Inverse seesaw mechanism in nonsupersymmetric $SO(10)$, proton lifetime, nonunitarity effects, and a low-mass Z' boson

Ram Lal Awasthi[2] and Mina K. Parida[1]

[1] *Centre of Excellence in Theoretical and Mathematical Sciences, SOA University, Khandagiri Square, Bhubaneswar 751030, India*

[2] *Harish-Chandra Research Institute, Chhatnag Road, Jhusi, Allahabad 211019, India*

(Received 7 December 2011; published 2 November 2012)

Recently realization of TeV scale inverse seesaw mechanism in supersymmetric $SO(10)$ framework has led to a number of experimentally verifiable predictions including low-mass $W_R^{\pm}$ and Z' gauge bosons and nonunitarity effects. Using nonsupersymmetric $SO(10)$ grand unified theory, we show how a TeV scale inverse seesaw mechanism for neutrino masses is implemented with a low-mass Z' boson accessible to Large Hadron Collider. We derive renormalization group equations for fermion masses and mixings in the presence of the intermediate symmetries of the model and extract the Dirac neutrino mass matrix at the TeV scale from successful grand unified theory-scale parametrization of fermion masses. We estimate leptonic nonunitarity effects measurable at neutrino factories and lepton flavor violating decays expected to be probed in the near future. While our prediction on the nonunitarity matrix element $\eta_{\mu\tau}$ for degenerate right-handed neutrinos is similar to the supersymmetric $SO(10)$ case, we find new predictions with significantly enhanced value of its phase $\delta_{\mu\tau} \simeq 10^{-4}$–$10^{-2}$ when partial degeneracy among these neutrino masses is adequately taken into account by a constraint relation that emerges naturally in this approach. Other predictions on branching ratios and CP-violating parameters are discussed. An important distinguishing characteristic as another test of the minimal model is that the threshold corrected two-loop prediction of the proton lifetime with maximum value $(\tau_p)_{\max} \simeq 10^{35}$ yrs. is accessible to ongoing search experiments for the decay $p \to e^+\pi^0$ in the near future. Simple model extensions with longer proton lifetime predictions are also discussed.

DOI: 10.1103/PhysRevD.86.093004 PACS numbers: 14.60.St, 12.10.Dm, 12.15.Lk, 12.60.Cn

I. INTRODUCTION

Supersymmetric grand unified theories (GUTs) provide a very attractive framework for representing particles and forces of nature as they solve the gauge hierarchy problem, unify three forces of nature, and also explain tiny neutrino masses through seesaw paradigm [1] while providing possible cold dark matter candidates of the universe. An evidence of supersymmetry at the Large Hadron Collider (LHC) would be a landmark discovery which would certainly change the future course of physics. But, in the absence of any evidence of supersymmetry so far, it is worthwhile to explore new physics prospects of nonsupersymmetric (non-SUSY) GUTs [2–4] and, particularly, those based upon $SO(10)$ which has grown in popularity as it unifies all fermions of one generation including the right-handed (RH) neutrino into a single spinorial representation. It provides spontaneous origins of P(= Parity) and CP violations [5–7]. Most interestingly, in addition to predicting the right order of tiny neutrino masses through mechanisms called the canonical (≡ type-I) [8] seesaw and type-II [9] seesaw, it has high potentiality to explain all fermion masses [10,11] including large mixings in the neutrino sector [12] with type-II seesaw dominance [13–15]. In fact neither seesaw mechanism, nor grand unification require supersymmetry per se. Although gauge couplings automatically unify in the minimal supersymmetric standard model [16], and they fail to unify through the minimal particle content of the standard model (SM) in one-step breaking of non-SUSY $SU(5)$ or $SO(10)$, they do unify once intermediate symmetries are included to populate the grand desert in case of non-SUSY $SO(10)$ [7,17–19]. In addition, with intermediate gauge symmetries $SO(10)$ also predicts signals of new physics which can be probed at low or accelerator energies.

A hallmark of $SO(10)$ grand unification is its underlying quark-lepton symmetry [2] because of which the canonical seesaw scale is pushed closer to the GUT scale making it naturally inaccessible to direct tests by low-energy experiments or collider searches. The energy scale of type-II seesaw mechanism in $SO(10)$ is also too high for direct experimental tests. In contrast to these high scale seesaw mechanisms, an experimentally verifiable and attractive mechanism that has been recently introduced into $SO(10)$ [20] is the radiative seesaw [21] where the quark-lepton unification has no role to play and additional suppression to light neutrino mass prediction occurs by loop mediation proportional to a small Higgs quartic coupling that naturally emerges from a Plank-scale induced term in the GUT Lagrangian. The model predicts a rich structure of prospective dark matter candidates also verifiable by ongoing search experiments. It has been further noted that this embedding of the radiative seesaw in $SO(10)$ may have a promising prospect for representing all fermion masses.

A number of other interesting neutrino mass generation mechanisms including type-III seesaw, double seesaw, linear seesaw, scalar- triplet seesaw have been suggested and some of them are also experimentally verifiable [1].

In the context of non-SUSY $SO(10)$ in this work our purpose is to explore the prospects of another neutrino mass generation mechanism called the inverse seesaw [22] which is also different from canonical or type-II seesaw mechanism and has the potentiality to be experimentally verifiable because of the low scale at which it can operate although higher scale inverse seesaw models have been suggested [23,24]. In a large class of models [25–28], the implementation requires the introduction of fermionic singlets under the gauge group of the model. Likewise, its implementation in $SO(10)$ introduces a new mass scale μ_S into the Lagrangian corresponding to the mass matrix of the additional singlet fermions of three generations and the TeV-scale seesaw requires this parameter to be small. There is an interesting naturalness argument in favor of its smallness based upon exact lepton number conservation symmetry [25,29]. Below the TeV scale, in the limit $\mu_S \to 0$, the corresponding Lagrangian has a leptonic global $U(1)$ symmetry which guarantees left-handed (LH) neutrinos to remain massless. The small value of μ_S essentially needed to match the neutrino oscillation data with the TeV-scale inverse seesaw formula may be taken as a consequence of very mild breaking of the exact global symmetry. Thus the small value of the parameter protected by the exact lepton number conservation is natural in the 't Hooft sense [30]. However, in spite of such interesting naturalness argument, there has been no dynamical understanding of its origin so far, although an interpretation using Higgs mechanism has been given in the context of a model with extended gauge, fermion and Higgs sectors [31]. In a different class of $SO(10)$ models, the small singlet fermion mass parameter has been generated radiatively [32] where more nonstandard fermions have been found to be necessary. In SUSY $SO(10)$ singlet fermions have also been used to derive new forms of fermion mass matrix while predicting standard fermion mass ratios [33] and to obtain new seesaw formula for neutrinos while explaining baryon asymmetry of the universe through leptogenesis [34]. While most of the inverse seesaw models need gauge singlet fermions under the SM gauge group or its extensions [22–29,31,32] and the use of $SO(10)$-singlet fermions may point to the disadvantages of the corresponding GUT-based models, extended electroweak theory based upon $SU(3)_L \times SU(3)_R \times U(1)$ gauge symmetry [35] contains such singlets in its fundamental representations. To give some examples of GUTs, one $SO(10)$-singlet fermion per generation is automatically contained in the 27—dimensional fermion representation of E_6 [36] where $27 = 16 + 10 + 1$ under $SO(10)$ but $27 = (3, 3^*, 1) + (3, 1, 3^*) + (1, 3, 3^*)$ under $SU(3)_L \times SU(3)_R \times SU(3)_C$ and, in the latter case, an additional discrete Z_3 symmetry is needed to qualify it as a trinification GUT model [37]. Interesting properties of $SU(3)^3$ gauge theory including experimentally verifiable predictions at accelerator energies have been discussed [38,39]. Gauge boson mediated proton decays are suppressed in $SU(3)^3$ type of models. In addition to the RH neutrino and the other singlet fermion needed for inverse seesaw, these models ($SU(3)^3$ and E_6) also contain 10 nonstandard fermions per generation and no experimental data are yet available on their masses at low energies so as to pursue the present bottom-up approach to derive the Dirac neutrino mass matrix from fermion mass fits at the GUT scale. The same argument holds against any other model that may contain additional nonstandard fermions beyond the RH neutrino and the singlet-fermion needed for inverse seesaw.

Regarding the potentiality of $SO(10)$ motivated inverse seesaw in the visible sector, the same quark-lepton symmetry that forces the canonical seesaw scale to be far beyond the experimentally accessible range, makes the TeV-scale inverse seesaw predict observable nonunitarity effects as new physics signals verifiable at low and accelerator energies and at neutrino factories [29].

Recently in a series of interesting investigations, using inverse seesaw mechanism, Bhupal Dev and Mohapatra [29] have shown that SUSY $SO(10)$, besides admitting a low spontaneous breaking scale of $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C (\equiv G_{2213})$ gauge symmetry with right-handed gauge bosons $W_R^{\pm}$ and Z' accessible to LHC, is also capable of fitting all fermion masses and mixings at the GUT scale while predicting observable nonunitarity effects. The model has been also shown to account for the observed baryon asymmetry of the universe through leptogenesis caused due to the decay of TeV scale masses of the pseudo-Dirac RH neutrinos [40]. Currently considerable attention has been devoted to propose models with an extra neutral Z' gauge boson which may also emerge from Pati-Salam or left-right gauge theories, or $SO(10)$ and E_6 grand unified theories, or also from string inspired models [41].

Different from the supersymmetric $SO(10)$ model of Ref. [29], here we adopt the view that there may not be any manifestation of supersymmetry at accelerator energies and that the actual parity restoration scale may be high. Instead of both the $W_R^{\pm}$ and the Z' boson masses being low, there may be only some remnants of high scale left-right symmetry or quark lepton symmetry manifesting at low and accelerator energies as smoking gun signatures such as the Z' [41,42] gauge boson and the associated nonunitarity effects of the TeV-scale inverse seesaw. With this point of view in this work we show that a non-SUSY $SO(10)$ with $SU(2)_L \times U(1)_R \times U(1)_{(B-L)} \times SU(3)_C (\equiv G_{2113})$ gauge symmetry at the TeV scale and left-right gauge theory at higher intermediate scale, with or without D-parity, achieves precision gauge coupling unification, and predicts a low mass Z' making them suitable for implementation of TeV-scale inverse seesaw mechanism. The model can also be verified or falsified through its predictions on observable nonunitarity effects and additional contributions to lepton flavor violations. Another testing ground for the model could be through the $SO(10)$ prediction on gauge boson mediated proton decay on which dedicated search experiments are ongoing.

We derive renormalization group equations in the presence of two intermediate gauge symmetries for running fermion masses and mixings, and determine the Dirac neutrino mass at the TeV scale from successful fits to the fermion masses at the GUT scale. In this approach we find a simple relation between the RH neutrino masses in the model. We also point out a different type of relation in the partial degenerate case that permits much lower values of RH neutrino masses resulting in a CP-violating phase increased by 2–4 orders larger than the degenerate case. Some of our predictions include branching ratios for $\mu \rightarrow e\gamma$ enhanced by 1–2 orders. Out of the two minimal models, while the intermediate scale D-parity conserving model is ruled out by proton decay constraint, the proton lifetime for $p \rightarrow e^+\pi^0$ in the intermediate scale D-parity nonconserving model is predicted to be well within the accessible range of ongoing search experiments. We have also discussed simple extensions of the two models with longer proton lifetime predictions. This method can also be implemented using Pati-Salam model or left-right models [2,5].

This paper is organized in the following manner. In Sec. II we briefly discuss the model and carry out gauge coupling unification and proton lifetime predictions in Sec. III. With a brief explanation of inverse seesaw mechanism in Sec. IV we summarize relevant formulas encoding nonunitarity effects and lepton flavor violations. In Sec. V we discuss renormalization group evolution of fermion masses and mixings to the GUT scale in the presence of nonsupersymmetric gauge theories G_{2113} and G_{2213} at intermediate scales. In this section we also show how fermion masses are fitted at the GUT scale and information on the Dirac neutrino mass matrix is obtained. Nonunitarity effects are discussed in Sec. VI with predictions on the moduli of relevant matrix elements. In Sec. VII we give predictions on CP-violating parameters and lepton flavor violation where we also discuss possible limitations of the present models. In Sec. VIII we provide a brief summary and discussion along with conclusion. In the Appendix A we provide beta function coefficients for gauge coupling unification while in Appendix B we summarize derivations of renormalization group equations (RGEs) for fermion masses and mixings.

