

Cosmic strings as emitters of extremely high energy neutrinosCecilia Lunardini^{1,2,*} and Eray Sabancilar^{1,†}¹*Physics Department, Arizona State University, Tempe, Arizona 85287, USA*²*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

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We study massive particle radiation from cosmic string kinks, and its observability in extremely high energy neutrinos. In particular, we consider the emission of moduli—weakly coupled scalar particles predicted in supersymmetric theories—from the kinks of cosmic string loops. Since kinks move at the speed of light on strings, moduli are emitted with large Lorentz factors, and eventually decay into many pions and neutrinos via hadronic cascades. The produced neutrino flux has energy $E \gtrsim 10^{11}$ GeV, and is affected by oscillations and absorption (resonant and nonresonant). It is observable at upcoming neutrino telescopes such as JEM-EUSO, and the radio telescopes LOFAR and SKA, for a range of values of the string tension, and of the mass and coupling constant of the moduli.

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I. INTRODUCTION

Theories with spontaneous symmetry breaking usually have topologically nontrivial vacuum configurations. Depending on the topology of the vacuum after the symmetry breaking, stable relics called topological defects—such as monopoles, strings or domain walls—could be formed in the early Universe [1]. Strings can form if the vacuum manifold is not simply connected. Although monopoles and domain walls are generally problematic for cosmology, cosmic strings are compatible with the observed Universe, provided that their tension is not too large (Sec. II; see, e.g., Refs. [2–6] for reviews). Cosmic strings are predicted in grand unified theories and superstring theory, and their existence can be revealed through their effects on the cosmic microwave background (CMB), large scale structure and 21 cm line observations, and—more directly—by detecting their radiation, such as gravitational waves and cosmic rays.

Since cosmic strings have grand unified theories or superstring scale energy densities in their core, they can be significant sources of ultrahigh energy ($E \gtrsim 10^{11}$ GeV) cosmic rays [7–13], either as isolated objects, or possibly in combination with other topological defects, like in monopole-string bound states [14–19]. Among the cosmic rays, neutrinos are especially interesting. Their weak coupling to matter makes them extremely penetrating, so they are the only form of radiation (together with gravitational waves) that can reach us from very early cosmological times, namely, all the way from redshift $z \sim 200$ (see Sec. IV B). Moreover, in the spectral region of interest, $E \gtrsim 10^{11}$ GeV, the neutrino sky is very quiet, since this region is beyond the range of neutrinos from even the most extreme hadron accelerators (gamma ray bursts, supernova remnants, active galactic nuclei, etc.). Therefore, even a

low statistics neutrino signal beyond this energy would constitute a clean indication of a fundamentally different mechanism at play, such as a top-down scenario involving strings or other topological defects. Experimentally, the technologies to detect ultrahigh energy neutrinos are mature: they look for radio or acoustic signals produced by the neutrinos as they propagate in air, water/ice, or rock. After the successful experiences of ANITA [20], FORTE [21], RICE [22] and NuMoon [23]—the latter using radio waves from the lunar regolith via the so-called Askaryan effect [24]—a new generation of experiments is being planned that can probe neutrinos from cosmic strings with unprecedented sensitivity. Of these, the space-based fluorescent light telescope JEM-EUSO [25] and radio telescopes LOFAR [26] and SKA [27] seem especially promising.

One of the distinguishing effects of cosmic strings as cosmic ray emitters is that they can produce bursts from localized features called cusps and kinks (Sec. II), where ultrarelativistic velocities are reached. The radiation from cusps and kinks is very efficient, whereas the emission from cusp/kink-free string segments is exponentially suppressed. This enhanced emission has been studied in connection with gravitational waves [28–30], and electromagnetic radiation [28,29,31,32] like gamma ray bursts [33–35] and radio transients [36–38], as well as neutrino bursts [10].

Among the several scenarios considered, there are a few that predict cosmic ray and neutrino fluxes at an observable level, e.g., Refs. [10–13]. One of these, Ref. [13], involves the decay of moduli—massive scalar fields that arise in supersymmetric and superstring theories—that can have various masses and couplings to matter. Moduli with coupling stronger than gravity are fairly natural [39–43] and relatively unconstrained due to their very short lifetimes [44], compared to gravitationally coupled ones [45–47]. By decaying into hadrons, the moduli eventually generate a neutrino flux. In Ref. [13] the emission of such moduli from string cusps and the corresponding neutrino flux were discussed.

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In this paper, we elaborate on the theme of moduli-mediated neutrino production from strings, and study modulus emission from kinks. We show that the emission from kinks is very efficient, and is the dominant energy loss mechanism for the cosmic string loops for a wide range of the parameters. We calculate the neutrino flux expected at Earth after a number of propagation effects, mainly absorption due to resonant (Z^0 resonance channel) and nonresonant neutrino-neutrino scattering. We find that the flux might be observable at near future surveys, JEM-EUSO, LOFAR and SKA, depending on the parameters.

The structure of the paper is as follows. After discussing some generalities on strings and kinks in Sec. II, the modulus emission from a cosmic string kink is calculated in Sec. III. In Sec. IV, we discuss the decay of moduli, the properties of the hadronic cascade initiated by their decay into gluons, and propagation of extremely high energy neutrinos in the Universe. In Sec. V, estimates are given for the kink event rate, the neutrino flux, and its detectability by the existing and future neutrino detectors. We also discuss the constraint from high energy gamma ray observations. Finally, in Sec. VI, we give our conclusions.

II. COSMIC STRINGS

Much of the phenomenology of a cosmic string depends on its *tension* (or mass per unit length), μ . It is often expressed in Planck units, as $G\mu$, where G is the Newton's constant. Several cosmological and astrophysical observations place upper limits on $G\mu$; we briefly review them here.

Since cosmic strings can create scalar, vector and tensor perturbations, they were initially considered as seeds for structure formation [48–50]. Hence, they contribute to the anisotropy of the CMB [51–63] and B -mode polarization of the CMB [64–67]. However, the current measurements of the CMB anisotropies at small angular scales by WMAP [68] and SPT [69], reveal that the cosmic string contribution to the total power is less than 1.75%, which translates into a constraint on the string tension, $G\mu \lesssim 1.7 \times 10^{-7}$ [63].

Although their contribution to the density perturbations is small, strings can still have effects on the early structure formation [70], early reionization due to early structure formation [71,72], formation of dark matter clumps [73], and might yield detectable signal in the 21 cm measurements [74–78]. Cosmic strings also produce gravitational waves [79] in a wide range of frequencies, both as localized bursts and as stochastic background, which can be detected by LIGO, eLISA and pulsar timing array projects [30,80–83]. The most stringent bound comes from the pulsar timing measurements, which put an upper bound on the long wavelength stochastic gravitational wave background, $h^2\Omega_{\text{GW}} \lesssim 5.6 \times 10^{-9}$ yielding the constraint $G\mu \lesssim 4 \times 10^{-9}$ [80]. However, this upper bound is obtained by ignoring the kinetic energy of the cosmic string loops, and by assuming that cosmic strings only decay by

emitting gravitational waves. Thus, the pulsar timing bound is expected to be somewhat relaxed by taking these effects into account.

Cosmic string loops can emit moduli efficiently in the early Universe when the length of the loop is of the order of the Compton wavelength of the emitted particle [45]. If moduli are gravitationally coupled to cosmic strings, very stringent cosmological constraints can be put on the string tension, $G\mu$, and the mass of the modulus, m [45–47]. On the other hand, if their coupling is stronger than gravitational strength, modulus radiation becomes the dominant energy loss mechanism for the loops, and the lifetime of moduli becomes a lot shorter. These relax the cosmological constraints on moduli significantly [44]. In this paper, we shall adopt the parameter space consistent with all the constraints mentioned above.

