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Texture zeros and weak basis transformations in the quark sector of the standard model

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Stimulated by the recent attention given to the texture zeros found in the quark mass matrices sector of the standard model, an analytical method for identifying (or excluding) texture zeros models will be implemented here. Starting from arbitrary quark mass matrices and making a suitable weak basis transformation, we are be able to find an equivalent quark mass matrix. It is shown that the number of nonequivalent quark mass matrix representations is finite. We give exact numerical results for parallel and nonparallel four-texture zeros models. We find that some five-texture zeros *Ansätze* are in agreement with all present experimental data, and we confirm definitively that six-texture zeros of Hermitian quark mass matrices are not viable models anymore.

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I. INTRODUCTION

Although the gauge sector of the standard model (SM) with the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry is very successful, the Yukawa sector of the SM is still poorly understood. The origin of the fermion masses, the mixing angles, and the CP violation remain as open problems in particle physics. There have been a lot of studies of possible fundamental symmetries in the Yukawa coupling matrices of the SM [1–3]. In the absence of a more fundamental theory of interactions, an independent phenomenological model approach to search for possible textures or symmetries in the fermion mass matrices is still playing an important role.

In the SM, the mass term is given by

$$-\mathcal{L}_M = \bar{u}_R M_u u_L + \bar{d}_R M_d d_L + \text{H.c.}, \qquad (1.1)$$

where the mass matrices M_u and M_d are three-dimensional complex matrices. In the most general case, they contain 36 real parameters. A first simplification, without losing generality, makes use of the polar decomposition theorem of matrix algebra, by which one can always express a general mass matrix as a product of a Hermitian and unitary matrix. Therefore, we can consider quark mass matrices to be Hermitian as the unitary matrix can be absorbed in the right- handed quark fields. This immediately brings down the number of free parameters from 36 to 18.

A simple and instructive *Ansatz* of Hermitian quark mass matrices with six-texture zeros was first proposed in Ref. [1]. An additional nonparallel six-texture zeros was given in Ref. [4]. Both textures are currently ruled out [5] because, among other things, they do not reproduce some entries of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix V. Specifically, in both cases, the magnitude of $|V_{ub}/V_{cb}|$ predicted by $\sqrt{m_u/m_c}$ is too low ($V_{ub}/V_{cb} \approx 0.06$ or smaller for reasonable values of the quark masses m_u and m_c [6,7]) to agree with the present experimental result ($|V_{ub}/V_{cb}|_{\text{ex}} \approx 0.09$ [6]). Because of this, some authors have highly recommended the use of four-texture zeros [5,8,9]. It is shown in this work that four-texture zeros is readily feasible, and we can even get five-texture zeros.

We would, therefore, present an analytical method to calculate models containing various texture zeros in the quark mass matrix sector, taking into account the latest experimental data provided [6]. We use simultaneously in our research two very common approaches: one approach, which is used in conjunction with the second approach, consists of placing zeros (called texture zeros) at certain entries of quark mass matrices that can predict self-consistent and experimentally favored relations between quark masses and flavor mixing parameters [4,10,11]; the second approach involves the WB transformation (weak basis transformation), which transforms the quark mass matrix representations into new equivalent ones [8].

This paper is organized as follows: in Sec. II we discuss some issues related to the WB transformation method and its utilities. Section III is dedicated to obtaining some numerical parallel and nonparallel four-texture zeros quark mass matrices using special techniques, which we then use in Sec. IV to find five-texture zeros in quark mass matrices compatible with the present experimental data. This configuration is studied from an analytical point of view in Sec. V, and our conclusions are presented in Sec. VI. The method used extensively throughout this paper to find texture zeros is verified in Appendix .

II. WB TRANSFORMATIONS

The most general WB transformation [8], that leaves the physical content invariable and the mass matrices Hermitian, is

$$M_u \to M'_u = U^\dagger M_u U, \qquad M_d \to M'_d = U^\dagger M_d U, \quad (2.1)$$

where U is an arbitrary unitary matrix. We say that the two representations $M_{u,d}$ and $M'_{u,d}$ are equivalent to each other. Besides, it implies that the number of equivalent representations is infinity. This kind of transformation will be used extensively in the calculations below.

First, let us show that the WB transformation is exhaustive in generating all possible mass matrix representations. Let us first consider the representation of Hermitian quark mass matrices indicated by (M_u, M_d) and diagonalize them as follows

$$U_u^\dagger M_u U_u = D_u \quad \text{and} \quad U_d^\dagger M_d U_d = D_d. \tag{2.2}$$

The CKM mixing matrix is given by

$$V_{\text{ckm}} = U_u^\dagger U_d. \tag{2.3}$$

On the other hand, the prime representation (M'_u, M'_d) gives

$$U_u'^\dagger M'_u U'_u = D_u \quad \text{and} \quad U_d'^\dagger M'_d U'_d = D_d \tag{2.4}$$

and

$$V_{\text{ckm}} = U_u'^\dagger U'_d. \tag{2.5}$$

Equating the expressions (2.3) and (2.5) yields

$$U_u^\dagger U_d = U_u'^\dagger U'_d \Rightarrow U'_u U_u^\dagger = U'_d U_d^\dagger. \tag{2.6}$$

And equating (2.2) and (2.4) gives, respectively,

$$U_u'^\dagger M'_u U'_u = U_u^\dagger M_u U_u \ \text{ and } \ U_d'^\dagger M'_d U'_d = U_d^\dagger M_d U_d, \tag{2.7}$$

where we find that the mass matrices M_u and M_d can be expressed in terms of the mass matrices M'_u and M'_d as follows

$$M_u = U_u U_u'^\dagger M'_u U'_u U_u^\dagger, \tag{2.8}$$

$$M_d = U_d U_d'^\dagger M'_d U'_d U_d^\dagger. \tag{2.9}$$

Using (2.6) into (2.9), we have

$$M_d = U_u U_u'^\dagger M'_d U'_u U_u^\dagger, \tag{2.10}$$

where $U = U_u U_u'^\dagger$ is a unitary matrix that allows us to state,

"In the SM, any two pairs of Hermitian quark mass matrices, given by (M_u, M_d) and (M'_u, M'_d), with identical eigenvalues and flavor mixing parameters, to a specific scale energy, are related through a WB transformation", (2.11)

i.e., there is no quark mass matrix representation outside the set (2.1). In this reasoning, we have assumed that both representations generate the same entries, including the phases, for the CKM mixing matrix (V_{ckm}), something validated by the fact that a WB transformation makes them equal, as will be shown in Sec. II A.

The importance of the WB transformation as a calculation tool can be appreciated from the following results.

A. The preliminary matrix representation

In the quark-family basis, it is more convenient to use the following quark mass matrix representation [8,12]

$$M_u = D_u = \begin{pmatrix} \lambda_{1u} & 0 & 0 \\ 0 & \lambda_{2u} & 0 \\ 0 & 0 & \lambda_{3u} \end{pmatrix}, \quad M_d = V D_d V^\dagger, \tag{2.12}$$

which comes from a WB transformation and we call the "u-diagonal representation." We call the other possibility

$$M_u = V^\dagger D_u V, \quad M_d = D_d = \begin{pmatrix} \lambda_{1d} & 0 & 0 \\ 0 & \lambda_{2d} & 0 \\ 0 & 0 & \lambda_{3d} \end{pmatrix}, \tag{2.13}$$

the "d-diagonal representation". One advantage of using representations (2.12) [or (2.13)] is to be able to use simultaneously the CKM mixing matrix V and the quark mass eigenvalues $|\lambda_{iu,d}|$ $(i = 1, 2, 3)$, where $\lambda_{iu,d}$ may be either positive or negative and satisfy the hierarchy

$$|\lambda_{1u,d}| \ll |\lambda_{2u,d}| \ll |\lambda_{3u,d}|. \tag{2.14}$$

It is usually said that the CKM matrix is an arbitrary unitary matrix with five phases rotated away through the phase redefinition of the left-handed up and down quark fields [13]. This can be shown by using the following unitary matrix

$$\begin{pmatrix} e^{ix} & & \\ & e^{iy} & \\ & & 1 \end{pmatrix}$$

in order to make a WB transformation on (2.12). The up matrix

$$M_u = \begin{pmatrix} e^{ix} & & \\ & e^{iy} & \\ & & 1 \end{pmatrix} D_u \begin{pmatrix} e^{ix} & & \\ & e^{iy} & \\ & & 1 \end{pmatrix}^\dagger = D_u, \tag{2.15}$$

remains equal, while the down matrix takes the form

$$M_d = \begin{pmatrix} e^{ix} & & \\ & e^{iy} & \\ & & 1 \end{pmatrix} (V D_d V^\dagger) \begin{pmatrix} e^{ix} & & \\ & e^{iy} & \\ & & 1 \end{pmatrix}^\dagger, \tag{2.16}$$

$$M_d = \left[\begin{pmatrix} e^{ix} & & \\ & e^{iy} & \\ & & 1 \end{pmatrix} V \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix}\right] \times D_d \left[\begin{pmatrix} e^{ix} & & \\ & e^{iy} & \\ & & 1 \end{pmatrix} V \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix}\right]^\dagger, \tag{2.17}$$

where in the last step we have used the identity (2.15) applied to the diagonal down mass matrix. The expression into the square brackets is precisely the most general way to write a unitary matrix [13].

In this representation, the matrix M_d, in (2.16), contains two free parameters x and y, which play an important role in obtaining texture zeros, as we shall see later.

