S_3 model for lepton mass matrices with nearly minimal texture

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We propose a simple extension of the electroweak standard model based on the discrete S_3 symmetry that is capable of realizing a nearly minimal Fritzsch-type texture for the Dirac mass matrices of both charged leptons and neutrinos. This is achieved with the aid of additional Z_5 and Z_3 symmetries, one of which can be embedded in $U(1)_{B-L}$. Five complex scalar singlet fields are introduced in addition to the standard model with right-handed neutrinos. Although more general, the modified texture of the model retains the successful features of the minimal texture without fine-tuning; namely, it accommodates the masses and mixing of the leptonic sector and relates the emergence of large leptonic mixing angles with the seesaw mechanism. For large deviations of the minimal texture, both quasidegenerate spectrum or inverted hierarchy are allowed for neutrino masses.

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I. INTRODUCTION

As experimental efforts improve our knowledge about the masses and the mixing pattern of neutrinos, the small neutrino masses and the large mixing angles still await a natural explanation. The former is successfully accommodated by the seesaw mechanism but its testability is usually out of reach of our present experiments. As for the mixing angles, the puzzle is to explain the large angles and the great difference from the Cabibbo-Kobayashi-Maskawa mixing for quarks, which is governed by small mixing angles.

Concerning the quark sector, a scheme relating small mass ratios with small mixing angles can be devised by assuming a simple texture for the quark mass matrix [1]. More precisely, within two families, a Hermitian mass matrix with a vanishing (1,1) element yields the correct mixing angle $\theta_C \sim (m_d/m_s)^{1/2}$. An extension to three families was proposed by Fritzsch assuming a minimal texture of the 3×3 matrix, with vanishing (1,1), (1,3), (3,1), (2,2) elements and a Hermitian form [2]. Such a texture determines the elements of the Cabibbo-Kobayashi-Maskawa matrix as functions of the ratios of the quark masses. This minimal texture, however, does not accommodate the present data of quark masses and mixing structure.

For the lepton sector, it was shown that it is possible to obtain the neutrino masses and mixing with the same Fritzsch-type texture described above, with the Hermitian form replaced by a symmetric form [3] (see also Ref. [4]). The texture is applied to the Dirac mass matrices for charged leptons and neutrinos, but light neutrino masses arise from the seesaw mechanism. Consequently, light neutrino masses depend *quadratically* on the Dirac mass matrix. This quadratic dependence, in turn, determines that the elements of the Pontercorvo-Maki-Nakagawa-Sakata (PMNS) matrix depend on the ratio of charged lepton masses but on the *square root* of the ratio of neutrino masses. This property is what enables large mixing angles to emerge from moderately hierarchical neutrino masses within this Fritzsch-type texture [3].

More recently, using this minimal texture, Ref. [5] succeeded in predicting all observables in neutrino sector from the known values of the squared-mass differences and mixing angles. The recently observed θ_{13} [6] was predicted in Ref. [3]b in the right range. This information could then be used in Ref. [5] to make more precise predictions for the effective mass of double beta decay and the CP violation measure. There is no ambiguity in hierarchy since the texture only allows the normal hierarchy for the neutrino masses, excluding either inverse hierarchy or quasidegenerate masses.

It is interesting to note that the Fritzsch-type texture proposed in Ref. [3] is a particular example of textures with vanishing matrix elements (*texture zeros*), which was extensively studied in the context of neutrino mixing; e.g., see Ref. [7] for a detailed study on two-zero textures. Usually, these special textures are supposed to be apparent for the neutrino mass matrix in the basis where the charged lepton mass matrix is diagonal [8]. However, the case in Ref. [3] is different, once the texture proposed is shared by the Dirac mass matrices of both charged leptons and neutrinos.

On the other hand, from the theoretical point of view, the phenomenologically successful textures should originate as a consequence of an underlying exact or approximate flavor symmetry acting at energies above the electroweak scale. For example, Abelian symmetries can be systematically used to justify texture zeros [9]. More specifically, one can obtain the necessary texture zeros of the Fritzsch ansatz for quarks from a Z_4 symmetry within a 2-Higgs-doublet model [10]. Some further relations between the nonzero entries of the mass matrices may arise from an underlying unifying gauge theory, which naturally accommodates a

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symmetric or Hermitian mass matrix such as in $SU(5)$ constructions [11] (symmetric M_u) or the original left-right proposal [2] (Hermitian $M_{u,d}$).

In contrast to Abelian symmetries, the use of discrete non-Abelian symmetries is particularly appealing [12] to generate *mass-independent* mixing patterns [13], the most interesting of which is the tribimaximal mixing. In this context, models using the simple S_3 symmetry can be found abundantly in the literature [14].

In this work, we take a different perspective and use the non-Abelian S_3 flavor symmetry, together with some Abelian symmetries, to impose texture zeros, and at the same time, relate some nonzero elements of the mass matrices. The latter cannot be accomplished within the gauge structure of the standard model (SM) if we only impose Abelian symmetries. The resulting form for the mass matrices is a modified form of the ansatz of Ref. [3]. The texture for the mass matrices of charged leptons and neutrinos has in fact a common origin according to our proposal.

Other approaches to the mass and mixing problem as we deal with here can be found in the literature as, e.g., the use of S_3 symmetry aiming at explaining a Fritzsch-type texture [15]. The latter, however, is obtained only for the neutrino mass matrix *itself*; *videlicet*, a seesaw mechanism is not considered and the charged lepton mass matrix is also not of the Fritzsch type but nearly diagonal.

The outline of the paper is the following. In Sec. II, we review the minimal texture hypothesis of Ref. [3]. In Sec. III, we present the model. We show in Sec. IV an analysis of the modified form for the ansatz and some numerical examples. The conclusions are shown in Sec. V. In the Appendix, we discuss the scalar potential and justify the necessary alignment for the vacuum expectation values (VEVs) of some scalar fields.

II. THE MINIMAL TEXTURE ANSATZ

The minimal texture hypothesis of Refs. [3,5] consists of assuming that the mass matrix for charged leptons and the Dirac mass matrix for neutrinos have the simple form [2]

$$M = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix}, \tag{1}$$

where A, B, C are complex numbers.

