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Semirealistic bouncing domain wall cosmology

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In this paper we constructed a semirealistic cosmological model in a dynamic domain wall framework. Our universe is considered to be a (3 + 1) dimensional dynamic domain wall in a higher dimensional Einstein-Maxwell-Born-Infeld background. One of our interesting outcomes from the effective Hubble equation for the domain wall dynamics is that it contains an additional component of "dark matter" which is induced from the charge of the bulk Born-Infeld gauge field. In this background spacetime we have studied the cosmological dynamics of the domain wall. In addition to the Born-Infeld gauge field if we consider additional pure gauge field, a nonsingular bounce happens at the early stage with a smooth transition between the contracting and the expanding phase.

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I. INTRODUCTION

The standard model of cosmology has already been proved to be one of the most successful models in physics. In spite of its success in accounting various cosmological as well astrophysical observations, the model is plagued with some basic fundamental problems. One of those is the famous big-bang singularity problem. In the standard big-bang model if one goes backward in time, it hits the singularity at finite time. Many different approaches have been proposed over the years to avoid this problem. One of the approaches that has gained considerable interest is in the framework of braneworld. In this approach our universe is identified with a four dimensional hypersurface [1–9] moving in the extra dimensional spacetime. A codimension one hypersurface is technically called domain wall. Throughout our paper we will consider the dynamics of a domain wall. In this framework it has been shown that dynamics of a domain wall in the extra dimension mimics the usual Hubble equation of standard cosmology with the additional components of induced invisible energy. This gives us a possibility of studying the cosmology in a new perspective [10,11]. One of the important aspects of this framework is that the Hubble equation of motion for the domain wall emerges from the boundary condition across its position in the extra dimension which is known as the Israel junction condition [12]. Furthermore different parameters of the bulk spacetime solution effectively act as a source of invisible energy density with different equations of state on the domain wall. By tuning those parameters in a model under consideration, one can in principle construct viable cosmologies with a bounce which avoids the usual big-bang singularity. Furthermore, it is an interesting point to note that by tuning those bulk parameters one can also construct a model universe with an induced "dark radiation" and "dark matter" component in addition to the bounce with a transition from the contracting phase followed by the standard expanding phase of the universe [13,14]. Motivated by our previous study, in this paper we constructed such a semirealistic bouncing domain wall cosmological model without introducing a standard dark matter component on the domain wall [15].

As a follow-up of our previous study we will construct a simple cosmological model of dynamic domain walls in the background of Maxwell and Born-Infeld gauge fields along the line of Ref. [16]. Let us mention at this point that we consider two types of gauge fields. One corresponds to the standard Maxwell field $\mathcal{A}_A$ and other one is Born-Infeld gauge field $\mathcal{B}_B$. The purpose of taking those two different types of gauge fields will be apparent as we proceed. Motivation to consider both kinds of gauge fields could be coming from string theory. The Born-Infeld—type higher derivative action naturally arises in string theories in their low-energy effective action. In addition to the the gauge field the effective action also contains an infinite series of higher curvature terms in the gravity sector. For our present purpose in this paper, we will ignore those higher spacetime curvature terms. For simplicity, in this paper we consider the gauge field higher derivative terms like Born-Infeld gauge field. We have solved analytically the equations of motion with the appropriate junction condition at the position of the domain wall. There exists three different types of solutions depending upon the choice of parameters. We have already discussed in detail about part of those solutions in our previous works [15]. For the present purpose, we have chosen the simplest but phenomenologically appealing solution which we find has interesting cosmological implications with regard to our aforementioned motivation to construct a domain wall cosmology.

We structured this report as follows: In Sec. II, we will start with a generic action corresponding to a domain wall moving in the Maxwell-Born-Infeld-dilaton background. In order for our paper to be self-contained, we will give the

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general analysis with the dilaton field in this section. In the subsequent Sec. III as we mentioned before we will consider a particular bulk background with a trivial dilaton configuration. We take the static bulk metric ansatz and study the dynamics of the domain wall in this static background. We get semirealistic bouncing domain wall cosmology with dark radiation and dark matter like energy components induced from the bulk black hole charges. In Sec. IV, we consider a more realistic case where we have matter field localized on the brane. This has changed the effective Hubble equation significantly. We find the corresponding constraints on the bulk spacetime parameters so that we have a nonsingular bouncing cosmology even with the standard matter field. We also discussed the possible constraints on the parameters of our solution from the cosmological observations. In Sec. V we will discuss the perturbation equations across the domain wall. Finally, in Sec. VI, we do some concluding remarks and describe some future directions for our work.

II. EINSTEIN EQUATIONS AND BOUNDARY CONDITIONS

We start with a general action of the Einstein-Maxwell-Born-Infeld-dilaton system in an arbitrary spacetime dimension n. The action takes the form

$$S = \int d^n x\sqrt{-g}\bigg(\frac{1}{2}R - \frac{1}{2}\partial_A\phi\partial^A\phi - V(\phi) - \frac{1}{4}e^{-2\zeta\phi}F_{AB}F^{AB} + \mathcal{L}(G,\phi)\bigg) + S_{\rm DW}, \tag{1}$$

where action for the domain wall is

$$S_{\rm DW} = -\int d^{n-1}x\sqrt{-h}(\{K\} + \bar{V}(\phi)).$$

The expression for $\mathcal{L}(G,\phi)$ is

$$L(G,\phi) = 4\lambda^2 e^{2\gamma\phi}\left(1 - \sqrt{1 + \frac{e^{-4\gamma\phi}G^{AB}G_{AB}}{2\lambda^2}}\right), \tag{2}$$

where λ is a constant parameter with the dimension of mass. $G_{AB} = \partial_A\mathcal{B}_B - \partial_B\mathcal{B}_A$ is the Born-Infeld field strength and $F_{BD} = \partial_B\mathcal{A}_D - \partial_D\mathcal{A}_B$ is the field strength of the Maxwell field A_D. h is the determinant of the induced metric h_{AB} on the domain wall. K is the trace of the extrinsic curvature K_{ab} of the domain wall.