B. A unique negative eigenvalue

The result (2.11) permits us to use the u-diagonal representation (2.12) [or the d-diagonal representation (2.13)] as the starting point to generate any other representation. If they exist, by this method, important texture zeros in the mass matrix can be found.

Because some texture zeros must lie along its diagonal entries of both up and down Hermitian quark mass matrices, it implies that at least one, and at most two, of its eigenvalues must be negative [8]. Furthermore, for the case of two negative eigenvalues, these mass matrices can be reduced to having only one negative eigenvalue, by factoring out a minus sign that can be included, for instance, into the mass matrix basis (2.12). Thus, without loss of generality, the texture zeros models can be deduced considering that

"each one of the quark mass matrices M_u and M_d contains exactly one negative eigenvalue." (2.18)

III. NUMERICAL FOUR-TEXTURE ZEROS

There is a wide variety of four-texture zeros representations. Using a specific approach, some nonparallel textures are easy to obtain, but more laborious methods are required in parallel cases. In our analysis we will use the next physical quantities.

A. Quark masses and CKM

For quark mass matrix phenomenology, values of $m_q(\mu)$ at $\mu = m_Z$ are useful because the observed CKM matrix parameters $|V_{ij}|$ are given at $\mu = m_Z$. We summarize quark masses at $\mu = m_Z$ [7,12,14]

$$\begin{aligned} m_u &= 1.38^{+0.42}_{-0.41}, \qquad m_c = 638^{+43}_{-84}, \\ m_t &= 172100 \pm 1200, \qquad m_d = 2.82 \pm 0.48, \\ m_s &= 57^{+18}_{-12}, \qquad m_b = 2860^{+160}_{-60}, \end{aligned} \tag{3.1}$$

given in units of MeV.

The CKM matrix [7,15,16] is a 3×3 unitary matrix,

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$

which can be parametrized by three mixing angles and the CP-violating Kobayashi-Maskawa phase [16]. Of the many possible conventions, a standard choice has become [17]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{3.2}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, and δ is the phase responsible for all CP-violating phenomena in flavor-changing processes in the SM. The angles θ_{ij} can be chosen to lie in the first quadrant, so s_{ij}, $c_{ij} \geq 0$.

It is known experimentally that $s_{13} \ll s_{23} \ll s_{12} \ll 1$, and it is convenient to exhibit this hierarchy using the Wolfenstein parametrization. We define [18,19]

$$\begin{aligned} s_{12} &= \lambda, \qquad s_{23} = A\lambda^2, \\ s_{13}e^{i\delta} &= \frac{A\lambda^3(\bar\rho + i\bar\eta)\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar\rho + i\bar\eta)]}. \end{aligned} \tag{3.3}$$

The constraints implied by the unitarity of the three-generation CKM matrix significantly reduce the allowed range of some of the CKM elements. The fit for the Wolfenstein parameters defined in Eq. (3.3) gives

$$\begin{aligned} \lambda &= 0.22535 \pm 0.00065, \qquad A = 0.811^{+0.022}_{-0.012}, \\ \bar\rho &= 0.131^{+0.026}_{-0.013}, \qquad \bar\eta = 0.345^{+0.013}_{-0.014}. \end{aligned} \tag{3.4}$$

These values are obtained using the method of Refs. [18,20]. The fit results for the values of all nine CKM elements are.

$$V = \begin{pmatrix} 0.974272 & 0.225349 & 0.00351322e^{-i1.20849} \\ 0.225209e^{-i3.14101} & 0.97344e^{-i3.13212\times10^{-5}} & 0.0411845 \\ 0.00867944e^{-i0.377339} & 0.0404125e^{-i3.12329} & 0.999145 \end{pmatrix}, \quad (3.5)$$

with magnitudes

$$|V| = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}, \quad (3.6)$$

and the Jarlskog invariant is

$$J = (2.96^{+0.20}_{-0.16}) \times 10^{-5}. \quad (3.7)$$

B. Nonparallel four-texture zeros

It is the most simple case. For instance, let us take the eigenvalues signs pattern as follow

$$\lambda_{1u} = -m_u, \qquad \lambda_{2u} = m_c, \qquad \lambda_{3u} = m_t, \quad (3.8)$$

$$\lambda_{1d} = m_d, \qquad \lambda_{2d} = -m_s, \qquad \lambda_{3d} = m_b. \quad (3.9)$$

Then, for this case, the numerical values in the u-diagonal representation (2.12) are

$$M_u = \begin{pmatrix} -1.38 & & \\ & 638 & \\ & & 172100 \end{pmatrix} \text{MeV},$$

$$M_d = \begin{pmatrix} -0.2 \pm 0.8 & -12.9758 - 0.386978i & 4.09941 - 9.38819i \\ -12.9758 + 0.386978i & -49.0183 & 119.924 - 0.043146i \\ 4.09941 + 9.38819i & 119.924 + 0.043146i & 2855.02 \end{pmatrix} \text{MeV}, \quad (3.10)$$

$$= \begin{pmatrix} 0 & -12.9758 - 0.386978i & 4.09941 - 9.38819i \\ -12.9758 + 0.386978i & -49.0183 & 119.924 - 0.043146i \\ 4.09941 + 9.38819i & 119.924 + 0.043146i & 2855.02 \end{pmatrix} \text{MeV},$$

where we have used the numerical CKM matrix (3.5) and errors of (3.6). In the second mass matrix above, in the entry $M_d(1,1) = -0.2 \pm 0.8$ calculated, since the uncertainty (± 0.8) in determining this element exceeds the value of 0.2 it is obviously reasonable to call the (1, 1) entry zero ($M_d(1,1) = 0$). Something pointed out in Ref. [12].

Making a WB transformation on (3.10) using the following unitary matrix

$$U = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \quad (3.11)$$

with $\tan\theta = \sqrt{\frac{m_u}{m_t}}$, the matrices (3.10) transform into a form, where the entries (1, 1), (1, 2) and (2, 3) of matrix M_u become zero. Then, we have

$$M'_u = UM_uU^\dagger = \begin{pmatrix} 0 & 0 & 487.338 \\ 0 & 638 & 0 \\ 487.338 & 0 & 172099 \end{pmatrix} \text{MeV} \quad (3.12)$$

and

$$M'_d = UM_dU^\dagger = \begin{pmatrix} 0 & -12.6361 - 0.386854i & 12.1844 - 9.38819i \\ -12.6361 + 0.386854i & -49.0183 & 119.96 - 0.0442417i \\ 12.1844 + 9.38819i & 119.96 + 0.0442417i & 2854.97 \end{pmatrix} \text{MeV}, \quad (3.13)$$

where the element $M'_d(1,1)$ is zero for the same reason given in (3.10). We finally obtain a nonparallel four-texture zeros mass matrix representation.

$$M'_u = \begin{pmatrix} 0 & 0 & 487.338 \\ 0 & 638 & 0 \\ 487.338 & 0 & 172099 \end{pmatrix} \text{MeV},$$
$$M'_d = \begin{pmatrix} 0 & 12.6421e^{-3.11099i} & 15.3817e^{-0.656498i} \\ 12.6421e^{3.11099i} & -49.0183 & 119.96e^{-0.000368804i} \\ 15.3817e^{0.656498i} & 119.96e^{0.000368804i} & 2854.97 \end{pmatrix} \text{MeV}. \tag{3.14}$$

New equivalent four-texture zeros representations can be obtained using the former representation. For example, if we use unitary matrices looking like

$$U_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{3.15}$$

and apply them to (3.14), it allows us to obtain new nonparallel four-texture zeros representations. For the case (3.15), we have

$$M_u = \begin{pmatrix} 0 & 487.338 & 0 \\ 487.338 & 172099 & 0 \\ 0 & 0 & 638 \end{pmatrix} \text{MeV},$$
$$M_d = \begin{pmatrix} 0 & 15.3817e^{-0.656498i} & 12.6421e^{-3.11099i} \\ 15.3817e^{0.656498i} & 2854.97 & 119.96e^{0.000368804i} \\ 12.6421e^{3.11099i} & 119.96e^{-0.000368804i} & -49.0183 \end{pmatrix} \text{MeV}, \tag{3.16}$$

where some of their entries have been permuted.

We have found typical nonparallel four-texture zeros quark mass matrix representations. The WB was applied by using simple unitary matrices like (3.11). The process is more difficult if we want to find parallel texture zeros in quark mass matrices.