II. THE MODEL

There has been extensive investigation on physically appealing intermediate scale models [6,7,17,19,43] in non-SUSY $SO(10)$. Although in the minimal two step-breaking of non-SUSY $SO(10)$ models [19] we found no suitable chain with a sufficiently low scale to implement the inverse seesaw, the following chain with two intermediate gauge symmetries appears to be quite suitable,

$$\begin{aligned} SO(10) &\overset{(M_U)}{\rightarrow} G_I \\ &\overset{(M_R^+)}{\rightarrow} SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C[G_{2113}] \\ &\overset{(M_R^0)}{\rightarrow} SU(2)_L \times U(1)_Y \times SU(3)_C[\mathrm{SM}] \\ &\overset{(M_Z)}{\rightarrow} SU(3)_C \times U(1)_Q, \end{aligned} \tag{1}$$

where we will consider two possibilities for G_I.

As model-I $G_I = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C[\equiv G_{2213}](g_{2L} \neq g_{2R})$ is realized by breaking the GUT-symmetry and by assigning vacuum expectation value (VEV) to the D-parity odd singlet in 45_H [6]. As the left-right discrete symmetry is spontaneously broken at the GUT scale, the Higgs sector becomes asymmetric below $\mu = M_U$ causing inequality between the gauge couplings g_{2L} and g_{2R}. This model does not have the cosmological domain wall problem. The second step of symmetry breaking takes place by the RH Higgs triplet $\sigma_R(1, 3, 0, 1) \subset 45_H$ whereas the third step of breaking to SM takes place by the G_{2113}-submultiplet $\chi_R^0(1, 1/2, -1/2, 1)$ contained in the RH doublet of 16_H. It is well known that SM breaks to low energy symmetry by the SM Higgs doublet contained in the bidoublet $(2, 2, 0, 1)$ under G_{2213} which originates from 10_H of $SO(10)$. This is the minimal particle content for the model to carry out the spontaneous breaking of GUT symmetry to low-energy theory. But a major objective of the present work is to explore the possibility of observable nonunitarity effects for which it is required to extract information on the Dirac neutrino mass matrix (M_D) from a fit to the fermion masses at the GUT scale and this is possible by including two Higgs doublets instead of one [29]. We assume these doublets to originate from two separate bidoublets contained in 10_H^a $(a = 1, 2)$. Implementation of inverse seesaw also requires the minimal extension by adding three $SO(10)$-singlet fermions S_i $(i = 1, 2, 3)$, one for each generation [22].

As model-II, we treat the GUT symmetry to be broken by the VEV of the G_{2213}-singlet $(1, 1, 0, 1) \subset 210_H$ which is even under D-parity [6]. This causes the Higgs sector below GUT-scale to be left-right symmetric resulting in equal gauge couplings in $G_I = G_{2213D}(g_{2L} = g_{2R})$. For the sake of simplicity we treat the rest of the symmetry breaking patterns of model II similar to model I and we assume the presence of three singlet fermions. We call these two models, model I and model II, as minimal models with two low scale Higgs doublets in each. We now examine precision gauge coupling unification for these two models.

III. GAUGE COUPLING UNIFICATION AND PROTON LIFETIME

In this section we examine gauge coupling unification in the minimal model I and the minimal model II and make predictions on proton lifetimes while we also predict the corresponding quantities in their simple extensions.

A. Unification in minimal models

It was shown in Ref. [7] that with G_{2113} gauge symmetry at the lowest intermediate scale in $SO(10)$ there is substantial impact of two-loop effects on mass scale predictions in a number of cases. The one-loop and the two-loop beta-function coefficients for the evolution of gauge couplings [44,45] for model I and model II with two Higgs doublets for each case are given in Appendix A. We have also included small mixing effects [43,46] due to two Abelian gauge factors $U(1)_R \times U(1)_{(B-L)}$ in both the models below the M_R^+ scale. Using $\sin^2\theta_W(M_Z) = 0.23116 \pm 0.00013$, $\alpha^{-1}(M_Z) = 127.9$ and $\alpha_S(M_Z) = 0.1184 \pm 0.0007$ [47] we find that with $M_{Z'} \sim M_{R^0} \sim 1$ TeV precision unification of gauge couplings occurs for the following values of masses at one-loop and two-loop levels for the model I,

$$\begin{aligned} &M_U^{\rm ol} = 10^{15.978}\ \text{GeV}, \quad M_{R^+}^{\rm ol} = 10^{10.787}\ \text{GeV}, \quad \alpha_G^{\rm ol} = 0.02253, \quad \text{(one-loop)},\\ &M_U = 10^{15.530}\ \text{GeV}, \quad M_{R^+} = 10^{11.15}\ \text{GeV}, \quad \alpha_G = 0.02290 \quad \text{(two-loop)}. \end{aligned} \tag{2}$$

The RG evolution of gauge couplings at two-loop level is shown in Fig. 1 exhibiting precision unification at $M_U = 10^{15.53}$ GeV. In model II coupling unification occurs with similar precision but at $M_U = 10^{15.17}$ GeV.

The decay width of the proton for $p \to e^+\pi^0$ is [48]

$$\Gamma(p \to e^+\pi^0) = \frac{m_p}{64\pi f_\pi^2}\left(\frac{g_G^4}{M_U^4}\right)|A_L|^2|\bar{\alpha}_H|^2(1+D+F)^2 \times R, \tag{3}$$

where $R = [(A_{\rm SR}^2 + A_{\rm SL}^2)(1+|V_{ud}|^2)^2]$ for $SO(10)$, $V_{ud} = 0.974 =$ the (1, 1) element of $V_{\rm CKM}$ for quark mixings, $A_{\rm SL}(A_{\rm SR})$ is the short-distance renormalization factor in the left (right) sectors and $A_L = 1.25 =$ long distance renormalization factor. $M_U =$ degenerate mass of 24 superheavy gauge bosons in $SO(10)$, $\bar{\alpha}_H =$ hadronic matrix element, $m_p =$ proton mass $= 938.3$ MeV, $f_\pi =$ pion decay constant $= 139$ MeV, and the chiral Lagrangian parameters are $D = 0.81$ and $F = 0.47$. With $\alpha_H = \bar{\alpha}_H(1+D+F) = 0.012\ \text{GeV}^3$ estimated from lattice gauge theory computations, we obtain $A_R \simeq A_L A_{\rm SL} \simeq A_L A_{\rm SR} \simeq 2.726$ for model I. The expression for the inverse decay rates for both the minimal models is expressed as

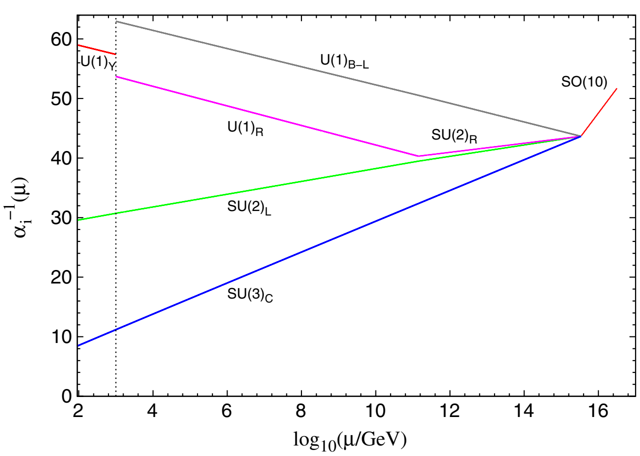

FIG. 1 (color online). Gauge coupling unification in model I with two-loop values $M_U = 10^{15.53}$ GeV and $M_{R^+} = 10^{11.15}$ GeV with a low mass Z' gauge boson at $M_{R^0} \sim 1$ TeV.

$$\Gamma^{-1}(p \to e^+\pi^0) = (1.01 \times 10^{34}\ \text{Yrs})\left(\frac{0.012\ \text{GeV}^3}{\alpha_H}\right)^2\left(\frac{2.726}{A_R}\right)^2 \times \left(\frac{1/43.6}{\alpha_G}\right)^2\left(\frac{7.6}{F_q}\right)\left(\frac{M_U}{2.98 \times 10^{15}\ \text{GeV}}\right)^4, \tag{4}$$

where the factor $F_q = 2(1+|V_{ud}|^2)^2 \simeq 7.6$ for $SO(10)$. Now using the estimated values of the model parameters in each case the predictions on proton lifetimes for both models are given in Table I where the uncertainties in unification scale and proton lifetime have been estimated by enhancing the error in α_S to 3σ level. It is clear that with maximal value $(\tau_p)_{\max} = 7 \times 10^{34}$ Yrs, model I predicts the proton lifetime closer to the current experimental lower bound $(\tau_p)_{\rm expt}(p \to e^+\pi^0) \geq 1.2 \times 10^{34}$ Yrs. [49] which is accessible to ongoing proton decay searches in the near future [50]. On the other hand, model II is ruled out at two-loop level as it predicts lifetime nearly two orders smaller. The reduction of lifetime by nearly two-orders compared to one-loop predictions in both cases is due to the corresponding reduction in the unification scale by a factor of $\simeq 1/3$.

The fact that the model I admits a low $(B-L)$ breaking scale corresponding to a light Z' accessible to accelerator searches makes this non-SUSY model suitable to accommodate inverse seesaw mechanism. Unlike the SUSY $SO(10)$ model [29], here the $W_R^\pm$ bosons are far beyond the LHC accessible range.

TABLE I. GUT scale, intermediate scale and proton lifetime predictions for nonsupersymmetric $SO(10)$ models with TeV scale Z' boson and two Higgs doublets as described in the text. The uncertainty in the proton lifetime has been estimated using 3σ uncertainty in $\alpha_S(M_Z)$.

Model	$M_U^{\rm ol}$ (GeV)	$M_{R^+}^{\rm ol}$ (GeV)	M_U (GeV)	M_{R^+} (GeV)	α_G^{-1}	A_R	τ_p^o (yrs.)	τ_p (yrs.)
I	$10^{15.978}$	$10^{10.787}$	$10^{15.530}$	$10^{11.150}$	43.67	2.726	$1.08\times 10^{36\pm 0.32}$	$2\times 10^{34\pm 0.32}$
II	$10^{15.56\pm 0.08}$	$10^{11.475}$	$10^{15.17\pm 0.08}$	$10^{11.750}$	42.738	2.670	$2.44\times 10^{34\pm 0.32}$	$6.3\times 10^{32\pm 0.32}$

B. Unification in simple model extensions

Although the minimal model I clearly satisfies the proton decay constraint to accommodate TeV scale seesaw, we study simple extensions of both models to show that they can evade proton lifetime constraint in case future experiments show τ_p to be substantially longer than 10^{35} Yrs. We use an additional real color octet scalar $C_8(1,0,8)\subset 45_H$ where the quantum numbers are under the SM gauge group and allow its mass to vary between 1 TeV and the GUT scale. Making it light would require additional fine tuning of parameters. Recently such a light scalar has been used in models with interesting phenomenological consequences and if the particle mass is in the accessible range, it may be produced at LHC with new physics signatures beyond the standard model [51].

The presence of this scalar octet with lower mass makes the evolution of $\alpha_{3c}^{-1}(\mu)$ flatter thereby pushing the GUT scale to higher values. In Fig. 2 we plot predicted proton lifetimes in the extended G_{2213} and G_{2213D} models as a function of the octet mass m_8. It is clear that such a simple extension of the two models can easily satisfy proton lifetime requirements in future experimental measurements even if they are found to be much longer than the current limit.

Then while the minimal model I can be easily chosen for inverse seesaw, both the models with such simple extension and possessing TeV scale $U(1)_{(B-L)}$ breaking scale qualify for the same purpose.

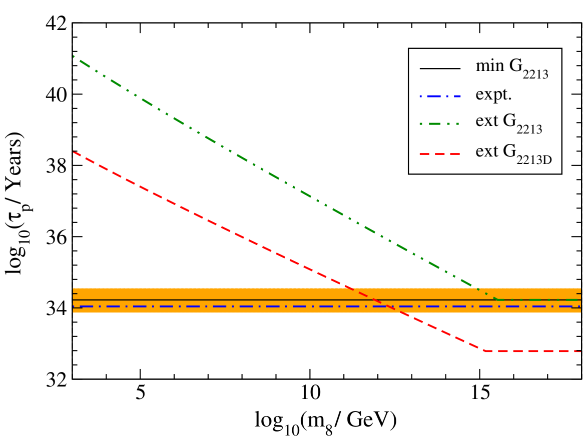

FIG. 2 (color online). Variation of proton lifetime as a function of color octet mass in simple extensions of model I (double dot-dashed line) and model II (dashed line). The horizontal solid line with error band is the prediction of the minimal model I while the horizontal dot-dashed line is the experimental lower bound for $p\to e^+\pi^0$.