With a rephasing of lepton fields we can transform (1) to a real symmetric matrix

$$|M| = \begin{pmatrix} 0 & |A| & 0 \\ |A| & 0 & |B| \\ 0 & |B| & |C| \end{pmatrix}. \tag{2}$$

We will denote the matrix $|M|$ in (2) as the *real form* of the matrix M in (1). The matrix in the real form can be diagonalized by a real orthogonal matrix. It follows that M can be diagonalized by

$$M^{\text{diag}} = U^{\mathsf{T}} M U, \tag{3}$$

where

$$U = d^{\dagger} O, \tag{4}$$

d is a diagonal rephasing matrix, and O a real orthogonal matrix [the transformation in (3) is a special case of a biunitary transformation]. The three parameters $|A|$, $|B|$, $|C|$ are completely determined by the three eigenvalues of M, $(m_1,\ -m_2,\ m_3)$, where $0 \leq m_1 < m_2 < m_3$. When $m_1 \ll m_2 \ll m_3$, this ansatz implements the idea that small mixing angles are consequences of large hierarchies in mass.

Let us attribute the simple form (1) for the mass matrix of charged leptons and for the Dirac mass matrix for neutrinos,

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l & 0 & B_l \\ 0 & B_l & C_l \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & 0 & B_\nu \\ 0 & B_\nu & C_\nu \end{pmatrix}, \tag{5}$$

with both being diagonalized by matrices of the form (4).

It is assumed here that the light neutrino masses are generated through the seesaw mechanism from the mass matrix

$$M_\nu = -M_D^{\mathsf{T}} M_R^{-1} M_D, \tag{6}$$

where M_R is the right-handed Majorana neutrinos mass matrix. For simplicity, it is further assumed that M_R, in the basis where M_D is diagonal, is proportional to the identity [Ref. [3], b], i.e.,

$$U_\nu^{\mathsf{T}} M_R U_\nu = M_0 \mathbb{1}_3. \tag{7}$$

M_0 is real, positive, and much larger than the Dirac neutrino mass scale. This means that in the basis where M_D has the form (5),

$$M_R = M_0 d_\nu^2, \tag{8}$$

where d_ν is the rephasing matrix for M_D. In this case, the mass matrix M_ν is still diagonalized by

$$U_\nu = d_\nu^{\dagger} O_\nu, \tag{9}$$

$$M_\nu^{\text{diag}} = -\frac{1}{M_0}[M_D^{\text{diag}}]^2 = U_\nu^{\mathsf{T}} M_\nu U_\nu, \tag{10}$$

i.e., the same matrix that diagonalizes M_D. This conclusion is not significantly modified for an M_R different from (8) if the eigenvalues of M_D are hierarchical [Ref. [3]b]; in particular, the effects of additional phases in M_R are only minor [Ref. [3]a].

The PMNS matrix is then given by

$$V_{\text{MNS}} = U_l^{\dagger} U_\nu = O_l^{\mathsf{T}} Q O_\nu, \tag{11}$$

where $Q = \text{diag}(1, e^{i\sigma}, e^{i\tau})$ is a matrix of phases coming from $d_l d_\nu^{\dagger}$ subtracting a global phase.

With this minimal texture, Ref. [5] succeeds in predicting all the relevant observables in the neutrino sector, such as the Jarlskog invariant or the absolute mass scale, from the presently known mass-squared differences and mixing angles. Our main interest in this paper is to propose a model realization of this ansatz.

III. THE MODEL

We propose a model based on S_3 symmetry that is capable of naturally generating the simple form (1) for the Dirac mass matrices for charged leptons and neutrinos.

We enlarge the symmetry group of the SM by including a flavor group $G_F = S_3 \otimes Z_5 \otimes Z_3$. The three families of left-handed lepton doublets L_i and right-handed singlets l_{iR} and ν_{iR}, $i = 1, 2, 3$ (or $l_i = e, \mu, \tau$), transform nontrivially under G_F. The Higgs doublet ϕ is a singlet of G_F. We also assume that the G_F symmetry is valid only on a scale above the electroweak scale, where new physics effects can be described by some nonrenormalizable G_F-symmetric interactions which depend on five complex scalars η, χ, χ', and ζ_i, $i = 1, 2$, that are complete singlets of the SM.

We arrange the fields of the model on multiplets transforming under irreducible representations of G_F. There are only three such representations of S_3: two singlets and a doublet, which are denoted as **1**, **1**$'$, **2**. We assign them as

$$\begin{aligned}
&\mathbf{2}\colon L_D \equiv (L_e, L_\mu), \qquad E_D \equiv (e_R, \mu_R),\\
&\quad N_D \equiv (\nu_{1R}, \nu_{2R}), \qquad \zeta_D \equiv (\zeta_1, \zeta_2);\\
&\mathbf{1}\colon L_S \equiv L_\tau, \quad E_S \equiv \tau_R, \quad N_S \equiv \nu_{3R},\\
&\quad \chi, \chi';\\
&\mathbf{1}'\colon \eta.
\end{aligned} \tag{12}$$

The complete assignment of representations of G_F is shown in Table I.

The relevant branching rule for S_3 is $\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}$. For two doublets $x = (x_1, x_2)^\mathsf{T}$ and $y = (y_1, y_2)^\mathsf{T}$, the decomposition can be performed explicitly as [12]

$$\begin{aligned}
[x \times y]_1 &= x_1 y_1 + x_2 y_2,\\
[x \times y]_{1'} &= x_1 y_2 - x_2 y_1,\\
[x \times y]_2 &= \begin{pmatrix} x_2 y_2 - x_1 y_1 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}.
\end{aligned} \tag{13}$$

Notice the following products for three doublets x, y, z:

$$\begin{aligned}
[x \times y \times z]_1 &\equiv [[x \times y]_2 \times z]_1 = [x \times [y \times z]_2]_1,\\
[x \times y \times z]_{1'} &\equiv [[x \times y]_2 \times z]_{1'} = -[x \times [y \times z]_2]_{1'},
\end{aligned} \tag{14}$$

are uniquely defined.