C. Parallel four-texture zeros

Let us begin implementing a method that we shall apply later to special cases. Let us start by giving the following structure for the up matrix elements[1]

$$M_u = \begin{pmatrix} 0 & |C_u| & 0 \\ |C_u| & \tilde{B}_u & |B_u| \\ 0 & |B_u| & A_u \end{pmatrix}, \tag{3.17}$$

where $\tilde{B}_u$ and A_u are real numbers. The mass matrix M_u can be diagonalized using the transformation

$$O_u^\dagger M_u O_u = \begin{pmatrix} \lambda_{1u} & & \\ & \lambda_{2u} & \\ & & \lambda_{3u} \end{pmatrix}, \tag{3.18}$$

where the exact analytical result of O_u is [5]

$$O_u = \begin{pmatrix} e^{ix}\rho\sqrt{\frac{\lambda_{2u}\lambda_{3u}(A_u-\lambda_{1u})}{A_u(\lambda_{2u}-\lambda_{1u})(\lambda_{3u}-\lambda_{1u})}} & e^{iy}\eta\sqrt{\frac{\lambda_{1u}\lambda_{3u}(\lambda_{2u}-A_u)}{A_u(\lambda_{2u}-\lambda_{1u})(\lambda_{3u}-\lambda_{2u})}} & \sqrt{\frac{\lambda_{1u}\lambda_{2u}(A_u-\lambda_{3u})}{A_u(\lambda_{3u}-\lambda_{1u})(\lambda_{3u}-\lambda_{2u})}} \\ -e^{ix}\eta\sqrt{\frac{\lambda_{1u}(\lambda_{1u}-A_u)}{(\lambda_{2u}-\lambda_{1u})(\lambda_{3u}-\lambda_{1u})}} & e^{iy}\sqrt{\frac{\lambda_{2u}(A_u-\lambda_{2u})}{(\lambda_{2u}-\lambda_{1u})(\lambda_{3u}-\lambda_{2u})}} & \rho\sqrt{\frac{\lambda_{3u}(\lambda_{3u}-A_u)}{(\lambda_{3u}-\lambda_{1u})(\lambda_{3u}-\lambda_{2u})}} \\ e^{ix}\eta\sqrt{\frac{\lambda_{1u}(A_u-\lambda_{2u})(A_u-\lambda_{3u})}{A_u(\lambda_{2u}-\lambda_{1u})(\lambda_{3u}-\lambda_{1u})}} & -e^{iy}\rho\sqrt{\frac{\lambda_{2u}(A_u-\lambda_{1u})(\lambda_{3u}-A_u)}{A_u(\lambda_{2u}-\lambda_{1u})(\lambda_{3u}-\lambda_{2u})}} & \sqrt{\frac{\lambda_{3u}(A_u-\lambda_{1u})(A_u-\lambda_{2u})}{A_u(\lambda_{3u}-\lambda_{1u})(\lambda_{3u}-\lambda_{2u})}} \end{pmatrix}, \tag{3.19}$$

where $\eta \equiv \lambda_{2u}/m_c = +1$ or -1 and $\rho \equiv \lambda_{3u}/m_t = +1$ or -1 corresponding to the possibility $(\lambda_{1u}, \lambda_{2u}, \lambda_{3u}) = (-m_u, m_c, m_t)$, $(\lambda_{1u}, \lambda_{2u}, \lambda_{3u}) = (m_u, -m_c, m_t)$, or $(\lambda_{1u}, \lambda_{2u}, \lambda_{3u}) = (m_u, m_c, -m_t)$. The imaginary phases in (3.19) were included, in order that given them appropriated values, the generated CKM matrix becomes compatible with the chosen convention (3.2).[2] Note that $\tilde{B}_u$, $|B_u|$, and $|C_u|$ can be expressed in terms of λ_{iu} $(i = 1, 2, 3)$ and A_u, using invariant matrix functions as follows

$$\operatorname{tr} M_u \Rightarrow \tilde{B}_u = \lambda_{1u} + \lambda_{2u} + \lambda_{3u} - A_u, \tag{3.20}$$

[1]It is sufficient to consider that the mass matrix be real and symmetric, since the phases may be included later by means of a WB process.

[2]It is not necessary to include an imaginary phase in the third column of O_u, since we can factor it out.

$$\mathrm{tr}\, M_u^2 \Rightarrow |B_u| = \sqrt{\frac{(A_u - \lambda_{1u})(A_u - \lambda_{2u})(\lambda_{3u} - A_u)}{A_u}}, \quad (3.21)$$

$$\det M_u \Rightarrow |C_u| = \sqrt{\frac{-\lambda_{1u}\lambda_{2u}\lambda_{3u}}{A_u}}, \quad (3.22)$$

where "tr" and "det" are the trace and the determinant, respectively. The matrix O_u can be seen as the unitary matrix such that the WB transformation transforms the representation (2.12) into the form

$$M_u' = O_u \begin{pmatrix} \lambda_{1u} & & \\ & \lambda_{2u} & \\ & & \lambda_{3u} \end{pmatrix} O_u^\dagger = \begin{pmatrix} 0 & |C_u| & 0 \\ |C_u| & \tilde{B}_u & |B_u| \\ 0 & |B_u| & A_u \end{pmatrix}, \quad (3.23)$$

$$M_d' = O_u (V D_d V^\dagger) O_u^\dagger = \begin{pmatrix} X_{(A_u,x,y)} & C_d & Y_{(A_u,x,y)} \\ C_d^* & \tilde{B}_d & B_d \\ Y^*_{(A_u,x,y)} & B_d^* & A_d \end{pmatrix}, \quad (3.24)$$

where the elements of M_d' depend on three parameters A_u, x, and y. To complete the analysis, we must obtain neglected values at the entries (1, 1) and (1, 3) compared with the remaining elements of the matrix M_d'. Then we have to solve three equations

$$X_{(A_u,x,y)} = 0, \quad \mathrm{Re}[Y_{(A_u,x,y)}] = 0, \quad \text{and} \quad \mathrm{Im}[Y_{(A_u,x,y)}] = 0, \quad (3.25)$$

where "Re" refers to the real part and "Im," the imaginary part of the function. In the process, the following details must be taken into account:

(1) The formulas (3.20) through (3.22) must be real numbers. Therefore, the parameter A_u is restricted to lie into an interval. Let us see the different possibilities, where the hierarchy (2.14) was considered.

(a) If $\lambda_{1u} = -m_u$, $\lambda_{2u} = m_c$ and $\lambda_{3u} = m_t$ then
$$m_c < A_u < m_t. \quad (3.26)$$

(b) If $\lambda_{1u} = m_u$, $\lambda_{2u} = -m_c$ and $\lambda_{3u} = m_t$ then
$$m_u < A_u < m_t. \quad (3.27)$$

(c) If $\lambda_{1u} = m_u$, $\lambda_{2u} = m_c$ and $\lambda_{3u} = -m_t$ then
$$m_u < A_u < m_c. \quad (3.28)$$

(2) The phases given in (3.19) could have been included initially in the transformation (2.16), instead of writing them explicitly in the matrix O_u. The validity of this point of view is checked by observing that the matrix (3.19) can be decomposed as the product of two matrices, where the right-hand side contains the imaginary phases as follows

$$O_u = O_{u(x=0,y=0)} \begin{pmatrix} e^{ix} & & \\ & e^{iy} & \\ & & 1 \end{pmatrix}, \quad (3.29)$$

such that, after replacing this decomposition into (3.24) and comparing with (2.16), we conclude that both points of view concur.

In Appendix , we will work a case previously studied in Ref. [8] and replicate the results presented there by using the techniques implemented here.

1. Example 1: parallel four-texture zeros

We are mainly concerned to find four-texture zeros with the recent data given in Sec. III A. Let us take the following case

$$\lambda_{1u} = -m_u, \qquad \lambda_{2u} = m_c, \qquad \lambda_{3u} = m_t, \quad (3.30)$$

$$\lambda_{1d} = -m_d, \qquad \lambda_{2d} = m_s, \qquad \lambda_{3d} = m_b. \quad (3.31)$$

We have, in the u-diagonal representation, the following mass matrix representation.

$$M_u = \begin{pmatrix} -1.38 & 0 & 0 \\ 0 & 638 & 0 \\ 0 & 0 & 172100 \end{pmatrix} \mathrm{MeV},$$
$$M_d = V D_d V^\dagger = \begin{pmatrix} 0.253114 & 13.2691 - 0.386919i & 3.01706 - 9.38676i \\ 13.2691 + 0.386919i & 58.7203 & 115.45 + 0.043146i \\ 3.01706 + 9.38676i & 115.45 - 0.043146i & 2855.21 \end{pmatrix} \mathrm{MeV}. \quad (3.32)$$

Making a WB transformation on (3.32), using the unitary matrix O_u (Eq. (3.19)), the following conditions

$$\begin{aligned} M'_{d(1,1)}(A_u, x_1, x_2, y_1, y_2) &= 0, \\ \mathrm{Re}[M'_{d(1,3)}(A_u, x_1, x_2, y_1, y_2)] &= 0, \\ \mathrm{Im}[M'_{d(1,3)}(A_u, x_1, x_2, y_1, y_2)] &= 0 \end{aligned} \quad (3.33)$$

are established in order to find zero entries (1, 1), (1, 3), and (3, 1) of the resulting matrix $M_d' = O_u M_d O_u^\dagger$, where the imaginary phases given in O_u have been defined as $e^{ix} = \cos x + i \sin x = x_1 + i x_2$ and $e^{iy} = \cos y + i \sin y = y_1 + i y_2$, such that

$$x_1^2 + x_2^2 = 1 \quad \text{and} \quad y_1^2 + y_2^2 = 1. \quad (3.34)$$

Equations (3.33) and (3.34) give the following exact solution:

$$A_u = 153231\,\text{MeV}, \quad x_1 = 0.883194, \quad x_2 = -0.469007, \quad y_1 = 0.202996, \quad y_2 = 0.97918. \tag{3.35}$$

Finally, we obtain an exact parallel four-texture zeros mass matrix representation.

$$M'_u = O_u M_u O_u^\dagger = \begin{pmatrix} 0 & 31.4461 & 0 \\ 31.4461 & 19505.7 & 53659.2 \\ 0 & 53659.2 & 153231 \end{pmatrix} \text{MeV}, \tag{3.36}$$

$$M'_d = O_u M_d O_u^\dagger, = \begin{pmatrix} 0 & -1.43578 - 13.3956i & 0 \\ -1.43578 + 13.3956i & 381.367 & 893.365 + 113.383i \\ 0 & 893.365 - 113.383i & 2532.81 \end{pmatrix} \text{MeV}. \tag{3.37}$$

In the same way, we can find other nonequivalent parallel four-texture zeros representations. Let us look at another case.

