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Self-dual soliton solution in a generalized Jackiw-Pi model

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We consider a generalization of the Jackiw-Pi model by introducing a nonstandard kinetic term. We present a Bogomolnyi framework for this theory and as a particular case we show that the Bogomolnyi equations of Chern-Simons Higgs theory can be obtained. Finally, the dimensionally reduced theory is analyzed and novel solitonic equations emerge.

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I. INTRODUCTION

The two-dimensional matter field interacting with gauge fields whose dynamics is governed by a Chern-Simons term support soliton solutions [1,2]. These models have the particularity to become auto-dual when the self-interactions are suitably chosen [3,4]. When this occurs, the model presents particular mathematical and physics properties, such as the supersymmetric extension of the model [5], and the reduction of the motion equation to first order derivative equation [6]. The Chern-Simons gauge field inherits its dynamics from the matter fields to which it is coupled, so it may be either relativistic [3] or nonrelativistic [4]. In addition, the soliton solutions are of topological and nontopological nature [7].

In recent years, theories with nonstandard kinetic term, named k-field models, have received much attention. The k-field models are mainly in connection with effective cosmological models [8,9] as well as the strong interaction physics, strong gravitational waves [10] and dark matter [11]. One interesting aspect to analyze in these models concern its topological structure. In this context several studies have been conducted, showing that the k-theories can support topological soliton solutions both in models of matter and in gauged models [12,13]. These solitons have certain features, such as their characteristic size, which are not necessarily those of the standard models [14]. Other interesting aspects are that they do not interact at large distances and they are, in general, not self-dual.

In this paper, we are interested in studying the Jackiw-Pi model with generalized dynamics. The so call Jackiw-Pi model is a nonrelativistic and Galilean invariant model which is also self-dual [4]. Here, we will show that introducing nonstandard dynamics in the Jackiw-Pi Lagrangian, via a function ω depending on the Higgs field, we can obtain self-dual or Bogomolnyi equations by minimizing the energy functional of the model. As a particular case, we will show that choosing a suitable ω, the Bogomolnyi equations of Chern-Simons Higgs theory can be obtained. Finally, we will study the dimensional reduction of the model to $(1 + 1)$-dimensions and we will arrive to the existence of interesting soliton solutions in the system.

II. THE MODEL

Let us start by considering the model proposed by Jackiw and Pi [4]:

$$S = \int d^3x\left(\frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + i\phi^* D_0\phi - \frac{1}{2m}|D_i\phi|^2 + \lambda|\phi|^4\right). \tag{1}$$

This is a nonrelativistic model where the gauge fields dynamic is dictated by a Chern-Simons term and matter is represented by scalar field $\phi(x)$. The covariant derivative is defined as $D_\mu = \partial_\mu + ieA_\mu$ $(\mu = 0, 1, 2)$. The metric tensor is $g^{\mu\nu} = (1, -1, -1)$ and $\epsilon^{\mu\nu\lambda}$ is the totally antisymmetric tensor such that $\epsilon^{012} = 1$.

The field equations corresponding to this action are

$$iD_0\phi = -\frac{1}{2m}D_i^2\phi - 2\lambda|\phi|^2\phi$$
$$B = \frac{e}{\kappa}\rho \qquad E^i = -\frac{1}{\kappa}\epsilon^{ij}j_i, \tag{2}$$

where $\rho = |\phi|^2$ and $j^i = -\frac{i}{2m}(\phi^* D^i\phi - (D^i\phi)^*\phi)$. The second of these equations is the Chern-Simons Gauss law which can be integrated, over the entire plane, to obtain the important consequence that any object with charge $Q = e\int d^2x\rho$ also carries magnetic flux $\Phi = \int Bd^2x$ [15]:

$$\Phi = \frac{1}{\kappa}Q. \tag{3}$$

Here, we are interested in time-independent soliton solutions that ensure the finiteness of the action (1). These are the stationary points of the energy which for the static field configuration reads

$$E = \int d^2x\left(\frac{1}{2m}|D_i\phi|^2 - \lambda|\phi|^4\right). \tag{4}$$

In order to find the minimum of the energy, the expression (4) can be rewritten as

