t^2u^4+171tu^5+42u^6) + M^8(126t^7+577t^6u+1128t^5u^2+1513t^4u^3+1513t^3u^4+1128t^2u^5+577tu^6 + 126u^7) - M^6(84t^8+432t^7u+905t^6u^2+1287t^5u^3+1436t^4u^4+1287t^3u^5+905t^2u^6+432tu^7+84u^8) + M^4(35t^9+204t^8u+468t^7u^2+700t^6u^3+819t^5u^4+819t^4u^5+700t^3u^6+468t^2u^7+204tu^8+35u^9) - M^2(t^2+tu+u^2)^2(7t^6+36t^5u+45t^4u^2+28t^3u^3+45t^2u^4+36tu^5+7u^6) + 3tu(t+u)(t^2+tu+u^2)^4]/[M^3tu(M^2-t)^3(M^2-u)^3(t+u)^3(-M^2+t+u)], \tag{A5}$$

$$\frac{G_{gg}(^3P_J^{[8]})}{m_c^6} = \frac{1}{16\pi s^2}\frac{1}{64}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\times(-1024)[140M^{22}(t^3+2t^2u+2tu^2+u^3) - M^{20}(725t^4+2095t^3u+2596t^2u^2 + 2095tu^3+725u^4) + 6M^{18}(235t^5+978t^4u+1599t^3u^2+1599t^2u^3+978tu^4+235u^5) - M^{16}(705t^6+6528t^5u+16050t^4u^2+20350t^3u^3+16050t^2u^4+6528tu^5+705u^6) + M^{14}(-2190t^7-3022t^6u+5603t^5u^2+15689t^4u^3+15689t^3u^4+5603t^2u^5-3022tu^6-2190u^7) + M^{12}(5400t^8+19278t^7u+25697t^6u^2+19598t^5u^3+14174t^4u^4 + 19598t^3u^5+25697t^2u^6+19278tu^7+5400u^8) - M^{10}(6110t^9+28087t^8u+52760t^7u^2+62879t^6u^3 + 60308t^5u^4+60308t^4u^5+62879t^3u^6+52760t^2u^7+28087tu^8+6110u^9) + M^8(4055t^{10}+22235t^9u + 50834t^8u^2+74420t^7u^3+83867t^6u^4+84706t^5u^5+83867t^4u^6+74420t^3u^7+50834t^2u^8+22235tu^9 + 4055u^{10}) - M^6(1530t^{11}+10029t^{10}u+27765t^9u^2+49691t^8u^3+67682t^7u^4+76683t^6u^5+76683t^5u^6 + 67682t^4u^7+49691t^3u^8+27765t^2u^9+10029tu^{10}+1530u^{11}) + M^4(255t^{12}+2250t^{11}u+8158t^{10}u^2 + 18865t^9u^3+32387t^8u^4+43880t^7u^5+48446t^6u^6+43880t^5u^7+32387t^4u^8+18865t^3u^9+8158t^2u^{10} + 2250tu^{11}+255u^{12}) - M^2tu(t^2+tu+u^2)^2(150t^7+726t^6u+1575t^5u^2+2117t^4u^3+2117t^3u^4 + 1575t^2u^5+726tu^6+150u^7) + 31t^2u^2(t+u)^2(t^2+tu+u^2)^4]/[M^5tu(M^2-t)^4(M^2-u)^4(t+u)^4 \times(-M^2+t+u)]. \tag{A6}$$

The short distance coefficients of $^1S_0^{[8]}$ for $q\bar{q}\to J/\psi g$ subprocess were

$$\frac{F_{q\bar{q}}(^1S_0^{[8]})}{m_c^2} = -\frac{1}{16\pi s^2}\frac{1}{9}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{160(t^2+u^2)}{3M(t+u)^2(-M^2+t+u)}, \tag{A7}$$

$$\frac{G_{q\bar{q}}(^1S_0^{[8]})}{m_c^4} = \frac{1}{16\pi s^2}\frac{1}{9}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{1600(t^2+u^2)}{9M^3(t+u)^2(-M^2+t+u)}. \tag{A8}$$