2. Example 2: another parallel four-texture zeros model

Another possibility that works well is

$$\lambda_{1u} = m_u, \qquad \lambda_{2u} = m_c, \qquad \lambda_{3u} = -m_t, \tag{3.38}$$

$$\lambda_{1d} = m_d, \qquad \lambda_{2d} = m_s, \qquad \lambda_{3d} = -m_b, \tag{3.39}$$

from which, we have $A_u = 7.34102$ MeV, $x_1 = 0.998393$, $x_2 = -0.0566637$, $y_1 = 0.999664$, and $y_2 = 0.0259074$. Thus, the corresponding parallel four-texture zeros mass matrix representation is

$$M'_u = O_u M_u O_u^\dagger = \begin{pmatrix} 0 & 4543.2 & 0 \\ 4543.2 & -171468. & 9388.13 \\ 0 & 9388.13 & 7.34102 \end{pmatrix} \text{MeV}, \tag{3.40}$$

$$M'_d = O_u M_d O_u^\dagger = \begin{pmatrix} 0 & 123.93 + 10.0184i & 0 \\ 123.93 - 10.0184i & -2829.92 & 267.035 + 1.39152i \\ 0 & 267.035 - 1.39152i & 29.738 \end{pmatrix} \text{MeV}. \tag{3.41}$$

IV. NUMERICAL FIVE-TEXTURE ZEROS

Now, let us try to find five-texture zeros for the quark mass matrix sector. If this cannot be achieved, we can conclude that five- and six-texture zeros are not viable models. For that, we will use the mathematical tools previously implemented in Sec. III C. We shall begin as usual by proposing a texture-zeros configuration. In this case with three zeros for the up/down quark mass matrix,[3] we will see how many zeros can be reached for the down/up quark mass matrix. In principle, there are many possibilities, but many of them are equivalent ones. In total, there are two nonequivalent cases, depending on the number of zeros included in their diagonal entries. Therefore, we have only two possibilities: one-zero or two-zero in diagonal entries. Let us name them as *one-zero family* and *two-zero family*, respectively. With an appropriated unitary matrix and performing the corresponding WB transformation, the other possibilities are obtained. In Table I both families are indicated, which summarizes the equivalent possibilities for each case. Let us study each family.

A. Two-zero family

In what follows, we work the u-diagonal and d-diagonal cases simultaneously. The standard representation for the two-zero family is

$$M_{u,d} = \begin{pmatrix} 0 & |C_{u,d}| & 0 \\ |C_{u,d}| & 0 & |B_{u,d}| \\ 0 & |B_{u,d}| & A_{u,d} \end{pmatrix}, \tag{4.1}$$

and its diagonalization matrix satisfies the following relation

[3] A model with four zeros in the up/down quark mass matrix is not realistic.

TABLE I. One- and two-zero family.

Unitary matrix	Two-zero family $(p_i M_{u,d} p_i^T)$	One-zero family $(p_i M_{u,d} p_i^T)$
$p_1 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert C_{u,d}\rvert & 0 \\ \lvert C_{u,d}\rvert & 0 & \lvert B_{u,d}\rvert \\ 0 & \lvert B_{u,d}\rvert & A_{u,d} \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert B_{u,d}\rvert & 0 \\ \lvert B_{u,d}\rvert & C_{u,d} & 0 \\ 0 & 0 & A_{u,d} \end{pmatrix}$
$p_2 = \begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \lvert C_{u,d}\rvert \\ 0 & A_{u,d} & \lvert B_{u,d}\rvert \\ \lvert C_{u,d}\rvert & \lvert B_{u,d}\rvert & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \lvert B_{u,d}\rvert \\ 0 & A_{u,d} & 0 \\ \lvert B_{u,d}\rvert & 0 & C_{u,d} \end{pmatrix}$
$p_3 = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}$	$\begin{pmatrix} A_{u,d} & \lvert B_{u,d}\rvert & 0 \\ \lvert B_{u,d}\rvert & 0 & \lvert C_{u,d}\rvert \\ 0 & \lvert C_{u,d}\rvert & 0 \end{pmatrix}$	$\begin{pmatrix} A_{u,d} & 0 & 0 \\ 0 & C_{u,d} & \lvert B_{u,d}\rvert \\ 0 & \lvert B_{u,d}\rvert & 0 \end{pmatrix}$
$p_4 = \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert C_{u,d}\rvert & \lvert B_{u,d}\rvert \\ \lvert C_{u,d}\rvert & 0 & 0 \\ \lvert B_{u,d}\rvert & 0 & A_{u,d} \end{pmatrix}$	$\begin{pmatrix} \lvert C_{u,d}\rvert & \lvert B_{u,d}\rvert & 0 \\ \lvert B_{u,d}\rvert & 0 & 0 \\ 0 & 0 & A_{u,d} \end{pmatrix}$
$p_5 = \begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}$	$\begin{pmatrix} A_{u,d} & 0 & \lvert B_{u,d}\rvert \\ 0 & 0 & \lvert C_{u,d}\rvert \\ \lvert B_{u,d}\rvert & \lvert C_{u,d}\rvert & 0 \end{pmatrix}$	$\begin{pmatrix} A_{u,d} & 0 & 0 \\ 0 & 0 & \lvert B_{u,d}\rvert \\ 0 & \lvert B_{u,d}\rvert & C_{u,d} \end{pmatrix}$
$p_6 = \begin{pmatrix} & 1 & \\ & & 1 \\ 1 & & \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert B_{u,d}\rvert & \lvert C_{u,d}\rvert \\ \lvert B_{u,d}\rvert & A_{u,d} & 0 \\ \lvert C_{u,d}\rvert & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} C_{u,d} & 0 & \lvert B_{u,d}\rvert \\ 0 & A_{u,d} & 0 \\ \lvert B_{u,d}\rvert & 0 & 0 \end{pmatrix}$

$$O_{u,d}^{\dagger} M_{u,d} O_{u,d} = \begin{pmatrix} \lambda_{1u,d} & & \\ & \lambda_{2u,d} & \\ & & \lambda_{3u,d} \end{pmatrix}, \tag{4.2}$$

where one and only one $\lambda_{iu,d}$ is assumed to be a negative number. The invariant quantities "det" and "trace" applied on (4.1) and (4.2)

$$\mathrm{tr}\, M_{u,d} = A_{u,d} = \lambda_{1u,d} + \lambda_{2u,d} + \lambda_{3u,d}, \tag{4.3}$$

$$\mathrm{tr}\, M_{u,d}^2 = A_{u,d}^2 + 2|B_{u,d}|^2 + 2|C_{u,d}|^2 = \lambda_{1u,d}^2 + \lambda_{2u,d}^2 + \lambda_{3u,d}^2, \tag{4.4}$$

$$\det M_{u,d} = -A_{u,d}|C_{u,d}|^2 = \lambda_{1u,d}\lambda_{2u,d}\lambda_{3u,d} \tag{4.5}$$

allow us to express the parameters of (4.1) in terms of its eigenvalues

$$A_{u,d} = \lambda_{1u,d} + \lambda_{2u,d} + \lambda_{3u,d}, \tag{4.6}$$

$$|B_{u,d}| = \sqrt{-\frac{(\lambda_{1u,d} + \lambda_{2u,d})(\lambda_{1u,d} + \lambda_{3u,d})(\lambda_{2u,d} + \lambda_{3u,d})}{A_{u,d}}}, \tag{4.7}$$

$$|C_{u,d}| = \sqrt{-\frac{\lambda_{1u,d}\lambda_{2u,d}\lambda_{3u,d}}{A_{u,d}}}. \tag{4.8}$$

From expression (4.8), together with (2.18), we have that

$$A_{u,d} > 0, \tag{4.9}$$

and using (4.7) and the hierarchy (2.14), we found that only one possibility is permitted

$$\lambda_{1u,d}, \lambda_{3u,d} > 0 \quad \text{and} \quad \lambda_{2u,d} < 0. \tag{4.10}$$

For the u-diagonal case, the diagonalization matrix (3.19) becomes

$$O_u = \begin{pmatrix} 0.99892 e^{ix} & -0.0464583 e^{iy} & 0.0000104863 \\ 0.0463719 e^{ix} & 0.997078 e^{iy} & 0.0607083 \\ -0.00283086 e^{ix} & -0.0606422 e^{iy} & 0.998156 \end{pmatrix}, \tag{4.11}$$

and for the d-diagonal case, the diagonalization matrix is given by

$$O_d = \begin{pmatrix} 0.980856 e^{ix} & -0.194731 e^{iy} & 0.000682127 \\ 0.19251 e^{ix} & 0.970182 e^{iy} & 0.147267 \\ -0.0293392 e^{ix} & -0.144316 e^{iy} & 0.989097 \end{pmatrix}. \tag{4.12}$$

As you can see, in both cases, we are using quasidiagonal matrices.

Performing the WB transformation using the unitary matrix $O_{u,d}$, we have

$$M'_{u,d} = O_{u,d} \begin{pmatrix} \lambda_{1u,d} & & \\ & \lambda_{2u,d} & \\ & & \lambda_{3u,d} \end{pmatrix} O_{u,d}^{\dagger}, \tag{4.13}$$

$$= \begin{pmatrix} 0 & |C_{u,d}| & 0 \\ |C_{u,d}| & 0 & |B_{u,d}| \\ 0 & |B_{u,d}| & A_{u,d} \end{pmatrix} \tag{4.14}$$

and

$$M'_{d,u} = O_{d,u} M_{d,u} O_{d,u}^{\dagger}, \tag{4.15}$$

where the matrices

$$M_d = V D_d V^\dagger \quad \text{and} \quad M_u = V^\dagger D_u V \tag{4.16}$$

depend on whether we work with the u-diagonal or the d-diagonal case.