Cosmic strings are born as smooth objects, but afterwards they undergo crossings and self-crossings, which lead to truncations and successive reconnections. Every crossing produces a kink on the string after reconnection. The result of such processes are string loops with a few kinks [84]. Kinks are discontinuities in the vector tangent to the world sheet characterizing the string motion, and gravitational and particle radiation is very efficient at kinks yielding waveforms with power law behavior in the momenta of the emitted particles [28,30]. There are also transient features on loops called cusps, where a part of the string doubles on itself, that reach the speed of light momentarily. Cusps also produce radiation in bursts, with waveforms that have a similar power law behavior. On the other hand, radiation from cosmic string loops with no cusps or kinks is exponentially suppressed, leaving the kink and cusp radiation as an interesting window on the observable effects from strings. In the next section, we shall study massive particle radiation from cosmic string kinks.

III. MASSIVE PARTICLE RADIATION FROM KINKS

The free part of the action has the Nambu-Goto term for a string of tension μ , and the massive scalar field term for the modulus of mass m

$$S = -\mu \int d^2\sigma \sqrt{-\gamma} - \int d^4x \sqrt{-g} \left(\frac{1}{2} |\partial_\mu \phi|^2 + \frac{1}{2} m^2 \phi^2 \right), \quad (1)$$

where g is the determinant of the spacetime metric $g_{\mu\nu}$ and γ is the determinant of the induced metric on the world sheet, $X^\mu(\sigma, \tau)$, given by $\gamma_{ab} = g_{\mu\nu} X_{,a}^\mu X_{,b}^\nu$. The interaction Lagrangian for the modulus field and the string has the form [13]

$$\mathcal{L}_{\text{int}} = -\frac{\sqrt{4\pi}\alpha\mu}{m_p} \phi \int d^2\sigma \sqrt{-\gamma}, \quad (2)$$

where α is the modulus coupling constant, μ is the string tension and m_p is the Planck mass. Ignoring backreaction effects, the equation of motion for the world sheet in the flat background $g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, and in the conformal gauge, where $\sigma^0 = \tau$ and $\sigma^1 = \sigma$, is

$$\ddot{\mathbf{X}} - \mathbf{X}'' = 0, \quad (3)$$

with the gauge conditions $\dot{\mathbf{X}} \cdot \mathbf{X}' = 0$ and $\dot{\mathbf{X}}^2 + \mathbf{X}'^2 = 1$. The general solution can be obtained in terms of the right and left moving waves as

$$\mathbf{X}(\sigma, \tau) = \frac{1}{2}[\mathbf{X}_+(\sigma_+) + \mathbf{X}_-(\sigma_-)], \quad (4)$$

where $\sigma_{\pm} \equiv \sigma \pm \tau$, and the gauge conditions are now given by

$$\mathbf{X}'_{\pm}{}^2 = 1, \quad (5)$$

where the prime refers to the derivative with respect to the corresponding light cone coordinate σ_{\pm} .

The total power of particle radiation is

$$P = \sum_n P_n, \quad (6)$$

where the power spectrum P_n can be calculated by using [45,85]

$$\frac{dP_n}{d\Omega} = \frac{G\alpha^2}{2\pi} \omega_n k |T(\mathbf{k}, \omega_n)|^2, \quad (7)$$

where $G = m_p^{-2}$ is the Newton's constant, α is the modulus coupling constant, k is the momentum of the emitted particle, $\omega_n = 4\pi n/L = \sqrt{k^2 + m^2}$ is the energy, L is the loop length, and $T(\mathbf{k}, \omega_n)$ is the Fourier transform of the trace of the energy-momentum tensor of the cosmic string loop given by

$$\begin{aligned} T(\mathbf{k}, \omega_n) &= -\frac{4\mu}{L} \int d^4x \int d\sigma d\tau \sqrt{-\gamma} \delta^4(x^\mu - X^\mu(\sigma, \tau)) e^{ik \cdot x}. \end{aligned} \quad (8)$$

Using Eq. (4) and the light cone coordinates σ_{\pm} , Eq. (8) can be factorized as

$$\begin{aligned} T(\mathbf{k}, \omega_n) &= -\frac{\mu}{L} \int_{-L}^L d\sigma_+ \\ &\quad \times \int_{-L}^L d\sigma_- (1 + \mathbf{X}'_+ \cdot \mathbf{X}'_-) e^{i\frac{1}{2}(\Phi_+ - \Phi_-)}, \end{aligned} \quad (9)$$

where $\Phi_{\pm} \equiv \omega_n \sigma_{\pm} \mp \mathbf{k} \cdot \mathbf{X}_{\pm}$.

The integral in Eq. (9) is exponentially suppressed¹ for a smooth loop of cosmic string of length $L \gg 1/m$ [45]. However, the phase becomes stationary if the string has

cusps—saddle points on the world sheet where the derivative of the phase vanishes—or kinks—points where the vector tangent to the world sheet has a discontinuity. The cusp case has been studied for massive particle emission in Ref. [13]. At the cusp both Φ_{\pm} are saddle points; hence their derivatives with respect to the corresponding light cone coordinates vanish. On the other hand, for the case of a kink, either Φ_+ or Φ_- has a saddle point, and the other one has a discontinuity. In what follows we assume that Φ_+ has a saddle point and Φ_- has a discontinuity. Then, assuming the kink is at $\sigma_{\pm} = 0$, the world sheet can be expanded about $\sigma_{\pm} = 0$ as follows:

$$\mathbf{X}_+(\sigma_+) \approx \mathbf{X}_+^{(0)} + \mathbf{X}_+^{(1)} \sigma_+ + \frac{1}{2} \mathbf{X}_+^{(2)} \sigma_+^2 + \frac{1}{6} \mathbf{X}_+^{(3)} \sigma_+^3, \quad (10)$$

$$\mathbf{X}_-(\sigma_-) \approx \begin{cases} \sigma_- \hat{\mathbf{n}}_1 & \text{for } \sigma_- < 0 \\ \sigma_- \hat{\mathbf{n}}_2 & \text{for } \sigma_- > 0. \end{cases} \quad (11)$$

Using the gauge conditions (5), one can show that $\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2$ and $\mathbf{X}_+^{(1)}$ are unit vectors, $\mathbf{X}_+^{(1)} \cdot \mathbf{X}_+^{(2)} = 0$ and $\mathbf{X}_+^{(1)} \cdot \mathbf{X}_+^{(3)} = -|\mathbf{X}_+^{(2)}|^2$. The curvature of the string can be approximated as $|\mathbf{X}_+^{(2)}| \sim 2\pi/L$ if the string is not too wiggly. Using the expansions (10) and (11), and the gauge conditions (5), the phases Φ_+ and Φ_- can be obtained as

$$\Phi_+ \approx (\omega_n - k)\sigma_+ + \frac{2\pi^2}{3L^2} k \sigma_+^3, \quad (12)$$

$$\Phi_- \approx \begin{cases} \omega_n \sigma_- + k \sigma_- s_1 & \text{for } \sigma_- < 0 \\ \omega_n \sigma_- + k \sigma_- s_2 & \text{for } \sigma_- > 0, \end{cases} \quad (13)$$

where s_1, s_2 are constants of order 1, $|\mathbf{k}| \equiv k$ and we assumed that $\mathbf{k} // \mathbf{X}_+^{(1)}$. It can be shown that [30–32] when moduli are emitted at a small angle rather than being parallel to the direction of \mathbf{X}_+ at the saddle point, the expansion (10) still applies provided that the angle satisfies

$$\theta_k \lesssim (kL)^{-1/3}; \quad (14)$$

otherwise, the emissions lead to an exponentially suppressed power.