IV. INVERSE SEESAW AND FORMULAS FOR *CP* AND LEPTON FLAVOR VIOLATIONS

For the phenomenological study of nonunitarity effects we confine to the model I and all our analyses are similar for model II. Introducing additional $SO(10)$-singlet fermions (S) for three generations, the Yukawa Lagrangian at the GUT scale gives rise to the effective Lagrangian near the second intermediate scale $\mu = M_R^0 \sim 1$ TeV,

$$\begin{aligned}\mathcal{L}_{\rm Yuk} &= Y^a \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10_H}^a + y_\chi \mathbf{16}\cdot\mathbf{1}\cdot\mathbf{16_H}^\dagger + \mu_S \mathbf{1}\cdot\mathbf{1}\\ &\supset (Y^a\bar{\psi}_L\psi_R\Phi^a + y_\chi\bar{\psi}_R S\chi_R^0 + \text{H.c.}) + S^T\mu_S S,\end{aligned} \tag{5}$$

where the first (second) equation is invariant under $SO(10)$ (G_{2113}) gauge symmetry. The LH and the RH fermion fields $\psi_L(2,0,-1/2,1)$, $\psi_R(1,1/2,-1/2,1)$ with their respective quantum numbers under G_{2113} are contained in the spinorial representation $\mathbf{16}\subset SO(10)$ and the two Higgs doublets $\Phi(2,\pm 1/2,0,1)\subset 10_H\subset SO(10)$. The Lagrangian has a new mass scale μ_S corresponding to the mass matrix of the $SO(10)$-singlet fermions. Denoting the RH neutrino mass as $M_R = y_\chi v_\chi$ where $v_\chi = \langle\chi_R^0\rangle$ and the Dirac mass matrix for neutrino as $M_D = Y_\nu v_u$ where v_u is the VEV of the up-type Higgs doublet, Eq. (4) gives the mass part of the neutrino sector in the Lagrangian in the flavor basis after the symmetry breaking $G_{2113}\to$ SM

$$\mathcal{L}_{\rm mass} = (\bar{\nu}M_D N + \bar{N}M_R S + \text{H.c.}) + S^T\mu_S S, \tag{6}$$

which, in the $(\nu, N, S)_L$ basis, leads to a mass matrix [22,29]

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D & 0\\ M_D^T & 0 & M_R\\ 0 & M_R^T & \mu_S\end{pmatrix}. \tag{7}$$

Denoting $X = M_D M_R^{-1}$, block diagonalization of Eq. (7) under the condition $M_R \gg M_D \gg \mu_S$ leads to the inverse seesaw formula for light neutrino mass matrix,

$$m_\nu = M_D M_R^{-1}\mu_S (M_R^T)^{-1} M_D^T \equiv X\mu_S X^T. \tag{8}$$

It is clear that the TeV-scale inverse seesaw formula is tenable and appropriate to fit the light neutrino masses provided μ_S is the smallest of the three mass scales occurring in Eq. (7). Based upon symmetry, there exist interesting naturalness arguments in the literature in favor of smallness of μ_S. In the limit $\mu_S \to 0$, a leptonic $U(1)$ global symmetry is restored in the Lagrangian signifying exact conservation of lepton number that guarantees left-handed neutrinos to be massless [25,29,34]. In particular, a small and nonvanishing value of μ_S can be viewed as a slight breaking of the global $U(1)$ symmetry. Thus the smallness of μ_S, desired in the TeV-scale inverse seesaw mechanism, which is protected by the global symmetry in the 't Hooft sense [30], is natural even though there is no dynamical understanding for such a small parameter. This view for the naturally small parameter μ_S being followed in the present work has been adopted in Ref. [29] and by a number of authors earlier pursuing inverse seesaw mechanism [25] although its interpretation through Higgs mechanism has been discussed in a model with extended gauge, fermion and Higgs sectors [31] and possibility of its radiative origin has been explored [32].

The physics underlying nonunitarity effects have been discussed at length in several recent papers [52–59] where relevant formulas have been utilized. Although the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix U diagonalizes the light neutrino mass matrix of three generations where

$$U^\dagger m_\nu U^* = \mathrm{diag}(m_1, m_2, m_3) \equiv \hat{m}_\nu, \tag{9}$$

the appropriate diagonalizing mixing matrix for the inverse seesaw matrix of Eq. (8) is a 9×9 matrix V,

$$V^\dagger \mathcal{M}_\nu V^* = \hat{M} = \mathrm{diag}(m_i, m_{R_j}, m_{\tilde{R}_k}), \quad (i, j, k = 1, 2, 3), \tag{10}$$

and this can be expressed in block partitions,

$$V = \begin{pmatrix} V_{3\times3} & V_{3\times6} \\ V_{6\times3} & V_{6\times6} \end{pmatrix}, \tag{11}$$

where the nonunitary $V_{3\times3}$ matrix now represents the equivalent of the full PMNS matrix,

$$\mathcal{N} \equiv V_{3\times3} \simeq \left(1 - \frac{1}{2}XX^\dagger\right)U \simeq (1-\eta)U. \tag{12}$$

Denoting the corresponding nine component eigenstate as $(\hat{\nu}_i, N_i, \tilde{N}_i)^T$, the six component heavy eigenstate as $P^T = (N_1, N_2, N_3, \tilde{N}_1, \tilde{N}_2, \tilde{N}_3)^T$ and $\mathcal{K} \equiv V_{3\times6} \simeq (0, X)V_{6\times6}$, in the leading order approximation in X, the light neutrino flavor eigenstate and the charged current Lagrangian in the mass basis are

$$\begin{aligned} \nu^T &= \mathcal{N}\hat{\nu}^T + \mathcal{K}P^T, \\ \mathcal{L}_{CC} &= -\frac{g_{2L}}{\sqrt{2}}\bar{l}_L\gamma^\mu \nu W_\mu^- + \mathrm{H.c.} \\ &\simeq -\frac{g_{2L}}{\sqrt{2}}\bar{l}_L\gamma^\mu(\mathcal{N}\hat{\nu}^T + \mathcal{K}P^T)W_\mu^- + \mathrm{H.c.} \end{aligned} \tag{13}$$

The parameter $\eta = XX^\dagger/2$ characterizing nonunitarity of the neutrino mixing matrix can have dramatic impact on leptonic CP-violation and branching ratios for processes with lepton flavor violation (LFV),

$$\begin{aligned} \mathcal{J}^{ij}_{\alpha\beta} &= \mathrm{Im}(\mathcal{N}_{\alpha i}\mathcal{N}_{\beta j}\mathcal{N}^*_{\alpha j}\mathcal{N}^*_{\beta i}), \\ &\simeq \mathcal{J} + \Delta\mathcal{J}^{ij}_{\alpha\beta}, \end{aligned} \tag{14}$$

where $\mathcal{J}$ is the well-known CP-violating parameter due to unitary PMNS matrix U

$$\mathcal{J} = \cos\theta_{12}\cos^2\theta_{13}\cos\theta_{23}\sin\theta_{12}\sin\theta_{13}\sin\theta_{23}\sin\delta, \tag{15}$$

and the nonunitarity contributions are

$$\begin{aligned} \Delta\mathcal{J}^{ij}_{\alpha\beta} \simeq -\sum_{\gamma=e,\mu,\tau} \mathrm{Im}(&\eta_{\alpha\gamma}U_{\gamma i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i} \\ &+ \eta_{\beta\gamma}U_{\alpha i}U_{\gamma j}U^*_{\alpha j}U^*_{\beta i} + \eta^*_{\alpha\gamma}U_{\alpha i}U_{\beta j}U^*_{\gamma j}U^*_{\beta i} \\ &+ \eta^*_{\beta\gamma}U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\gamma i}). \end{aligned} \tag{16}$$

Very recently $\sin\theta_{13}$ has been measured [60] to be small and nonvanishing although no experimental information is available on the leptonic CP-phase δ. Even in the limiting case of vanishing unitarity CP-violation corresponding to $\sin\theta_{13} \to 0$, or $\delta \to 0$, π for nonvanishing θ_{13} nonunitarity effects caused due to η may not vanish. In the modified charged current interaction in Eq. (13), the heavy neutrinos contribute to LFV decays with branching ratios [61]

$$\begin{aligned} \mathrm{BR}(l_\alpha \to l_\beta\gamma) &= \frac{\alpha_w^3 s_w^2 m_{l_\alpha}^5}{256\pi^2 M_w^4 \Gamma_\alpha}\left|\sum_{i=1}^{6}\mathcal{K}_{\alpha i}\mathcal{K}^*_{\beta i} I\left(\frac{m^2_{R_i}}{M_w^2}\right)\right|^2, \\ I(x) &= -\frac{2x^3 + 5x^2 - x}{4(1-x)^3} - \frac{3x^3\ln x}{2(1-x)^4}. \end{aligned} \tag{17}$$

In Eq. (17) the total decay width Γ_α for lepton species l_α with lifetime τ_α is evaluated using $\Gamma_\alpha = \frac{\hbar}{\tau_\alpha}$ where $\tau_\mu = (2.197019 \pm 0.000021) \times 10^{-6}$ sec and $\tau_\tau = (290.6 \pm 1.0) \times 10^{-15}$ sec.

The matrix element $(\mathcal{K}\mathcal{K}^\dagger)_{\alpha\beta} \propto \eta_{\alpha\beta}$ may lead to significant LFV decays in the TeV scale seesaw whereas LFV decays are drastically suppressed in type-I seesaw in $SO(10)$. The procedure for estimating these effects has been outlined in Ref. [29] which we follow. The Dirac neutrino mass matrix at the TeV scale which we derive in the next section is central to the determination of nonunitarity effects.

V. RG EVOLUTION OF FERMION MASSES AND DETERMINATION OF M_D

The determination of the Dirac neutrino mass matrix $M_D(M_{R^0})$ at the TeV seesaw scale is done in three steps [29]: (1) Derivation of RGEs for the specific model and extrapolation of masses to the GUT scale, (2) Fitting the masses at the GUT scale and determination of $M_D(M_{\rm GUT})$, (3) Determination of $M_D(M_{R^0})$ by top-down approach.

A. RGEs and extrapolation to GUT scale

At first RGEs for Yukawa coupling matrices and fermion mass matrices are set up from which RGEs for mass eigenvalues and Cabibbo-Kobayashi-Moskawa (CKM) mixings are derived in the presence of G_{2113} and G_{2213} symmetries. RGEs in dynamical left-right breaking model has been derived earlier [62].

Denoting $\Phi_{1,2}$ as the corresponding bidoublets under G_{2213} they acquire VEVs

$$\langle\Phi_1\rangle = \begin{pmatrix} v_u & 0 \\ 0 & 0 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \begin{pmatrix} 0 & 0 \\ 0 & v_d \end{pmatrix}. \tag{18}$$

Defining the mass matrices

$$\begin{aligned} &M_u = Y_u v_u, \quad M_D = Y_\nu v_u, \quad M_d = Y_d v_d, \\ &M_e = Y_e v_d, \quad M_R = y_\chi v_\chi, \end{aligned} \tag{19}$$

we have derived the new RGEs in the presence of non-SUSY G_{2113} and G_{2213} gauge symmetries for matrices Y_i, M_i, $i = u, d, e, N$, the mass eigenvalues m_i, $i = u, c, t, d, s, b, e, \mu, \tau, N_1, N_2, N_3$, and the CKM mixing matrix elements as given in the Appendix B. We use the input values of running masses and quark mixings at the electroweak scale as in Refs. [47,63] and the resulting CKM matrix with the CKM Dirac phase $\delta^q = 1.20 \pm 0.08$

$$V_{\rm CKM} = \begin{pmatrix} 0.9742 & 0.2256 & 0.0013 - 0.0033i \\ -0.2255 + 0.0001i & 0.9734 & 0.04155 \\ 0.0081 - 0.0032i & -0.0407 - 0.0007i & 0.9991 \end{pmatrix}. \tag{20}$$

We use RGEs of the standard model for $\mu = M_Z$ to $M_R^0 = 1$ TeV. With two Higgs doublets at $\mu \geq M_R^0$ we use the starting value of $\tan\beta = v_u/v_d = 10$ at $\mu = 1$ TeV which evolves to reach the value $\tan\beta \simeq 6.9$ at the GUT scale. Using the bottom-up approach discussed earlier [63] and the RGEs of Appendix B, the resulting quantities including the mass eigenvalues m_i and the $V_{\rm CKM}$ at the GUT scale are [64]

