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Emergent universe by tunneling

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In this work we propose an alternative scheme for an emergent universe scenario where the universe is initially in a static state supported by a scalar field located in a false vacuum. The universe begins to evolve when, by quantum tunneling, the scalar field decays into a state of true vacuum. The emergent universe models are interesting since they provide specific examples of nonsingular inflationary universes.

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I. INTRODUCTION

Cosmological inflation has become an integral part of the standard model of the universe. Apart from being capable of removing the shortcomings of the standard cosmology, it gives important clues for large scale structure formation. The scheme of inflation [1–4] (see Ref. [5] for a review) is based on the idea that there was an early phase in which the universe evolved through accelerated expansion in a short period of time at high energy scales. During this phase, the universe was dominated by the potential $V(\phi)$ of a scalar field ϕ, which is called the inflaton.

Singularity theorems have been devised that apply in the inflationary context, showing that the universe necessarily had a beginning (according to classical and semiclassical theory) [6–10]. In other words, according to these theorems, the quantum gravity era cannot be avoided in the past even if inflation takes place. However, recently, models that escape this conclusion have been studied in Refs. [11–18]. These models do not satisfy the geometrical assumptions of these theorems. Specifically, the theorems assume that either (i) the universe has open space sections, implying $k = 0$ or -1, or (ii) the Hubble expansion rate H is bounded away from zero in the past, $H > 0$.

In particular, Refs. [11–18] consider closed models in which $k = +1$ and H can become zero, so that both assumptions (i) and (ii) of the inflationary singularity theorems are violated. In these models the universe is initially in a past eternal classical Einstein static (ES) state which eventually evolves into a subsequent inflationary phase. Such models, called emergent universe (EU), are appealing since they provide specific examples of nonsingular (geodesically complete) inflationary universes.

Normally in the emergent universe scenario, the universe is positively curved and initially it is in a past eternal classical Einstein static state which eventually evolves into a subsequent inflationary phase, see Refs. [11–18].

For example, in the original scheme [11,12], it is assumed that the universe is dominated by a scalar field (inflaton) ϕ with a scalar potential $V(\phi)$ that approach a constant V_0 as $\phi \longrightarrow -\infty$ and monotonically rise once the scalar field exceeds a certain value ϕ_0, see Fig. 1.

During the past-eternal static regime it is assumed that the scalar field is rolling on the asymptotically flat part of the scalar potential with a constant velocity, providing the conditions for a static universe. But once the scalar field exceeds some value, the scalar potential slowly droops from its original value. The overall effect of this is to distort the equilibrium behavior breaking the static solution. If the potential has a suitable form in this region, slow-roll inflation will occur, thereby providing a graceful entrance to early universe inflation.

This scheme for an emergent universe has been used not only on models based on general relativity [11,12], but also on models where nonperturbative quantum corrections of the Einstein field equations are considered [13,17,18], in the context of a scalar tensor theory of gravity [19,20] and recently in the framework of the so-called two measures field theories [21–24].

Another possibility for the emergent universe scenario is to consider models in which the scale factor asymptotically tends to a constant in the past [14,15,25–30].

We can note that both schemes for an emergent universe are not truly static during the static regime. For instance, in the first scheme during the static regime the scalar field is rolling on the flat part of its potential. On the other hand,

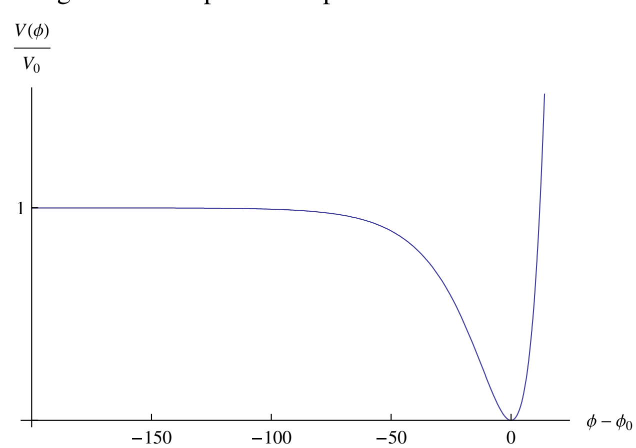

FIG. 1 (color online). Schematic representation of a potential for a standard emergent universe scenario.