The term in the integrand of Eq. (9) can be found as

$$1 + \mathbf{X}'_+ \cdot \mathbf{X}'_- \approx c + \frac{c'}{L} \sigma_+ + O(\sigma_+^2), \quad (15)$$

where c and c' are constants of order 1, which we will take as 1 in what follows. Using Eq. (15), to the leading order we obtain

$$T(\mathbf{k}, \omega_n) \approx -\frac{\mu}{L} I_+ I_-, \quad (16)$$

where

$$I_{\pm} \equiv \int_{-L}^L d\sigma_{\pm} e^{i\frac{1}{2}\Phi_{\pm}}. \quad (17)$$

¹The $L \leq 1/m$ condition is satisfied in the very early Universe when the cosmic string loops can be quite small. Hence, moduli produced efficiently at earlier epochs can have cosmological effects, and are subject to constraints [44–47].

These integrals can be written explicitly by using Eqs. (12) and (13) as follows:

$$I_+ = \int_{-\infty}^{\infty} d\sigma_+ e^{\frac{i}{2}[(\omega_n - k)\sigma_+ + \frac{2\pi^2}{3L^2}k\sigma_+^3]}. \quad (18)$$

After a change of variables, one obtains [13]

$$I_+ = L \left(\frac{\omega_n}{k} - 1 \right)^{1/2} \int_{-\infty}^{\infty} dx e^{i\frac{2}{3}u(x + \frac{3}{5})}, \quad (19)$$

where $u \equiv Lk(\frac{\omega_n}{k} - 1)^{3/2}$. The imaginary part of the integral vanishes, and the real part is given in terms of the modified Bessel function of order 1/3:

$$I_+ = \frac{2}{\sqrt{3}} L \left(\frac{\omega_n}{k} - 1 \right)^{1/2} K_{1/3}(u). \quad (20)$$

The function $K_{1/3}(u)$ exponentially dies out at large u , and it can be approximated as a power law in the limit $u \ll 1$ as $K_{1/3}(u) \approx u^{-1/3}$. This limit corresponds to $k \gg m$, and in this regime, we can write $u \approx Lm^3/16k^2$. Then, we obtain

$$I_+ \sim L^{2/3} k^{-1/3}, \quad (21)$$

where this formula is valid when $u \lesssim 1$, i.e., $k \gtrsim k_c$, where

$$k_c \sim \frac{1}{4} m \sqrt{mL}. \quad (22)$$

For smaller values of k , I_+ is exponentially suppressed; thus we are only interested in the above regime for practical purposes.

Using Eqs. (13) and (17), the integral I_- can be similarly written as

$$I_- = \int_{-\infty}^0 d\sigma_- e^{\frac{i}{2}[\omega_n \sigma_- + k\sigma_- s_1]} + \int_0^{\infty} d\sigma_- e^{\frac{i}{2}[\omega_n \sigma_- + k\sigma_- s_2]}, \quad (23)$$

which results in

$$I_- \sim \sqrt{\psi} k^{-1}, \quad (24)$$

where the sharpness of a kink is defined as

$$\psi \equiv \frac{1}{2} (1 - \hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2). \quad (25)$$

Using Eqs. (21) and (24), we find the power spectrum from Eq. (7) as

$$\frac{d^2 P}{dk d\Omega} \sim \frac{\psi \alpha^2 G \mu^2}{8\pi^2} L^{1/3} k^{-2/3}, \quad k \gtrsim \frac{1}{4} m \sqrt{mL}. \quad (26)$$

Integrating over a solid angle gives a factor

$$\Omega_k \sim 2\pi \theta_k \sim 2\pi (kL)^{-1/3}, \quad (27)$$

where θ_k given by Eq. (14) is used. Then, the total power can be obtained as

$$P \sim \frac{\ln(\mu^{1/2}/m)^{3/2}}{4\pi} \psi \alpha^2 G \mu^2, \quad (28)$$

where we used the cutoff for the upper limit for the integral over momenta [12,86]

$$k_{\max} \sim \mu^{3/4} L^{1/2}, \quad (29)$$

and the lower limit $k_{\min} \sim k_c$ from Eq. (22). Note that for typical values of the modulus mass m and the string tension μ , the logarithmic factor is about 20. Then, we can simply write the total power as

$$P \sim \bar{\alpha}^2 G \mu^2, \quad (30)$$

where we define $\bar{\alpha} \equiv \sqrt{\psi} \alpha$. The number of particles emitted from a kink with momenta k in the interval $(k, k + dk)$ can be found from Eq. (26) as

$$dN(k) \sim \frac{dP(k)L}{k} \sim \bar{\alpha}^2 G \mu^2 L k^{-2} dk, \quad k \gtrsim \frac{1}{4} m \sqrt{mL}. \quad (31)$$

In addition to moduli, cosmic string loops also produce gravitational radiation with the power [2]

$$P_g \sim 50 G \mu^2. \quad (32)$$

It is convenient to define the power as

$$P = \Gamma G \mu^2, \quad (33)$$

where

$$\Gamma \approx \begin{cases} \bar{\alpha}^2 & \text{for } \bar{\alpha}^2 \gtrsim 50 \\ 50 & \text{for } \bar{\alpha}^2 \lesssim 50. \end{cases} \quad (34)$$

The dominant energy loss mechanism for loops determines the lifetime of a loop as

$$\tau_L \sim \frac{\mu L}{P} \sim \frac{L}{\Gamma G \mu}. \quad (35)$$

Then, the minimum loop size that survives at cosmic time t is

$$L_{\min} \sim \Gamma G \mu t. \quad (36)$$

In the next section, we shall discuss the decay of the moduli produced from cosmic string kinks into neutrinos via hadronic cascades, and the propagation of these neutrinos in the Universe.

IV. PARTICLE DECAY AND PROPAGATION

For simplicity, throughout this paper we will assume a matter dominated flat universe model, which lets us carry out the calculations analytically. We assume cosmological constant $\Lambda = 0$, and the total density parameter $\Omega_m + \Omega_r = 1$ has matter and radiation components. We use the following values of the cosmological parameters: age of the Universe $t_0 = 4.4 \times 10^{17}$ s, time of radiation-matter equality $t_{\text{eq}} = 2.4 \times 10^{12}$ s, $1 + z_{\text{eq}} = 3200$ [68]. The

scale factor in the radiation and matter eras are respectively given by $a_r \propto t^{1/2}$ and $a_m \propto t^{2/3}$. Using $a/a_0 = 1/(1+z)$, where $a_0 = 1$, the cosmic time can be written in terms of redshift as $t = t_0(1+z_{\text{eq}})^{1/2}(1+z)^{-2}$ in the radiation era, and $t = t_0(1+z)^{-3/2}$ in the matter era.