$\mu = {\rm M_{GUT}}$:

$$\begin{aligned} &m_e = 0.48\,{\rm MeV}, \quad m_\mu = 97.47\,{\rm MeV}, \quad m_\tau = 1.8814\,{\rm GeV}, \quad m_d = 1.9\,{\rm MeV}, \quad m_s = 38.9\,{\rm MeV}, \\ &m_b = 1.4398\,{\rm GeV}, \quad m_u = 1.2\,{\rm MeV}, \quad m_c = 0.264\,{\rm GeV}, \quad m_t = 83.04\,{\rm GeV}, \end{aligned} \tag{21}$$

$$V_{\rm CKM}(M_{\rm GUT}) = \begin{pmatrix} 0.9748 & 0.2229 & -0.0003 - 0.0034i \\ -0.2227 - 0.0001i & 0.9742 & 0.0364 \\ 0.0084 - 0.0033i & -0.0354 + 0.0008i & 0.9993 \end{pmatrix}. \tag{22}$$

B. Determination of M_D

With Higgs representations 45_H, 16_H, 10_H, the dim. 6 operator [29]

$$\frac{\mathbf{f_{ij}}}{\mathbf{M}^2}\mathbf{16_i 16_j 10_H 45_H 45_H}, \tag{23}$$

with $M \simeq M_{\rm Pl}$ or $M \simeq M_{\rm string}$, is suppressed by $(M_U/M)^2 \simeq 10^{-3} - 10^{-5}$ for GUT-scale VEV of 45_H and acts as an effective 126_H operator to fit the fermion masses at the GUT scale where the formulas for mass matrices are

$$\begin{aligned} &M_u = G_u + F, \qquad M_d = G_d + F, \\ &M_e = G_d - 3F, \qquad M_D = G_u - 3F. \end{aligned} \tag{24}$$

In Eq. (24) the matrices $G_k = Y_k.16.16\langle 10_H^k\rangle$, $k = u, d$ and F are derived from Eq. (23). Using a charged-lepton diagonal mass basis and Eqs. (21) and (24) we get,

$$\begin{aligned} &M_e(M_{\rm GUT}) = {\rm diag}(0.0005, 0.098, 1.956)\ {\rm GeV}, \\ &G_{d,ij} = 3F_{ij}, \qquad (i \neq j). \end{aligned} \tag{25}$$

Assuming for the sake of simplicity that the matrix F is diagonal leads to the conclusion that the matrix G_d is also diagonal. This gives relations between the diagonal elements which, in turn, determine the diagonal matrices F and G_d completely

$$\begin{aligned} &G_{d,ii} + F_{ii} = m_i, \qquad (i = d, s, b), \\ &G_{d,jj} - 3F_{jj} = m_j, \qquad (j = e, \mu, \tau), \end{aligned} \tag{26}$$

$$\begin{aligned} F &= {\rm diag}\frac{1}{4}(m_d - m_e, m_s - m_\mu, m_b - m_\tau), \\ &= {\rm diag}(3.75 \times 10^{-4}, -0.0145, -0.3797)\ {\rm GeV}, \\ G_d &= {\rm diag}\frac{1}{4}(3m_d + m_e, 3m_s + m_\mu, 3m_b + m_\tau), \\ &= {\rm diag}(0.0016, 0.0544, 1.6709)\ {\rm GeV}, \end{aligned} \tag{27}$$

where we have used the RG extrapolated values of Eq. (21). Then using Eqs. (24) and (27) and the assumed basis gives the mass matrices M_u and G_u,

$$M_u(M_{\mathrm{GUT}}) = \begin{pmatrix} 0.0153 & 0.0615 - 0.0112i & 0.1028 - 0.2706i \\ 0.0615 + 0.0112i & 0.3933 & 3.4270 + 0.0002i \\ 0.1028 + 0.2706i & 3.4270 - 0.0002 & 82.90 \end{pmatrix} \mathrm{GeV}, \tag{28}$$

$$G_u(M_{\mathrm{GUT}}) = \begin{pmatrix} 0.0150 & 0.0615 - 0.0112i & 0.1028 - 0.2706i \\ 0.0615 + 0.0112i & 0.4079 & 3.4270 + 0.0002i \\ 0.1028 + 0.2706i & 3.4270 - 0.0002i & 83.01 \end{pmatrix} \mathrm{GeV}. \tag{29}$$

Now using Eqs. (27) and (29) in Eq. (24) gives the Dirac neutrino mass matrix M_D at the GUT scale

$$M_D(M_{\mathrm{GUT}}) = \begin{pmatrix} 0.0139 & 0.0615 - 0.0112i & 0.1029 - 0.2707i \\ 0.0615 + 0.0112i & 0.4519 & 3.4280 + 0.0002i \\ 0.1029 + 0.2707i & 3.4280 - 0.0002i & 83.340 \end{pmatrix} \mathrm{GeV}. \tag{30}$$

We then use the RGE for M_D given in Appendix A to evolve $M_D(M_{\mathrm{GUT}})$ to $M_D(M_{R^+})$ and then from $M_D(M_{R^+})$ to $M_D(M_{R^0})$ in two steps and obtain,

$$M_D(M_{R^0}) = \begin{pmatrix} 0.0151 & 0.0674 - 0.0113i & 0.1030 - 0.2718i \\ 0.0674 + 0.0113i & 0.4758 & 3.4410 + 0.0002i \\ 0.1030 + 0.2718i & 3.4410 - 0.0002i & 83.450 \end{pmatrix} \mathrm{GeV}. \tag{31}$$

VI. NONUNITARITY DEVIATIONS IN LEPTON MIXING MATRIX

From Eq. (12) it is clear that any nonvanishing value of η is a measure of deviation from the unitarity of the PMNS matrix. Using the TeV scale mass matrix for M_D from Eq. (31) and assuming

$$M_R = \mathrm{diag}(m_{R_1}, m_{R_2}, m_{R_3}), \tag{32}$$

results in

$$\eta = \frac{1}{2} X.X^\dagger = \frac{M_D M_R^{-2} M_D^\dagger}{2}, \qquad \eta_{\alpha\beta} = \frac{1}{2} \sum_{k=1,2,3} \frac{M_{D_{\alpha k}} M^*_{D_{\beta k}}}{m^2_{R_k}}. \tag{33}$$

For the sake of simplicity assuming degeneracy of RH neutrinos masses $m_R = m_{R_i}$ $(i = 1, 2, 3)$ gives

$$\eta = \frac{1\ \mathrm{GeV}^2}{m_R^2} \begin{pmatrix} 0.0447 & 0.1937 - 0.4704i & 4.4140 - 11.360i \\ 0.1937 + 0.4704i & 6.036 & 144.40 - 0.0002i \\ 4.4140 + 11.360i & 144.40 + 0.0002i & 3488.0 \end{pmatrix}. \tag{34}$$

The deviations from unitarity in the leptonic mixing is constrained, for example, by deviations from universality tests in weak interactions, rare leptonic decays, invisible width of Z boson and neutrino oscillation data. The bounds derived at 90% confidence level from the current data on the elements of the symmetric matrix are summarized in Ref. [52],

$$\begin{aligned} &|\eta_{\tau\tau}| \le 2.7 \times 10^{-3}, && |\eta_{\mu\mu}| \le 8.0 \times 10^{-4}, \\ &|\eta_{ee}| \le 2.0 \times 10^{-3}, && |\eta_{e\mu}| \le 3.5 \times 10^{-5}, \\ &|\eta_{e\tau}| \le 8.0 \times 10^{-3}, && |\eta_{\mu\tau}| \le 5.1 \times 10^{-3}. \end{aligned} \tag{35}$$

In the degenerate case the largest element in Eq. (34) when compared with $|\eta_{\tau\tau}|$ of Eq. (35) gives the lower bound on the RH neutrino mass,

$$m_R \ge 1.1366\ \mathrm{TeV}, \tag{36}$$

which is only 7% higher than the SUSY $SO(10)$ bound $(m_R)_{\mathrm{SUSY}} \ge 1.06$ TeV [29]. Using this lower bound for other elements in Eq. (34) yields

$$\begin{aligned} &|\eta_{\mu\mu}| \le 4.672 \times 10^{-6}, && |\eta_{ee}| \le 3.460 \times 10^{-8}, \\ &|\eta_{e\mu}| \le 3.938 \times 10^{-7}, && |\eta_{e\tau}| \le 9.436 \times 10^{-6}, \\ &|\eta_{\mu\tau}| \le 1.1178 \times 10^{-4}. \end{aligned} \tag{37}$$

TABLE II. Variation of third generation RH neutrino mass m_{R_3} as a function of first or second generation RH neutrino mass in the partially degenerate case $m_{R_1} = m_{R_2}$ predicted by nonunitarity through nonsupersymmetric $SO(10)$.

$m_{R_{1,2}}$ (GeV)	m_{R_3} (GeV)	$m_{R_{1,2}}$ (GeV)	m_{R_3} (GeV)
48.0	5572.83	500.0	1140.66
50.0	3324.69	600.0	1139.11
100.0	1286.51	700.0	1138.18
150.0	1195.81	800.0	1137.57
200.0	1168.32	900.0	1137.16
300.0	1149.80	1000.0	1136.87
400.0	1143.53	1136.58	1136.58

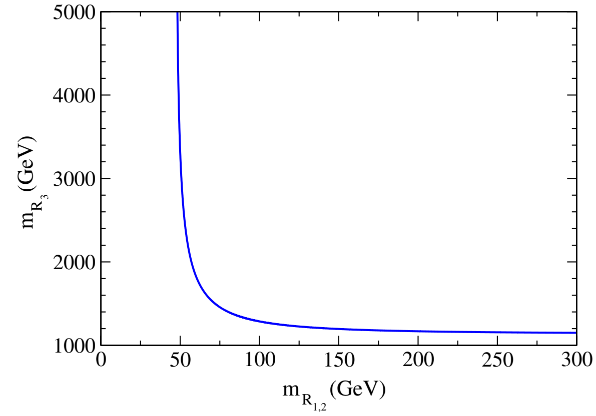

FIG. 3 (color online). Variation of the third generation RH neutrino mass m_{R_3} as a function of first or second generation neutrino mass m_{R_1} or m_{R_2} in the partially degenerate case for which $m_{R_1} = m_{R_2}$.

As in SUSY $SO(10)$ [29], these predicted bounds are several orders lower than the current experimental bounds and they might be reached provided corresponding LFV decays are probed with much higher precision. But compared to SUSY $SO(10)$, in this model the upper bound is nearly 2 times larger for $|\eta_{\mu\tau}|$, 3 times larger for $|\eta_{\mu\mu}|$, and nearly 40% smaller in the case of $|\eta_{e\tau}|$. It is interesting to note that in the present non-SUSY $SO(10)$ model while some of the nonunitarity effects are comparable to the results of Ref. [29], others are distinctly different as shown in the next section.

We note in this model that when RH neutrino masses are nondegenerate, they are also constrained by the experimental lower bound on $\eta_{\tau\tau}$ and the corresponding relation obtained by saturating the bound is

$$\frac{1}{2}\left[\frac{0.0845}{m_{R_1}^2} + \frac{11.8405}{m_{R_2}^2} + \frac{6963.9}{m_{R_3}^2}\right] = 2.7 \times 10^{-3}, \quad (38)$$

where the numerators inside the square bracket are in GeV^2. Using partial degeneracy, $m_{R_1} = m_{R_2} \neq m_{R_3}$ leads to the relation between the RH neutrino masses as given in Table II. A plot of m_{R_3} vs m_{R_i} ($i = 1, 2$) is shown in Fig. 3 exhibiting increase of m_{R_3} with decrease of m_{R_i}. The two asymptotes in the hyperbolic curve are at $m_{R_1} = m_{R_2} \simeq$ 47 GeV and $m_{R_3} \simeq 1136.6$ GeV.

VII. ESTIMATIONS OF *CP* AND LEPTON FLAVOR VIOLATIONS AND DISCUSSIONS

Two important physical applications of inverse seesaw are leptonic CP and flavor violation effects reflected through the elements, both moduli and phases, of the η-matrix and the relevant formulas have been discussed in Sec. IV. The inverse seesaw formula of Eq. (8) has three matrices out of which M_D has been determined by fitting the charged fermion masses and mixings, but since the other two matrices, M_N and μ_S, cannot be completely determined by using the neutrino oscillation data alone, we make plausible assumptions. In addition to the fully degenerate case we also examine consequences of partial degeneracy with $m_{R_1} = m_{R_2}$.