Corresponding Einstein equations turn out to be

$$R_{AB} = T^{\phi}_{AB} + T^{\mathcal{A}}_{AB} + T^{\mathcal{B}}_{AB}, \tag{3}$$

$$D_C\partial^C\phi - \frac{\partial(\phi)}{\partial\phi} + 8\lambda^2\gamma e^{2\gamma\phi}\left\{2\mathcal{Y}\frac{\partial\mathcal{L}}{\partial\mathcal{Y}} - \mathcal{Y}\right\} + \frac{1}{2}\zeta e^{-2\zeta\phi}G_{AB}G^{AB} = 0, \tag{4}$$

$$D_A\left(e^{-2\gamma\phi}\frac{\partial\mathcal{L}}{\partial\mathcal{Y}}F^{AB}\right) = 0, \tag{5}$$

$$D_A(e^{-2\zeta\phi}G^{AB}) = 0, \tag{6}$$

where various energy momentum tensor components are

$$T^{\phi}_{AB} = \partial_A\phi\partial^A\phi + \frac{2}{n-2}V(\phi)g_{AB};$$

$$T^{\mathcal{A}}_{AB} = \frac{1}{2}e^{-2\zeta\phi}\left(2G_{AC}G^C_B - \frac{1}{n-2}G_{CD}G^{CD}g_{AB}\right),$$

$$T^{\mathcal{B}}_{AB} = \frac{8\lambda^2}{(n-2)}e^{2\gamma\phi}\left\{2\mathcal{Y}\frac{\partial\mathcal{L}}{\partial\mathcal{Y}} - \mathcal{Y}\right\}g_{AB} - 8e^{-2\gamma\phi}\frac{\partial\mathcal{L}}{\partial\mathcal{Y}}F_{AC}F^C_B.$$

D_A is a covariant derivative with respect to the bulk metric and $\mathcal{Y} = \frac{e^{-4\gamma\phi}G^{AB}G_{AB}}{2\lambda^2}$. In addition to the above equation we need to satisfy the following Israel junction conditions:

$$\{K_{MN}\} = -\frac{1}{n-2}\bar{V}(\phi)h_{MN}, \tag{7}$$

$$\{n^M\partial_M\phi\} = \frac{\partial\bar{V}(\phi)}{\partial\phi}, \tag{8}$$

where n^M is the unit normal to the domain wall. R is the curvature scalar.

In the subsequent analysis we will consider our model enjoying reflection symmetry (Z_2) across the domain wall. Considering a static spherically symmetric bulk metric

$$ds^2 = -N(r)dt^2 + \frac{1}{N(r)}dr^2 + R(r)^2 d\Omega^2_\kappa, \tag{9}$$

with $d\Omega^2_\kappa$ being a metric on a $(n-2)$ dimensional space with a constant curvature $\tilde{R}_{ij} = k(n-3)\tilde{g}_{ij}$ with $k \in \{-1, 0, 1\}$, we are interested to study induced cosmological dynamics on the domain wall with a Friedmann-Robertson-Walker metric

$$ds^2_{\rm wall} = -d\tau^2 + R(\tau)^2 d\Omega^2_\kappa. \tag{10}$$

τ is the domain wall proper time. As one can clearly see from the above construction, the radial direction along the extra dimension plays the role of the scale factor of our domain wall universe.

By considering the unit normal to be pointing towards the $r < r(t)$ region, one can find the following equations consistent with the dynamic domain wall in the extra dimension:

$$R' = C\bar{V}(\phi). \tag{11}$$

Using the above equation in the boundary condition for the scalar field one gets

$$\frac{\partial\phi}{\partial R} = -\frac{n-2}{R}\frac{1}{\bar{V}}\frac{\partial\bar{V}}{\partial\phi}. \tag{12}$$

In the above derivation we have used the expression for K_{ij} and K_{00}.

So, one can solve the above equation for ϕ as a function of scale factor R without referring to the bulk scalar field potential. This is inconsistent with the dynamic domain wall coupled with a bulk scalar field we mentioned before. Now we will solve the full equation of motion to be consistent with the above equations.

III. BULK SOLUTIONS AND DOMAIN WALL COSMOLOGY

In our previous papers [15] we have already solved for the Born-Infeld and the Maxwell field coupled with dilaton separately. In this paper, we will solve them together and then try to study their cosmologies.

We consider a class of solutions for both the Born-Infeld and the Maxwell fields where all the components of F^{AB} and G^{AB} are zero except the F^{rt} and G^{rt} components. The solution looks like

$$G^{rt} = \frac{2Q\lambda e^{2\gamma\phi}}{\sqrt{4Q^2 + \lambda^2 R^{2n-4}}}; \qquad F^{rt} = \frac{Q^\dagger e^{2\zeta\phi}}{2R^{n-2}}, \tag{13}$$

where Q and $Q^\dagger$ are the integration constants and related to the Born-Infeld and electromagnetic charge, respectively. Born-Infeld and electromagnetic charges can be expressed as follows:

$$q = \frac{1}{4\pi}\int_{\Sigma_\infty} e^{-2\gamma\phi\,*}G = \frac{Q\omega_{n-1}}{4\pi}; \qquad q^\dagger = \frac{1}{4\pi}\int_{\Sigma_\infty} e^{-2\zeta\phi\,*}F = \frac{Q^\dagger \omega_{n-1}}{4\pi}, \tag{14}$$

where ${}^*F_{AB} = \frac{1}{2\sqrt{-g}}\varepsilon^{ABCD}F^{CD}$ and ${}^*G_{AB} = \frac{1}{2\sqrt{-g}}\varepsilon^{ABCD}G^{CD}$. Σ_∞ is a hypersurface at $R\to\infty$. ω_{n-1} is volume of unity n sphere.