In order to facilitate the calculus, we define the following new variables

$$\begin{aligned} e^{ix} &= x_1 + i x_2, \quad \text{with} \quad x_1^2 + x_2^2 = 1, \\ e^{iy} &= y_1 + i y_2, \quad \text{with} \quad y_1^2 + y_2^2 = 1, \end{aligned} \tag{4.17}$$

where their norms satisfy

$$|x_1|, |x_2| \leq 1, \quad \text{and} \quad |y_1|, |y_2| \leq 1. \tag{4.18}$$

With the former definitions, the elements of the matrix $M'_{d,u}$ defined in (4.15) now have a polynomial form in each case considered: $\lambda_{1d} = -m_d$, $\lambda_{2d} = -m_s$, or $\lambda_{3d} = -m_b$ for the u-digonal case (or $\lambda_{1u} = -m_d$, $\lambda_{2u} = -m_s$, or $\lambda_{3u} = -m_b$ for the d-diagonal case). The results are summarized in Tables II and III.

1. Analysis of "down" mass matrix

Table II summarizes the components of M'_d for the u-diagonal case. By simple inspection, using (4.18) shows that it is not possible to find zeros at entries (2, 2), (2, 3), and (3, 3), and no solutions were found for either

$$\begin{aligned} &\mathrm{Re}[M'_d(1,2)] = 0, \qquad \mathrm{Im}[M'_d(1,2)] = 0, \\ \text{or} \quad &\mathrm{Re}[M'_d(1,3)] = 0, \qquad \mathrm{Im}[M'_d(1,3)] = 0, \end{aligned}$$

equations. Therefore, it is impossible to find two-texture zeros in the down quark mass matrix coming from an u-diagonal representation for the two-zero family case.

2. Analysis of "up" mass matrix and a model with five-texture zeros

Let us consider the d-diagonal case. The entries of matrix M'_u, after the WB transformation is made, are given in Table III. According to the Table, only entries (1, 2) and (1, 3) deserve some attention, and of these only the cases $\lambda_{1u} = -m_u$ and $\lambda_{2u} = -m_c$ give an acceptable solution.

For the first case, with $\lambda_{1u} = -m_u$, we have

$$M'_u(1,2) = 0, \tag{4.19}$$

$$M'_u(1,1) \approx 0, \tag{4.20}$$

where

$$\begin{aligned} x_1 &= 0.706984, \qquad y_1 = -0.540778, \\ x_2 &= 0.70723, \qquad y_2 = -0.841165. \end{aligned} \tag{4.21}$$

The corresponding five-texture zeros representation obtained is

$$M'_u = \begin{pmatrix} 0 & 0 & -92.3618 + 157.694i \\ 0 & 5748.17 & 28555.1 + 5911.83i \\ -92.3618 - 157.694i & 28555.1 - 5911.83i & 166988 \end{pmatrix} \text{MeV}, \tag{4.22a}$$

$$M'_d = \begin{pmatrix} 0 & 13.9899 & 0 \\ 13.9899 & 0 & 424.808 \\ 0 & 424.808 & 2796.9 \end{pmatrix} \text{MeV}. \tag{4.22b}$$

The other possibility that works well is the following numerical five-texture zeros in the two-zero family case.

$$M'_u = \begin{pmatrix} 0 & 0 & 123.038 - 285.496i \\ 0 & 1430.03 & 18632.8 - 2336.25i \\ 123.038 + 285.496i & 18632.8 + 2336.25i & 170033 \end{pmatrix} \text{MeV}, \tag{4.23a}$$

$$M'_d = \begin{pmatrix} 0 & 13.2473 & 0 \\ 13.2473 & 0 & 425.817 \\ 0 & 425.817 & 2796.6 \end{pmatrix} \text{MeV}, \tag{4.23b}$$

with $\lambda_{2u} = -m_c$.

TABLE II. The u-diagonal representation: the "down" mass matrix entries for the two-zero family case.

M'_d entries	Negative mass eigenvalue Case 1. $\lambda_{1d} = -m_d$ (MeV)	Case 2. $\lambda_{2d} = -m_s$ (MeV)	Case 3. $\lambda_{3d} = -m_b$ (MeV)
$M'_d(1,1)$	$0.758616 + 0.0000632072x_1 + 0.000196652x_2$ $-0.000112489y_1 - 1.23159x_1y_1 - 0.0359124x_2y_1$ $+0.0359124x_1y_2 - 1.23159x_2y_2$	$-0.575839 + 0.0000858823x_1 + 0.000196682x_2$ $-0.000116848y_1 + 1.20436x_1y_1 - 0.0359178x_2y_1$ $+0.0359178x_1y_2 + 1.20436x_2y_2$	$11.261 - 0.0000849534x_1 - 0.00019705x_2$ $+0.000116858y_1 - 1.0895x_1y_1 + 0.0359851x_2y_1$ $-0.0359851x_1y_2 - 1.0895x_2y_2$
$M'_d(1,2)$	$-5.41488 + 0.182964x_1 + 0.569243x_2$ $-0.324408y_1 + 13.1875x_1y_1 + 0.384538x_2y_1$ $+0.000121238y_2 - 0.384538x_1y_2 + 13.1875x_2y_2$ $+i(-0.569234x_1 + 0.182961x_2 - 0.00012214y_1$ $-0.386206x_1y_1 + 13.2447x_2y_1 - 0.326823y_2$ $-13.2447x_1y_2 - 0.386206x_2y_2)$	$4.52621 + 0.248601x_1 + 0.56933x_2$ $-0.336979y_1 - 12.8959x_1y_1 + 0.384597x_2y_1$ $-0.000121238y_2 - 0.384597x_1y_2 - 12.8959x_2y_2$ $+i(-0.569321x_1 + 0.248597x_2 + 0.00012214y_1$ $-0.386264x_1y_1 - 12.9519x_2y_1 - 0.339487y_2$ $+12.9519x_1y_2 - 0.386264x_2y_2)$	$-4.05674 - 0.245913x_1 - 0.570396x_2$ $+0.337008y_1 + 11.6661x_1y_1 - 0.385317x_2y_1$ $+0.000109807y_2 + 0.385317x_1y_2 + 11.6661x_2y_2$ $+i(0.570386x_1 - 0.245909x_2 - 0.000110625y_1$ $+0.386987x_1y_1 + 11.7166x_2y_1 + 0.339516y_2$ $-11.7166x_1y_2 + 0.386987x_2y_2)$
$M'_d(1,3)$	$0.359323 + 3.00824x_1 + 9.35933x_2$ $-5.35379y_1 - 0.802056x_1y_1 - 0.0233874x_2y_1$ $+0.00200082y_2 + 0.0233874x_1y_2 - 0.802056x_2y_2$ $+i(-9.35933x_1 + 3.00824x_2 - 0.00200077y_1$ $+0.0234892x_1y_1 - 0.805546x_2y_1 - 5.35364y_2$ $+0.805546x_1y_2 + 0.0234892x_2y_2)$	$-0.245286 + 4.08743x_1 + 9.36075x_2$ $-5.56125y_1 + 0.784325x_1y_1 - 0.023391x_2y_1$ $-0.00200082y_2 + 0.023391x_1y_2 + 0.784325x_2y_2$ $+i(-9.36075x_1 + 4.08743x_2 + 0.00200077y_1$ $+0.0234928x_1y_1 + 0.787738x_2y_1 - 5.56109y_2$ $-0.787738x_1y_2 + 0.0234928x_2y_2)$	$0.216621 - 4.04322x_1 - 9.37828x_2$ $+5.56172y_1 - 0.709524x_1y_1 + 0.0234348x_2y_1$ $+0.00181218y_2 - 0.0234348x_1y_2 - 0.709524x_2y_2$ $+i(9.37828x_1 - 4.04322x_2 - 0.00181213y_1$ $-0.0235368x_1y_1 - 0.712611x_2y_1 + 5.56157y_2$ $+0.712611x_1y_2 - 0.0235368x_2y_2)$
$M'_d(2,2)$	$127.279 + 0.016987x_1 + 0.0528505x_2 + 13.9766y_1$ $+1.22703x_1y_1 + 0.0357795x_2y_1 - 0.00522333y_2$ $-0.0357795x_1y_2 + 1.22703x_2y_2$	$-86.9431 + 0.023081x_1 + 0.0528585x_2$ $+14.5182y_1 - 1.19991x_1y_1 + 0.035785x_2y_1$ $+0.00522333y_2 - 0.035785x_1y_2 - 1.19991x_2y_2$	$87.5349 - 0.0228314x_1 - 0.0529575x_2$ $-14.5194y_1 + 1.08547x_1y_1 - 0.035852x_2y_1$ $-0.00473086y_2 + 0.035852x_1y_2 + 1.08547x_2y_2$
$M'_d(2,3)$	$165.914 + 0.13913x_1 + 0.432866x_2$ $+114.475y_1 - 0.0747673x_1y_1$ $-0.00218017x_2y_1 - 0.0427818y_2 + 0.00218017x_1y_2$ $-0.0747673x_2y_2 + i(-0.436093x_1$ $+0.140167x_2 + 0.0430994y_1$ $+0.000139144x_2y_1 + 115.325y_2 - 0.000139144x_1y_2)$	$178.932 + 0.189042x_1 + 0.432932x_2$ $+118.911y_1 + 0.0731144x_1y_1 - 0.0021805x_2y_1$ $+0.0427818y_2 + 0.0021805x_1y_2 + 0.0731144x_2y_2$ $+i(-0.436159x_1 + 0.190451x_2 - 0.0430994y_1$ $-0.000136068x_2y_1 + 119.794y_2 + 0.000136068x_1y_2)$	$-178.968 - 0.186998x_1 - 0.433742x_2$ $-118.921y_1 - 0.0661415x_1y_1 + 0.00218458x_2y_1$ $-0.0387482y_2 - 0.00218458x_1y_2 - 0.0661415x_2y_2$ $+i(0.436975x_1 - 0.188392x_2 + 0.0390359y_1$ $+0.000123091x_2y_1 - 119.804y_2 - 0.000123091x_1y_2)$
$M'_d(3,3)$	$2845.12 - 0.0170502x_1 - 0.0530471x_2$ $-13.9765y_1 + 0.00455581x_1y_1 + 0.000132844x_2y_1$ $+0.00522329y_2 - 0.000132844x_1y_2 + 0.00455581x_2y_2$	$2844.14 - 0.0231669x_1 - 0.0530552x_2$ $-14.5181y_1 - 0.00445509x_1y_1 + 0.000132865x_2y_1$ $-0.00522329y_2 - 0.000132865x_1y_2 - 0.00445509x_2y_2$	$-2844.14 + 0.0229163x_1 + 0.0531545x_2$ $+14.5193y_1 + 0.00403021x_1y_1 - 0.000133113x_2y_1$ $+0.00473083y_2 + 0.000133113x_1y_2 + 0.00403021x_2y_2$