From Eq. (8), the nonunitary PMNS matrix $\mathcal{N} = (1 - \eta)U$ and the relation $m_\nu = \mathcal{N}\hat{m}_\nu\mathcal{N}^T$ give

$$\mu_S = X^{-1}\mathcal{N}\hat{m}_\nu\mathcal{N}^T(X^T)^{-1}. \quad (39)$$

We construct the unitary matrix U using standard parametrization,

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{13}s_{12}e^{i\delta} & c_{13}c_{23} \end{pmatrix}, \quad (40)$$

and the neutrino oscillation data at 3σ level [60,65] and assuming hierarchical neutrino masses,

$$\begin{gathered} \Delta m_{21}^2 = (7.09\text{–}8.19) \times 10^{-5}\ \text{eV}^2, \qquad \Delta m_{31}^2 = (2.18\text{–}2.73) \times 10^{-3}\ \text{eV}^2, \\ \sin^2\theta_{12} = 0.27\text{–}0.36, \qquad \sin^2\theta_{23} = 0.39\text{–}0.64, \qquad \sin^2\theta_{13} = 0.092 \pm 0.06. \end{gathered} \quad (41)$$

We take the leptonic Dirac phase δ in the U matrix to be zero for which the predicted CP-violation from unitarity vanishes irrespective of the values of θ_{13}. We have also checked that inclusion of larger values of $\theta_{13} \simeq 8^\circ$–$9^\circ$ [60] do not alter our results significantly. Similar results are obtained with $\delta = \pi$.

Taking the light neutrino mass eigenvalues $m_1 = 0.001$ eV, $m_2 = 0.0088$ eV, $m_3 = 0.049$ eV, and the constructed U matrix, we utilize the η matrix of Eq. (34) for the degenerate case and Eq. (12) to obtain the nonunitary matrix $\mathcal{N}$. Using Eq. (39) we also get the μ_S matrix. Once the matrices η and U are determined as discussed above and in Sec. IV, the CP-violating parameters are computed using Eq. (16). Even though U has no imaginary part because of assumed vanishing value of its Dirac phase, CP-violation would arise from the imaginary parts of the corresponding components of η matrix. We also estimate branching ratios for different LFV decay modes using Eq. (17).

For the degenerate case with $m_R = 1.1366$ TeV we get

$$\mu_S = \begin{pmatrix} 0.9932 - 0.0124i & -0.1908 + 0.0022i & 0.0066 - 0.0033i \\ -0.1908 + 0.0022i & 0.0370 - 0.0004i & -0.0013 + 0.0006i \\ 0.0066 - 0.0033i & -0.0013 + 0.0006i & 0.00003 - 0.00004i \end{pmatrix} \text{GeV},$$

$$\begin{aligned} \Delta\mathcal{J}^{12}_{e\mu} &= -1.3082 \times 10^{-6}, \\ \Delta\mathcal{J}^{23}_{e\mu} &= -1.5573 \times 10^{-6}, \\ \Delta\mathcal{J}^{23}_{\mu\tau} &= 1.5574 \times 10^{-6}, \\ \Delta\mathcal{J}^{31}_{\mu\tau} &= 1.5572 \times 10^{-6}, \\ \Delta\mathcal{J}^{12}_{\tau e} &= 4.0144 \times 10^{-6}, \end{aligned} \tag{42}$$

and the branching ratios

$$\begin{aligned} \mathrm{BR}(\mu \to e\gamma) &= 2.0025 \times 10^{-16}, \\ \mathrm{BR}(\tau \to e\gamma) &= 2.1586 \times 10^{-14}, \\ \mathrm{BR}(\tau \to \mu\gamma) &= 3.0290 \times 10^{-12}. \end{aligned} \tag{43}$$

Thus we find that in this non-SUSY $SO(10)$ model for the degenerate RH neutrino masses, like the SUSY $SO(10)$ prediction [29], although all the five CP violating parameters are just one order smaller than the corresponding parameter in the quark sector where $\mathcal{J}_{\mathrm{CKM}} = (3.05^{+0.19}_{-0.20}) \times 10^{-5}$, there are certain quantitative differences. The magnitudes of predicted CP-violations for all the five parameters in the non-SUSY $SO(10)$ model are reduced by nearly 50% compared to their corresponding SUSY $SO(10)$ values.

When compared with the predicted values in SUSY $SO(10)$ [29] the present results on branching ratios satisfy

$$\begin{aligned} \frac{\mathrm{BR}(\mu \to e\gamma)_{\mathrm{susy}}}{\mathrm{BR}(\mu \to e\gamma)_{\text{non-susy}}} &\simeq \frac{3}{2}, \\ \frac{\mathrm{BR}(\tau \to e\gamma)_{\mathrm{susy}}}{\mathrm{BR}(\tau \to e\gamma)_{\text{non-susy}}} &\simeq 5, \\ \frac{\mathrm{BR}(\tau \to \mu\gamma)_{\mathrm{susy}}}{\mathrm{BR}(\tau \to \mu\gamma)_{\text{non-susy}}} &\simeq \frac{2}{3}, \end{aligned} \tag{44}$$

which can be tested by next generation experiments on LFV decays.

Our predictions for the partially degenerate RH neutrinos on different elements $\eta_{\alpha\beta}$ and their phases are given in Table III and those for CP-violating parameters $\Delta\mathcal{J}^{ij}_{\alpha\beta}$ and branching ratios are summarized in Table IV.

Compared to the predictions in the degenerate case, $|\eta_{\mu\tau}| \simeq 10^{-4}$, $\delta_{\mu\tau} \simeq 10^{-6}$, for the partially degenerate case we find that while $|\eta_{\mu\tau}|$ is of the same order, but

TABLE III. Predictions of moduli and phases of nonunitarity parameters as a function of RH neutrino masses.

$m_{R_1} = m_{R_2}$ (GeV)	m_{R_3} (GeV)	$\lvert\eta_{e\mu}\rvert$	$\delta_{e\mu}$	$\lvert\eta_{e\tau}\rvert$	$\delta_{e\tau}$	$\lvert\eta_{\mu\tau}\rvert$	$\delta_{\mu\tau}$
1136	1136	3.938×10^{-7}	1.180	9.436×10^{-6}	1.20	1.118×10^{-4}	1.3×10^{-6}
500	1141	4.222×10^{-7}	1.071	9.576×10^{-6}	1.166	1.136×10^{-4}	2.0×10^{-4}
100	1286	1.848×10^{-6}	0.308	1.687×10^{-5}	0.563	1.691×10^{-4}	5.0×10^{-3}
50	3325	6.733×10^{-6}	0.172	4.806×10^{-5}	0.202	3.424×10^{-4}	1.0×10^{-2}

TABLE IV. Nonunitarity predictions of leptonic CP-violating parameters and branching ratios for lepton flavor violating decays $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ as a function RH neutrino masses.

m_{R_1,R_2} (GeV)	m_{R_3} (GeV)	BR($\mu \to e\gamma$)	BR($\tau \to e\gamma$)	BR($\tau \to \mu\gamma$)	$\Delta\mathcal{J}^{12}_{e\mu}$	$\Delta\mathcal{J}^{23}_{e\mu}$	$\Delta\mathcal{J}^{23}_{\mu\tau}$	$\Delta\mathcal{J}^{31}_{\mu\tau}$	$\Delta\mathcal{J}^{12}_{\tau e}$
1136	1136	2.0×10^{-16}	2.1×10^{-14}	3.0×10^{-12}	-1.3×10^{-6}	-1.6×10^{-6}	1.6×10^{-6}	1.6×10^{-6}	4.0×10^{-6}
500	1140	2.0×10^{-16}	1.9×10^{-14}	2.7×10^{-12}	-1.3×10^{-6}	-1.6×10^{-6}	1.6×10^{-6}	1.6×10^{-6}	4.0×10^{-6}
100	1286	1.4×10^{-15}	2.2×10^{-14}	2.2×10^{-12}	-1.2×10^{-6}	-1.6×10^{-6}	2.2×10^{-6}	1.3×10^{-6}	4.1×10^{-6}
50	3325	1.1×10^{-14}	1.1×10^{-13}	5.5×10^{-12}	-1.0×10^{-6}	-1.8×10^{-6}	4.1×10^{-6}	7.4×10^{-7}	4.3×10^{-6}

$\delta_{\mu\tau} \simeq 10^{-2}$, 10^{-3} and 10^{-4} for $m_{R_{1,2}} = 50$ GeV, 100 GeV, and 500 GeV, respectively. These parameters enter into the neutrino oscillation probability in the "golden channel" [53],

$$P_{\mu\tau} \simeq 4|\eta_{\mu\tau}|^2 + 4s_{23}^2 c_{23}^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) - 4|\eta_{\mu\tau}| \sin\delta_{\mu\tau} \sin 2\theta_{23} \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right), \quad (45)$$

leading to the CP-asymmetry,

$$\mathcal{A}_{\mu\tau}^{CP} = \frac{P_{\mu\tau} - P_{\bar{\mu}\bar{\tau}}}{P_{\mu\tau} + P_{\bar{\mu}\bar{\tau}}} \simeq \frac{-4|\eta_{\mu\tau}| \sin\delta_{\mu\tau}}{\sin 2\theta_{23} \sin(\frac{\Delta m_{31}^2 L}{4E})}, \quad (46)$$

when the first term in Eq. (45) is much smaller compared to the other two terms. Our results in the partial degenerate case satisfy the condition that gives Eq. (46) from Eq. (45). The nonunitarity CP violating effects are predicted to be much more pronounced by noting that the strength of the third term in Eq. (45) is enhanced by 100–10,000 times compared to the prediction in the degenerate case. Crucial to this prediction is our constraint Eq. (38) between RH neutrino masses which plays an important role in estimating the phase of $\eta_{\mu\tau}$ in the partially degenerate case that takes into account the increasing behavior of m_{R_3} for decreasing values of $m_{R_1} = m_{R_2}$.

Among other significant differences in the model predictions are Br($\mu \to e\gamma$) values higher by two orders or by one order for $m_{R_1} = m_{R_2} = 50$ GeV or 100–500 GeV while Br($\tau \to e\gamma$) is predicted to be one order lower for the RH neutrino masses $m_{R_1} = m_{R_2} = 100$–1180 GeV. Presently the experimental limits on branching ratios are Br($\mu \to e\gamma$) $\leq 2.4 \times 10^{-12}$ [66], Br($\tau \to e\gamma$) $\leq 1.2 \times 10^{-7}$ [67], and Br($\tau \to \mu\gamma$) $\leq 4.5 \times 10^{-8}$ [67]. The projected reach of future sensitivities are up to Br($\tau \to e\gamma$) $\sim 10^{-9}$, Br($\tau \to \mu\gamma$) $\sim 10^{-9}$, but Br($\mu \to e\gamma$) $\sim 10^{-14}$ [68,69].

In Table V we show predictions of mass eigenvalues of the μ_S matrix that signifies masses of three fermion singlets S_i ($i = 1, 2, 3$) for degenerate and partially degenerate cases of RH neutrino masses. These mass eigenvalues are noted to vary starting from the lightest $\sim$1 eV to the heaviest $\sim$1 GeV which may have interesting phenomenological consequences that need further investigation. It is to be noted that the smallest mass eigenvalue is also predicted directly by the inverse seesaw formula from the TeV scale value of $(M_D)_{33} \sim 100$ GeV in a manner similar to the type-I seesaw case.

TABLE V. Mass eigenvalues of μ_S signifying masses of singlet fermions predicted by the inverse seesaw in $SO(10)$.

$m_{R_{1,2}}$ (GeV)	m_{R_3} (GeV)	Mass eigenvalues μ_{S_i} (MeV)
50	3324.7	(2.4583, 3.23×10^{-3}, 1.18×10^{-6})
100	1286.5	(8.0423, 2.60×10^{-3}, 1.07×10^{-6})
500	1140.7	(199.37, 5.29×10^{-2}, 1.05×10^{-6})
1136.6	1136.6	(1030.0, 2.72×10^{-1}, 1.04×10^{-6})

We have also examined the consequences of quasidegenerate light neutrino masses expected to manifest through tritium beta decay or neutrinoless double beta decay searches. For example with $m_1 = 0.09923$ eV, $m_2 = 0.09965$ eV, and $m_3 = 0.111$ eV, which are consistent with neutrino oscillation data, the three eigenvalues of the resulting μ_S matrix are $\mu_S^{(i)} = (30.110$ GeV, 1.2 MeV, 20.6 eV) with three pairs of heavy pseudo-Dirac neutrinos having almost degenerate masses (1151.7, 1121.6) GeV, (1139.5, 1139.5) GeV, and (1136.5, 1136.5) GeV. The predictions for LFV decays, CP-violating parameters and the nonunitarity effects are similar to the case of the degenerate pseudo-Dirac neutrinos with hierarchical light neutrino masses as discussed above. However, the heaviest eigenvalue of the fermion singlet mass matrix increases to $\mu_S^{(1)} \simeq 30$ GeV compared to the corresponding value of $\mu_S^{(1)} \simeq 1$ GeV in the hierarchical case of light neutrinos as shown in Table V.