We can now easily write the Yukawa Lagrangian for charged leptons,

$$\begin{aligned}
-\mathcal{L}_l^Y = {}& \frac{a_l^*}{\Lambda}[\bar{L}_D \phi E_D]_{1'} \eta^* + \frac{b_l'^*}{\Lambda} \bar{L}_S \phi [E_D \zeta_D]_1\\
&+ \frac{b_l^*}{\Lambda}[\bar{L}_D \zeta_D]_1 \phi E_S + c_l^* \bar{L}_S \phi E_S + \text{H.c.},
\end{aligned} \tag{15}$$

where we have used (14) and only retained operators of order up to $1/\Lambda$ [16]. Notice the tau lepton is the only one that receives mass through renormalizable interactions, hence its large mass. In the basis $\bar{l}_{iR} l_{jL}$, we obtain the mass matrix

$$M_l = \begin{pmatrix} 0 & -A_l & 0 \\ A_l & 0 & B_l' \\ 0 & B_l & C_l \end{pmatrix}, \tag{16}$$

where the elements are

$$\begin{aligned}
A_l &= a_l v_\phi \frac{u_\eta}{\Lambda}, & B_l &= b_l v_\phi \frac{u_2^*}{\Lambda},\\
B_l' &= b_l' v_\phi \frac{u_2^*}{\Lambda}, & C_l &= c_l v_\phi.
\end{aligned} \tag{17}$$

These elements depend on the VEVs of the neutral fields which we assume have the form

$$\langle \zeta_D \rangle = (0, u_2)^\mathsf{T}, \qquad \langle \eta \rangle = u_\eta, \qquad \langle \phi^0 \rangle = v_\phi, \tag{18}$$

where $\sqrt{2} v_\phi = v = 246$ GeV is the electroweak scale; u_2 and u_η may be complex *a priori*. Notice the minus sign in the (1,2) entry of (16) can be eliminated by rephasing the appropriate right-handed lepton field.

Analogously, the Dirac mass matrix for neutrinos is generated by the effective Yukawa Lagrangian

$$\begin{aligned}
-\mathcal{L}_\nu^Y = {}& \frac{a_\nu^*}{\Lambda}[\bar{L}_D \tilde{\phi} N_D]_{1'} \eta^* + \frac{b_\nu'^*}{\Lambda} \bar{L}_S \tilde{\phi} [N_D \zeta_D]_1\\
&+ \frac{b_\nu^*}{\Lambda}[\bar{L}_D \zeta_D]_1 \tilde{\phi} N_S + c_\nu^* \bar{L}_S \tilde{\phi} N_S + \text{H.c.}
\end{aligned} \tag{19}$$

By assuming the same VEVs of (18), we obtain the same form as (16),

$$M_D = \begin{pmatrix} 0 & -A_\nu & 0 \\ A_\nu & 0 & B_\nu' \\ 0 & B_\nu & C_\nu \end{pmatrix}, \tag{20}$$

with the identification

TABLE I. Transformation properties under $G_F = S_3 \otimes Z_5 \otimes Z_3$ where $\omega = e^{i2\pi/3}$ and $\omega_5 = e^{i2\pi/5}$.

	L_D	L_S	E_D	E_S	N_D	N_S	ϕ	ζ_D	η	χ	χ'
S_3	2	1	2	1	2	1	1	2	$1'$	1	1
Z_5	ω_5^4	1	ω_5	1	ω_5	1	1	ω_5^4	ω_5^2	1	ω_5^3
Z_3	ω	ω	ω	ω	ω	ω	1	1	1	ω	ω

$$A_\nu = a_\nu v_\phi \frac{u_\eta}{\Lambda}, \qquad B_\nu = b_\nu v_\phi \frac{u_2^*}{\Lambda},$$
$$B'_\nu = b'_\nu v_\phi \frac{u_2^*}{\Lambda}, \qquad C_\nu = c_\nu v_\phi. \tag{21}$$

The Majorana mass terms are generated by

$$-\mathcal{L}_\nu^M = \frac{1}{2}\lambda_1[\bar{N}_D N_D^c]_1 \chi'^* + \frac{1}{2}\lambda_2 \bar{N}_S N_S^c \chi^* + \text{H.c.} \tag{22}$$

These terms generate the Majorana mass matrix

$$M_R = \text{diag}(\mu_1, \mu_1, \mu_2), \tag{23}$$

where $\mu_1 = \lambda_1 u_S'^*$, $\mu_2 = \lambda_2 u_S^*$, and

$$\langle \chi' \rangle = u'_S, \qquad \langle \chi \rangle = u_S. \tag{24}$$

The VEVs u_S, u'_S may be complex.

The mass matrix for light neutrinos is given by the seesaw formula (6), which leads to

$$M_\nu = -\begin{pmatrix} \frac{A_\nu^2}{\mu_1} & 0 & \frac{A_\nu B'_\nu}{\mu_1} \\ 0 & \frac{A_\nu^2}{\mu_1} + \frac{B_\nu^2}{\mu_2} & \frac{B_\nu C_\nu}{\mu_2} \\ \frac{A_\nu B'_\nu}{\mu_1} & \frac{B_\nu C_\nu}{\mu_2} & \frac{B'^2_\nu}{\mu_1} + \frac{C_\nu^2}{\mu_2} \end{pmatrix}. \tag{25}$$

From Eqs. (16), (20), and (23), we recover the minimal texture of Ref. [3] if (i) $|\mu_1| = |\mu_2| = M_0$ in M_R [and $M_R = M_0 \mathbb{1}_3$ in the basis where M_D has the real form (2)], (ii) $B'_l = B_l$ in M_l, and (iii) $B_\nu = B'_\nu$ in M_D. It is argued in Ref. [3] that (i) is not essential as long as M_D has hierarchical eigenvalues. We can confirm this by applying a rephasing transformation to (25) to obtain

$$d_\nu^\dagger M_\nu d_\nu^\dagger = \begin{pmatrix} a^2 e^{-i2\delta_2} & 0 & ab' \\ 0 & a^2 e^{i2\delta_1} + b^2 & bc \\ ab' & bc & b'^2 e^{i2\delta_2} + c^2 \end{pmatrix}. \tag{26}$$

We have used the shorthands for the positive real numbers $a^2 = |A_\nu^2/\mu_1|$, $b^2 = |B_\nu^2/\mu_2|$, $b'^2 = |B'^2_\nu/\mu_1|$, $c^2 = |C_\nu^2/\mu_2|$, and for the phases $\delta_1 = \arg(A_\nu B_\nu^*/\sqrt{\mu_1\mu_2^*})$, $\delta_2 = \arg(B'_\nu C_\nu^*/\sqrt{\mu_1\mu_2^*})$. Given the hierarchy $c \gg b$, $b' \gg a$, we can see that each element of $M_\nu^\dagger M_\nu$, calculated from (26), does not depend on the phases δ_1, δ_2 in the leading terms. Thus, we can consider M_ν to be real in the first approximation.