TABLE III. The d-diagonal representation: the "up" mass matrix entries for the two-zero family case.

M'_u entries		Negative mass eigenvalue	
	Case 1. $\lambda_{1u} = -m_u$ (MeV)	Case 2. $\lambda_{2u} = -m_c$ (MeV)	Case 3. $\lambda_{3u} = -m_t$ (MeV)
$M'_u(1,1)$	$151.93 + 1.84869x_1 - 0.735842x_2$ $+1.839y_1 + 74.8244x_1y_1 - 8.85442x_2y_1$ $+0.0337901y_2 + 8.85442x_1y_2 + 74.8244x_2y_2$	$-59.245 + 1.86453x_1 - 0.735821x_2$ $+1.85259y_1 - 32.2673x_1y_1 - 8.92032x_2y_1$ $+0.0337892y_2 + 8.92032x_1y_2 - 32.2673x_2y_2$	$64.2966 - 1.86453x_1 + 0.735833x_2$ $-1.85259y_1 + 32.0358x_1y_1 + 8.92032x_2y_1$ $-0.0337897y_2 - 8.92032x_1y_2 + 32.0358x_2y_2$
$M'_u(1,2)$	$-300.727 + 199.742x_1 - 79.504x_2$ $+193.933y_1 - 179.051x_1y_1 + 21.1882x_2y_1$ $+3.56336y_2 - 21.1882x_1y_2 - 179.051x_2y_2$ $+i(79.3596x_1 + 199.379x_2 - 3.73171y_1$ $-22.926x_1y_1 - 193.736x_2y_1 + 203.095y_2$ $+193.736x_1y_2 - 22.926x_2y_2)$	$132.633 + 201.453x_1 - 79.5017x_2$ $+195.366y_1 + 77.2138x_1y_1 + 21.3458x_2y_1$ $+3.56326y_2 - 21.3458x_1y_2 + 77.2138x_2y_2$ $+i(79.3573x_1 + 201.087x_2 - 3.7316y_1$ $-23.0966x_1y_1 + 83.5468x_2y_1 + 204.596y_2$ $-83.5468x_1y_2 - 23.0966x_2y_2)$	$-131.697 - 201.453x_1 + 79.503x_2$ $-195.366y_1 - 76.6599x_1y_1 - 21.3458x_2y_1$ $-3.56332y_2 + 21.3458x_1y_2 - 76.6599x_2y_2$ $+i(-79.3586x_1 - 201.087x_2 + 3.73166y_1$ $+23.0966x_1y_1 - 82.9474x_2y_1 - 204.596y_2$ $+82.9474x_1y_2 + 23.0966x_2y_2)$
$M'_u(1,3)$	$163.157 + 1340.29x_1 - 533.481x_2$ $+1333.97y_1 + 26.6073x_1y_1 - 3.1486x_2y_1$ $+24.5107y_2 + 3.1486x_1y_2 + 26.6073x_2y_2$ $+i(533.503x_1 + 1340.34x_2 - 24.4856y_1$ $+3.41346x_1y_1 + 28.8455x_2y_1 + 1332.61y_2$ $-28.8455x_1y_2 + 3.41346x_2y_2)$	$98.7777 + 1351.77x_1 - 533.466x_2$ $+1343.83y_1 - 11.4741x_1y_1 - 3.17203x_2y_1$ $+24.51y_2 + 3.17203x_1y_2 - 11.4741x_2y_2$ $+i(533.488x_1 + 1351.83x_2 - 24.4849y_1$ $+3.43886x_1y_1 - 12.4393x_2y_1$ $+1342.46y_2 + 12.4393x_1y_2 + 3.43886x_2y_2)$	$-98.9206 - 1351.77x_1 + 533.475x_2$ $-1343.83y_1 + 11.3918x_1y_1 + 3.17203x_2y_1$ $-24.5103y_2 - 3.17203x_1y_2 + 11.3918x_2y_2$ $+i(-533.497x_1 - 1351.83x_2 + 24.4853y_1$ $-3.43886x_1y_1 + 12.3501x_2y_1 - 1342.46y_2$ $-12.3501x_1y_2 - 3.43886x_2y_2)$
$M'_u(2,2)$	$5396.4 + 78.3341x_1 - 31.1797x_2$ $-1978.06y_1 - 73.1657x_1y_1 + 8.65814x_2y_1$ $-36.3452y_2 - 8.65814x_1y_2 - 73.1657x_2y_2$	$3115.84 + 79.0053x_1 - 31.1788x_2$ $-1992.67y_1 + 31.552x_1y_1 + 8.72257x_2y_1$ $-36.3441y_2 - 8.72257x_1y_2 + 31.552x_2y_2$	$-3115.38 - 79.0051x_1 + 31.1793x_2$ $+1992.67y_1 - 31.3256x_1y_1 - 8.72257x_2y_1$ $+36.3447y_2 + 8.72257x_1y_2 - 31.3256x_2y_2$
$M'_u(2,3)$	$24777.1 + 257.091x_1 - 102.331x_2$ $-6495.55y_1 + 11.0171x_1y_1 - 1.30373x_2y_1$ $-119.35y_2 + 1.30373x_1y_2 + 11.0171x_2y_2$ $+i(107.083x_1 + 269.029x_2 + 124.757y_1$ $-0.0158108x_1y_1 - 0.13361x_2y_1 - 6789.79y_2$ $+0.13361x_1y_2 - 0.0158108x_2y_2)$	$25116.1 + 259.294x_1 - 102.328x_2$ $-6543.55y_1 - 4.75103x_1y_1 - 1.31343x_2y_1$ $-119.347y_2 + 1.31343x_1y_2 - 4.75103x_2y_2$ $+i(107.08x_1 + 271.334x_2 + 124.753y_1$ $-0.0159285x_1y_1 + 0.0576178x_2y_1 - 6839.96y_2$ $-0.0576178x_1y_2 - 0.0159285x_2y_2)$	$-25116.1 - 259.293x_1 + 102.33x_2$ $+6543.55y_1 + 4.71695x_1y_1 + 1.31343x_2y_1$ $+119.349y_2 - 1.31343x_1y_2 + 4.71695x_2y_2$ $+i(-107.081x_1 - 271.334x_2 - 124.755y_1$ $+0.0159285x_1y_1 - 0.0572044x_2y_1 + 6839.96y_2$ $+0.0572044x_1y_2 + 0.0159285x_2y_2)$
$M'_u(3,3)$	$168118 - 80.1828x_1 + 31.9155x_2$ $+1976.22y_1 - 1.6587x_1y_1 + 0.196283x_2y_1$ $+36.3114y_2 - 0.196283x_1y_2 - 1.6587x_2y_2$	$168065. - 80.8698x_1 + 31.9146x_2$ $+1990.82y_1 + 0.715295x_1y_1 + 0.197744x_2y_1$ $+36.3103y_2 - 0.197744x_1y_2 + 0.715295x_2y_2$	$-168065. + 80.8696x_1 - 31.9151x_2$ $-1990.82y_1 - 0.710164x_1y_1 - 0.197744x_2y_1$ $-36.3109y_2 + 0.197744x_1y_2 - 0.710164x_2y_2$

B. One-zero family

A typical representation of this family is given by

$$M_{u,d} = \begin{pmatrix} 0 & |B_{u,d}| & 0 \\ |B_{u,d}| & C_{u,d} & 0 \\ 0 & 0 & A_{u,d} \end{pmatrix}. \quad (4.24)$$

The mass matrix $M_{u,d}$ is diagonalized as follows

$$O^{\dagger}_{u,d} M_{u,d} O_{u,d} = O^{\dagger}_{u,d} \begin{pmatrix} 0 & |B_{u,d}| & 0 \\ |B_{u,d}| & C_{u,d} & 0 \\ 0 & 0 & A_{u,d} \end{pmatrix} O_{u,d}, \quad (4.25)$$

$$= \begin{pmatrix} \lambda_{1u,d} & & \\ & \lambda_{2u,d} & \\ & & \lambda_{3u,d} \end{pmatrix}. \quad (4.26)$$