The introduction of three additional fermion singlets under $SO(10)$ needed for the implementation of inverse seesaw mechanism may be argued to be a limitation of the related GUT models. For that matter, the other $SO(10)$ models of Refs. [23,24,29,33,34,40] have utilized these singlets to obtain different interesting results. More recently, the superpartners of two out of these three fermion singlets have been demonstrated to be acting as components of inelastic dark matter [70]. There is another $SO(10)$—based radiative inverse seesaw model which has been designed to explain the smallness of the μ_S parameter with the symmetry breaking chain $SO(10) \to SU(5) \times U(1)_\chi \to SM \times U(1)_\chi$ where more nonstandard fermions and singlets have been found to be necessary [32]. These indicate the popularity of $SO(10)$-singlet fermion models in spite of the stated limitation.

In this respect the E_6 [36] or $SU(3)^3$ [37–39] type GUT models do not have this limitation as they contain the necessary fermion singlets within their fundamental representations but they also contain a number of additional nonstandard fermions. In the absence of any experimental data on the masses of these additional fermions at low energies, the determination of the Dirac neutrino mass matrix from fermion mass fits at the GUT scale using the bottom-up approach adopted here is not possible. As one major objective of the present work is the prediction on the lifetimes of gauge boson mediated proton decay $p \to e^+\pi^0$ on which dedicated search experiments are ongoing [49], $SU(3)^3$ type of GUTs do not serve this objective as the corresponding decays are suppressed [37,38]. This model may be important if, ultimately, proton decay search experiments observe a very large lower limit on the lifetime.

It has been also argued that because of large size of Higgs representations such as 210_H and 126_H needed in

$SO(10)$ models employing type-I and type-II seesaw mechanisms, GUT-threshold corrections may give rise to larger uncertainties in $\sin^2\theta_W$ predictions and associated mass scale(s) [71]. Counterexamples of this result in $SO(10)$ having Pati-Salam intermediate symmetry $(SU(2)_L \times SU(2)_R \times SU(4)_C \times D \equiv G_{224D})$ with unbroken D-parity have been derived with exactly vanishing GUT-threshold corrections on $\sin^2\theta_W$ as well as on the intermediate scale [72]. It has been also shown how threshold corrections can be reduced substantially in other $SO(10)$ models with naturally plausible constraint that all superheavy components of a $SO(10)$ Higgs representation are degenerate in masses [18,19]. Noting that the Higgs representation 126_H is needed for the implementation of the type-I and type-II seesaw mechanisms, and the inverse seesaw needs comparatively much smaller Higgs representation like 16_H, the possibilities of threshold uncertainties are expected to be correspondingly reduced in our models. In particular our minimal model I contains neither of the larger Higgs representations 210_H and 126_H; it requires only the smaller representations 45_H, 16_H and 10_{H_1}, 10_{H_2}. Also in the case of the model II and its extension, the GUT-threshold effects due to superheavy components of Higgs representations 210_H, 16_H and 10_{H_1}, 10_{H_2} are expected to be substantially reduced compared to the $SO(10)$ model of Ref. [19] with G_{2213D} intermediate symmetry because of the absence of the large representation 126_H. The maximal value of proton lifetime is found to increase by a factor 2(4) due to GUT threshold effects in our model I (model II) over the two-loop predictions.

Regarding other possibilities of inverse seesaw motivated non-SUSY $SO(10)$, we find that the minimal single-step breaking scenario to the TeV scale gauge symmetry, $SO(10) \to G_{2113}$, is ruled out by renormalization group and coupling unification constraints. One of the two-step breaking chains, $SO(10) \to G_{224D} \to G_{2113}$ gives a low value of the unification scale $M_U = 10^{14.7}$ GeV whereas $SO(10) \to G_{214} \to G_{2113}$ also yields an almost similar value, $M_U = 10^{14.8}$ GeV where we have used $SU(2)_L \times U(1)_R \times SU(4)_C \equiv G_{214}$. The third remaining chain, $SO(10) \to G_{224} \to G_{2113}$, where D-parity is broken at the GUT scale, gives $M_U = 10^{15.15}$ GeV. Thus all the three minimal chains at two-loop level are ruled out by the existing lower bound on proton lifetime [49]. As the large representation 126_H is absent in these models, the GUT-threshold effects [19] are smaller in the corresponding minimal models than the required values to make them compatible with the lower limit on proton lifetime unless the splitting among the superheavy components is too large. In view of these, the minimal model I turns out to be the best among all possible single and two-step breaking minimal models of $SO(10)$ with the TeV scale G_{2113} gauge symmetry.

One of the appealing features which has been noted [6] in $SO(10)$ breaking chains under the category of model I is that they do not have the cosmological domain wall problem [73] because of spontaneous breaking of D-parity along with the gauge symmetry at the GUT scale. When this criteria is included while searching for equally good models, there are only two possible chains with three step breakings and only one chain with four step breaking to the TeV-scale symmetry G_{2113}. However, if utilization of large Higgs representations is excluded, the minimal model I emerges to be unique from among all possible $SO(10)$ breaking chains. Investigation of prospects for these longer symmetry breaking chains along with others which is beyond the scope of the present work will be addressed elsewhere.

VIII. SUMMARY AND CONCLUSION

We have investigated the prospects of inducting TeV-scale inverse seesaw mechanism for neutrino masses into nonsupersymmetric $SO(10)$ grand unification and found that it can be successfully implemented with a low-mass Z' gauge boson accessible to experimental detection at LHC and planned accelerators. By setting up RGEs in the presence of G_{2213} and G_{2113} gauge symmetries we have extrapolated fermion masses and mixings to the GUT scale using bottom-up approach and determined the Dirac neutrino mass matrix from a successful fit at the GUT scale. We have found a relation between the RH neutrino masses which, in the partially degenerate case, predicts the third generation RH neutrino mass to increase substantially with the decrease of first or second generation RH neutrino masses. Although the predicted branching ratios in the case of degenerate RH neutrinos show less than one order variations from the corresponding SUSY $SO(10)$ predictions, in the partially degenerate case, the branching ratio $\mathrm{Br}(\mu \to e\gamma)$ is predicted to be larger by 1–2 orders while $\mathrm{Br}(\tau \to e\gamma)$ is predicted to be lower by one order for all values of allowed RH neutrino masses. For the nonunitarity matrix element $\eta_{\mu\tau}$ an important model prediction is its enhanced phase $\delta_{\mu\tau}$ larger by 2–4 orders which is expected to play a dominant role in the experimental detection of the nonunitary CP-violation effects at neutrino factories. We have also shown that the models accommodate quasidegenerate light neutrino masses relevant for neutrinoless double beta decay or the tritium beta decay searches with predictions on the LFV, CP-violation, and nonunitarity effects similar to the case of hierarchical light neutrinos and degenerate pseudo-Dirac neutrinos while the heaviest mass of the fermion singlets increases from $\mu_S^{(1)} \simeq 1$ GeV to $\mu_S^{(1)} \simeq 30$ GeV.

Interestingly, the two-loop prediction on proton lifetime in the minimal model (model I) turns out to be $[\tau_p(p \to e^+\pi^0)]_{\max} = 7 \times 10^{34}$ Yrs. which increases by a factor of 2 when GUT threshold effects are included. While providing a possibility of verification of the underlying GUT hypothesis, this offers another opportunity for testing the minimal model by ongoing search experiments regarding its validity or falsifiability. We have also identified this model to be the best among all involving single or two-step breakings of $SO(10)$ to the TeV scale gauge symmetry G_{2113} which is essential for low mass Z' and prominent nonunitarity effects. But if utilization of large Higgs representations is excluded, the minimal model I emerges to be unique from among all possible $SO(10)$ breaking chains.

Too fast proton decay in another model (model II) has been shown to be evaded by a simple extension where some of the predictions on τ_p should be within the reach of future experiments. On the other hand, if the actual proton lifetime is too large, this is also shown to be accommodated in model extensions along with associated nonunitarity and lepton flavor violation effects with the prospect of detection of a color octet scalar at accelerator energies.

In conclusion we find that induction of TeV-scale inverse seesaw mechanism into nonsupersymmetric $SO(10)$ predicts pronounced nonunitarity and CP-violating effects measurable at accelerator energies and neutrino factories for hierarchical as well as partially degenerate spectra of light neutrino masses. In the TeV scale inverse seesaw mechanism motivated GUT model, these effects are mainly due to predominance of the Dirac neutrino mass matrix in $SO(10)$ because of its underlying quark-lepton symmetry and this holds even if only an experimentally verifiable low-mass Z' gauge boson is present as one of the smoking gun signatures of asymptotic parity restoration.

ACKNOWLEDGMENTS

M. K. P. thanks Harish-Chandra Research Institute for a visiting position. The authors thank Sandhya Choubey for discussion.

APPENDIX A

In the standard notation of two-loop evolution equations for gauge couplings,

$$\frac{\mu \partial g_i}{\partial \mu} = \frac{1}{16\pi^2} a_i g_i^3 + \frac{1}{(16\pi^2)^2} \sum_j b_{ij} g_i^3 g_j^2, \tag{A1}$$

the one- and two-loop beta function coefficients are given in Table VI. We have noted a small contribution of $U(1)_R \times U(1)_{B-L}$ mixing effect [46] especially in the case of model I.

APPENDIX B

Each of the two $SO(10)$ models we have considered for inverse seesaw has two types of nonstandard gauge symmetries, G_{2213} or G_{2213D} and G_{2113}. Here we derive RGEs for running Yukawa and fermion mass matrices from

TABLE VI. One-loop and two-loop beta function coefficients for gauge coupling evolutions in model I and model II described in the text taking the second Higgs doublet mass at 1 TeV.

MODEL	Symmetry	a_i	b_{ij} (GeV)
I, II	G_{213}	$(-19/6, 41/10, -7)$	$\begin{pmatrix} 199/50, 27/10, 44/5 \\ 9/10, 35/6, 12 \\ 11/10, 9/2, -26 \end{pmatrix}$
I, II	G_{2113}	$(-3, 53/12, 33/8, -7)$	$\begin{pmatrix} 8, 1, 3/2, 12 \\ 3, 17/4, 15/8, 12 \\ 9/2, 15/8, 65/16, 4 \\ 9/2, 3/2, 1/2, -26 \end{pmatrix}$
I	G_{2213}	$(-8/3, -13/6, 17/4, -7)$	$\begin{pmatrix} 37/3, 6, 3/2, 12 \\ 6, 143/6, 9/4, 12 \\ 9/2, 27/4, 37/8, 4 \\ 9/2, 9/2, 1/2, -26 \end{pmatrix}$
II	G_{2213D}	$(-13/6, -13/6, 17/4, -7)$	$\begin{pmatrix} 143/6, 6, 9/4, 12 \\ 6, 143/6, 9/4, 12 \\ 27/4, 27/4, 23/4, 4 \\ 9/2, 9/2, 1/2, -26 \end{pmatrix}$

which, following the earlier approach [63], we derive RGEs for the mass eigenvalues and mixing angles. We define the rescaled β-functions

$$16\pi^2\mu\frac{\partial F_i}{\partial\mu}=\beta_{F_i}. \tag{B1}$$

With G_{2113} symmetry the scalar field $\Phi_d(2,1/2,0,1)$ through its VEV v_d gives masses to down quarks and charged leptons while $\Phi_u(2,-1/2,0,1)$ through its VEV v_u gives Dirac masses to up quarks and neutrinos. These fields are embedded into separate bidoublets in the presence of G_{2213} and their vacuum structure has been specified in Sec. IV. We have derived the beta functions for RG evolution of Yukawa matrices (Y_i), fermion mass matrices (M_i), and the vacuum expectation values ($v_{u,d}$). The rescaled beta functions are given below in both cases,