The short distance coefficients of $^3S_1^{[8]}$ for $q\bar{q}\to J/\psi g$ subprocess were

$$\frac{F_{q\bar{q}}(^3S_1^{[8]})}{m_c^2} = -\frac{1}{16\pi s^2}\frac{1}{9}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{1}{3}\frac{64(4t^2-tu+4u^2)(2M^4-2M^2(t+u)+t^2+u^2)}{3M^3tu(t+u)^2}, \tag{A9}$$

$$\frac{G_{q\bar{q}}(^3S_1^{[8]})}{m_c^4} = \frac{1}{16\pi s^2}\frac{1}{9}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{1}{3}(-128)[24M^6(4t^2-tu+4u^2) - 14M^4(4t^3+3t^2u+3tu^2+4u^3) - 8M^2(5t^4+11t^3u+3t^2u^2+11tu^3+5u^4) + 11(4t^5+3t^4u+7t^3u^2+7t^2u^3+3tu^4+4u^5)]/[9M^5tu(t+u)^3]. \tag{A10}$$

The short distance coefficients of $^3P_J^{[8]}$ for $q\bar{q}\to J/\psi g$ subprocess were

$$\frac{F_{q\bar{q}}({}^3P_J^{[8]})}{m_c^4} = -\frac{1}{16\pi s^2}\frac{1}{9}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{640(8M^4(t+u)-4M^2(t^2+4tu+u^2)+3(t^3+t^2u+tu^2+u^3))}{3M^3(t+u)^3(-M^2+t+u)}, \tag{A11}$$

$$\frac{G_{q\bar{q}}({}^3P_J^{[8]})}{m_c^6} = \frac{1}{16\pi s^2}\frac{1}{9}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}256[160M^6(t+u)-16M^4(5t^2+17tu+5u^2)+4M^2(t^3-11t^2u-11tu^2+u^3) + 31(t+u)^2(t^2+u^2)]/[3M^5(t+u)^4(-M^2+t+u)]. \tag{A12}$$

The short distance coefficients of ${}^1S_0^{[8]}$ for $gq(\bar{q}) \to J/\psi q(\bar{q})$ subprocess were

$$\frac{F_{gq(\bar{q})}({}^1S_0^{[8]})}{m_c^2} = -\frac{1}{16\pi s^2}\frac{1}{24}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{160(s^2+u^2)}{3M(s+u)^2(-M^2+s+u)}, \tag{A13}$$

$$\frac{G_{gq(\bar{q})}({}^1S_0^{[8]})}{m_c^4} = \frac{1}{16\pi s^2}\frac{1}{24}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{320(M^2(11s^3+23s^2u-su^2+11u^3)-5s(s^3+s^2u+su^2+u^3))}{9M^3(M^2-s)(s+u)^3(M^2-s-u)}. \tag{A14}$$

The short distance coefficients of ${}^3S_1^{[8]}$ for $gq(\bar{q}) \to J/\psi q(\bar{q})$ subprocess were

$$\frac{F_{gq(\bar{q})}({}^3S_1^{[8]})}{m_c^2} = -\frac{1}{16\pi s^2}\frac{1}{24}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{1}{3}\frac{64(4s^2-su+4u^2)(2M^4-2M^2(s+u)+s^2+u^2)}{3M^3su(s+u)^2}, \tag{A15}$$

$$\frac{G_{gq(\bar{q})}({}^3S_1^{[8]})}{m_c^4} = \frac{1}{16\pi s^2}\frac{1}{24}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{1}{3}128[2M^6(20s^3+69s^2u-39su^2+20u^3)-2M^4(40s^4+113s^3u+27s^2u^2 + 10su^3+20u^4)+M^2(108s^5+193s^4u+41s^3u^2+225s^2u^3+su^4+20u^5)-11s(4s^5+3s^4u+7s^3u^2 + 7s^2u^3+3su^4+4u^5)]/[9M^5su(M^2-s)(s+u)^3]. \tag{A16}$$

The short distance coefficients of ${}^3S_J^{[8]}$ for $gq(\bar{q}) \to J/\psi q(\bar{q})$ subprocess were

$$\frac{F_{gq(\bar{q})}({}^3P_J^{[8]})}{m_c^4} = \frac{1}{16\pi s^2}\frac{1}{24}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}\frac{640(8M^4(s+u)-4M^2(s^2+4su+u^2)+3(s^3+s^2u+su^2+u^3))}{3M^3(s+u)^3(-M^2+s+u)}, \tag{A17}$$

$$\frac{G_{gq(\bar{q})}({}^3P_J^{[8]})}{m_c^6} = -\frac{1}{16\pi s^2}\frac{1}{24}\frac{1}{4}\frac{(4\pi\alpha_s)^3}{N_c^2-1}256[8M^6(5s^2+26su+25u^2)+4M^4(s^3-23s^2u-111su^2-19u^3) + M^2(57s^4+226s^3u+166s^2u^2+58su^3+61u^4)-31s(s+u)^2(s^2+u^2)]/[3M^5(M^2-s) \times (s+u)^4(-M^2+s+u)]. \tag{A18}$$

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