The following matricial functions allow us to write the elements of $M_{u,d}$ in terms of its eigenvalues $\lambda_{iu,d}$. They are

$$\mathrm{tr}\, M_{u,d} = A_{u,d} + C_{u,d} = \lambda_{1u,d} + \lambda_{2u,d} + \lambda_{3u,d}, \quad (4.27)$$

$$\mathrm{tr}\, M^2_{u,d} = A^2_{u,d} + 2|B_{u,d}|^2 + C^2_{u,d}, = \lambda^2_{1u,d} + \lambda^2_{2u,d} + \lambda^2_{3u,d}, \quad (4.28)$$

$$\det M_{u,d} = -A_{u,d}|B_{u,d}|^2 = \lambda_{1u,d}\lambda_{2u,d}\lambda_{3u,d}, \quad (4.29)$$

from which we have various solutions

(a)

$$A_{u,d} = \lambda_{1u,d}, \qquad |B_{u,d}| = \sqrt{-\lambda_{2u,d}\lambda_{3u,d}}, \qquad C_{u,d} = \lambda_{2u,d} + \lambda_{3u,d}, \quad (4.30)$$

(b)

$$A_{u,d} = \lambda_{2u,d}, \qquad |B_{u,d}| = \sqrt{-\lambda_{1u,d}\lambda_{3u,d}}, \qquad C_{u,d} = \lambda_{1u,d} + \lambda_{3u,d}, \quad (4.31)$$

(c)

$$A_{u,d} = \lambda_{3u,d}, \qquad |B_{u,d}| = \sqrt{-\lambda_{1u,d}\lambda_{2u,d}}, \qquad C_{u,d} = \lambda_{1u,d} + \lambda_{2u,d}. \quad (4.32)$$

Each one of these former cases was analyzed. Both representations u-diagonal and d-diagonal were worked. The Eqs. (4.30), (4.31), and (4.32) give two possibilities for each case (a), (b), and (c), depending of which eigenvalue is negative. In turn, each one of these cases contains three possibilities depending of the negative eigenvalue assigned for the down (up) mass matrix. In total, there are 36 possibilities. Neither of these cases was able to give models with five-texture or six-texture zeros.

V. ANALYTICAL FIVE-TEXTURE ZEROS AND THE CKM MATRIX

The five-texture zeros form of Eq. (4.23), derived under the conditions given in Sec. IV A 2, is especially interesting because the latest low-energy data shows that it is a viable model, something not considered or ruled out in Refs. [5,8,21]. Therefore, let us assume the following five-texture zeros model

$$M_u = P^{\dagger} \begin{pmatrix} 0 & 0 & |C_u| \\ 0 & A_u & |B_u| \\ |C_u| & |B_u| & \tilde{B}_u \end{pmatrix} P, \qquad M_d = \begin{pmatrix} 0 & |C_d| & 0 \\ |C_d| & 0 & |B_d| \\ 0 & |B_d| & A_d \end{pmatrix}, \quad (5.1)$$

where up and down quark mass matrices are given in the most general way, $P = \mathrm{diag}(e^{-i\phi_{c_u}}, e^{-i\phi_{b_u}}, 1)$ with $\phi_{b_u} \equiv \arg(B_u)$ and $\phi_{c_u} \equiv \arg(C_u)$, where the phases for M_d were not considered because they can be absorbed, through a WB transformation, into P. Considering $\lambda_{2u} = -m_c$, we have from (3.20) through (3.22) that

$$\tilde{B}_u = m_u + m_t - m_c - A_u, \qquad |B_u| = \frac{\sqrt{A_u + m_c}\sqrt{m_t - A_u}\sqrt{A_u - m_u}}{\sqrt{A_u}}, \qquad |C_u| = \frac{\sqrt{m_c}\sqrt{m_t}\sqrt{m_u}}{\sqrt{A_u}}, \quad (5.2)$$

where (3.27) was considered.

Taking into account (4.9) and (4.10), for the down mass matrix we have that

$$A_d = m_d + m_b - m_s, \qquad |B_d| = \frac{\sqrt{m_d + m_b}\sqrt{m_b - m_s}\sqrt{m_s - m_d}}{\sqrt{m_d + m_b - m_s}}, \qquad |C_d| = \frac{\sqrt{m_b}\sqrt{m_d}\sqrt{m_s}}{\sqrt{m_d + m_b - m_s}}. \quad (5.3)$$

The unitary matrix U_u, which diagonalizes M_u, is given by

$$U_u = P^\dagger \cdot p_2 \cdot O_u \approx \begin{pmatrix} \frac{\sqrt{Au - m_u} e^{i(\phi_{c_u} + x_u)}}{\sqrt{Au}} & -\frac{\sqrt{A_u + m_c}\sqrt{m_u} e^{i(\phi_{c_u} + y_u)}}{\sqrt{A_u}\sqrt{m_c}} & \frac{\sqrt{m_c}\sqrt{m_t - A_u}\sqrt{m_u} e^{i(\phi_{c_u} + z_u)}}{\sqrt{A_u m_t}} \\ -\frac{\sqrt{A_u + m_c}\sqrt{m_t - A_u}\sqrt{m_u} e^{i(\phi_{b_u} + x_u)}}{\sqrt{A_u}\sqrt{m_c}\sqrt{m_t}} & -\frac{\sqrt{m_t - A_u}\sqrt{A_u - m_u} e^{i(\phi_{b_u} + y_u)}}{\sqrt{m_t}\sqrt{A_u}} & \frac{\sqrt{A_u + m_c}\sqrt{A_u - m_u} e^{i(\phi_{b_u} + z_u)}}{\sqrt{m_t}\sqrt{A_u}} \\ \frac{\sqrt{A_u - m_u}\sqrt{m_u} e^{i x_u}}{\sqrt{m_c}\sqrt{m_t}} & \frac{\sqrt{A_u + m_c} e^{i y_u}}{\sqrt{m_t}} & \frac{\sqrt{m_t - A_u} e^{i z_u}}{\sqrt{m_t}} \end{pmatrix}, \quad (5.4)$$

where an additional imaginary phase iz_u in the third column of O_u [Eq. (3.19)] was added in order to reproduce all phases present in the CKM matrix. The 3×3 matrix $p_2 = [(1,0,0), (0,0,1), (0,1,0)]$ and the hierarchy (2.14) together with (3.27) were considered.

The unitary matrix U_d, which diagonalizes M_d, is given by

$$U_d \approx \begin{pmatrix} e^{ix_d} & -\frac{\sqrt{m_d} e^{iy_d}}{\sqrt{m_s}} & \frac{\sqrt{m_d m_s}}{(m_b)^{3/2}} \\ \frac{\sqrt{m_d} e^{ix_d}}{\sqrt{m_s}} & e^{iy_d} & \frac{\sqrt{m_s}}{\sqrt{m_b}} \\ -\frac{\sqrt{m_d} e^{ix_d}}{\sqrt{m_b}} & -\frac{\sqrt{m_s} e^{iy_d}}{\sqrt{m_b}} & 1 \end{pmatrix}, \quad (5.5)$$

where, in the process, an imaginary phase in the third column was not necessary to be included. Now, we can easily find the CKM matrix $V = U_u^\dagger U_d$. In particular, using the matrix form (5.4) and (5.5) for U_u, U_d, respectively, can survive current experimental tests. To leading order, we obtain.

$$|V_{ud}| \approx |V_{cs}| \approx |V_{tb}| \approx 1, \quad (5.6a)$$

$$|V_{us}| \approx |V_{cd}| \approx \left| \sqrt{\frac{A_u + m_c}{A_u}} \sqrt{\frac{m_u}{m_c}} + e^{\pm i(\phi_{b_u} - \phi_{c_u})} \sqrt{\frac{m_d}{m_s}} \right|, \quad (5.6b)$$

$$|V_{cb}| \approx |V_{ts}| \approx \left| \sqrt{\frac{m_s}{m_b}} - e^{\pm i\phi_{b_u}} \sqrt{\frac{A_u + m_c}{m_t}} \right|, \quad (5.6c)$$

$$\frac{|V_{ub}|}{|V_{cb}|} \approx \sqrt{\frac{m_u}{m_c}} \left| \frac{\sqrt{\frac{A_u}{m_t}} - e^{-i\phi_{b_u}} \sqrt{\frac{A_u + m_c}{A_u}} \sqrt{\frac{m_s}{m_b}}}{\sqrt{\frac{A_u + m_c}{m_t}} - e^{-i\phi_{b_u}} \sqrt{\frac{m_s}{m_b}}} \right|, \quad (5.6d)$$

$$\frac{|V_{td}|}{|V_{ts}|} \approx \sqrt{\frac{m_d}{m_s}}, \quad (5.6e)$$

where we assume that $m_u \ll A_u \ll m_t$. The sign "+" for V_{us}, V_{cb} and "−" for V_{cd}, V_{ts}. Note that if $A_u \gg m_c$, then $\frac{|V_{ub}|}{|V_{cb}|} \approx \sqrt{\frac{m_u}{m_c}}$, but this is not our case.