Using the solution for the Born-Infeld and Maxwell field and the ansatz for the metric Eq. (9), the remaining equations of motion turn out to be

$$\frac{R''}{R} = -\frac{1}{n-2}\phi'^2, \tag{15a}$$

$$\frac{1}{2R^{n-2}}\{N(R^{n-2})'\}' - \frac{k(n-3)(n-2)}{2R^2} = -V - \mathcal{T}_{22}(R,Q) - \frac{Q^{\dagger 2}}{R^{2n-4}}e^{2\zeta\phi}, \tag{15b}$$

$$\frac{n-2}{4R^{n-2}}(N'R^{n-2})' = -V - \mathcal{T}_{00}(R,Q) + \frac{(n-3)Q^{\dagger 2}}{2R^{2n-4}}e^{2\zeta\phi}, \tag{15c}$$

$$\frac{1}{R^{n-2}}(\phi' N R^{n-2})' = \frac{\partial V(\phi)}{\partial\phi} + 8\lambda^2\gamma e^{2\gamma\phi}\mathcal{E}(r,Q) + \frac{\zeta Q^{\dagger 2}}{R^{n-2}}e^{2\zeta\phi}, \tag{15d}$$

where

$$\mathcal{T}_{22}(R,Q) = 4\lambda^2 e^{2\gamma\phi}\mathcal{E}(R,Q);$$
$$\mathcal{T}_{00}(R,Q) = 4(n-2)\lambda^2 e^{2\gamma\phi}\left[\frac{\mathcal{E}(R,Q)}{n-2} + \frac{\mathcal{G}(R,Q)}{2}\right], \tag{16}$$

and

$$\mathcal{E}(R,Q) = \frac{\sqrt{4Q^2+\lambda^2 R^{2n-4}}}{\lambda R^{n-2}} - 1;$$
$$\mathcal{G}(R,Q) = -\frac{4Q^2}{\sqrt{4Q^2+\lambda^2 R^{2n-4}}}\frac{1}{\lambda R^{n-2}}. \tag{17}$$

$\mathcal{T}_{00}$ and $\mathcal{T}_{22}$ are tt and xx components of the energy-momentum tensor for the Born-Infeld Lagrangian, respectively.

In order to solve, we choose the following Liouville-type brane potential:

$$\bar{V}(\phi) = \bar{V}_0 e^{\alpha\phi}, \tag{18}$$

which provides a straightforward solution for the scalar field ϕ and the scale factor R without any specific form of the the bulk potential:

$$\phi = \phi_0 - \frac{\alpha(n-2)}{\alpha^2(n-2)+1}\log(r), \tag{19a}$$

$$R(r) = C\bar{V}_0 e^{\alpha\phi_0} r^{\frac{1}{\alpha^2(n-2)+1}}, \tag{19b}$$

where ϕ_0 and C are integration constants. Now, what we need to check is how the above solutions for the scalar field and the scale factor are constraining our solution for the bulk spacetime. For this we further specify our bulk potential for the scalar field as

$$V(\phi) = V_0 e^{\theta\phi}, \tag{20}$$

where V_0 is constant. By using Eq. (19) for R and ϕ as solutions of the ansatz and the bulk potential for the scalar field, one obtains different types of solutions [15] which are characterized by the bulk parameters and suitable boundary conditions. We will study those solutions and their cosmological implication in detail elsewhere. In this paper we will take one particularly simple solution and study its cosmological behavior. The solution we are considering is for a simple choice of parameters ζ, γ, θ and α setting to zero. We, therefore, do not have any nontrivial dilaton field in our background. We are also interested in the domain wall universe with a spatially flat i.e $k=0$ section. In our framework, therefore, a spatially flat domain wall is moving in a black brane background. One also notes that for the aforementioned value of the parameters the bulk and brane potential turn into simple cosmological constant and brane tension, respectively.

Our bulk solution looks like

$$\begin{aligned}N(r) &= -2Mr^{-(n-3)} - \left(\frac{2V_0}{(n-2)(n-1)} - \frac{8\lambda^2}{(n-2)(n-1)}\right)r^2\\ &+ \frac{Q^{\dagger 2}}{(n-3)(n-2)}r^{-2(n-3)} + \frac{8\lambda r^{-(n-4)}}{(n-1)(n-2)}\\ &\times\left(-\sqrt{4Q^2+\lambda^2 r^{2n-4}} + \frac{4(n-2)Q^2 r^{-(n-2)}}{\lambda(n-3)}\mathcal{D}(r,Q)\right)\\ &\times R(r) = r; \quad \phi = \phi_0, \end{aligned} \tag{21}$$

where M and ϕ_0 are integration constants and

$$\mathcal{D}(r,Q) = {}_2F_1\left[\frac{n-3}{2n-4}, \frac{1}{2}, \frac{3n-7}{2n-4}, -\frac{4Q^2 r^{-(2n-4)}}{\lambda^2}\right]. \tag{22}$$

The solution itself is complicated. For simplicity we study our solution in various limits along the radial coordinate and study its behavior. If we expand our solution in large r, the expression for the above solution becomes

$$\begin{aligned}N(r)|_{r\to\infty} &= -\frac{2V_0}{(n-2)(n-1)}r^2 - 2Mr^{-(n-3)}\\ &+ \frac{2\tilde{Q}^2}{(n-3)(n-2)}r^{-(2n-6)} + \mathcal{O}(r^{10-4n}),\end{aligned} \tag{23}$$

where $\tilde{Q}^2 = 8Q^2 + \beta Q^{\dagger 2}$ and, for the small r limit,

$$\begin{aligned}N(r)|_{r\to 0} &= \frac{Q^{\dagger 2}}{(n-3)(n-2)}r^{-2(n-3)} - 2\mathcal{M}r^{-(n-3)}\\ &- \frac{16\lambda\mathcal{Q}}{(n-1)(n-2)}r^{-(n-4)} + \mathcal{O}(r^2),\end{aligned}$$

where

$$\begin{aligned}\mathcal{M} &= M - \frac{16Q^2\Gamma\left[\frac{3n-7}{2n-4}\right]\Gamma\left[\frac{1}{2n-4}\right]}{\sqrt{\pi}(n-1)(n-3)}\left(\frac{4Q^2}{\lambda^2}\right)^{-\frac{n-3}{2n-4}},\\ \mathcal{Q} &= Q\left(1 - \frac{(n-2)\Gamma\left[\frac{3n-7}{2n-4}\right]\Gamma\left[\frac{-1}{2n-4}\right]}{(n-3)\Gamma\left[\frac{n-3}{2n-4}\right]\Gamma\left[\frac{2n-5}{2n-4}\right]}\right).\end{aligned} \tag{24}$$

It is, therefore, clear from the above limits that the full complicated solution for the bulk metric can be cast into the following simple form:

$$N(r) = \frac{Q^{\dagger 2}}{6}r^{-4} - 2\mathcal{M}r^{-2} - \frac{4\lambda\mathcal{Q}}{3}r^{-1} - \mathcal{H}(r) - \frac{V_0}{6}r^2, \tag{25}$$

where $\mathcal{H}(r)$ is some complicated function of radial distance r. But it is important to note that in both limits of r the function is regular, i.e.,

$$\mathcal{H}(r)\overset{r\to 0}{\to}\mathcal{O}(r^2); \qquad \mathcal{H}(r)\overset{r\to\infty}{\to}2Mr^{-2} - \frac{\tilde{Q}^2}{3}r^{-4}. \tag{26}$$

In the above expressions we considered the number of spacetime dimension to be $n = 5$ which is of our particular interest. As one can imagine the expression for $\mathcal{Q}$ depends upon the sign of Q but originally the metric always depends on Q^2. So, for the subsequent discussions, we will take Q to be positive. The limiting expressions for $\mathcal{H}(r)$ give us the total charge density related to $\tilde{Q}$ and mass density M of the black hole. The solution has a timelike singularity at $r = 0$.