A. Modulus decay

The decay channel for moduli with the largest branching ratio is the decay into gauge bosons with the interaction of the form [40]

$$\mathcal{L} \sim \frac{\alpha}{m_p} \phi F_{\mu\nu} F^{\mu\nu}, \quad (37)$$

for a modulus field ϕ and a gauge field of field strength $F_{\mu\nu}$. For the gauge bosons in the standard model, the modulus lifetime is estimated as

$$\tau \sim 8 \times 10^{-6} m_4^{-3} \alpha_3^{-2} \text{ s}, \quad (38)$$

where $m_4 \equiv m/(10^4 \text{ GeV})$ and $\alpha_3 \equiv \alpha/10^3$. Since most of the moduli are emitted from kinks with momenta $k \sim m\sqrt{mL}/4$ [because of the decreasing power law given by Eq. (31)], their lifetime is boosted by a factor of $\gamma \sim \sqrt{mL}/4$. For the fiducial values of the parameters, the Lorentz factor of a modulus emitted at redshift z and surviving at present epoch is given by

$$\gamma(z) \sim \frac{\sqrt{m\Gamma G \mu t}}{4(1+z)} \sim 6.5 \times 10^{13} \frac{\Gamma^{1/2} \mu_{-17}^{1/2} m_4^{1/2}}{(1+z)^{7/4}}, \quad (39)$$

where $\mu_{-17} \equiv G\mu/(10^{-17})$, we have used the fact that loops of size L_{min} given by Eq. (36) yield the dominant contribution to the observable events; the factor of $(1+z)$ in the denominator takes into account the redshifting of the energy of the moduli emitted at epoch z . Thus, the ratio of the lifetime of a modulus emitted at redshift $z \lesssim z_{\text{eq}}$ and decaying at redshift z_d to the cosmic time at epoch z_d is

$$\frac{\tau(z, z_d)}{t(z_d)} \sim 10^{-3} \frac{\mu_{-17}^{1/2}}{\Gamma^{1/2} m_4^{5/2}} \frac{(1+z_d)^{5/2}}{(1+z)^{7/4}}. \quad (40)$$

Note that $\Gamma \geq 50$ from Eq. (34) and $z_d \leq z$. Hence, moduli will decay in the same epoch, $z_d \lesssim z$, as they are produced. Therefore, we assume that all the moduli decay before they reach Earth.

The most efficient channel for neutrino production from modulus decays is the decay into gauge bosons. In particular, gluons decaying into hadrons produce neutrinos with the largest multiplicity [10,13]. The interaction of a modulus with a gluon field is of the form (37), and the hadronic cascade from these gluons produces numerous pions of either sign, which eventually decay into neutrinos and antineutrinos. For both, we expect a flavor composition in the ratio $\nu_\mu : \nu_e : \nu_\tau = 2:1:0$, from the pion decay chain.

The number of neutrinos per unit energy can be found by using the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi method. Monte Carlo simulations for the hadronic decay

of a very massive particle show a power law behavior in energy as E^{-n} with $n = 1.9$ [87]. For simplicity, we approximate the index as $n \approx 2$. Then, the fragmentation function has the form [10,13]

$$\frac{dN_\nu}{dE} \equiv \xi(E, z) \approx 0.05 \frac{k}{(1+z)E^2}, \quad (41)$$

where $k/(1+z)$ and E are the modulus and neutrino energies at the present epoch respectively. Here $E_{\text{min}} < E < E_{\text{max}}$ [13], where

$$E_{\text{min}} \sim \epsilon\gamma(z) \sim 6.5 \times 10^{13} \frac{\Gamma^{1/2} \mu_{-17}^{1/2} m_4^{1/2} \epsilon_{\text{GeV}}}{(1+z)^{7/4}} \text{ GeV}, \quad (42)$$

and $E_{\text{max}} \sim 0.1k$. We take $\epsilon_{\text{GeV}} \equiv \epsilon/(1 \text{ GeV}) \sim 1$ [13]. Since the neutrino spectrum has the form E^{-2} , most of the neutrinos will have the energy $E \sim E_{\text{min}}$. This introduces a lower bound on the redshift, below which no neutrinos are produced with a given energy $E \lesssim E_{\text{min}}$. For our estimates, we are interested in energies $E \gtrsim 10^{11} \text{ GeV}$ corresponding to the minimum redshift in the matter era

$$z_{\text{min}}(E) \sim 40\Gamma^{2/7} \mu_{-17}^{2/7} m_4^{2/7} E_{11}^{-4/7}, \quad (43)$$

where $E_{11} \equiv E/(10^{11} \text{ GeV})$. Since the maximum redshift from which neutrinos can propagate to us is set by the neutrino horizon $z_\nu \sim 200$ (see Sec. IV B), requiring $z_{\text{min}} \lesssim z_\nu$, we have the constraint on the parameters

$$G\mu \lesssim 2.8 \times 10^{-15} \Gamma^{-2} m_4^{-1} E_{11}^2. \quad (44)$$

B. Neutrino propagation

The neutrino flux at Earth is affected by a number of propagation effects: the redshift of energy, flavor oscillation, quantum decoherence and absorption. The redshift of energy will be included as we carry out the flux calculation in the next sections; the other effects, instead, warrant a separate discussion, which is the subject of this section.

The oscillations of very high energy neutrinos have been discussed in detail (see, e.g., Ref. [88]). Oscillations in vacuum are a good approximation, as the refraction potentials due to the intergalactic gas and to the cosmological relic neutrino background (which is assumed to be CP -symmetric here) are negligible [89]. For the large propagation distances we consider, the flavor conversion probabilities are energy independent, as the energy-dependent oscillatory terms average out [90]. For our predicted initial flavor composition, $\nu_\mu : \nu_e : \nu_\tau = 2:1:0$ (Sec. IV A), the effect of oscillations is to equilibrate the flavors [88]; therefore the composition at Earth should be $\nu_\mu : \nu_e : \nu_\tau = 1:1:1$, for both neutrinos and antineutrinos.

A neutrino oscillates as long as its wave packet remains a coherent superposition of mass eigenstates. Depending on the size of the produced wave packet, decoherence can occur as the neutrino propagates, due to the different

propagation velocities of the mass states. Dedicated analyses [90,91] have shown that neutrinos of the energies of interest here remain coherent over cosmological distances; therefore we do not consider decoherence effects.

Absorption effects are largely dominated by scattering on the relic cosmological background [92,93], with negligible contribution from other background species. In first approximation, absorption can be modeled as a simple disappearance of the neutrino flux; secondary neutrinos generated by scattering are degraded in energy and therefore they are negligible compared to primary flux.

The survival probability for the primary neutrinos, of observed energy E and production redshift z , is defined as [92–95]:

$$P(E, z) = e^{-\tau_\nu(E, z)}, \quad (45)$$

where the optical depth for the relic neutrino background is

$$\tau_\nu(E, z) = \int_{t(z)}^{t_0} dt' \sigma_{\nu\nu}(E, \tilde{z}) n_\nu(\tilde{z}), \quad (46)$$

and

$$dt' = -\frac{3}{2} \frac{d\tilde{z}}{(1 + \tilde{z})^{5/2}}, \quad (47)$$

in the matter era. Here $n_\nu(z) = 56(1 + z)^3 \text{ cm}^{-3}$ is the number density of relic neutrinos in each of the six species (neutrinos and antineutrinos of each flavor), and $\sigma_{\nu\nu}(E, z)$ is the neutrino-neutrino cross section, evaluated at the production energy $E' = E(1 + z)$, and summed over all the neutrino species in the background. For the energies of interest here, and at the leading order, this cross section is the same for neutrinos and antineutrinos, and is practically flavor independent:

$$\begin{aligned} \sigma_{\nu\nu} &= \sigma_e \approx \sigma_\mu \approx \sigma_\tau \\ \sigma_\alpha &= \sum_{\beta=e,\mu,\tau} [\sigma(\nu_\alpha + \nu_\beta \rightarrow \text{any}) + \sigma(\nu_\alpha + \bar{\nu}_\beta \rightarrow \text{any})]. \end{aligned} \quad (48)$$

In the limit of massless neutrinos, $m_\nu \approx 0$, the Z^0 -resonance effects can be ignored and the maximum cross section is attained at $E \gtrsim 10^{11} \text{ GeV}$ [13]:

$$\sigma_{\text{max}} \approx \frac{N}{\pi} G_F^2 m_W^2, \quad (49)$$

where $N \sim 10\text{--}15$, $G_F = 1.17 \times 10^{-5} \text{ GeV}^2$, and $m_W \approx 80.39 \text{ GeV}$. Using Eq. (49) in Eq. (46), and requiring $\tau_\nu = 1$ for absorption, the neutrino horizon—the maximum redshift from which the neutrinos with energy E can propagate to us—is given by [92,93]

$$z_\nu \sim 200, \quad (50)$$

for energies $E \gtrsim 10^{11} \text{ GeV}$. In this regime, $P(E, z)$ can be approximated as a step function

$$P(E, z) \approx 1 - \Theta(z - z_\nu), \quad (51)$$

which becomes handy when estimating the neutrino flux analytically.