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for the second scheme the scale factor is only asymptotically static.

In this paper we propose an alternative scheme for an emergent universe scenario, where the universe is initially in a truly static state. This state is supported by a scalar field which is located in a false vacuum ($\phi = \phi_F$), see Fig. 2. The universe begins to evolve when, by quantum tunneling, the scalar field decays into a state of true vacuum. Then, a small bubble of a new phase of field value ϕ_W can form, and expand as it converts volume from high to low vacuum energy and feeds the liberated energy into the kinetic energy of the bubble wall. This process was first studied by Coleman and De Luccia in Refs. [31,32].

Inside the bubble, spacelike surfaces of constant ϕ are homogeneous surfaces of constant negative curvature. One way of describing this situation is to say that the interior of the bubble always contains an open Friedmann-Robertson-Walker universe [32]. If the potential has a suitable form, inflation and reheating may occur in the interior of the bubble as the field rolls from ϕ_W to the true minimum at ϕ_T, in a similar way to what happens in models of open inflationary universes, see for example Refs. [33–37].

The advantage of this scheme (and of the emergent universe in general), over the eternal inflation scheme is that it corresponds to a realization of a singularity-free inflationary universe. In fact, eternal inflation is usually future eternal but it is not past eternal, because in general space-time that allows for inflation to be future eternal, cannot be past null complete [6–10]. On the other hand, emergent universes are geodesically complete.

Notice that in our scheme for an emergent universe, the metastable state which supports the initial static universe could exist only a finite amount of time. In our scheme of emergent universe, the principal point is not that the universe could have existed an infinite period of time, but that in our model the universe is nonsingular because the background where the bubble materializes is geodesically complete. This implies that we have to consider the problem of the initial conditions for a static universe. With respect to this point, there are very interesting possibilities discussed for example in the early works on EU, see Ref. [12].

One of these options is to explore the possibility of an emergent universe scenario within a string cosmology context, where it has been shown that the Einstein static universe is one of only two asymptotic solutions of the Ramond-Ramond sector of superstring cosmology [38].

Another possibility is that the initial Einstein static universe is created from *nothing* [39,40]. With respect to this, recently the possibility of a static universe created from *nothing* has been discussed by Mithani and Vilenkin in Ref. [41], where it is shown an explicit example.

The study of the Einstein static (ES) solution as a preferred initial state for our universe have been considered in the past, where it has been proposed that entropy considerations favor the ES state as the initial state for our universe, see Refs. [42,43].

In this paper we consider a simplified version of the emergent universe by tunneling, where the focus is on studying the process of creation and evolution of a bubble of true vacuum in the background of an ES universe.

This is motivated because we are mainly interested in the study of new ways of leaving the static period and beginning the inflationary regime in the context of emergent universe models.

In particular, in this paper we consider an inflaton potential similar to Fig. 3 and study the process of tunneling of the scalar field from the false vacuum ϕ_F to the true vacuum ϕ_T and the consequent creation and evolution of a bubble of true vacuum in the background of an ES universe.

The simplified model studied here contains the essential elements of the scheme we want to present (EU by tunneling), so we postpone the detailed study of the inflationary period, which occurs after the tunneling, for future work.