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$$E = \int d^3x \left(\frac{1}{2m} |D_\pm \phi|^2 + \left(-\lambda \mp \frac{e^2}{2m\kappa} \right) |\phi|^4 \right), \quad (5)$$

where we have used the Chern-Simons Gauss law and the identity

$$|D_i\phi|^2 = |(D_1 \pm iD_2)\phi|^2 \mp eB|\phi|^2 \pm m\epsilon^{ij}\partial_i J_j. \quad (6)$$

Thus, with the self-dual coupling

$$\lambda = \mp \frac{e^2}{2m\kappa}, \quad (7)$$

and sufficiently well-behaved fields so that the integral over all space of $\epsilon^{ij}\partial_i J_j$ vanishes, the energy becomes

$$E = \int d^3x \frac{1}{2m} |D_\pm \phi|^2. \quad (8)$$

Thus, the energy is bounded below by zero, and this lower bound is saturated by fields obeying the first order self-duality equations

$$(D_1 \pm iD_2)\phi = 0 \qquad B = \frac{e}{\kappa}\rho. \quad (9)$$

Following the same idea of the works cited in Ref. [12], we will consider here a generalization of the Jackiw-Pi model described by the action

$$S = \int d^3x \left(\frac{\kappa}{2} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + i\omega(\rho)\phi^* D_0 \phi - \frac{1}{2m} |D_i \phi|^2 - V(\rho) \right), \quad (10)$$

where we have replaced the usual kinetic term $i\phi^* D_0 \phi$ by a more generalized term $i\omega(\rho)\phi^* D_0 \phi$. Here, $\omega(\rho)$ is, in principle, an arbitrary function of the complex scalar field ϕ and $V(\rho)$ is the scalar field potential to be determined below.

The equations of motion for this system are given by

$$i\left(\frac{\partial \omega(\rho)}{\partial \phi^*} \phi^* D_0 \phi + \omega(\rho) D_0 \phi \right) = -\frac{1}{2m} D_i^2 \phi + \frac{\partial V(\rho)}{\partial \phi^*}$$
$$B = \frac{e}{\kappa}\omega(\rho)\rho \qquad E^i = -\frac{1}{\kappa}\epsilon^{ij} j_i, \quad (11)$$

where the two first equations differ from those present in Eq. (2) by the presence of the function $\omega(\rho)$.

The theory may be descried in terms of the Hamiltonian formulation as

$$H = \int d^2x \left(\frac{1}{2m} |D_i \phi|^2 + V(\rho) \right), \quad (12)$$

which may be rewritten using the Gauss Law and the identity (6) in the form

$$E = \int d^3x \left(\frac{1}{2m} |D_\pm \phi|^2 \mp \frac{e^2}{2m\kappa} \omega(\rho)\rho^2 + V(\rho) \right). \quad (13)$$

In order to relate the solutions in this theory with those present in the Chern-Simons Higgs theory, we may choose, as a particular case, the following $\omega(\rho)$ function

$$\omega(\rho) = 2m \frac{e^2}{\kappa}(\rho - 1). \quad (14)$$

Then, the energy functional (13) is written as

$$E = \int d^3x \left(\frac{1}{2m} |D_\pm \phi|^2 \mp \frac{e^4}{\kappa^2} \rho^2(\rho - 1) + V(\rho) \right), \quad (15)$$

and the Gauss law of the equation (11) takes the form

$$B = 2m \frac{e^3}{\kappa^2}(\rho - 1)\rho. \quad (16)$$

The form of the potential $V(\rho)$ that we choose is motivated by the desire to find the self-dual soliton solution. Thus, if we choose the potential as

$$V(\rho) = \pm \frac{e^4}{\kappa^2} \rho(\rho - 1)^2, \quad (17)$$

and replace it in the expression (15), we arrive at the following expression of the energy functional:

$$E = \int d^3x \left(\frac{1}{2m} |D_\pm \phi|^2 \mp \frac{e}{2m} B \right), \quad (18)$$

which is bounded below by a multiple of the magnitude of the magnetic flux (for positive flux we choose the lower signs, and for negative flux we choose the upper signs):

$$E \geq \frac{e}{2m} |\Phi|. \quad (19)$$

Here, the magnetic flux is determined by the requirement of finite energy. This implies that the covariant derivative must vanish asymptotically, which fixes the behavior of the gauge field A_i. Then we have

$$\Phi = \int d^2x B = \oint_{|x|=\infty} A_i dx^i = 2\pi N, \quad (20)$$

where N is a topological invariant which takes only integer values. It is interesting to remark, here, the existence of the topological bound which is not present in the Jackiw-Pi model. So, this a nonrelativistic model with a topological bound and therefore we shall expect to find topological solitons.