G_{2113} symmetry:

$$\begin{aligned}
\beta_{Y_u}&=\Big[\frac{3}{2}Y_uY_u^\dagger+\frac{1}{2}Y_dY_d^\dagger+T_u-\sum_i C_i^q g_i^2\Big]Y_u,\\
\beta_{Y_d}&=\Big[\frac{3}{2}Y_dY_d^\dagger+\frac{1}{2}Y_uY_u^\dagger+T_d-\sum_i C_i^q g_i^2\Big]Y_d,\\
\beta_{Y_\nu}&=\Big[\frac{3}{2}Y_\nu Y_\nu^\dagger+\frac{1}{2}Y_eY_e^\dagger+T_u-\sum_i C_i^l g_i^2\Big]Y_\nu,\\
\beta_{Y_e}&=\Big[\frac{3}{2}Y_eY_e^\dagger+\frac{1}{2}Y_\nu Y_\nu^\dagger+T_d-\sum_i C_i^l g_i^2\Big]Y_e,\\
\beta_{M_u}&=\Big[\frac{3}{2}Y_uY_u^\dagger+\frac{1}{2}Y_dY_d^\dagger-\sum_i \tilde{C}_i^q g_i^2\Big]M_u,\\
\beta_{M_d}&=\Big[\frac{3}{2}Y_dY_d^\dagger+\frac{1}{2}Y_uY_u^\dagger-\sum_i \tilde{C}_i^q g_i^2\Big]M_d,\\
\beta_{M_D}&=\Big[\frac{3}{2}Y_\nu uY_\nu^\dagger+\frac{1}{2}Y_eY_e^\dagger-\sum_i \tilde{C}_i^l g_i^2\Big]M_D,\\
\beta_{M_e}&=\Big[\frac{3}{2}Y_e uY_e^\dagger+\frac{1}{2}Y_\nu Y_\nu^\dagger-\sum_i \tilde{C}_i^l g_i^2\Big]M_e,
\end{aligned} \tag{B2}$$

where the beta-functions for VEVs are

$$\begin{aligned}
\beta_{v_u}&=\Big[\sum_i C_i^v g_i^2-T_u\Big]v_u,\\
\beta_{v_d}&=\Big[\sum_i C_i^v g_i^2-T_d\Big]v_d,
\end{aligned} \tag{B3}$$

with

$$T_u=\mathrm{Tr}(3Y_u^\dagger Y_u+Y_\nu^\dagger Y_\nu),\qquad T_d=\mathrm{Tr}(3Y_d^\dagger Y_d+Y_e^\dagger Y_e). \tag{B4}$$

The parameters occurring in these equations, and also in Eqs. (B9) and (B10) given below are

$$\begin{aligned}
&a=\frac{3}{2},\qquad b=\frac{1}{2},\qquad a'=b'=0,\\
&C_i^q=(9/4,3/4,1/4,8),\qquad C_i^l=(9/4,3/4,9/4,0),\\
&\tilde{C}_i^q=(0,0,1/4,8),\qquad \tilde{C}_i^l=(0,0,9/4,0),\\
&C_i^v=(9/4,3/4,0,0),\qquad (i=2L,1R,BL,3C).
\end{aligned} \tag{B5}$$

G_{2213} symmetry: Following definitions of Sec. IV in the presence of left-right symmetry, the rescaled beta functions for RGEs of the Yukawa and fermion mass matrices are

$$\begin{aligned}
\beta_{Y_u}&=(Y_uY_u^\dagger+Y_dY_d^\dagger)Y_u+Y_u(Y_u^\dagger Y_u+Y_d^\dagger Y_d)+T_uY_u\\
&\quad+\hat{T}_1Y_d-\sum_i C_i^q g_i^2Y_u,\\
\beta_{Y_d}&=(Y_dY_d^\dagger+Y_uY_u^\dagger)Y_d+Y_d(Y_d^\dagger Y_d+Y_u^\dagger Y_u)+T_dY_d\\
&\quad+\hat{T}_2Y_u-\sum_i C_i^q g_i^2Y_d,\\
\beta_{Y_\nu}&=(Y_\nu Y_\nu^\dagger+Y_eY_e^\dagger)Y_\nu+Y_\nu(Y_\nu^\dagger Y_\nu+Y_e^\dagger Y_e)+T_uY_\nu\\
&\quad+\hat{T}_1Y_e-\sum_i C_i^l g_i^2Y_\nu,\\
\beta_{Y_e}&=(Y_eY_e^\dagger+Y_\nu Y_\nu^\dagger)Y_e+Y_e(Y_e^\dagger Y_e+Y_\nu^\dagger Y_\nu)+T_dY_e\\
&\quad+\hat{T}_2Y_\nu-\sum_i C_i^l g_i^2Y_e,\\
\beta_{M_u}&=(Y_uY_u^\dagger+Y_dY_d^\dagger)M_u+M_u(Y_u^\dagger Y_u+Y_d^\dagger Y_d)\\
&\quad-\sum_i\tilde{C}_i^q g_i^2M_u+\hat{T}_1\tan\beta M_d,\\
\beta_{M_d}&=(Y_dY_d^\dagger+Y_uY_u^\dagger)M_d+M_d(Y_d^\dagger Y_d+Y_u^\dagger Y_u)\\
&\quad-\sum_i\tilde{C}_i^q g_i^2]M_d+\frac{\hat{T}_2}{\tan\beta}M_u,\\
\beta_{M_D}&=(Y_\nu uY_\nu^\dagger+Y_eY_e^\dagger)M_D+M_D(Y_\nu^\dagger Y_\nu+Y_e^\dagger Y_e)\\
&\quad-\sum_i\tilde{C}_i^l g_i^2M_D+\hat{T}_1\tan\beta M_e,\\
\beta_{M_e}&=(Y_eY_e^\dagger+Y_\nu Y_\nu^\dagger)M_e+M_e(Y_e^\dagger Y_e+Y_\nu^\dagger Y_\nu)\\
&\quad-\sum_i\tilde{C}_i^l g_i^2M_e+\frac{\hat{T}_2}{\tan\beta}M_D,
\end{aligned} \tag{B6}$$

where the rescaled beta functions for VEVs β_{v_u}, β_{v_d} are the same as in Eq. (B3) with different coefficients C_i^v defined below and functions T_u and T_d are the same as in Eq. (B4). Other two traces entering in this case are

$$\hat{T}_1=\mathrm{Tr}(3Y_d^\dagger Y_u+Y_e^\dagger Y_\nu),\qquad \hat{T}_2=\mathrm{Tr}(3Y_u^\dagger Y_d+Y_\nu^\dagger Y_e). \tag{B7}$$

The parameters occurring in these equations and also in Eqs. (B9) and (B10) given below are

$$\begin{aligned} & a = b = 2, \qquad a' = b' = 1, \\ & C_i^q = (9/4, 9/4, 1/4, 8), \qquad C_i^l = (9/4, 9/4, 9/4, 0), \qquad \tilde{C}_i^q = (0, 0, 1/4, 8), \\ & \tilde{C}_i^l = (0, 0, 9/4, 0), \qquad C_i^\nu = (9/4, 9/4, 0, 0), \qquad (i = 2L, 2R, BL, 3C). \end{aligned} \tag{B8}$$

Then following the procedure described in Ref. [63], and using the definition of parameters in the two different mass ranges, given above we obtain RGEs for mass eigenvalues and elements of CKM mixing matrix $V_{\alpha\beta}$ which can be expressed in the generalized form for both cases,

Mass eigenvalues:

$$\begin{aligned} \beta_{m_i} &= \Big[-\sum_k \tilde{C}_k^{(q)} g_k^2 + a y_i^2 + 2b \sum_{j=d,s,b} |V_{uj}|^2 y_j^2 + a' \frac{\hat{T}_1 \tan\beta}{m_i} \sum_{j=d,s,b} |V_{uj}|^2 m_j \Big] m_i, \qquad i = u, c, t, \\ \beta_{m_i} &= \Big[-\sum_k \tilde{C}_k^{(q)} g_k^2 + a y_i^2 + 2b \sum_{j=u,c,t} |V_{dj}|^2 y_j^2 + b' \frac{\hat{T}_2}{\tan\beta m_i} \sum_{j=u,c,t} |V_{dj}|^2 m_j \Big] m_i, \qquad i = d, s, b, \\ \beta_{m_i} &= \Big[-\sum_k \tilde{C}_k^{(l)} g_k^2 + a y_i^2 + 2b \sum_{j=N_1,N_2,N_3} y_j^2 + b' \frac{\hat{T}_2}{\tan\beta m_i} \sum_{j=N_1,N_2,N_3} m_j \Big] m_i, \qquad i = e, \mu, \tau, \\ \beta_{m_i} &= \Big[-\sum_k \tilde{C}_k^{(l)} g_k^2 + a y_i^2 + a' \frac{\hat{T}_1 \tan\beta}{m_i} \sum_{j=e,\mu,\tau} m_j \Big] m_i, \qquad i = N_1, N_2, N_3. \end{aligned} \tag{B9}$$

CKM matrix elements:

$$\begin{aligned} \beta_{V_{\alpha\beta}} = & \sum_{\gamma=u,c,t;\gamma\neq\alpha} \Big[a' \frac{\hat{T}_1 \tan\beta}{m_\alpha - m_\gamma} (V \hat{M}_d V^\dagger)_{\alpha\gamma} + \frac{b}{v_d^2} \frac{m_\alpha^2 + m_\gamma^2}{m_\alpha^2 - m_\gamma^2} (V \hat{M}_d^2 V^\dagger)_{\alpha\gamma} \Big] V_{\gamma\beta} \\ & - \sum_{\gamma=d,s,b;\gamma\neq\beta} V_{\alpha\gamma} \Big[b' \frac{\hat{T}_2}{\tan\beta(m_\gamma - m_\beta)} (V^\dagger \hat{M}_u V)_{\gamma\beta} + \frac{b}{v_u^2} \frac{m_\gamma^2 + m_\beta^2}{m_\gamma^2 - m_\beta^2} (V^\dagger \hat{M}_u^2 V)_{\gamma\beta} \Big]. \end{aligned} \tag{B10}$$

Then using third generation dominance, the beta functions for all the 9 elements are easily obtained for respective mass ranges where in addition to the parameters in the respective cases in Eqs. (B5) and (B8), $a' = b' = 0$ in the mass range $M_{R^0} \to M_{R^+}$ with G_{2113} symmetry, but $a' = b' = 1$ in the mass range $M_{R^+} \to M_U$ with G_{2213} or G_{2213D} symmetry and, in the latter case, the nonvanishing traces $\hat{T}_{1,2}$ are easily evaluated in the mass basis.

[1] For reviews on different neutrino mass generation mechanisms with or without GUTs, supersymmetry, or flavor symmetry see, for instance, G. Altarelli, arXiv:0905.2350; arXiv:1011.5342; arXiv:1107.1980; E. Ma, in *Proceedings of the 9th Workshop in High Energy Physics Phenomenology (WHEPP9), Bhubaneswar, India, 2006* (unpublished); Pramana J. Phys. **67**, 803 (2006); R. N. Mohapatra *et al.*, Rep. Prog. Phys. **70**, 1757 (2007); R. N. Mohapatra, Prog. Part. Nucl. Phys. **64**, 307 (2010); S. Raby *et al.*, arXiv:0810.4551; E. Ma, arXiv:0905.0221; AIP Conf. Proc. **1116**, 239 (2009); H. Hettamansperger, M. Lindner, and W. Rodejohann, J. High Energy Phys. 04 (2011) 123; G. Senjanovic, Riv. Nuovo Cimento **034**, 1 (2011).

[2] J. C. Pati and A. Salam, Phys. Rev. D **8**, 1240 (1973); **10**, 275 (1974).

[3] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[4] H. Georgi, *Proceedings of APS Division of Particles and Fields,* edited by C. Carlson (AIP, New York, 1975), p. 575; H. Fritzsch and P. Minkowski, Ann. Phys. (Berlin) **93**, 193 (1975).

[5] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975); G. Senjanović and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975).

[6] D. Chang, R. N. Mohapatra, and M. K. Parida, Phys. Rev. Lett. **52**, 1072 (1984); Phys. Rev. D **30**, 1052 (1984).

[7] D. Chang, R. N. Mohapatra, J. Gipson, R. E. Marshak, and M. K. Parida, Phys. Rev. D **31**, 1718 (1985).

[8] P. Minkowski, Phys. Lett. **67B**, 421 (1977); T. Yanagida in *Workshop on Unified Theories*, KEK Report No. 79-18, 1979, p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity* (North Holland, Amsterdam, 1979), p. 315; S. L. Glashow, *1979 Cargese Summer Institute on*

Quarks and Leptons (Plenum, New York, 1980), p. 687; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[9] M. Magg and C. Wetterich, Phys. Lett. **94B**, 61 (1980); G. Lazaridis, Q. Shafi, and C. Wetterich, Nucl. Phys. **B181**, 287 (1981); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).

[10] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993).

[11] B. Brahmachari and R. N. Mohapatra, Phys. Rev. D **58**, 015001 (1998); T. Fukuyama and N. Okada, J. High Energy Phys. 11 (2002) 011; N. Oshimo, Phys. Rev. D **66**, 095010 (2002).