It is obvious that Eqs. (5.6a), (5.6b), and (5.6e), are consistent with the previous results [5,22]. A good fit of Eqs. (5.6) and the CKM to the experimental data suggests

$$\begin{aligned} A_u &= 1430.03\ \text{MeV}, \quad \phi_{b_u} = -0.124733, \quad \phi_{c_u} = -1.16389, \quad x_u = -1.83392, \\ y_u &= -2.68335, \quad z_u = 0.00200664, \quad x_d = -3.00697, \quad y_d = 0.344676, \end{aligned} \quad (5.7)$$

which differ from the values given in Refs. [5,22], $\phi_1 \approx \pi/3 \sim (\phi_{b_u} - \phi_{c_u})$, such that it is an important contribution term of CP violation in the context of present mass matrices, and $\phi_2 \approx \pi/25 \sim -\phi_{b_u} \to 0$. The numerical analysis shows that by plugging for the quark masses the values given in (3.1) and the input parameters in (5.7), we obtain the following absolute values for the mixing matrix:

$$|V_{\text{ckm}}| = \begin{pmatrix} 0.993 & 0.255 \pm 0.030 & 0.00334 \pm 0.00094 \\ 0.255 \pm 0.030 & 1.004 & 0.034 \pm 0.014 \\ 0.0079 \pm 0.0020 & 0.035 \pm 0.014 & 1.011 \end{pmatrix} \quad (5.8)$$

in good agreement with the experimental measured values presented in (3.6). For the Wolfenstein parameters we find that

$$\lambda' = 0.247 \pm 0.027, \quad A' = 0.55^{+0.43}_{-0.31}, \quad \bar{\rho}' = 0.117 \pm 0.061, \quad \bar{\eta}' = 0.361 \pm 0.070, \quad (5.9)$$

which is in quite good agreement with the fit experimental values (3.4). The inner angles of the CKM unitarity triangle, $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, are

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = 24.4114^\circ, \quad \alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = 82.6294^\circ, \quad \gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = 72.9592^\circ, \quad (5.10)$$

which are in the constraint established by [7]. The Jarlskog invariant obtained is

$$J' = \mathrm{Im}(V_{us}V_{ub}^{*}V_{cs}^{*}V_{cb}) = 2.8322 \times 10^{-5}, \qquad (5.11)$$

which can be found in the interval given in (3.7).

VI. CONCLUSIONS

Within the standard model framework, we have investigated texture zeros for quark mass matrices that reproduce the quark masses and experimental mixing parameters. To simplify the problem, without loss of generality, we consider that the quark mass matrices are Hermitian, since the right chirality fields are singlets under the gauge symmetry $SU(2)$. So, for any model where the fields are right chiral singlet under the local gauge symmetry, we may consider that their mass matrices are Hermitian. Specific six-texture zeros in quark mass matrices, including the Fritzsch model [1] and others like Ref. [4], have already been discarded because they cannot adjust their results to the experimental data known at present. In Sec. II, together with the definition of WB transformation, it is shown that the number of nonequivalent representations for the quark mass matrices is finite, which greatly simplifies the problem. Through WB transformations, it was relatively easy to find nonparallel four-texture zeros mass matrices. More difficult, but feasible, was the case for parallel four-texture zeros mass matrices, which were found in an exact way. Significant was the consistent five-texture zeros quark mass matrix found by us. Similarly, we show the impossibility, under any circumstances, to find mass matrices with six-texture zeros consistent with experimental data. This is a generalization of six-texture zeros mass matrices discarded by Fritzsch *et al.*

Throughout this letter, into the SM, we have used the fact that all WB are equivalent. The opposite case is valid too, i.e., two quark mass matrices representations giving the same physical quantities must be related through a WB transformation, which is condensed in statement (2.11).

By making appropriated WB transformations, numerical parallel and nonparallel four-texture zeros were found. An exhaustive deduction process allows us to find a five-texture zeros numerical structure compatible with the experimental data, Eqs. (4.22) and (4.23). This representation was found in the two-zero family case. Equivalent representations are given in Table I.

We have determined the impossibility of finding quark mass matrices having a total of six-texture zeros which are consistent with the measured values of the quark masses and mixing angles, although a consistent model with five-texture zeros was successful. The five-texture zeros *Ansatz* of Eq. (5.1) (with $\lambda_{2u} = -m_c$), together with some assumptions that include appropriated values for A_u, ϕ_{b_u}, ϕ_{c_u}, x_u, y_u, z_u, x_d, and y_d does lead to successful predictions for V_{CKM}, such as those of Eqs. (5.6), (5.8), (5.9), (5.10), and (5.11).[4] One nice thing about five-texture zeros quark mass matrices (5.1) is that no hierarchies on quark masses need to be imposed to make correct predictions, although expressions (5.6) come in a more complex notation.

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APPENDIX: VERIFICATION OF THE METHOD

Reference [8] uses the following quark mass data:

$$m_u = 2.50\ \mathrm{MeV}, \qquad m_c = 600\ \mathrm{MeV}, \qquad m_t = 174000\ \mathrm{MeV}, \qquad (A1)$$

$$m_d = 4.00\ \mathrm{MeV}, \qquad m_s = 80\ \mathrm{MeV}, \qquad m_b = 3000\ \mathrm{MeV}, \qquad (A2)$$

and the numerical CKM matrix used is

$$V = \begin{pmatrix} 0.036195 + 0.97493i & -0.057798 + 0.21177i & 0.00037188 - 0.0035669i \\ -0.21247 + 0.054471i & 0.97351 + 0.050582i & -0.0044010 - 0.039760i \\ 0.0043605 + 0.0083871i & 0.0086356 - 0.038067i & 0.99836 + 0.040693i \end{pmatrix}. \qquad (A3)$$

We assume the following case:

$$\lambda_{1u} = -m_u, \qquad \lambda_{2u} = m_c, \qquad \lambda_{3u} = m_t, \qquad (A4)$$

$$\lambda_{1d} = -m_d, \qquad \lambda_{2d} = m_s, \qquad \lambda_{3d} = m_b. \qquad (A5)$$

Then, the quark mass matrices (2.12) are

[4] In the case $\lambda_{1u} = -m_u$, similar results can be found.

$$M_u = \begin{pmatrix} -2.5 & & \\ & 600 & \\ & & 174000 \end{pmatrix} \text{MeV}, \tag{A6}$$

$$M_d = \begin{pmatrix} 0.086447 & -3.4055 + 17.655i & -0.039835 - 10.774i \\ -3.4055 - 17.655i & 80.631 & -17.515 - 115.56i \\ -0.039835 + 10.774i & -17.515 + 115.56i & 2995.3 \end{pmatrix} \text{MeV}. \tag{A7}$$

Let us use the diagonalization matrix (3.19) with $x = \pi$ and $y = \pi$,

$$O_u \approx 10^{-3} \begin{pmatrix} -997.92 & -64.527\sqrt{\frac{A_u - 600}{A_u}} & 0.22297\sqrt{\frac{174000 - A_u}{A_u}} \\ 0.15442\sqrt{A_u} & -2.3965\sqrt{A_u - 600} & 2.4014\sqrt{174000 - A_u} \\ -0.15442\sqrt{\frac{(174000 - A_u)(A_u - 600)}{A_u}} & 2.3965\sqrt{174000 - A_u} & 2.4014\sqrt{A_u - 600} \end{pmatrix}, \tag{A8}$$

where the approximation $A_u \gg m_u$ was assumed because of the restriction (3.26). The matrix O_u now plays the role of a unitary matrix to make the WB transformation on (A6) and (A7). The entries, in the new representation, depend on A_u. In order to have texture zeros at the entry (1, 3), we need to solve

$$\begin{aligned} M_d(1,3) = Y(A_u) \propto\ & (8144.2 - 42221i)A_u\sqrt{174000 - A_u} + (95.463 + 25819i)A_u\sqrt{A_u - 600} - (10852) \\ & \times \sqrt{A_u(A_u - 600)(174000 - A_u)} - (9.3588 - 61.745i)(174000 - A_u)\sqrt{A_u} + (2714.0 + 17906i)(A_u - 600) \\ & \times \sqrt{A_u} + (0.0013716 - 0.37097i)(174000 - A_u)\sqrt{A_u - 600} - (33.934 + 175.92i)(A_u - 600) \\ & \times \sqrt{174000 - A_u} \approx 0, \end{aligned} \tag{A9}$$

whose solution is $A_u \approx 84621$ MeV, which agrees perfectly with the value given in the aforementioned paper. The quark mass matrices (A6) and (A7) take the form

$$M'_u = O_u M_u O_u^T = \begin{pmatrix} 0 & 55.537 & 0 \\ 55.537 & 89977 & 86660 \\ 0 & 86660 & 84621 \end{pmatrix} \text{MeV}, \tag{A10}$$

$$M'_d = O_u M_d O_u^T \tag{A11}$$

$$\approx \begin{pmatrix} 0 & 2.5792 + 25.325i & 0 \\ 2.5792 - 25.325i & 1600.5 & 1456.0 + 114.63i \\ 0 & 1456.0 - 114.63i & 1475.5 \end{pmatrix} \text{MeV}. \tag{A12}$$

At the present stage we have not yet obtained the matrices given in (25) and (26) of paper [8]. But we can make an additional WB transformation using the following imaginary phase unitary matrix

$$P = \begin{pmatrix} 1 & & \\ & e^{i4.4984} & \\ & & e^{-i0.063300} \end{pmatrix}. \tag{A13}$$

We finally get the desired matrices

$$M''_u = P^\dagger M'_u P = \begin{pmatrix} 0 & -11.794 - 54.270i & 0 \\ -11.794 + 54.270i & 89977 & -13009 + 85678i \\ 0 & -13009 - 85678i & 84621 \end{pmatrix} \text{MeV}, \tag{A14}$$

$$M''_d = P^\dagger M'_d P = \begin{pmatrix} 0 & 24.199 - 7.8983i & 0 \\ 24.199 + 7.8983i & 1600.5 & -331.91 + 1422.3i \\ 0 & -331.91 - 1422.3i & 1475.5 \end{pmatrix} \text{MeV}. \tag{A15}$$

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