This bound is saturated by fields satisfying the first-order self-duality equations

$$D_\pm \phi = (D_1 \pm iD_2)\phi = 0, \quad (21)$$

$$B = 2m \frac{e^3}{\kappa^2}(\rho - 1)\rho. \quad (22)$$

These equations may be compared with the self-duality equations of the Chern-Simons Higgs theory. We can note that if we fix $m = 1$ and choose the plus sign in the potential expression (17), we arrive to the Chern-Simons Higgs self-duality equations

$$(D_1 + iD_2)\phi = 0, \quad (23)$$

$$B = 2m\frac{e^3}{\kappa^2}(\rho - 1)\rho. \tag{24}$$

On the other hand, the anti-self-duality equations may be obtained by choosing the function $\omega(\rho)$ as

$$\omega(\rho) = -2m\frac{e^2}{\kappa}(\rho - 1). \tag{25}$$

In that case, if we desire to arrive at the expression (18), we must choose the following potential term:

$$V(\rho) = \mp\frac{e^4}{\kappa^2}\rho(\rho - 1)^2. \tag{26}$$

Then, the Bogomolnyi equations become

$$D_\pm\phi = (D_1 \pm iD_2)\phi = 0, \tag{27}$$

$$B = -2\frac{e^3}{\kappa^2}(\rho - 1)\rho. \tag{28}$$

Choosing $V(\rho) = \frac{e^4}{\kappa^2}\rho(\rho - 1)^2$, we obtain the anti-self-duality equations

$$D_-\phi = (D_1 - iD_2)\phi = 0, \tag{29}$$

$$B = -2\frac{e^3}{\kappa^2}(\rho - 1)\rho. \tag{30}$$

Thus, we can obtain the same Bogomolnyi equations as those present in the Chern-Simons Higgs model. The difference lies in the fact that in our case, we are dealing with a nonrelativistic model and we have imposed the constraint $m = 1$. Another interesting fact is that, here, we expect to find both topological and nontopological soliton solutions, just as in Chern-Simons Higgs theory. It is also worth noting that the Bogomolnyi equations in Chern-Simons Higgs theory are neither solvable nor integrable. However, numerical solutions can be found using a radial vortex-like ansatz [3,7].

An important comment is that the generalized Jackiw-Pi model, studied here, is a self-dual model. This is important because the generalized pure Chern-Simons system previously explored are not self-dual (For instance, see Ref. [12]).

III. DIMENSIONAL REDUCTION AND THE SOLITONIC SOLUTION

In this section, we are interested in analyzing the dimensional reduction of the model (10) as well as in studying the soliton solution in the dimensionally reduced model. In order to analyze the lineal problem [16,17], it is natural to consider a dimensional reduction of the action (10) by suppressing dependence on the second spacial coordinate, renaming A_y as B. Then, the action (10) becomes

$$S = \int d^2x\Big(\kappa(A_0\partial_x B + B\partial_0 A_1) + i\omega(\rho)\phi^* D_0\phi - \frac{1}{2m}|D_x\phi|^2 - \frac{e^2}{2}2mB^2\rho - V(\rho)\Big). \tag{31}$$

Notice that the Gauss law constraint for this action is

$$\partial_x B = \frac{e}{\kappa}\omega(\rho)\rho, \tag{32}$$

which can be solved as

$$B(x) = \frac{e}{2\kappa}\int dz\epsilon(x - z)\omega(\rho(z))\rho(z), \tag{33}$$

where $\epsilon(x) = \theta(x) - \theta(-x)$ is the odd step function.

By using the Gauss law and the explicit form of B given by Eq. (33), the action can be written simply as

$$S = \int d^2x\Big[\Big(\frac{e}{2}\int dz\epsilon(x - z)\omega(\rho(z))\rho(z)\Big)\partial_0 A_1) + i\omega(\rho)\phi^*\partial_0\phi - \frac{1}{2m}|D_x\phi|^2 - \frac{e^2}{2m}B^2\rho - V(\rho)\Big]. \tag{34}$$