We will show in the next section how (ii) and (iii) affect the relations between the parameters of M_l, M_ν, and the masses.

IV. DEVIATIONS FROM THE MINIMAL TEXTURE

Let us analyze the consequences of the deviation of M_l and M_D from the minimal texture (1).

The texture obtained for M_l and M_D in our model, Eqs. (16) and (20), has the modified form

$$M' = \begin{pmatrix} 0 & -A & 0 \\ A & 0 & B' \\ 0 & B & C \end{pmatrix}. \tag{27}$$

It generalizes the minimal texture (1) in that B' and B are not necessarily equal. We expect, however, that $|B| \sim |B'|$ such that the hierarchy $|A| \ll |B|, |B'| \ll |C|$ is maintained in conformity to the hierarchy of masses $m_1 \ll m_2 \ll m_3$. If we also had the elements (1,2) and (2,1) distinct in (27), we would obtain the nearest-neighbor interaction form [17], which is always achievable by a weak basis change.

Since M' in (27) is no longer symmetric, even if we eliminate the minus sign in the (1,2) entry, the relevant mass matrix that furnishes the (squared) masses is $M'^\dagger M'$. This matrix can still be transformed to the real form

$$|M'^\dagger M'| = \begin{pmatrix} a^2 & 0 & b'a \\ 0 & a^2 + b^2 & bc \\ b'a & bc & b'^2 + c^2 \end{pmatrix} \tag{28}$$

by rephasing the appropriate fields. We have also used the shorthands $a \equiv |A|$, $b \equiv |B|$, $b' \equiv |B'|$, $c \equiv |C|$. Note that (26) has the same form as (28) when the phases are neglected.

The characteristic equation for (28) is

$$\lambda^3 - (2a^2 + b^2 + b'^2 + c^2)\lambda^2 + [(a^2 + b^2)(a^2 + b'^2) + 2a^2c^2]\lambda - a^4c^2 = 0. \tag{29}$$

This equation should be compared to the characteristic equation for $|M|^2$ which is obtained from (29) when $b' = b$. We know $|M|^2$ has eigenvalues (m_1^2, m_2^2, m_3^2) and the same eigenvectors of $|M|$ in (2). If we also identify the eigenvalues of $|M'^\dagger M'|$ as (m_1^2, m_2^2, m_3^2), we can still write a, c, and $\bar{b} \equiv \sqrt{\frac{1}{2}(b^2 + b'^2)}$ as functions of the masses and one remaining degree of freedom, quantified by $\Delta b^2 \equiv b^2 - b'^2$. We assume $m_3 > m_2 > m_1 \geq 0$ for the expressions below.

Let us analyze the relations of a, c, $\bar{b}$ with the masses and Δb^2. The relation between a and c is the same as in the minimal texture; i.e.,

$$a = \sqrt{\frac{m_1 m_2 m_3}{c}}, \tag{30}$$

which follows from $\det(|M'^\dagger M'|) = m_1^2 m_2^2 m_3^2$. The relation between $\bar{b}$ and c is given by

$$(\bar{b})^2 = \frac{1}{2}(m_1^2 + m_2^2 + m_3^2 - c^2) - \frac{m_1 m_2 m_3}{c}, \tag{31}$$

which follows from $\text{Tr}(|M'^\dagger M'|) = m_1^2 + m_2^2 + m_3^2$. At last, the parameter c can be obtained as a root of

$$(c^2 - m_1^2 - m_2^2 - m_3^2)^2 - (\Delta b^2)^2 = 4(m_1^2 m_2^2 + m_2^2 m_3^2 + m_1^2 m_3^2) - 8 m_1 m_2 m_3 c. \quad (32)$$

Once c is fixed by (32), a and $\bar{b}$ are known for a given Δb^2.

To obtain c, we should analyze Eq. (32). Among the possible multiple roots, we identify the physical root as the one that reduces to the known expression [2]

$$c_0 \equiv m_1 - m_2 + m_3, \quad (33)$$

in the limit $\Delta b^2 \to 0$. In general, we should write

$$c = c_0 + \delta c. \quad (34)$$

To quantify the deviation δc, we should rewrite Eq. (32) in terms of the relative deviation $\delta c / c_0$, which gives

$$\frac{1}{4}\left(\frac{\delta c}{c_0}\right)^4 + \left(\frac{\delta c}{c_0}\right)^3 + \alpha_2\left(\frac{\delta c}{c_0}\right)^2 + \alpha_1\left(\frac{\delta c}{c_0}\right) = \left(\frac{\Delta b^2}{2c_0^2}\right)^2. \quad (35)$$

We have used the shorthands

$$\alpha_1 = -2\frac{b_0^2}{c_0^2}, \qquad \alpha_2 = \frac{m_1^2 + m_2^2 + m_3^2 + 3[m_1 m_3 - m_2(m_1 + m_3)]}{c_0^2}, \quad (36)$$

and

$$b_0 \equiv \sqrt{\frac{(m_2 - m_1)(m_1 + m_3)(m_3 - m_2)}{m_1 - m_2 + m_3}}. \quad (37)$$

The latter corresponds to $b = b'$ in the minimal texture and can be obtained from (31) in the limit $c = c_0$. We can confirm from expression (37) that indeed the negative eigenvalue of (2) should be associated to the intermediate mass.