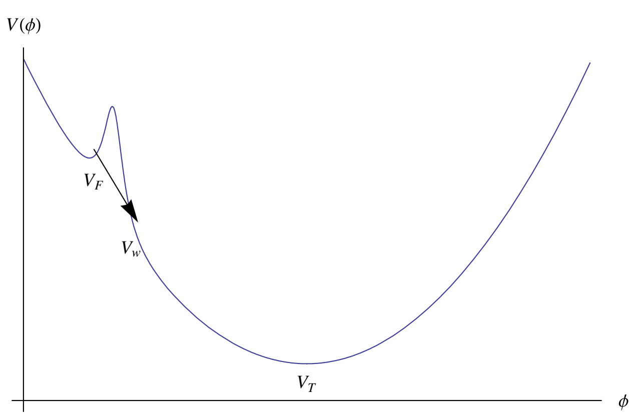

FIG. 2 (color online). A double-well inflationary potential $V(\phi)$. In the graph, some relevant values are indicated. They are the false vacuum $V_F = V(\phi_F)$ from which the tunneling begins, $V_W = V(\phi_W)$ where the tunneling stops and where the inflationary era begins, while $V_T = V(\phi_T)$ denote the true vacuum energy.

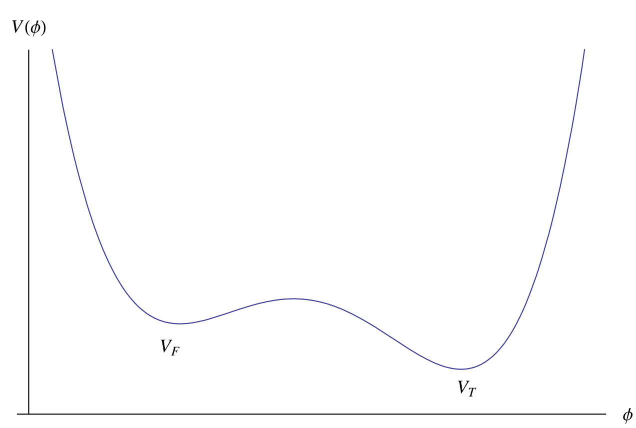

FIG. 3 (color online). Potential with a false and true vacuum.

Nevertheless, given the similarities, we expect that the behavior inside the bubble of the nonsimplified version of the EU by tunneling will be similar to the models of single-field open inflation. Then, if inflation inside the bubble is long, the universe will be almost exactly flat, see Refs. [33–37,44–53]. The density perturbations arise in these scheme in the usual way by the quantum fluctuation of the scalar field (inflaton) as the field slow-rolls to the true minimum. With respect to this, the general formula for the power spectrum for the single-field open inflation was given in Refs. [54,55]. Recently, in the context of the string landscape, the contribution to the cosmic microwave background radiation anisotropies of the perturbation in the open inflation scenario was studied in Ref. [56]. The detailed study of these topics is beyond the scope of this work, but we expect to return to these points in the near future.

The paper is organized as follows. In Sec. II we study an Einstein static universe supported by a scalar field located in a false vacuum. In Sec. III we study the tunneling process of the scalar field from the false vacuum to the true vacuum and the consequent creation of a bubble of true vacuum in the background of Einstein static universe. In Sec. IV we study the evolution of the bubble after its materialization. In Sec. V we summarize our results.

II. STATIC UNIVERSE BACKGROUND

Based on the standard emergent universe scenario, we consider that the universe is positively curved and it is initially in a past eternal classical Einstein static state. The matter of the universe is modeled by a standard perfect fluid $P = (\gamma - 1)\rho$ and a scalar field (inflaton) with energy density $\rho_\phi = \frac{1}{2}(\partial_t\phi)^2 + V(\phi)$ and pressure $P_\phi = \frac{1}{2}(\partial_t\phi)^2 - V(\phi)$. The scalar field potential $V(\phi)$ is depicted in Fig. 3. The global minimum of $V(\phi)$ is tiny and positive, at a field value ϕ_T, but there is also a local false minimum at $\phi = \phi_F$.

We have considered that the early universe is dominated by two fluids because in our scheme of the EU scenario, during the static regime the inflaton remains static at the false vacuum, in contrast to standard EU models where the scalar field rolls on the asymptotically flat part of the scalar potential. Then, in order to obtain a static universe we need to have another type of matter besides the scalar field. For this reason we have included a standard perfect fluid. For simplicity we are going to consider that there are no interactions between the standard perfect fluid and the scalar field.