So far we have discussed the analytic solution and its various limiting properties of our bulk spacetime. In what follows we will study the dynamics of a domain wall in that background. As is well known [16], dynamics of a domain wall satisfies a Hubble like equation of motion,

$$\dot{R}^2 + F(R) = 0, \tag{27}$$

where the "overdot" is the derivative with respect to the domain wall proper time τ. For the simple solution we considered, the expression for $F(R)$ turns out to be

$$F(R) = N(R) - \frac{\bar{V}_0^2}{36}R^2. \tag{28}$$

The form of the potential looks like the asymptotic modification of the metric function $N(R)$.

The Hubble equation of motion turns out to be

$$\begin{aligned}H^2 &= -\frac{Q^{\dagger 2}}{6}R^{-6} + 2\mathcal{M}R^{-4} + \frac{4\lambda\mathcal{Q}}{3}R^{-3}\\ &+ \mathcal{H}(R)R^{-2} + \left(\frac{V_0}{6} + \frac{\bar{V}_0^2}{36}\right).\end{aligned} \tag{29}$$

As mentioned before, the important point we want to emphasize here is that the effective domain wall equation of motion contains a so-called dark matter energy component in addition to the usual dark radiation term. This is our new finding which was not discussed in the previous domain wall study. So the novel feature of our model is that, even without the matter field localized on the brane it evolves like a standard cosmology. Interestingly this dark matter component is depending upon the charge (Q) of the Born-Infeld electric field. On the other hand, invisible dark radiation energy depends upon the linear combination of both mass (M) and Born-Infeld electric charge (Q) of the bulk black hole spacetime. So the evolution of the domain wall in the Born-Infeld background mimics the evolution of the standard cosmology. This is the reason we call our domain wall dynamics as semirealistic in nature.

In addition to this semirealistic evolution we also want to have a bounce in the domain wall dynamics at a finite value of its scale factor. This can be achieved by introducing a Maxwell gauge field in the bulk regarding which we have particularly emphasized in the Introduction. From the above Eq. (29), we see that due to the presence of

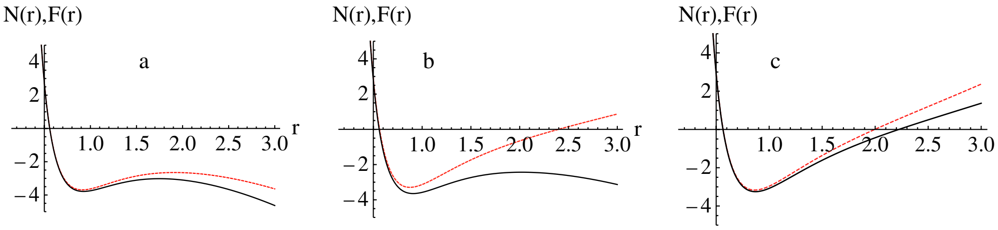

FIG. 1 (color online). Plot for metric function $N(r)$ (dotted red line) and potential $F(r)$ (solid black line). For these particular plots we set $Q^{\dagger} = 1$, $Q = 1$, $M = 3$, $\lambda = 2$ with plot (A) $V_0 = 2$, $\bar{V}_0 = 2$, plot (B) $V_0 = -1$, $\bar{V}_0 = 4$ and plot (C) $V_0 = -2$, $\bar{V}_0 = -2$.

negative energy component so-called "stiff matter" induced from the charge of bulk Maxwell field, we have a bounce followed by a standard cosmological evolution. It is very difficult to get an analytic expression for the solution of the above equation of motion. We, therefore, plotted the potential $F(R)$ for different values of the parameters of the model in Fig. 1 comparing with the bulk $G_{00} = N(R)$ metric component. It is clear from the plots that there exists a minimum value of the scale factor at which bounce occurs for three different cases. For the limiting case, near the bouncing point we can solve the above Hubble equation with the approximation that the domain wall dynamics is governed by the stiff matter and the dark radiation. In that limit the solution for the scale factor looks like [13]

$$R(\eta) = \sqrt{\frac{1}{12\mathcal{M}}(Q^{\dagger 2} + 24\mathcal{M}^2\eta^2)}, \tag{30}$$

where, for convenience, we use conformal time $d\tau = R(\eta)d\eta$ in the above expression. It is clear from the above solution that we have the minimum value of the scale factor

$$R(\eta)_{\min} = \sqrt{\frac{Q^{\dagger 2}}{12\mathcal{M}}}, \quad \text{where } \mathcal{M} = M - \frac{2.5}{\sqrt{\pi}}(4\lambda Q^2)^{\frac{2}{3}}.$$

In order to have a real solution, mass (M) and Born-Infeld charge (Q) of the black hole should satisfy $M > \frac{2.5}{\sqrt{\pi}}(4\lambda Q^2)^{\frac{2}{3}}$. As expected in the late time evolution is radiation dominated, $R(\tau) \sim \tau^{\frac{1}{2}}$.

In order for the completeness we also solve the above Hubble equation numerically as shown in Fig. 2. The qualitative feature of the scale factor is the same for the different parameter values at the bouncing point. So we only plotted the scale factor for the model A of Fig. 1. As Fig. 1 shows the effective potential of the domain wall has a minimum which leads to an exponential expansion phase of the domain wall after the bounce. We, therefore, have a natural inflationary phase after the bounce but for a very short period of time. Near the minimum of the potential the scale factor evolves like

$$R(\tau) \sim e^{\frac{\sqrt{-F''(R_0)}\tau}{2}},$$

where $F(R)$ has a minimum at R_0.