If the Z^0 resonance is realized in the annihilation channel $\nu_\alpha + \bar{\nu}_\alpha \rightarrow \text{any}$, at a redshift z_{res} along the neutrino path, a pronounced dip in the neutrino spectrum is expected at the resonance energy due to the strong enhancement of the cross section [90,93–97]:

$$\sigma_{\nu\nu}^{\text{res}} \propto \frac{s}{(s/M_Z^2 - 1)^2 + \Gamma_Z^2/M_Z^2}, \quad (52)$$

where $s \approx 2m_\nu E(1 + z_{\text{res}})$ if the background neutrinos are not relativistic. The effect of the resonance is especially transparent in this case [93,94]; we discuss it here in its essentials.

Considering their momentum, $p_\nu(z) \approx 6.104 \times 10^{-4}(1 + z) \text{ eV}$, the cosmological neutrinos are nonrelativistic today for masses exceeding $\sim 10^{-3} \text{ eV}$, and throughout the interval of redshift of interest, $z \lesssim z_\nu$, if $m_j \gg p_\nu(z_\nu) \approx 0.1 \text{ eV}$. From the data of oscillation experiments (see, e.g., Ref. [98]) it is known that, above this value, the neutrino mass spectrum becomes degenerate: $m_1 \approx m_2 \approx m_3$. Therefore, we can reason in terms of a single neutrino mass value, m_ν , and take $m_\nu = 0.3 \text{ eV}$ as reference. The degenerate case is optimal for the observability of the resonance effect, because the dip in the spectrum occurs at the same energy for all neutrinos and has a sharp shape. Furthermore, it is located in the region of the spectrum, $\sim 10^{11}\text{--}10^{13} \text{ GeV}$, where experiments have good sensitivity [90] (see Fig. 1).

For a neutrino of energy E at Earth, the Z^0 resonance is realized at redshift z_{res} if

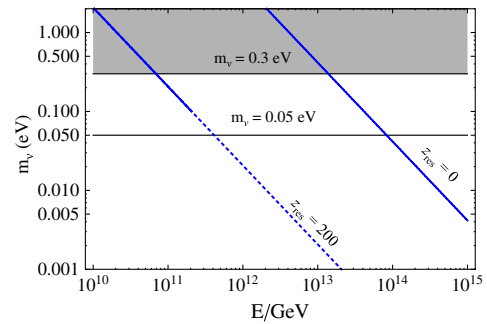


FIG. 1 (color online). Graphical representation of the interval of observed energy, Eq. (54), where suppression of the flux due to resonant $\nu - \bar{\nu}$ annihilation is expected (region between the diagonal lines). The dashed indicates the region where Eq. (54) is only indicative, due to thermal effects influencing the resonance. The horizontal shaded area refers to the interval of neutrino masses where the neutrino mass spectrum is strongly degenerate. We also mark the value $m_\nu \approx 0.05 \text{ eV}$, which is the highest mass expected for a nondegenerate (hierarchical) mass spectrum.

$$E = \frac{M_Z^2}{2m_\nu(1+z_{\text{res}})} \approx 1.4 \times 10^{13} \text{ GeV} \left(\frac{0.3 \text{ eV}}{m_\nu} \right) (1+z_{\text{res}})^{-1}, \quad (53)$$

with $M_Z \approx 91.19$ GeV the mass of the Z^0 boson. It follows that the flux of neutrinos of observed energy

$$E = (6.9 \times 10^{10} - 1.4 \times 10^{13}) \text{ GeV} \left(\frac{0.3 \text{ eV}}{m_\nu} \right), \quad (54)$$

is affected by the resonance between $z = z_\nu$ and the present epoch (see Fig. 1), and therefore should be strongly suppressed compared to the flux at energies outside this interval, where the smaller, nonresonant, absorption cross section is at play.

Following the detailed discussion in Ref. [95], we calculated $P(z, E)$ and used it to obtain the neutrino flux expected at Earth from all sources at all redshifts. This flux is calculated by convolving the flux per unit of production redshift with the probability $P(z, E)$; it exhibits the characteristic suppression dip in the interval given in Eq. (54), as expected (see Sec. V C).

The absorption pattern is more complicated if the neutrino mass spectrum is not degenerate, i.e., $m_1 \lesssim m_2 \ll m_3 \approx 0.05$ eV (or, $m_3 \ll m_1 \lesssim m_2 \approx 0.05$ eV). For this configuration the probability $P(z, E)$ has three distinct dips of resonant suppressions at separate resonance energies [90,95], corresponding to the three masses. These dips are broadened, in energy, by the integration over the production redshift, and, most importantly, by thermal effects, which are important for this range of neutrino masses [90,95,97]. We postpone a discussion of these effects to a forthcoming publication [99].

V. NEUTRINO FLUX AND DETECTION

As kinks move along a loop of cosmic string, they emit particles in a fanlike pattern, and scan a ribbon of solid angle $\Omega \sim 2\pi\theta_k$ [see Eq. (14)]. Thus, one can analogously visualize the radiation from a kink as a source of light emitted from a lighthouse passing by. An observer who happens to be within the beam direction sees particles as a burst event provided that the flux is detectable. In this section, we make order of magnitude estimates for the event rate for bursts and neutrino flux, and compare it with the existing and future neutrino experiments.

A. Loop distribution

The neutrino flux from cosmic string loops depends on the size of the loops chopped off from the string network and their number density. The population of cosmic string loops has been studied both analytically [100–104] and in simulations, following two main approaches, the field theoretical one (Abelian Higgs simulations on a lattice) [105,106] and the Nambu-Goto formalism [107–114].

Results vary vastly depending on the approach, with the Nambu-Goto type simulations giving a much richer network of loops. Even within a specific model, some uncertainties exist [110,114]—for example in the distribution of the loops in size—reflecting the limited computational means. While waiting for a clearer picture to emerge, the phenomenology of a loop network can only be worked out in a model-dependent way, with the understanding that updates will be necessary in the future.