[12] B. Bajc, G. Senjanovic, and F. Vissani, Phys. Rev. Lett. **90**, 051802 (2003).

[13] H. S. Goh, R. N. Mohapatra, and S. Nasri, Phys. Rev. D **70**, 075022 (2004); R. N. Mohapatra and M. K. Parida, Phys. Rev. D **84**, 095021 (2011).

[14] A. Melfo, A. Ramirez, and G. Senjanovic, Phys. Rev. D **82**, 075014 (2010).

[15] G. Altarelli and G. Blankenburg, J. High Energy Phys. 03 (2011) 133; P. S. Bhupal Dev, R. N. Mohapatra, and M. Severson, Phys. Rev. D **84**, 053005 (2011).

[16] W. J. Marciano and G. Senjanovic, Phys. Rev. D **25**, 3092 (1982); U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. B **260**, 447 (1991); P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991).

[17] N. G. Deshpande, E. Keith, and P. Pal, Phys. Rev. D **46**, 2261 (1992).

[18] R. N. Mohapatra and M. K. Parida, Phys. Rev. D **47**, 264 (1993).

[19] D. G. Lee, R. N. Mohapatra, M. K. Parida, and M. Rani, Phys. Rev. D **51**, 229 (1995).

[20] M. K. Parida, Phys. Lett. B **704**, 206 (2011).

[21] E. Ma, Phys. Rev. D **73**, 077301 (2006).

[22] R. N. Mohapatra, Phys. Rev. Lett. **56**, 561 (1986); R. N. Mohapatra and J. W. F. Valle, Phys. Rev. D **34**, 1642 (1986).

[23] M. Lindner, M. A. Schmidt, and A. Yu Smirnov, J. High Energy Phys. 07 (2005) 048.

[24] S. K. Kang and C. S. Kim, Phys. Lett. B **646**, 248 (2007); J. Ellis, D. V. Nanopoulos, and K. Olive, Phys. Lett. B **300**, 121 (1993); S. K. Majee, M. K. Parida, and A. Raychaudhuri, Phys. Lett. **668**, 299 (2008); M. K. Parida and A. Raychaudhuri, Phys. Rev. D **82**, 093017 (2010); S. K. Majee, M. K. Parida, A. Raychaudhuri, and U. Sarkar, Phys. Rev. D **75**, 075003 (2007).

[25] See, for instance, F. Deppisch, T. S. Kosmas, and J. W. F. Valle, Nucl. Phys. **B752**, 80 (2006); F. Deppisch and J. W. F. Valle, Phys. Rev. D **72**, 036001 (2005); J. Garoya, M. C. Gonjalez-Gartia, and N. Rius, J. High Energy Phys. 02 (2007) 021; C. Arina, F. Bazzochi, N. Forengo, J. C. Romao, and J. W. F. Valle, Phys. Rev. Lett. **101**, 161802 (2008); F. Bazzochi, D. G. Cerdeno, C. Munoz, and J. W. F. Valle, Phys. Rev. D **81**, 051701 (2010); M. B. Gavela, T. Hambye, D. Hernandez, and P. Hernandez, J. High Energy Phys. 09 (2009) 038; M. Malinsky, T. Ohlsson, Z.-z. Xing, and H. Zhang, Phys. Lett. B **679**, 242 (2009); M. Hirsch, T. Kernreiter, J. C. Romao, and A. Villanova del Moral, J. High Energy Phys. 01 (**2010**) 103; F. Bazzocchi, Phys. Rev. D **83**, 093009 (2011); M. E. Catano, R. Martinez, and R. Ochoa, arXiv:1206.1966; M. Hirsch, W. Porod, and F. Staub, arXiv:1206.3516; X. G. He and E. Ma, Phys. Lett. B **683**, 178 (2010).

[26] A. Abada, D. Das, and C. Weiland, J. High Energy Phys. 03 (2012) 100; A. Abada, D. Das, A. Vicente, and C. Weiland, J. High Energy Phys. 09 (2012) 015; P. S. B. Dev, S. Mondal, B. Mukhopadhyaya, and S. Roy, arXiv:1207.6542.

[27] S. Khalil, Phys. Rev. D **82**, 07702 (2010); S. Khalil, H. Okada, and T. Toma, J. High Energy Phys. 07 (2011) 026.

[28] P.-H. Gu and U. Sarkar, Phys. Lett. B **694**, 226 (2010).

[29] P. S. Bhupal Dev and R. N. Mohapatra, Phys. Rev. D **81**, 013001 (2010); **82**, 035014 (2010).

[30] G.'t Hooft, in *Proceedings of the 1979 Cargese Summer Institute on Recent Developments in Gauge Theories*, edited by G.'t Hooft *et al.* (Plenum Press, New York, 1980).

[31] E. Ma, Mod. Phys. Lett. A **24**, 2491 (2009); D. Ibanez, S. Morisi, and J. W. F. Valle, Phys. Rev. D **80**, 053015 (2009).

[32] E. Ma, Phys. Rev. D **80**, 013013 (2009).

[33] C. H. Albright and S. M. Barr, Phys. Rev. Lett. **85**, 244 (2000); Phys. Rev. D **62**, 093008 (2000); For a review see S. M. Barr, Int. J. Mod. Phys. A **18**, 3971 (2003).

[34] S. M. Barr, Phys. Rev. Lett. **92**, 101601 (2004); C. H. Albright and S. M. Barr, Phys. Rev. D **69**, 073010 (2004).

[35] A. G. Dias, C. A. de S. Pires, P. S. Rodrigues da Silva, and A. Sampieri, Phys. Rev. D **86**, 035007 (2012).

[36] F. Gursey, P. Ramond, and P. Sikivie, Phys. Lett. **60B**, 177 (1976); E. Ma, Phys. Rev. D **36**, 274 (1987).

[37] S. L. Glashow, in *Grand Unification. Proceedings, 5th Workshop, Providence, USA, 1984*, edited by K. Kang (World Scientific, Singapore, 1985).

[38] C. Cauet, H. Pas, and S. Weisenfeldt, Phys. Rev. D **83**, 093009 (2011).

[39] S. Heinemeyer and E. Ma, Fortschr. Phys. **58**, 729 (2010); S. Heinemeyer, E. Ma, M. Mondragon, and G. Zoupanos, AIP Conf. Proc. **1200**, 568 (2010); E. Ma, J. Phys. Conf. Ser. **315**, 012006 (2011).

[40] S. Blanchet, P. S. Bhupal Dev, and R. N. Mohapatra, Phys. Rev. D **82**, 115025 (2010).

[41] P. Langacker, Rev. Mod. Phys. **81**, 1199 (2009); AIP Conf. Proc. **1200**, 55 (2010); For earlier work on Z' boson with two-step breaking of left-right gauge symmetry and in GUTs see M. K. Parida and A. Raychaudhuri, Phys. Rev. D **26**, 2364 (1982); M. K. Parida and C. C. Hazra, Phys. Lett. **121B**, 355 (1983); Phys. Rev. D **40**, 3074 (1989).

[42] G. J. P. Eboli, J. Gonzalez-Fraile, and M. Gonzalez-Gartia, Phys. Rev. D **85**, 055019 (2012); A. Falkowski, C. Grojean, A. Kaminska, S. Pokorski, and A. Weiler, J. High Energy Phys. 11 (2011) 028.

[43] S. Bertolini, L. Di Luzio, and M. Malinsky, Phys. Rev. D **80**, 015013 (2009); **85**, 095014 (2012).

[44] H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[45] D. R. T. Jones, Phys. Rev. D **25**, 581 (1982).

[46] L. Lavoura, Phys. Rev. D **48**, 2356 (1993); F. del Aguila, G. D. Coughlan, and M. Quiros, Nucl. Phys. **B307**, 633

(1988); F. del Aguila, G. M. Masip, and M. Perez-Victoria, Nucl. Phys. **B456**, 531 (1995).
[47] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010); C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[48] P. Nath and P. Filviez Perez, Phys. Rep. **441**, 191 (2007); B. Bajc, I. Dorsner, and M. Nemevsek, J. High Energy Phys. 11 (2008) 007; P. Langacker, Phys. Rep. **72**, 185 (1981).
[49] H. Nishino *et al.*, Phys. Rev. Lett. **102**, 141801 (2009); Phys. Rev. D **85**, 112001 (2012);
[50] M. Shiozawa, neutrino.kek.jp/jhfnu/workshop2/ohp/shiozawa.pdf; K. Abe *et al.*, arXiv:1109.3262.
[51] B. A. Dobrescu, K. Kong, and R. Mahbubani, J. High Energy Phys. 07 (2007) 006; Y. Bai and B. A. Dobrescu, J. High Energy Phys. 07 (2011) 100; M. K. Parida, P. K. Sahu, and K. Bora, Phys. Rev. D **83**, 093004 (2011); M. K. Parida, Ref. [20]; R. N. Mohapatra and M. K. Parida, Ref. [13].
[52] S. Antusch, J. P. Baumann, and E. Fernandez-Martinez, Nucl. Phys. **B810**, 369 (2009); S. Antusch, M. Blennow, and E. Fernandez-Martinez, and J. Lopez-Pavon, Phys. Rev. D **80**, 033002 (2009); S. Antush, C. Biggio, E. Fernandez-Martinez, M. Belen Gavela, and J. Lopez-Pavon, J. High Energy Phys. 10 (2006) 084; D. V. Forero, S. Morisi, M. Tartola, and J. W. F. Valle, J. High Energy Phys. 09 (2011) 142.
[53] E. Fernandez-Martinez, M. B. Gavela, J. Lopez-Pavon, and O. Yasuda, Phys. Lett. B **649**, 427 (2007).
[54] K. Kanaya, Prog. Theor. Phys. **64**, 2278 (1980).
[55] J. Kersten and A. Y. Smirnov, Phys. Rev. D **76**, 073005 (2007).
[56] M. Malinsky, T. Ohlsson, and H. Zhang, Phys. Rev. D **79**, 073009 (2009).
[57] G. Altarelli and D. Meloni, Nucl. Phys. **B809**, 158 (2009).
[58] F. del Aguila and J. A. Aguilar-Saavedra, Phys. Lett. B **672**, 158 (2009); F. del Aguila, J. A. Aguilar-Saavedra, and J. de Blas, Acta Phys. Pol. B **40**, 2901 (2009).
[59] A. van der Schaaf, J. Phys. G **29**, 2755 (2003); Y. Kuno, Nucl. Phys. B, Proc. Suppl. **149**, 376 (2005).
[60] K. Abe *et al.* (T2K collaboration), Phys. Rev. Lett. **107**, 041801 (2011); F. P. An *et al.* (DAYA-BAY Collaboration), Phys. Rev. Lett. **108**, 171803 (2012); P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **107**, 181802 (2011); J. K. Ahn *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012).
[61] A. Ilakovac and A. Pilaftsis, Nucl. Phys. **B437**, 491 (1995).
[62] E. Akhmedov, M. Lindner, E. Schnapka, and J. W. F. Valle, Phys. Rev. D **53**, 2752 (1996);
[63] C. R. Das and M. K. Parida, Eur. Phys. J. C **20**, 121 (2001).
[64] Conrad Sanderson, *Armadillo: An Open Source C + + Linear Library for Fast Prototyping and Computationally Intensive Experiments*, Technical Report, NICTA (2010).
[65] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo, and A. M. Rotunno, Phys. Rev. D **86**, 013012 (2012); T. Schwetz, M. Tartola, and J. W. F. Valle, New J. Phys. **13**, 063004 (2011); D. V. Forero, M. Tartola, and J. W. F. Valle, arXiv:1205.4018.
[66] J. Adam *et al.* (MEG Collaboration), Phys. Rev. Lett. **107**, 171801 (2011).
[67] K. Hayasaka *et al.* (Belle Collaboration), Phys. Lett. B **666**, 16 (2008).
[68] For a review see F. R. Joaquim and A. Rossi, Nucl. Phys. **B765**, 71 (2007).
[69] Y. Kuno (PRIME Working Group), Nucl. Phys. B, Proc. Suppl. **149**, 376 (2005).
[70] H. An, P. S. Bhupal Dev, Y. Cai, and R. N. Mohapatra, Phys. Rev. Lett. **108**, 081806 (2012).
[71] V. V. Dixit and M. Sher, Phys. Rev. D **40**, 3765 (1989).
[72] M. K. Parida and P. K. Patra, Phys. Rev. Lett. **66**, 858 (1991); **68**, 754 (1992); M. K. Parida, Phys. Rev. D **57**, 2736 (1998).
[73] T. W. B. Kibble, G. Lazaridis, and Q. Shafi, Phys. Rev. D **26**, 435 (1982).