At this point we want to emphasize that, for the two horizon bulk black hole background, the bounce generically happens inside the inner horizon. The stability issue on this kind of bounce inside the Cauchy horizon has been raised in Ref. [17], although we think this issue needs further study to completely rule out this kind of bouncing cosmological models. But the general argument says that the inner horizon of a charged black is intrinsically unstable under small perturbation. This instability is related to the strong cosmic censorship conjecture of a black hole spacetime. However, we are not going to study this issue here any further. The point we want to emphasize in our study is that for a wide range of parameters of our solution we have charged under the Born-Infeld gauge field black hole which has no inner horizon. So, for those cases the stability issue is still not clearly understood. We defer it for our future study.

IV. DOMAIN WALL COSMOLOGY WITH BRANE MATTER FIELD

So far we have discussed the case where there is no realistic matter field localized on the brane. In this section we will study a more realistic situation where we have radiation as well as normal baryonic matter field localized on the domain wall. As we have discussed earlier, the dark matter component is induced on the domain wall through

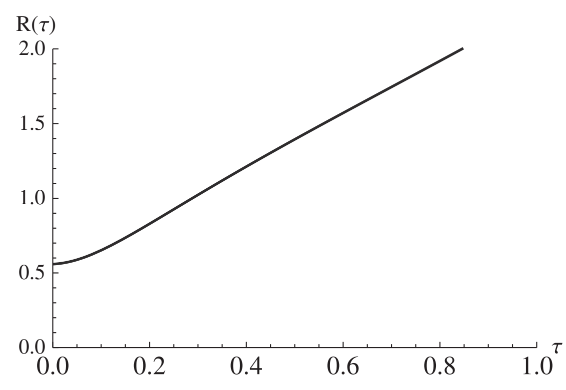

FIG. 2. Numerical plot for scale factor $R(\tau)$ for model A of Fig. 1. So, parameters are $Q^{\dagger} = 1$, $Q = 1$, $M = 3$, $\lambda = 2$, $V_0 = 2$, $\bar{V}_0 = 2$. The minimum value of the scale factor $R(\tau)_{\min} = 0.558303$.

the bulk field. So, the modified Hubble equation for the domain wall turns out to be

$$\begin{aligned} H^2 &= -\frac{Q^{\dagger 2}}{6R^6} + \frac{2\mathcal{M}}{R^4} + \frac{4\lambda\mathcal{Q}}{3R^3} + \frac{\mathcal{H}(R)}{R^2} \\ &\quad + \left(\frac{V_0}{6} + \frac{(\bar{V}_0 + \rho_{\rm rad}R^{-4} + \rho_m R^{-3})^2}{36}\right) \\ &= \frac{1}{36}\left(\frac{\rho_{\rm rad}^2}{R^8} + \frac{2\rho_{\rm rad}\rho_m}{R^7} + \frac{\rho_m^2}{R^6}\right) - \frac{6Q^{\dagger 2}}{36R^6} + \frac{36\mathcal{M} + \bar{V}_0\rho_{\rm rad}}{18R^4} \\ &\quad + \frac{24\lambda\mathcal{Q} + \bar{V}_0\rho_m}{18R^3} + \frac{\mathcal{H}(R)}{R^2} + \left(\frac{V_0}{6} + \frac{\bar{V}_0^2}{36}\right). \end{aligned} \tag{31}$$

As we can easily identify from the third and fourth terms of the second line of the above equation, the induced dark radiation ($\rho_{\rm drad}$) and the dark matter ($\rho_{\rm dm}$) component on our domain wall can be read off as

$$\begin{aligned} \rho_{\rm drad} &= \frac{36\mathcal{M}}{\bar{V}_0} = \frac{36}{\bar{V}_0}\left[M - \frac{2.5}{\sqrt{\pi}}(4\lambda Q^2)^{\frac{2}{3}}\right]; \\ \rho_{\rm dm} &= \frac{24\lambda\mathcal{Q}}{\bar{V}_0} = \frac{96\lambda Q}{\bar{V}_0}, \end{aligned} \tag{32}$$

with the standard normalization for the Hubble equation $\bar{V}_0 = \frac{48\pi}{M_p^2}$, where M_p is the four dimensional Planck constant.

Now big-bang nucleosynthesis in standard cosmological evolution during the radiation dominated era as well as the anisotropy in the cosmic microwave background spectrum [18] tell us that any nonstandard radiation like energy density must be very tiny in order to satisfy the observed relic abundance. So, the induced dark radiation energy ($\rho_{\rm drad}$) should be much smaller than that of the usual radiation density ($\rho_{\rm rad}$). As we mentioned before and is also clear from the above expression for the dark radiation, by suitably choosing the mass (M) and Born-Infeld charge (Q) of the bulk black hole, we can make it zero or very tiny. Furthermore, we know that about 23% of the total energy component in our universe is nonbaryonic dark matter in nature. With this consideration we can fix the Born-Infeld charge of the black hole to say $Q = Q_{\rm dm}$. This observation also fixes the mass of the bulk black hole to be

$$M \simeq \frac{2.5}{\sqrt{\pi}}(4\lambda Q_{\rm dm}^2)^{\frac{2}{3}}. \tag{33}$$

Considering the above mass of the black hole, we can ignore the dark radiation term in the effective Hubble equation in our subsequent discussions. At this point it is important to note the recent interests in the additional dark radiation component in the standard model of cosmology. There has been recent speculation that, in order to fit some cosmological observation such as WMAP, the effective number of relativistic degrees of freedom in our universe has to be larger than four [19,20], even though there is an active debate going on along the lines of this subject. In order to confirm this we need to wait for further precision observation such as PLANCK. A different particle physics model has already been considered in order to explain this extra dark radiation component. An axtra relativistic particle such as a sterile neutrino has been introduced as a dark radiation [19]. Interestingly brane world cosmological models naturally predict an effective dark radiation component induced from the bulk gravitation [21] as we also have seen in our current analysis. So by imposing the constraint coming from this extra cosmological dark radiation component in our model, we can in principle give a precise constraint on the bulk black hole charges. For our current study we will consider the dark radiation component to be negligibly small.