Here we adopt the results suggested from the latest Nambu-Goto simulation that has the largest dynamical range to date for the evolution of the cosmic string network [114], where it has been argued that the large loops form with size βt , where $\beta \sim 0.1$, and t is the cosmic time at which the loop is chopped off the network of long cosmic strings. The density of long strings is $\rho \sim \zeta\mu/t^2$, with $\zeta \sim 16$. Using this framework, we can estimate the number density of loops of length $(L, L + dL)$ that are formed in the radiation era and still survive in the matter era as

$$n(L, t)dL \sim p^{-1}\zeta(\beta t_{\text{eq}})^{1/2}t^{-2}L^{-5/2}dL, \quad (55)$$

where $\Gamma G\mu t \leq L \leq \beta t_{\text{eq}}$ and p is the reconnection probability. There are also loops formed in the matter era; however, we have verified that their number density is negligible compared to the loops surviving from the radiation era given by Eq. (55). The dependence of loop density on reconnection probability has not been resolved yet; however, it is expected that the loop density increases for decreasing reconnection probability as discussed in Refs. [30,115]. For ordinary cosmic strings, $p = 1$, and it has been estimated as $10^{-3} \leq p \leq 1$ for cosmic F and D strings [116]. Note that the most numerous loops have size of order $L_{\text{min}} \sim \Gamma G\mu t$. As we shall see in Sec. V, those will give the most dominant contribution to the observable effects, such as the diffuse neutrino flux.

When a loop of size βt_{eq} is formed, it will decay by the time [see Eq. (35)]

$$t \sim \frac{\beta t_{\text{eq}}}{\Gamma G\mu}. \quad (56)$$

This loop can survive until epoch $z < z_{\text{eq}}$ provided that

$$1 + z \gtrsim 0.07\Gamma^{2/3}\mu_{-8}^{2/3}, \quad (57)$$

where $\mu_{-8} \equiv G\mu/10^{-8}$, $\Gamma \gtrsim 50$ is given by Eq. (34), and we used $t = t_0(1+z)^{-3/2}$. Hence, we can conclude that even for the maximum $G\mu$ allowed by the current bounds, the loops can survive all the way to very recent epochs from which we can get observable effects unless Γ is too large.

B. Burst rate

The number of kink bursts per unit time can be estimated as [10,13]

$$d\dot{N} = \frac{d\Omega}{4\pi} \frac{n(L, t)dL}{L(1+z)/2} dV(z), \quad (58)$$

where $n(L, t)dL/(L/2)$ is the frequency of a kink event per physical volume per unit loop length, $L/2$ is the oscillation period of a loop of length L , $d\Omega/4\pi \sim \theta_k/2$ is the probability that an observer lies within the solid angle of kink radiation, and $dV(z)$ is the physical volume in the interval $(z, z + dz)$ in the matter era, given by

$$dV(z) = 54\pi t_0^3 (1+z)^{-11/2} [(1+z)^{1/2} - 1]^2 dz. \quad (59)$$

To find the total burst rate, we integrate Eq. (58) over L and z . Integral over L is dominated by its lower limit L_{\min} given by Eq. (36), and the integral over redshift is dominated by its upper limit $z_\nu \sim 200$. Numerically, we obtain the total event rate as

$$\dot{N} \sim 1.6 \times 10^{18} p^{-1} \Gamma^{-3} \mu_{-17}^{-3} m_4^{-1/2} \text{ yr}^{-1}. \quad (60)$$

Remember that $\Gamma \gtrsim 50$ is given by Eq. (34). Since experiments run for a few years, the event rate should be at least ~ 1 per year to get observable events. Requiring $\dot{N} \gtrsim 1 \text{ yr}^{-1}$ yields the constraint on the parameters

$$G\mu \lesssim 1.2 \times 10^{-11} p^{-1/3} \Gamma^{-1} m_4^{-1/6} \dot{N}_{\text{yr}}^{-1/3}, \quad (61)$$

where $\dot{N}_{\text{yr}} \equiv \dot{N}/(1 \text{ yr})$.

C. Neutrino flux

The diffuse neutrino flux is obtained using the flux from a single kink on a loop, and summing over all the loops in a volume constrained by the neutrino horizon $z_\nu \sim 200$. It can be estimated by

$$J_\nu(E) = \int \frac{d\dot{N}}{\Omega_k r^2} \xi(E, z) dN(k) P(E, z), \quad (62)$$

where

$$r(z) = 3t_0(1+z)^{-1/2} [(1+z)^{1/2} - 1] \quad (63)$$

is the physical distance to the source at redshift z in the matter era, $d\dot{N}$ is the kink event rate defined by Eq. (58) and Ω_k is the solid angle into which moduli are emitted given by Eq. (27). Here $\xi(E, z)$ is the fragmentation function given by Eq. (41), $dN(k)$ is the number of moduli emitted from a kink with momenta k in the interval $(k, k + dk)$ given by Eq. (31), and $P(E, z)$ is the survival probability of neutrinos defined in Eq. (45). Putting everything together, we obtain

$$E^2 J_\nu(E) \sim 0.05 p^{-1} \beta^{1/2} \zeta\left(\frac{t_{\text{eq}}}{t_0}\right)^{1/2} \bar{\alpha}^2 (G\mu)^2 m_p^2 t_0^{-1/2} \times \int_{z_{\min}}^{\infty} dz \frac{P(E, z)}{(1+z)^{7/2}} \int_{L_{\min}} \frac{dL}{L^{5/2}} \int_{k_{\min}}^{k_{\max}} \frac{dk}{k}. \quad (64)$$

Note that the integral over k gives a logarithmic factor $\ln(k_{\max}/k_{\min}) \sim \ln(\mu^{1/2}/m)^{3/2} \sim 20$ from Eqs. (22) and (29),

and the integral over L is dominated by its lower bound L_{\min} given by Eq. (36). The integral over z can be done numerically. However, it is useful to see the limiting case $m_\nu \approx 0$, where we can ignore the Z -resonance effects, and carry out the integral over redshift analytically. Using the approximate form of $P(E, z)$ given by Eq. (51) in Eq. (64), the neutrino flux can be calculated as

$$E^2 J_\nu(E) \sim 2 \times 10^{-4} \frac{\mu_{-17}^{1/2} \bar{\alpha}^2}{p \Gamma^{3/2}} [z_{\min}^{-1/4} - z_\nu^{-1/4}] \frac{\text{GeV}}{\text{cm}^2 \text{ s sr}}. \quad (65)$$

Using z_{\min} from Eq. (43), the predicted diffuse neutrino flux in the $m_\nu \approx 0$ limit is

$$E^2 J_\nu(E) \sim 8 \times 10^{-5} \frac{\mu_{-17}^{3/7} \bar{\alpha}^2 E_{11}^{1/7}}{p \Gamma^{11/7}} \times [1 - (z_{\min}/z_\nu)^{1/4}] \frac{\text{GeV}}{\text{cm}^2 \text{ s sr}}. \quad (66)$$

Taking into account the neutrino mass in the survival probability $P(E, z)$, and fixing $m_\nu = 0.3 \text{ eV}$ and reconnection probability $p = 1$, we evaluate Eq. (64) numerically. In Fig. 2 we show the predicted flux for a few different values of the parameters $G\mu$ and $\bar{\alpha}$, together with the detectability limits of the current and future neutrino experiments.