Following Ref. [17], the gauge field A_x may be eliminated from the action (34) by a gauge transformation. Indeed, after transforming the matter field as

$$\phi(x) \to e^{-i\alpha(x)}\phi(x), \tag{35}$$

with

$$\alpha(x) = \frac{e}{2}\int dz\epsilon(x - z)A_x(z), \tag{36}$$

we arrive to the following action:

$$S = \int d^2x\Big(i\omega(\rho)\phi^*\partial_0\phi - \frac{1}{2m}|\partial_x\phi|^2 - \frac{e^2}{2m}B^2\rho - V(\rho)\Big). \tag{37}$$

Consider, now, the derivation of the Bogomolnyi equations in the reduced model (37). As discussed in Refs. [18,19], the field B plays an important role in the derivation of the self-dual equations. Indeed the expression (33) of B involves the existence of a novel soliton solution. Using the relation

$$\int d^2x(|(\partial_x + \gamma eB)\phi|^2 + \gamma e\partial_x B\rho) = \int d^2x(|\partial_x\phi|^2 + e^2B^2\rho), \tag{38}$$

we can rewrite the action (37) as

$$S = \int d^2x\Big(i\omega(\rho)\phi^*\partial_0\phi - \frac{1}{2m}|(\partial_x + \gamma eB)\phi|^2 - \frac{\gamma e}{2m}\partial_x B\rho - V(\rho)\Big). \tag{39}$$

Here, $\gamma = \pm 1$. The Gauss law (32) may be used to replace the derivative of the field B in the action. Then we have, in the static field configuration, the Hamiltonian associated to the action written as

$$H = \int dx \Big(\frac{1}{2m} |(\partial_x + \gamma e B)\phi|^2 + \frac{e^2\gamma}{2m\kappa}\omega(\rho)\rho^2 + V(\rho) \Big). \tag{40}$$

As in the (2 + 1)-dimensional case, we can choose $\omega(\rho) = 2m\frac{e^2}{\kappa}(\rho - 1)$ and $V(\rho) = \mp \frac{e^4}{\kappa^2}\rho(\rho - 1)^2$ to obtain

$$H = \int dx \Big(\frac{1}{2m} |(\partial_x + \gamma e B)\phi|^2 + \frac{e\gamma}{2m}\partial_x B \Big). \tag{41}$$

Since the field B must be zero in the boundary, the last term in the Hamiltonian vanishes and we have

$$H = \int dx \frac{1}{2m} |(\partial_x + \gamma e B)\phi|^2. \tag{42}$$

This is non-negative and therefore takes the minimum when the ϕ satisfies

$$(\partial_x + \gamma B)\phi = 0. \tag{43}$$

We can write this equation in a more explicit form by using Eq. (33):

$$\partial_x \phi(x) + \frac{\gamma e}{2\kappa} \int dz \epsilon(x - z)\omega(\rho(z))\rho(z)\phi(x) = 0. \tag{44}$$

To solve (44), we can proceed as in Ref. [19]. Thus, we assume that ϕ may be written as $\phi = \sqrt{\rho}$, which leads to

$$\frac{1}{2}\partial_x(\log\rho(x)) + \frac{e\gamma}{2\kappa}\int dz \epsilon(x - z)\omega(\rho(z))\rho(z) = 0. \tag{45}$$

Differentiating the above equation with respect to x, we arrive at the following one-dimensional Liouville type equation:

$$\frac{1}{2}\partial_x^2(\log\rho(x)) + \frac{e\gamma}{\kappa}\omega(\rho(x))\rho(x) = 0. \tag{46}$$

This is the general Bogomolnyi equation corresponding to the dimensionally reduced model. In particular, we are analyzing the case $\omega(\rho) = 2m\frac{e^2}{\kappa}(\rho - 1)$, so that the previous equation becomes

$$\frac{1}{2}\partial_x^2(\log\rho(x)) + \frac{2e^3\gamma}{\kappa^2}(\rho - 1)\rho(x) = 0. \tag{47}$$