Equation (35) can be solved numerically, but we can seek an approximate solution that goes to zero as $\Delta b^2 \to 0$. To quantify Δb^2 better, we can parametrize Δb^2 by either

$$\Delta b^2 = 2\bar{b}^2 \beta \quad \text{or} \quad \Delta b^2 = 2 b_0^2 \beta_0, \quad (38)$$

depending on the choice of using the parameters $\{a, \bar{b}, c\}$ or the masses $\{m_i\}$ as input parameters. Since b, b' approach b_0 in the limit $\Delta b^2 \to 0$, $\beta \approx \beta_0$ for small values. For large values, β strictly obeys $|\beta| \leq 1$ by definition, whereas $|\beta_0|$ can assume values larger than unity, but we still need $|\beta| \lesssim 1$ to ensure $b \sim b'$ [18]. By noting from (36) that $|\alpha_1| \ll 1$, whereas $\alpha_2 \sim \mathcal{O}(1)$, we can drop the cubic and quartic terms in (35) and write an approximate solution as

$$\frac{\delta c}{c_0} \approx -\frac{b_0^2}{c_0^2} f(\beta_0), \quad (39)$$

where

$$f(\beta_0) \equiv \frac{1}{\alpha_2}\left[\sqrt{1 + \alpha_2 \beta_0^2} - 1\right]. \quad (40)$$

For a hierarchical spectrum $b_0^2/c_0^2 \approx m_2/m_3$, and the error for (39) is of the order of $\beta_0^6 b_0^6 / c_0^6 \sim \beta_0^6 m_2^3 / m_3^3$. We can see that the relative deviation $\delta c / c_0$ is small, and δc is at most of the order of m_2 as long as $b \sim b'$. We can also see from (30) that the relative deviation $\delta a / a_0$ is small and of the order of (39).

As b, b' are expected to be of the same order, they vary significantly with β_0 (or β) in (38). We can write b, b' as functions of the masses and β_0. Thus, for a given β_0 and masses $\{m_i\}$, the matrix (28) is fixed.

We should also comment on the dependence of the eigenvectors of (28) on β_0 as it deviates from 0. Let us denote by O the matrix of eigenvectors of (28), with eigenvectors associated to m_1^2, m_2^2, m_3^2 arranged in the columns 1,2,3, respectively. For the minimal texture ($\beta_0 = 0$), O is fixed [3]. With hierarchical masses, the diagonal elements O_{11}, O_{22}, O_{33} are the largest in modulus, and we use the convention that they are positive; O_{31}, O_{12}, O_{32} are thus negative. As β_0 varies for $|\beta_0| \lesssim 1$, one can check that the elements O_{21}, O_{12}, O_{13} are approximately constant with β_0 ($|O_{12}|$ increases mildly and $|O_{13}|$ decreases mildly with β_0). On the other hand, $|O_{31}|$, $|O_{32}|$, $|O_{23}|$ increase significantly with β_0. The diagonal elements decrease accordingly. We can see that the mixing angles can be significantly modified.

If we extend this discussion to M_l and M_D of Eqs. (16) and (20), their modified texture (27) introduces two more degrees of freedom that we can parametrize as β_{0l} and $\beta_{0\nu}$. Given that the charged lepton masses have large hierarchy, one can check that the elements (1,2), (1,3), (2,1), and (3,1) of O_l are small and do not change significantly for $|\beta_0| \lesssim 1$; $|(O_l)_{23}|$, $|(O_l)_{32}|$ increase substantially with β_{0l}. We can conclude that the space of solutions found in Ref. [5] is significantly broadened, as β_{0l}, $\beta_{0\nu}$ can be varied in the range $|\beta_{0\alpha}| \lesssim 1$, $\alpha = l, \nu$. Besides the solutions close to $\beta_{0l} = \beta_{0\nu} = 0$, which are necessarily present, one can easily find solutions away from that point.

As an example, we make a numerical comparison between two cases: (a) the minimal texture ($\beta_{0l} = \beta_\nu = 0$) and (b) $(\beta_{0l}, \beta_\nu) = (1, -0.418)$; the latter is substantially different from the minimal case. It is appropriate to use β_ν instead of $\beta_{0\nu}$ because we use the parameters a, $\bar{b}$, c as input parameters instead of the neutrino masses. The comparison is shown in two figures. In Fig. 1, we show $\sin^2\theta_{13}$ as a function of $\sin^2\theta_{23}$. We can see the two cases are indistinguishable. Figure 2 shows $\sin^2\theta_{12}$ as a function of the lightest neutrino mass m_1 where the two cases (a) and (b) can be almost completely separated. We also note that m_1 is in general nearly twice as heavy for case (b) compared to case (a).

To generate our points for the scatter plots, we employ the following procedure. We use as inputs for the charged lepton sector, the central values of the masses (m_e, m_μ, m_τ) [19] and β_{0l}, which determine the mass matrix *squared* (28). For the neutrino sector, we use $\{a, \bar{b}, c, \beta_\nu\}$ as inputs to fix the neutrino mass matrix (26), with $\delta_1 = \delta_2 = 0$.

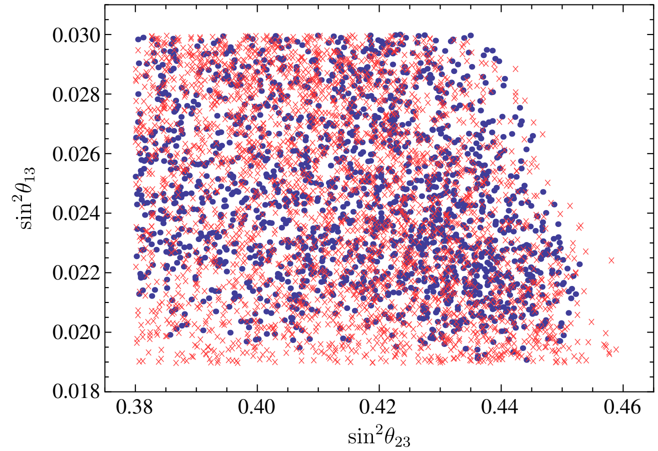

FIG. 1 (color online). $\sin^2\theta_{13}$ as a function of $\sin^2\theta_{23}$; the symbols $\times$ correspond to $\beta_{0l} = \beta_\nu = 0$ while dots correspond to $(\beta_{0l}, \beta_\nu) = (1, -0.418)$.