D. Diffuse gamma ray background constraint

As moduli decay via hadronic cascades, the pions from this process also decay into photons and electrons. These high energy photons and electrons interact with the CMB photons and extra galactic background light, producing an electromagnetic cascade, whose energy density is constrained by the measurements of diffuse gamma ray background [117]. The strongest upper bound on the cascade energy density comes from the highest energy end of the observed spectrum. The most recent data from Fermi-LAT observations reach $E \sim 100 \text{ GeV}$ [118]. The cascade photons with energy $E \gtrsim E_{\text{abs}}$ will be strongly absorbed due to interaction with the CMB photons, where E_{abs} due to pair production can be estimated as

$$E_{\text{abs}}(z) \sim \frac{m_e^2}{\epsilon_{\text{CMB}}(1+z)} \sim 5.6 \times 10^5 \frac{1}{1+z} \text{ GeV}, \quad (67)$$

where $\epsilon_{\text{CMB}} = 2.35 \times 10^{-4} \text{ eV}$ and $m_e = 0.511 \text{ MeV}$. This implies that a cascade photon of energy above $E_{\text{cas}} \sim 100(1+z) \text{ GeV}$, is efficiently absorbed at redshift

$$1 + z_{\text{cas}} \sim \left(\frac{E_{\text{abs}}(0)}{E_{\text{cas}}(0)} \right)^{1/2} \sim 70. \quad (68)$$

The electromagnetic energy density of the cascade from cosmic string kinks is [10,13]

$$\omega_{\text{cas}} = \frac{1}{2} f_\pi \int \frac{dt}{(1+z)^4} n(L, t) dL dP(k), \quad (69)$$

where f_π is the fraction of energy transferred to pions in the hadronic cascade initiated by a modulus decay, $1/2$ is the fraction of energy transferred to electron-positrons and photons from the pion decays, and $dP(k)$ is the power emitted from a kink, given by Eq. (26). Integrating over L , k and z [similar to the diffuse flux in Eq. (64)], and integrating over z up to $z_{\text{cas}} \sim 70$, we have

$$\omega_{\text{cas}} \sim 1.2 \times 10^{-5} \frac{\mu_{-17}^{1/2} \bar{\alpha}^2}{\rho \Gamma^{3/2}} \frac{\text{eV}}{\text{cm}^3}, \quad (70)$$

where $\Gamma \gtrsim 50$ is given by Eq. (34).

The maximum value of ω_{cas} allowed by Fermi-LAT diffuse gamma ray data is $\omega_{\text{cas}}^{\text{max}} \sim 5.8 \times 10^{-7} \text{ eV/cm}^3$ [119]. Therefore, $\omega_{\text{cas}} \leq \omega_{\text{cas}}^{\text{max}}$ is satisfied for

$$G\mu \lesssim 3 \times 10^{-15} \bar{\alpha}^{-4} \left(\frac{\Gamma}{50}\right)^3. \quad (71)$$

Note that $\omega_{\text{cas}} \gtrsim \omega_{\text{cas}}^{\text{max}}$ is not strictly ruled out. This bound only constrains the observed highest energy diffuse gamma ray photons at $E_\gamma \sim 100 \text{ GeV}$ that are originated at very large redshifts $z \lesssim 70$. The constraints on the energy density of cascade photons produced at redshifts larger than $z_{\text{cas}} \sim 70$ are much weaker since they are more efficiently absorbed. Besides, the radiation from kinks is not homogenous, but confined to a narrow ribbon of width $2\pi\theta_k \ll 1$. Unless the cosmic magnetic fields are strong enough, the beamed electromagnetic radiation from cosmic string kinks might not diffuse efficiently; hence, the constraint might be relaxed significantly. Nevertheless, the examples given in Fig. 2 respect the cascade upper bound given by Eq. (71).

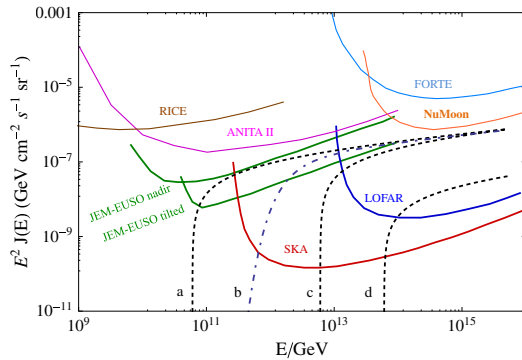


FIG. 2 (color online). Examples of neutrino flux from cosmic string kinks, via moduli decay. We took $G\mu = 10^{-17}$. The dashed curves, (a), (c) and (d) refer to $m_\nu = 0$ and $\bar{\alpha} = 1, 10^2, 10^3$ respectively. The dot-dashed line, (b), is the same as (a) but for $m_\nu = 0.3 \text{ eV}$, with resonant absorption effects included. Cases (c) and (d) are not affected by the Z^0 resonance for this value of the neutrino mass. The figure also shows existing limits from ANITA, FORTE, NuMoon and RICE [20–23], and expected sensitivities of the future detectors JEM-EUSO, LOFAR and SKA [25–27].

E. Neutrino bursts from individual kinks

Before closing, we comment briefly on the possibility to identify the neutrino emission of individual kinks, i.e., bursts, rather than the diffuse flux. The signature of a burst would be two or more time-coincident events in a detector.² Time coincidence at arrival is expected for neutrinos from a single burst, because the emission occurs with a very short, practically vanishing, time scale of order [33,37]

$$\Delta \sim \frac{1}{k_{\text{min}} \gamma^2} \sim 10^{-42} \text{ s}, \quad (72)$$

for the fiducial values of the parameters, where k_{min} and γ are respectively given by Eqs. (22) and (39). The time lag due to the spread in neutrino velocities is negligible for the Lorentz factors of interest here, $\gamma \gtrsim 10^{11}$ [see Eq. (42)].

The fluence of neutrinos with energy above E , from a kink on a cosmic string loop, can be estimated as [10,13]

$$F_\nu(>E) = \int \frac{\xi(E, z) E dN(k) P(E, z)}{\Omega_k r^2}. \quad (73)$$

Using Eqs. (27), (31), (41), (45), and (63) integrating over momenta yields

$$F_\nu(>E) \sim 4.3 \times 10^{-20} \frac{\bar{\alpha}^2 \mu_{-16}^{15/4} \Gamma^{3/2} E_{11}^{-1}}{[\sqrt{1+z}-1]^2 (1+z)^{9/4}} \text{ cm}^{-2}, \quad (74)$$

where we take the loop length to be L_{min} given by Eq. (36) since these loops are the most numerous, as was discussed in Sec. VC; hence it is more likely to get a burst from such loops.

We can now estimate how many neutrinos might be detected at a detector of effective area

$$A_{\text{det}} \sim \sigma_{\nu N} \frac{M}{m_N} \quad (75)$$

where the nucleon mass, $m_N \sim 1 \text{ GeV}$, M is the target mass, and $\sigma_{\nu N} \sim 10^{-31} \text{ cm}^2$ is the neutrino-nucleon cross section. The reference cross section is from recent calculations at $E \sim 10^{12} \text{ GeV}$ (see e.g., Ref. [120–122]) and is a reasonable approximation for higher energies as well, due to the slow rise of $\sigma_{\nu N}$ at these energies (less than $\propto E^{1/2}$). A typical value of the target mass for neutrino detection is $M \sim 10^{21} \text{ g}$, which applies to JEM-EUSO in its nadir mode at energy $E \sim 10^{11} \text{ GeV}$ [25].

We model a best case scenario by choosing the closest distance to the source, $z \sim z_{\text{min}}$ (neutrinos can only come from $z_{\text{min}} < z < z_\nu$), and the regime $\bar{\alpha}^2 \gtrsim 50$ (where $\Gamma \sim \bar{\alpha}^2$), for which the number of emitted neutrinos is larger.

²Due to the opening angle at emission, the coincident events would most likely appear in different regions of the detector field of view; this will allow us to distinguish them from one another.

The number of events in a detector due to a burst can be estimated as

$$\mathcal{N}_\nu \equiv F_\nu(>E)A_{\text{det}} \sim 1.7\bar{\alpha}_3^{3.14}\mu_{-17}^{2.82}E_{14}^{0.86}A_{16}, \quad (76)$$

where $A_{16} \equiv A_{\text{det}}/(10^{16} \text{ cm}^2)$. The fact that $\mathcal{N}_\nu \geq 1$ means that, for our parameters of reference, the identification of a burst by time coincidence of multiple events is possible in principle, although in practice instrumental backgrounds might be an obstacle.