Now, further constraint on the black hole parameters will come from the bounce for a particular value of the scale factor. Since we are considering the case where the domain wall is very close to the bouncing point, we can ignore the matter and cosmological constant part from the Hubble Eq. (31) and set it to zero right at the bounce. At the bouncing point we approximated Eq. (31) to be

$$\left(\frac{\rho_{\rm rad}^2}{R_b^8} + \frac{2\rho_{\rm rad}\rho_m}{R_b^7} + \frac{\rho_m^2}{R_b^6}\right) - \frac{6Q^{\dagger 2}}{R_b^6} + \frac{2\bar{V}_0\rho_{\rm rad}}{R_b^4} = 0. \tag{34}$$

It is very difficult to get an analytical expression for the scale factor. As we have checked if the condition below is satisfied then we can have a bounce at R_b satisfying the above equation:

$$\rho_{\rm rad}^2 + 2\rho_{\rm rad}\rho_m R_0 - (6Q^{\dagger 2} - \rho_m^2)R_0^2 + 2\bar{V}_0\rho_{\rm rad}R_0^4 \leq 0, \tag{35}$$

where

$$R_0 = \frac{2\cdot 3^{\frac{1}{3}}ab + 2^{\frac{1}{3}}\left(-9a^2c + \sqrt{3}\sqrt{a^3(-4b^3 + 27ac^2)}\right)^{\frac{2}{3}}}{6^{\frac{2}{3}}a\left(-9a^2c + \sqrt{3}\sqrt{a^3(-4b^3 + 27ac^2)}\right)^{\frac{1}{3}}},$$

with

$$a = 8\bar{V}_0\rho_{\rm rad}; \qquad b = 2(6Q^{\dagger 2} - \rho_m^2); \qquad c = 2\rho_{\rm rad}\rho_m.$$

If we consider only the radiation field on the brane then all the above expressions become simple. To simplify the subsequent analysis, let us consider $\rho_m = 0$; then in order to get a bounce one needs to satisfy [13]

$$Q^{\dagger 4} \geq \frac{2}{9}\bar{V}_0\rho_{\rm rad}^2 = \frac{32\pi\rho_{\rm rad}^3}{3M_p^2}. \tag{36}$$

So, the bounce restricts the value of electromagnetic charge of the bulk black hole. Then if the above bound is satisfied, the minimum value for the scale factor approximately amounts to

$$R_b = R_{\rm min} \simeq \left(\frac{384\pi\rho_{\rm rad}^3}{M_p^2}\right)^{\frac{1}{4}}. \tag{37}$$

In conclusion we have seen that dynamics of a domain wall in the Maxwell-Born-Infeld black hole background is semirealistic in nature. The dynamics of the domain wall is governed by the dark radiation, dark matter and cosmological constant, all of which can be induced from the above-mentioned static bulk black hole charges. In addition to the above semirealistic cosmological expansion, we also have seen that our model passes through a bouncing phase as well. All these interesting features give us hope that the brane domain wall model could be an interesting framework to construct singularity-free cosmological models. In the next section we will discuss the perturbation across the domain wall junction.

V. PERTURBATION

In this section we will try to set up the stage for the scalar perturbation in the dynamic domain wall scenario for our future study. A detailed study on the perturbation dynamics in the framework of the domain wall scenario has not been studied yet. The present paper is beyond the scope of this study. In this section, we will begin this program by first calculating how the perturbed Israel junction condition across the domain wall looks. Some part of this calculation can be found in many papers dealing with perturbation in a brane-world scenario (see the review [22]). First we will start with a general form of the bulk background metric,

$$\begin{aligned} ds^2 &= G_{ab}dX^a dX^b \\ &= -n(r)^2 dt^2 + b(r)^2 dr^2 + R(r)^2(dx^2 + dx^2 + dx^2), \end{aligned} \tag{38}$$

where for our particular case $n(r) = \frac{1}{b(r)} = N(r)$. We parametrize our brane as

$$X^a = \bar{X}^a(y^\mu) \text{ where } \bar{X}^0 = t \equiv t(\tau); \quad \bar{X}^1 = r \equiv r(\tau). \tag{39}$$

Four tangent vectors to the brane are

$$\bar{V}^{\,a}_{\mu} = \frac{\partial \bar{X}^a}{\partial y^\mu} \Rightarrow \bar{V}^a_\eta = \left(\frac{\partial t}{\partial \tau}, 0, 0, \frac{\partial r}{\partial \tau}\right); \quad \bar{V}^a_i = (0, \delta^a_i, 0). \tag{40}$$

The normal vector to the brane would be

$$\bar{n}_{\,a} = \frac{bn\dot{r}}{\gamma}\delta^t_a + \frac{1}{\gamma}\delta^r_a, \tag{41}$$

where $\gamma = \sqrt{n^2 - b^2\dot{r}^2}$. The normal vector satisfies $\bar{V}^a_\mu \bar{n}_a = 0$ and normalizability condition $\bar{n}_a \bar{n}^a = 1$.

The form of the induced brane metic is defined as before:

$$\begin{aligned} ds^2_{\text{brane}} &= G_{ab}V^a_\mu V^b_\nu dy^\mu dy^\nu \\ &= -d\tau^2 + R(\tau)^2(dx^2 + dy^2 + dz^2), \end{aligned} \tag{42}$$

with

$$\left(\frac{dt}{d\tau}\right) = \frac{1}{\sqrt{n^2 - b^2\dot{r}^2}} = \frac{1}{\gamma}. \tag{43}$$

Now, in this background setup, we will consider the linear perturbation with a dynamic domain wall. We will consider the scalar perturbation in our background. The linearly perturbed Einstein equations of motion take the following form:

$$\begin{aligned} \delta R_{AB} &= \delta T^{\phi}_{AB} + \delta T^{\mathcal{A}}_{AB} + \delta T^{\mathcal{B}}_{AB}; \\ \delta K_{MN} &= -\frac{1}{2(n-2)}\delta[\bar{V}(\phi)h_{MN}], \end{aligned} \tag{44}$$

with the Background metric perturbation $g_{ab} = \bar{G}_{ab} + \delta G_{ab}$, where

$$\delta G_{ab} = \begin{pmatrix} -2n^2\mathcal{W} & R^2\mathcal{X}_{,i} & n\mathcal{W}_r \\ R^2\mathcal{X}_{,i} & R^2[2\mathcal{S}\delta_{ij} + 2\mathcal{E}_{,ij}] & R^2\mathcal{X}_{r,i} \\ n\mathcal{W}_r & R^2\mathcal{X}_{r,j} & 2b^2\mathcal{W}_{rr} \end{pmatrix}. \tag{45}$$