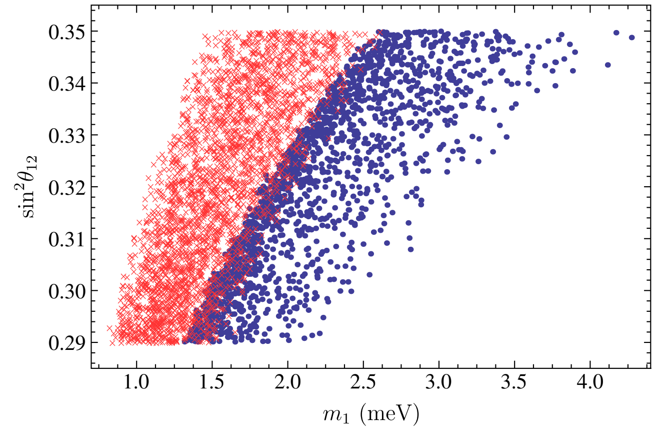

FIG. 2 (color online). $\sin^2\theta_{12}$ as a function of the lightest neutrino mass; the symbols $\times$ correspond to $\beta_{0l} = \beta_\nu = 0$ while dots correspond to $(\beta_{0l}, \beta_\nu) = (1, -0.418)$.

The two phases σ, τ in (11) complete the list of free parameters. Then, for fixed values of (β_{0l}, β_ν), we vary the parameters $\{a, \bar{b}, c, \sigma, \tau\}$ randomly and select only the points compatible, within 2-σ, with the values of the mixing angles, θ_{12}, θ_{13}, θ_{23}, and mass-squared differences Δm^2_{21}, Δm^2_{31} of Ref. [20]. No analytical approximations are employed in the diagonalization process. Although we use slightly different data, we can see our points are compatible with the predictions of Ref. [5] for the minimal texture. For the nonminimal case, we assume normal hierarchy, and only points with parameters $\{a, \bar{b}, c\}$ close to the minimal case are sought.

For completeness, we list some typical values for the parameters for the two cases:

$$\begin{aligned} &\text{(a)} \quad a = 0.07\ \text{eV}, \quad \bar{b} = 0.105\ \text{eV}, \quad c = 0.170\ \text{eV}, \quad \sigma = 1.5, \quad \tau = -1.78;\\ &\text{(b)} \quad a = 0.0783\ \text{eV}, \quad \bar{b} = 0.0984\ \text{eV}, \quad c = 0.175\ \text{eV}, \quad \sigma = 2.4, \quad \tau = -0.65. \end{aligned} \tag{41}$$

They are compatible within 1-σ of Ref. [20]. While for case (a), the phases in (20) are really typical, together with their opposite signs ($\sigma \to -\sigma$ and $\tau \to -\tau$), the phase τ for case (b) is well distributed in the whole range $(-\pi, \pi]$.

For an even larger departure from the minimal texture, one can find solutions which were not possible for the minimal texture [3], namely, neutrino masses with quasidegenerate spectrum (QD) or inverted hierarchy (IH). One example of solutions have parameters

$$\begin{aligned} &\text{(QD)} \quad \beta_{0l} = 2.5, \quad \beta_\nu = -0.41, \quad a = 0.225\,\text{eV}, \quad \bar{b} = 0.0115\,\text{eV}, \quad c = 0.265\,\text{eV}, \quad \sigma = 1.6, \quad \tau = -1.8;\\ &\text{(IH)} \quad \beta_{0l} = 5.2, \quad \beta_\nu = 0.107, \quad a = 0.220\,\text{eV}, \quad \bar{b} = 0.0349\,\text{eV}, \quad c = 0.0608\,\text{eV}, \quad \sigma = 1.6, \quad \tau = -1.8, \end{aligned} \tag{42}$$

which lead to the neutrino masses

$$\begin{aligned} (m_1, m_2, m_3)_{\text{QD}} &= (49.92, 50.68, 71.14)\ \text{meV},\\ (m_1, m_2, m_3)_{\text{IH}} &= (49.33, 50.10, 3.50)\ \text{meV}. \end{aligned} \tag{43}$$

In particular, these examples agree with the values of Ref. [20] within 1-σ. The large departure from the minimal texture for the charged lepton mass matrix can be seen from the fact that these points correspond $\beta_l \approx 0.928$ and $\beta_l \approx 0.982$ for the QD and IH cases, respectively. Therefore, $|B_l| \gg |B'_l|$ in (16). In this case, we lose some naturality in view of the structure (15). For the case of IH, although β_ν is small, it cannot be much larger [e.g., $\mathcal{O}(1)$], otherwise we lose the appropriate root of (35) or (32). It should be emphasized that a large set of solutions with parameters $\{\beta_{0l}, \beta_\nu, a, \bar{b}, c\}$ similar to (42) can be easily found, and (42) is not special in this regard. It is also evident that very different predictions arise from the extreme cases (42). For example, the effective mass that enters the neutrinoless double beta decay experiments is quite large for the following cases:

$$\text{(QD)} \quad m_{ee} \approx 50.1\,\text{meV}, \qquad \text{(IH)} \quad m_{ee} \approx 48.1\,\text{meV}. \tag{44}$$

As a remark, we should also note some similar analysis has been performed by Ref. [21] on the non-Hermitian departure from the Fritzsch form, although it considers Dirac neutrinos. Our modified form (27) is a particular case of the nearest-neighbor interaction form considered in Ref. [21] but the latter only analyzes the deviations perturbatively. The approximation (39) is better than the perturbative first-order approximation. Also, in our case we have Majorana neutrinos.

V. CONCLUSIONS

We have presented here a simple extension of SM based on the flavor group S_3, which is capable of generating the minimal texture proposed in Ref. [3] with three additional degrees of freedom: (i) M_R has two independent diagonal elements instead of one (we also have more phases), (ii) the (2,3) and (3,2) elements of M_l are independent, and (iii) the (2,3) and (3,2) elements of M_D are independent as well. If we equate the parameters in (i), (ii), and (iii) [and adjust the phases for (i)], we reproduce exactly the ansatz of Ref. [3].