Requiring $\mathcal{N}_\nu \geq 1$ implies a minimum value of $G\mu$:

$$G\mu \geq 7.8 \times 10^{-18} \bar{\alpha}_3^{-1.12} E_{14}^{-0.31} A_{16}^{-0.36}. \quad (77)$$

This has to be combined with the maximum value imposed by the condition that $z_{\text{min}} \lesssim z_\nu \sim 200$ [see Eq. (44)]: $G\mu \lesssim 2.8 \times 10^{-15} \bar{\alpha}_3^{-4} m_4^{-1} E_{14}^2$. The range of $G\mu$ is further restricted by imposing that burst detections are frequent enough, say one per year at least. Using Eq. (61), this gives $G\mu \lesssim 1.2 \times 10^{-17} \bar{\alpha}_3^{-2} m_4^{-1/6} \dot{N}_{\text{yr}}^{-1/3}$. Therefore, the burst detectability is possible for a somewhat narrow range of $G\mu$.

Note also that a detector's capability to see bursts depends on its energy sensitivity: for most of the parameter space, the neutrino emission is concentrated above the JEM-EUSO peak sensitivity, $E \sim 10^{11} \text{ GeV}$; therefore detection at JEM-EUSO might be hard. However, LOFAR and SKA are expected to surpass the JEM-EUSO sensitivity at higher energies (see Fig. 2), and therefore are more promising burst detectors. The detection of a burst would be an important signature of cosmic string kinks or cusps (see Refs. [10,13] for bursts from cusps), complementary to a possible diffuse flux observation. It would also help breaking the degeneracy between the two parameters, $\bar{\alpha}$ and $G\mu$, since the detected number of neutrinos from a burst, (76), and the diffuse flux, (66), have different dependences on the parameters. Besides, even if only single neutrinos are detected, the rate of events, (60), can be used to help distinguish cosmic strings as the source, and break the degeneracy of the parameters.

VI. SUMMARY AND DISCUSSION

Cosmic strings loops form as a result of reconnection of long strings, and self-intersection of large loops. Kinks arise naturally as a result of these processes. We studied how kinks can radiate moduli, particles that arise in the supersymmetric models of particle physics, and that can have various masses and couplings to matter. The decay of moduli into pions via hadronic cascades produces a flux of neutrinos, which can be observable depending on the parameters.

Specifically, we considered the string tension $G\mu$, the modulus coupling constant α and mass m as free parameters, and showed that neutrinos with energies $E \gtrsim 10^{11} \text{ GeV}$ can be easily produced by cosmic string loops via this mechanism, with flux

$$E^2 J_\nu(E) \sim 1.7 \times 10^{-7} \frac{\mu_{-17}^{3/7} \bar{\alpha}_3^2 E_{11}^{1/7}}{p(\Gamma/50)^{11/7}} \frac{\text{GeV}}{\text{cm}^2 \text{ s sr}}. \quad (78)$$

The hadronic cascade stops producing pions at the modulus rest frame energy of order $\epsilon \sim 1 \text{ GeV}$. In the rest frame of the loop, this energy is boosted by the Lorentz factor γ , so that the minimum observed energy of the neutrinos is

$$E_{\text{min}}(z) \sim 4.6 \times 10^{14} \frac{(\Gamma/50)^{1/2} \mu_{-17}^{1/2} m_4^{1/2} \epsilon_{\text{GeV}}}{(1+z)^{7/4}} \text{ GeV}. \quad (79)$$

The neutrino flux is shown in Fig. 2 for representative sets of parameters; the termination of the flux at E_{min} appears clearly. The figure also gives the flux sensitivity of various experiments, showing that the predicted flux is within reach for the next generation of neutrino detectors such as JEM-EUSO, LOFAR and SKA.

A distinctive feature of radiation from cosmic string kinks is that particles are emitted in a fanlike pattern, confined into a narrow ribbon; hence bursts from individual kinks can possibly be identified by timing and directional coincidence. In Eq. (76), we estimated the number of neutrinos emitted by a kink, and the corresponding number of events in a detector of a given effective area. We found that, for the fiducial values of the parameters used in our analysis, multiple neutrinos can be seen in the field of view of the detector.

If ultrahigh energy neutrinos are observed at future experiments, what would be possible to learn? Top-down mechanisms would offer natural explanations, and, among those, cosmic strings would be a favored candidate. Even in the framework of cosmic strings, however, data analyses will necessarily be model dependent, and various models would have to be considered. Our scenario involving moduli is a possibility among many, and other intermediate states leading to neutrino production are possible, e.g., modulus emission from string cusps [13] and heavy scalar particle emission from cusps of superconducting strings [10]. Another possible generation mechanism of extremely high energy neutrinos could be the Kaluza-Klein mode emission from cusps and kinks of cosmic F and D strings. The emission of Kaluza-Klein modes of gravitons from cusps was studied in Refs. [123,124], and various cosmological constraints have been put on the cosmic superstring tension. Depending on the parameters, observable neutrino fluxes might be produced by this mechanism as well.

A discrimination among different models will require the combination of complementary data, probably the detection of gravitational wave/electromagnetic counterparts of neutrino signals [125,126]. The identification of pointlike sources of extremely energetic neutrinos (bursts) would favor cosmic string kinks or cusps as sources, a hypothesis that would be substantiated further by the observation of accompanying gravitational wave and/or gamma ray bursts. To distinguish between kinks and cusps

could be possible since the event rate is larger for kinks for the given values of the parameters.

In addition to a possible discovery of topological defects, detecting a flux of ultrahigh energy neutrinos might reveal new pieces of the still incomplete puzzle of neutrino physics. Most interestingly, if the data show a Z^0 resonance dip, we might gather information on the neutrino mass and have other, perhaps more direct, evidence of the existence of the cosmological relic neutrinos. The information on the neutrino mass might be especially important if at least one neutrino is light enough to evade a direct mass measurement in the laboratory.

It is important to consider, however, that the extraction of any information from data would be complicated by many theoretical uncertainties. Let us comment on the uncertainties and simplifying assumptions of our calculation. First of all, we worked in a flat matter dominated universe, and ignored the recent accelerated expansion period of the Universe, whose effect can be at most about a factor of a few in our final estimates. We adopted the loop distribution suggested by the Nambu-Goto simulations [114] to arrive at our predicted neutrino flux. However, we would like to emphasize that the number density and the size distribution of loops are still subjects of debate, with large differences among different approaches and some unresolved questions within a given approach. As the situation becomes clearer, our work will serve as a template for updated studies. We also approximated the

neutrino fragmentation function for the moduli decays as $dN/dE \propto E^{-n}$, and used $n = 2$, whereas the numerical calculations yield $n \approx 1.9$ [87]. In our estimates we take into account the reconnection probability p . For cosmic strings of superstring theory, namely, F and D strings [116], $p \ll 1$, whereas for ordinary field theory strings $p = 1$. The flux, event rate and the chance of getting neutrino bursts is expected to be enhanced for cosmic superstrings with $p \lesssim 1$, compared to ordinary cosmic strings. We ignored the backreaction of modulus emission from kinks on the evolution of kinks. Since the total power from a kink is only logarithmically divergent [see Eq. (28)], the effect of radiation is expected to smooth out the sharpness of a kink slowly. Finally, our treatment of the neutrino absorption due to resonant scattering on the neutrino background is limited to relatively large masses, $m_\nu \gtrsim 0.1$ eV, for which thermal effects on the background are negligible. The generalization to include these effects is forthcoming [99].

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