In five dimension, the gauge transformation $x^a \longrightarrow x^a + \zeta^a$, contains three arbitrary scalar functions. Therefore, by appropriately choosing those functions one can set three scalar degrees of freedom of δG_{ab} to be zero. After choosing this gauge, the metric perturbation becomes

$$\delta G_{ab} = \begin{pmatrix} -2n^2\mathcal{W} & 0 & n\mathcal{W}_r \\ 0 & 0 & R^2\mathcal{X}_{r,i} \\ n\mathcal{W}_r & R^2\mathcal{X}_{r,j} & 2b^2\mathcal{W}_{rr} \end{pmatrix}. \tag{46}$$

Finally we, therefore, have four scalar degrees of freedom. Furthermore, since we have a domain wall which breaks the translational invariance along the radial direction, we have a brane fluctuating mode. Let us parametrize the perturbed brane position as $X^a = \bar{X}^a + \chi^a(y^\mu)$, where y^μ is the brane coordinate. The fluctuation vector field χ^a can be conveniently decomposed as

$$\chi^a = \xi^\mu \bar{V}^a_\mu + \zeta \bar{n}^a, \tag{47}$$

where (ζ, ξ^μ) are the five arbitrary functions defining the fluctuating domain wall coordinate. However, we also have a reparametrization invariance in domain wall coordinate y^μ. So, we can again fix this gauge by choosing $\xi^\mu = 0$. The domain wall fluctuation can, therefore, be parametrized by a single function ζ. The perturbed induced metric on the brane therefore would be

$$\delta h_{\mu\nu} = \delta G_{ab}\bar{V}^a_\mu \bar{V}^b_\nu + 2\zeta \bar{K}_{\mu\nu}, \tag{48}$$

where $\bar{K}_{\mu\nu} = \bar{V}^a_\mu \bar{V}^b_\nu \bar{\nabla}_a \bar{n}_b$ is the background extrinsic curvature of the brane.

The form of the perturbed normal vector to the domain wall δn_a takes the following form:

$$\begin{aligned} &\delta n_i = -\partial_i \zeta, \qquad \delta n_t = \frac{\bar{n}_t}{2}\mathcal{D}_1 - \frac{n^2}{\gamma^2}\mathcal{D}_2, \\ &\delta n_r = \frac{\bar{n}_r}{2}\mathcal{D}_1 - \frac{b^2\dot{r}}{\gamma^2}\mathcal{D}_2, \end{aligned} \tag{49}$$

where

$$\mathcal{D}_1 = \delta G_{ab}\bar{n}^a\bar{n}^b + \zeta\bar{n}^c\partial_c\bar{G}_{ab}\bar{n}^a\bar{n}^b;$$
$$\mathcal{D}_2 = \dot{\zeta} + \zeta\bar{G}_{ab}\bar{n}^a\bar{V}^c_\tau\gamma\partial_c\bar{n}^b.$$

Equipped with all the above relevant variations the expression for the perturbed extrinsic curvature becomes

$$\delta K_{ij} = -\zeta_{,ij} + \frac{RR'}{n^2}\delta n_r + \Delta\Gamma^a_{ij}\bar{n}_a, \tag{50}$$

$$\delta K_{0i} = \frac{1}{2}\frac{\partial_i\zeta\bar{n}^b}{\gamma}(\bar{\nabla}_b\bar{n}_t + \bar{\nabla}_t\bar{n}_b + \dot{r}\bar{\nabla}_b\bar{n}_r + \dot{r}\bar{\nabla}_r\bar{n}_b) + \bar{V}^b_\tau\left(\frac{1}{2}(\bar{\nabla}_i\delta n_b + \bar{\nabla}_b\delta n_i) + \Delta\Gamma^c_{ib}\bar{n}_c\right), \tag{51}$$

$$\delta K_{00} = \frac{\dot{\zeta}\bar{n}^b}{\gamma^2}(\bar{\nabla}_b\bar{n}_t + \bar{\nabla}_t\bar{n}_b + \dot{r}\bar{\nabla}_b\bar{n}_r + \dot{r}\bar{\nabla}_r\bar{n}_b) + \bar{V}^a_\tau\bar{V}^b_\tau\left(\frac{1}{2}(\bar{\nabla}_a\delta n_b + \bar{\nabla}_b\delta n_a) + \Delta\Gamma^c_{ab}\bar{n}_c\right), \tag{52}$$

where

$$\Delta\Gamma^c_{ab} = \delta\Gamma^c_{ab} + \zeta\bar{n}^r\partial_r\bar{\Gamma}^c_{ab}. \tag{53}$$

In this section we have computed the perturbing boundary condition across the junction of the dynamic domain wall. We will do the detailed analysis of this perturbation in our subsequent paper.

VI. CONCLUSION

The standard model of cosmology is one of the most successful models in successfully explaining the evolution of our universe. There are some important fundamental issues in this model which have been puzzling physicists for a long time. As we have been mentioning throughout our present paper, our universe under the standard model of cosmology encountered a singularity as we go backward in time. This is definitely unexpected for any physically meaningful theory. There have been many attempts to construct effective models which can avoid this big-bang singularity. As we have mentioned, a higher dimensional cosmological model has particularly gained considerable interest in this respect. In this paper we have studied dynamic domain wall cosmology where we can realize the bouncing cosmology. People have already found this kind of bouncing solution before [13], but the interesting finding in our model is the possibility of inducing a dark matter like energy component on the domain wall by considering simple well-known fields in the bulk. This aspect leads us to construct a semirealistic bouncing domain wall cosmology by introducing different types of gauge fields in the higher dimensional background. We have considered the Maxwell-Born-Infeld gauge field background in the bulk and studied the dynamics of the domain wall in those backgrounds. We found out the analytic bulk spacetime solutions taking into account the backreaction of those gauge fields and the dynamic domain walls. There exist many different types of solutions depending upon various choices of parameters [15]. In this paper we discussed a particularly simple solution in which the dynamics of the domain wall mimics a semirealistic cosmological evolution along the extra dimension compared to our standard cosmological scenario.