The model possesses an additional flavor symmetry $G_F = S_3 \otimes Z_5 \otimes Z_3$ and five additional complex scalars $\zeta_D = (\zeta_1, \zeta_2)$, η, χ, χ'. The scalars χ, χ' only couple to right-handed neutrinos through lepton number violating interactions. In fact, Z_3 in G_F can be extended to $U(1)_{B-L}$ in the lepton sector if we assign $B - L = 2$ to both χ, χ'. This symmetry, however, is softly broken to Z_3 (factoring out the group $\{\pm 1\}$ for leptons) by the term χ^3 in the scalar potential. Therefore, our flavor group could be a discrete remnant of a larger symmetry containing $U(1)_{B-L}$ at higher energies. We should emphasize that the symmetry $Z_5 \otimes Z_3$ highly constrains the model, naturally providing both a rationale for large tau mass and the necessary vacuum alignment.

On the phenomenological side, the freedom (i) does not lead to large deviations from the minimal texture. The freedoms (ii) and (iii), on the other hand, can modify significantly the mixing angles in the orthogonal matrices O_l and O_ν that diagonalize the respective squared mass matrices in the real form. The freedoms (ii) and (iii) were parametrized by two parameters $\beta_{0\alpha}$ (38), $\alpha = l, \nu$, that may vary in the range $|\beta_{0\alpha}| \lesssim 1$. The values $\beta_{0l} = \beta_{0\nu} = 0$ correspond to the minimal texture. Since the dependence of the PMNS matrix on β_{0l}, $\beta_{0\nu}$ is smooth, the solutions for $\beta_{0l} = \beta_{0\nu} = 0$ [5] are not disrupted as we relax $\beta_{0l} \approx \beta_{0\nu} \approx 0$. However, as we deviate from the minimal texture, very different solutions are possible. This was shown through various examples. In the first example, we have shown in Fig. 2 that the minimal case can be distinguished from the case $(\beta_{0l}, \beta_\nu) = (1, -0.418)$ with similar values for the rest of the parameters. For even larger deviations from the minimal texture, we have shown that both quasidegenerate spectrum and inverted hierarchy are possible. This contrasts sharply with the minimal texture where neither of them is possible [5].

In summary, the modified texture (27) arising from our model for the Dirac mass matrices of charged leptons and neutrinos easily accommodates the present data on the neutrinos sector but still allows a wide range of possibilities if we permit large deviations from the minimal texture.

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APPENDIX: ALIGNMENT OF $\langle \zeta_D \rangle$

We justify here the alignment $\langle \zeta_D \rangle = (0, u_2)^\mathsf{T}$ assumed in (18) [22]. For this alignment, the existence of the scalar η transforming as $\mathbf{1}'$ of S_3 and its coupling to ζ_D are essential. It is exactly $\langle \eta \rangle \neq 0$ that breaks S_3 to Z_3, whereas $\langle \zeta_D \rangle$ breaks this remaining symmetry. To be explicit, we can write the generators a, b of S_3 acting on ζ_D as [12]

$$D(a) = \mathrm{diag}(1, -1), \qquad D(b) = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}. \tag{A1}$$

Both are contained in $O(2)$: $D(a)$ is a reflection in the ζ_2 direction and $D(b)$ is a $2\pi/3$ rotation in the plane. The VEV $\langle \eta \rangle \neq 0$ breaks the Z_2 generated by $D(a)$, and S_3 is broken to the Z_3 generated by $D(b)$.

To analyze the alignment of ζ_D, we consider only the relevant terms that depend on ζ_D, which are

$$V_{\zeta_D} = \mu_d^2 \zeta_D^\dagger \zeta_D + \lambda_1 (\zeta_D^\dagger \zeta_D)^2 + \lambda_2 ([\zeta_D^* \zeta_D]_{1'})^2 + \{\lambda_3 [\zeta_D \zeta_D \zeta_D]_{1'} \eta^* + \lambda_3' [\zeta_D \zeta_D]_1 \chi \chi'^* + \mathrm{H.c.}\}, \tag{A2}$$

where we assume for our purposes that λ_3, λ_3' are real; the coefficient μ_d^2 effectively includes terms depending on other G_F invariant combination of fields such as $\phi^\dagger \phi$ or $\chi^* \chi$. The term $([\zeta_D^* \zeta_D]_2)^2$ does not introduce new terms. We note that the first two terms of the potential (A2) are $U(2)$ invariant, and we obviously need $\mu_d^2 < 0$ to obtain nontrivial $\langle \zeta_D \rangle$. The third, fourth, and fifth terms break the $U(2)$ invariance because

$$([\zeta_D^* \zeta_D]_{1'})^2 = -4 \mathrm{Im}^2 (\zeta_1^* \zeta_2), \tag{A3}$$

$$[\zeta_D \zeta_D \zeta_D]_{1'} = \zeta_2 (\zeta_2^2 - 3\zeta_1^2), \tag{A4}$$

$$[\zeta_D \zeta_D]_1 = \zeta_1^2 + \zeta_2^2. \tag{A5}$$

The term(A3) is still invariant by $O(2)$ and global rephasing. The $O(2)$ symmetry is reduced to S_3 by the term proportional to (A4), which is further reduced to Z_3 when η acquires a nonzero VEV.

To ensure the term proportional to (A3) is positive semidefinite, we choose $\lambda_2 < 0$. This choice tends to align the phases of $\langle \zeta_1 \rangle$ and $\langle \zeta_2 \rangle$, or otherwise make either $\langle \zeta_1 \rangle = 0$ or $\langle \zeta_2 \rangle = 0$. The term(A4) tends to make $\langle \zeta_1 \rangle = 0$. One can then check that the direction $\langle \zeta_D \rangle = (0, u_2)^\mathsf{T}$ is a local minimum in the shell (orbit) where $\zeta_D^\dagger \zeta_D$ is constant, provided that $\langle \eta \rangle \neq 0$. This check is most easily performed for the case where all the coefficients of the scalar potential are real and the VEVs of η, χ, χ' are real as well. Then, we can choose the sign of the coefficients of (A2) such that

$$\lambda_3\langle\eta\rangle < 0, \qquad \lambda_3'\langle\chi\chi'^*\rangle < 0. \tag{A6}$$

In this case, the u_2 corresponding to the deepest minimum in this direction will be real and positive.