This equation presents two types of solution, one derived from the topological solution and the other from the nontopological solution present in the two-dimensional Chern-Simons Higgs theory. Let us start by considering the solution derived from the topological case. For this case, we propose as a solution the following series:

$$\rho = 1 + \sum_{n=1}^{\infty} a_n \mathrm{sech}^{\mathrm{n}}(\mathrm{bx}), \tag{48}$$

where a_n are the real coefficients of series and b is a real constant. To check that this is really a solution, we rewrite Eq. (47) as

$$-(\partial_x\rho)^2 + (\partial_x^2\rho)\rho + v\rho^3(\rho - 1) = 0, \tag{49}$$

with $v = \frac{4e^3\gamma}{\kappa^2}$. Inserting the series (48) into Eq. (49), we obtain

$$\begin{aligned}&\sum_{n,m=1}^{\infty}[-a_n a_m m n b^2 + n^2 a_n a_m b^2 + 3v a_n a_m]\mathrm{sech}^{\mathrm{n+m}}(\mathrm{bx}) + \sum_{n,m=1}^{\infty}[a_n a_m n m b^2 - n^2 a_n a_m b^2 - n a_n a_m b^2]\mathrm{sech}^{\mathrm{n+m+2}}(\mathrm{bx})\\ &\quad - \sum_{n=1}^{\infty}(n^2 + n)a_n b^2 \mathrm{sech}^{2+\mathrm{n}}(\mathrm{bx}) + \sum_{n=1}^{\infty}[\mathrm{n^2 a_n b^2} + v\mathrm{a_n}]\mathrm{sech}^{\mathrm{n}}(\mathrm{bx}) + 3v\sum_{n,m,i=1}^{\infty} a_n a_m a_i \mathrm{sech}^{\mathrm{n+m+i}}(\mathrm{bx})\\ &\quad + v\sum_{\mathrm{n,m,i,j}=1}^{\infty}\mathrm{a_n a_m a_i a_j sech^{n+m+i+j}(bx)} = 0,\end{aligned} \tag{50}$$

where, in this last equation, we have used the relation

$$\tanh^2(bx) = 1 - \mathrm{sech}^2(\mathrm{bx}). \tag{51}$$

Since the expression (50) is an expansion of powers of sech(bx), each coefficient of different powers must vanish separately. This implies that for the coefficient of sech(bx), we have following relation:

$$b^2 = -v, \tag{52}$$

whereas that for the coefficients of $\mathrm{sech}^2(\mathrm{bx})$ and $\mathrm{sech}^3(\mathrm{bx})$, we deduce

$$a_2 = 2a_1^2, \tag{53}$$

and

$$a_3 = \frac{13}{8}a_1^3 - \frac{1}{2}a_1. \tag{54}$$

This procedure may be repeated in order to determine the successive coefficients.

There is another type of solution, which satisfies the boundary condition $\rho \to 0$ as $|x| \to \infty$. This case is more complicated and we are not able to propose an analytic expression for the solution. However, it is not difficult to obtain an approximate asymptotic solution for large $|x|$. In this limit, the Eq. (47) may be approximated by

$$\partial_x^2(\log\rho(x)) + v(\rho - 1)\rho(x) = 0. \tag{55}$$

As in the previous case, the solution exists for $v < 0$:

$$\rho = a\,\mathrm{sech}^2(\mathrm{bx}), \tag{56}$$

where the a and b are related by $b^2 = -\frac{av}{2}$. Thus if the solution exists, it should approach to zero exponentially.

IV. CONCLUSION

We have studied a generalized Jackiw-Pi model by introducing a nonstandard dynamic $\omega(\rho)$ in the original Jackiw-Pi Lagrangian. It was shown that this model supports Bogomolnyi equations and soliton solutions therein, which represent an important fact because the other soliton solutions found in generalized pure Chern-Simons models are not self-dual. In particular, we have shown that choosing a suitable function $\omega(\rho)$ and a sixth-order self-dual potential, the energy of the model is bounded below by a topological number. In addition, the resulting Bogomolnyi equations are those of the Chern-Simons Higgs theory.

The introduction of the function $\omega(\rho)$ also has consequences at the quantum level. Although the model may be viewed as the second quantized N-particle system interacting with a Chern-Simons gauge field, its dynamics are modified by $\omega(\rho)$, and therefore differs from the dynamics of the Jackiw-Pi model. In this case, the quantum field equation is

$$i\partial_t\Phi(x) = [\Phi(x), H], \tag{57}$$

and the gauge field should be subject to the constraint

$$B = \frac{e}{\kappa}\omega(\rho)\rho. \tag{58}$$

Finally, it is interesting to mention that the $(1 + 1)$-dimensional model, obtained by dimensional reduction of the generalized Jackiw-Pi, presents novel solitons solutions. One type of these solitons has a topological origin and we have been able to find the analytical expression, the other has nontopological origin and we have found its asymptotic behavior.

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