As we already mentioned, the important aspect of our model is the presence of a dark matter like energy component which is induced from the bulk Born-Infeld charge. In addition to this we have a standard dark radiation component coming from the black hole mass (M) and Born-Infeld charge (Q). In addition to the standard evolution an effective negative energy density is induced from the bulk usual electromagnetic charge ($Q^\dagger$) leading to a singularity free bounce of the domain wall at finite value of its scale factor. All these aspects provide us an interesting possibility to construct a realistic bouncing domain wall cosmology. Furthermore, it gives us a hint that maybe the domain wall framework could be an interesting playground to solve the long-standing dark matter and dark energy problem in our universe. Perturbation analysis in this kind of model is very important in regard to the stability of itself as well as the cosmic microwave background observation. We have just initiated this in our current paper which shows a fairly complicated set of equations only for the perturbed junction condition across the domain wall. In our next paper we will consider this in detail.

[1] D. W. Joseph, Phys. Rev. **126**, 319 (1962).

[2] K. Akama, Lect. Notes Phys. **176**, 267 (1982); Prog. Theor. Phys. **60**, 1900 (1978); **78**, 184 (1987); **79**, 1299 (1988); **80**, 935 (1988); K. Akama and T. Hattori, Mod. Phys. Lett. A **15**, 2017 (2000).

[3] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **125B**, 136 (1983); M. Visser, Phys. Lett. **159B**, 22 (1985).

[4] P. Laguna-Castillo and R. A. Matzner, Nucl. Phys. **B282**, 542 (1987); E. J. Squires, Phys. Lett. **167B**, 286 (1986); G. W. Gibbons and D. L. Wiltshire, Nucl. Phys. **B287**, 717 (1987); J. M. Overduin and P. S. Wesson, Phys. Rep. **283**, 303 (1997).

[5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).

[6] G. Dvali and M. Shifman, Phys. Lett. B **396**, 64 (1997); Nucl. Phys. **B504**, 127 (1997).

[7] M. Cvetic and H. H. Soleng, Phys. Rep. **282**, 159 (1997).

[8] J. Hughes, J. Liu, and J. Polchinski, Phys. Lett. B **180**, 370 (1986).

[9] A. Lukas, B. A. Ovrut, and D. Waldram, Phys. Rev. D **61**, 023506 (1999); **60**, 086001 (1999).

[10] P. Kraus, J. High Energy Phys. 12 (1999) 011.

[11] T. Nihei, Phys. Lett. B **465**, 81 (1999); C. Csaki, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. B **462**, 34 (1999); P. Binetruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 285 (2000); D. Ida, J. High Energy Phys. 09 (2000) 014; C. Bercelo and M. Visser, Phys. Lett. B **482**, 183 (2000); L. Anchordoqui, C. Nunez, and K. Olsen, J. High Energy Phys. 10 (2000) 050; P. Bowcock, C. Charmousis, and R. Gregory, Classical Quantum Gravity **17**, 4745 (2000); C. Csaki, J. Erlich, and C. Grojean, Nucl. Phys. **B604**, 312 (2001); Y. S. Myung, Mod. Phys. Lett. A **17**, 1915 (2002); D. H. Coule, Classical Quantum Gravity **18**, 4265 (2001); J. P. Gregory and A. Padilla, Classical Quantum Gravity **19**, 4071 (2002); S. Nojiri, S. D. Odintsov, and S. Ogushi, arXiv:hep-th/0205187.

[12] W. Israel, Nuovo Cimento B **44**, 1 (1966); **48**, 463(E) (1967).

[13] S. Mukherji and M. Peloso, Phys. Lett. B **547**, 297 (2002); A. Biswas, S. Mukherji, and S. Sekhar Pal, Int. J. Mod. Phys. A **19**, 557 (2004); A. Biswas and S. Mukherji, J. Cosmol. Astropart. Phys. 02 (2006) 002; S. Mukherji and S. Pal, Mod. Phys. Lett. A **25**, 35 (2010).

[14] M. Novello and S. E. Perez Bergliaffa, Phys. Rep. **463**, 127 (2008), and references therein; C. P. Burgess, F. Quevedo, R. Raba, G. Tasinato, and I. Zavala, J. Cosmol. Astropart. Phys. 02 (**2004**) 008; M. R. Setare and F. Darabi, Int. J. Mod. Phys. D **16**, 1563 (2007); G. De. Risi, Phys. Rev. D **77**, 044030 (2008).

[15] D. Maity, Phys. Rev. D **78**, 084008 (2008); **78**, 084023 (2008).

[16] H. A. Chamblin and H. S. Reall, Nucl. Phys. **B562**, 133 (1999); A. Chamblin, M. J. Perry, and H. S. Reall, J. High Energy Phys. 09 (1999) 014.

[17] J. L. Hovdebo and R. C. Myers, J. Cosmol. Astropart. Phys. 11 (2003) 012.

[18] E. Lisi, S. Sarkar, and F. L. Villante, Phys. Rev. D **59**, 123520 (1999); K. A. Olive, G. Steigman, and T. P. Walker, Phys. Rep. **333-334**, 389 (2000); S. H. Hansen, G. Mangano, A. Melchiorri, G. Miele, and O. Pisanti, Phys. Rev. D **65**, 023511 (2001).

[19] J. Hamann, S. Hannestad, G. G. Raffelt, I. Tamborra, and Y. Y. Y. Wong, Phys. Rev. Lett. **105**, 181301 (2010).

[20] A. G. Riess, L. Macri, S. Casertano, H. Lampeitl, H. C. Ferguson, A. V. Filippenko, S. W. Jha, W. Li, and R. Chornock, Astrophys. J. **730**, 119 (2011); Z. Hou *et al.*, arXiv:1104.2333; T. L. Smith, S. Das, and O. Zahn, arXiv:1105.3246.

[21] P. Binetruy, .C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 285 (2000); T. Shiromizu, K. i. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000); V. V. Flambaum and E. V. Shuryak, Europhys. Lett. **74**, 813 (2006); A. Hebecker and J. March-Russell, Nucl. Phys. **B608**, 375 (2001).

[22] R. Maartens, Living Rev. Relativity **7**, 7 (2004), and references therein.