We should remark that due to the discrete G_F symmetry, the potential (A2) possesses multiple degenerate minima. After η, χ, χ' have acquired nonzero VEVs, the $Z_3 \subset S_3$ symmetry guarantee that if $\langle\zeta_D\rangle = (0, u_2)^\mathsf{T}$ is a minimum, then

$$D(b)\langle\zeta_D\rangle, \qquad D^2(b)\langle\zeta_D\rangle, \tag{A7}$$

are also degenerate minima. These minima are not aligned as $(0, u_2)$ but they induce equivalent mass matrices since the form (16) is recovered after we apply $D^{-1}(b)$ or $D^{-2}(b)$ transformations on the fermions transforming as **2** of S_3.

Besides the last two terms in (A2), there is still one last non-Hermitian term invariant under G_F, which is contained in the potential involving only χ,

$$V_\chi = \mu_\chi^2\chi^\dagger\chi + \lambda_\chi(\chi^\dagger\chi)^2 + \{m\chi^3 + \text{H.c.}\}. \tag{A8}$$

The remaining renormalizable terms in the total potential are all Hermitian. We can see there are parameter ranges of the potential that guarantee nonzero and arbitrary VEVs; we keep $\langle\phi\rangle$ in the electroweak value and the rest of the VEVs may be pushed to higher energies.

It must be noted that we have a rephasing transformation $\zeta_D \to e^{i\alpha}\zeta_D$, $\eta \to e^{i3\alpha}\eta$, $\chi' \to e^{i2\alpha}\chi'$, with ϕ and χ transforming trivially, resulting in a $U_\alpha(1)$ continuous accidental symmetry, which contains Z_5 of G_F. This generates a Goldstone boson after scalar fields acquire VEVs. The Goldstone boson is given by the following combination:

$$G = \frac{u_2\text{Im}\zeta_2 + 3u_\eta\text{Im}\eta + 2u_s'\text{Im}\chi'}{\sqrt{u_2^2 + 9u_\eta^2 + 4u_s'^2}}. \tag{A9}$$

G would couple with the SM fields through nonrenormalizable interactions in (15), and they are suppressed by the scale Λ. Also, the coupling could be even suppressed if u_2, $u_\eta \ll u_s'$. The main interaction is with N_D in Eq. (22) and so G is harmless.

[1] S. Weinberg, Trans. N.Y. Acad. Sci. **38**, 185 (1977).
[2] H. Fritzsch, Phys. Lett. B **73**, 317 (1978); Nucl. Phys. **B155**, 189 (1979).
[3] M. Fukugita, M. Tanimoto, and T. Yanagida, Phys. Lett. B **562**, 273 (2003); Prog. Theor. Phys. **89**, 263 (1993).
[4] Z.-z. Xing, Phys. Lett. B **550**, 178 (2002).
[5] M. Fukugita, Y. Shimizu, M. Tanimoto, and T. T. Yanagida, Phys. Lett. B **716**, 294 (2012).
[6] F. P. An *et al.* (Daya Bay Collaboration), Phys. Rev. Lett. **108**, 171803 (2012); J. K. Ahn *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012).
[7] H. Fritzsch, Z.-z. Xing, and S. Zhou, J. High Energy Phys. 09 (2011) 083; A. Kageyama, S. Kaneko, N. Shimoyama, and M. Tanimoto, Phys. Lett. B **538**, 96 (2002); M. Honda, S. Kaneko, and M. Tanimoto, J. High Energy Phys. 09 (2003) 028; S. Goswami and A. Watanabe, Phys. Rev. D **79**, 033004 (2009); S. Kumar, Phys. Rev. D **84**, 077301 (2011); D. Meloni and G. Blankenburg, arXiv:1204.2706.
[8] P. H. Frampton, S. L. Glashow, and D. Marfatia, Phys. Lett. B **536**, 79 (2002).
[9] W. Grimus, A. S. Joshipura, L. Lavoura, and M. Tanimoto, Eur. Phys. J. C **36**, 227 (2004).
[10] G. C. Branco, D. Emmanuel-Costa, and C. Simoes, Phys. Lett. B **690**, 62 (2010).
[11] D. Emmanuel-Costa and C. Simoes, Phys. Rev. D **85**, 016003 (2012).
[12] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, Prog. Theor. Phys. Suppl. **183**, 1 (2010).
[13] C. S. Lam, Phys. Rev. D **74**, 113004 (2006).
[14] Y. Koide, Phys. Rev. D **69**, 093001 (2004); S. L. Chen, M. Frigerio, and E. Ma, Phys. Rev. D **70**, 073008 (2004); Phys. Rev. D **70**, 079905(E) (2004).S. Morisi and M. Picariello, Int. J. Theor. Phys. **45**, 1267 (2006); R. N. Mohapatra, S. Nasri, and H.-B. Yu, Phys. Lett. B **639**, 318 (2006); R. Allahverdi, B. Dutta, and R. N. Mohapatra, Phys. Lett. B **695**, 181 (2011); A. C. B. Machado and V. Pleitez, Phys. Lett. B **698**, 128 (2011).
[15] D. Meloni, S. Morisi, and E. Peinado, arXiv:1203.2535; D. Meloni, S. Morisi, and E. Peinado, J. Phys. G **38**, 015003 (2011).
[16] If allow order $1/\Lambda^2$ terms in (15), we have to add the terms $\bar{L}_D E_D \phi\zeta_D\zeta_D$, $\bar{L}_D E_D \phi\chi\chi'^*$, which induce nonzero terms in the (1,1), (2,2), (1,2), and (2,1) entries of M_l. These contributions are negligible if $|\langle\zeta_D\rangle|/\Lambda$, $|\langle\chi\rangle|/\Lambda$, $|\langle\chi'\rangle|/\Lambda \ll 1$.
[17] G. C. Branco, L. Lavoura, and F. Mota, Phys. Rev. D **39**, 3443 (1989).
[18] Strictly speaking, we need to ensure that $|\Delta b^2|$ is not so large as to make either b or b' go to zero.
[19] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
[20] D. V. Forero, M. Tortola, and J. W. F. Valle, arXiv:1205.4018v3.
[21] H. Fritzsch, Z.-z. Xing, and Y.-L. Zhou, Phys. Lett. B **697**, 357 (2011).
[22] We thank the anonymous referee for pointing out the alignment problem.