The metric for the static state is given by the closed Friedmann-Robertson-Walker metric:

$$ds^2 = dt^2 - a(t)^2\left[\frac{dr^2}{1 - \frac{r^2}{R^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad (1)$$

where $a(t)$ is the scale factor, t represents the cosmic time, and the constant $R > 0$. We have explicitly written R in the metric in order to make more clear the effects of the curvature on the bubble process (probability of creation and propagation of the bubble).

Given that there are no interactions between the standard fluid and the scalar field, they separately obey energy conservation and Klein Gordon equations,

$$\partial_t\rho + 3\gamma H\rho = 0, \quad (2)$$

$$\partial_t^2\phi + 3H\partial_t\phi = -\frac{\partial V(\phi)}{\partial\phi}, \quad (3)$$

where $H = \partial_t a/a$.

The Friedmann and the Raychaudhuri field equations become

$$H^2 = \frac{8\pi G}{3}\left(\rho + \frac{1}{2}(\partial_t\phi)^2 + V(\phi)\right) - \frac{1}{R^2a^2}, \quad (4)$$

$$\partial_t^2 a = -\frac{8\pi G}{3}a\left[\left(\frac{3}{2}\gamma - 1\right)\rho + \dot{\phi}^2 - V(\phi)\right]. \quad (5)$$

The static universe is characterized by the conditions $a = a_0 = Cte$, $\partial_t a_0 = \partial_t^2 a_0 = 0$, and $\phi = \phi_F = Cte$, $V(\phi_F) = V_F$ corresponding to the false vacuum.

From Eqs. (2) to (5), the static solution for a universe dominated by a scalar field placed in a false vacuum and a standard perfect fluid are obtained if the following conditions are met

$$\rho_0 = \frac{1}{4\pi G}\frac{1}{\gamma R^2 a_0^2}, \quad (6)$$

$$V_F = \left(\frac{3}{2}\gamma - 1\right)\rho_0, \quad (7)$$

where ρ_0 is energy density of the perfect fluid present in the static universe. Note that $\gamma > 2/3$ in order to have a positive scalar potential.

By integrating Eq. (2) we obtain

$$\rho = \frac{A}{a^{3\gamma}}, \quad (8)$$

where A is an integration constant. By using this result, we can rewrite the conditions for a static universe as follows

$$A = \frac{1}{4\pi G}\frac{a_0^{3\gamma-2}}{\gamma R^2}, \quad (9)$$

$$V_F = \left(\frac{3}{2}\gamma - 1\right)\frac{1}{4\pi G}\frac{1}{\gamma R^2 a_0^2}. \quad (10)$$

In a purely classical field theory if the universe is static and supported by the scalar field located at the false vacuum V_F, then the universe remains static forever. Quantum mechanics makes things more interesting because the field can tunnel through the barrier and by this process create a small bubble where the field value is ϕ_T.

Depending on the background where the bubble materializes, the bubble could expanded or collapsed [57,58].

III. BUBBLE NUCLEATION

In this section we study the tunneling process of the scalar field from the false vacuum to the true vacuum and the consequent creation of a bubble of true vacuum in the background of Einstein static universe.

Given that in our case the geometry of the background corresponds to an Einstein static universe and not a de Sitter space, we proceed following the scheme developed in Refs. [57,59], instead of the usual semiclassical calculation of the nucleation rate based on instanton methods [32].

In particular, we will consider the nucleation of a spherical bubble of true vacuum V_T within the false vacuum V_F. We will assume that the layer which separates the two phases (the wall) is of negligible thickness compared to the size of the bubble (the usual thin-wall approximation). The energy budget of the bubble consists of latent heat (the difference between the energy densities of the two phases) and surface tension.

In order to eliminate the problem of predicting the reaction of the geometry to an essentially acausal quantum jump, we neglect during this computation the gravitational backreaction of the bubble onto the space-time geometry.

The gravitational backreaction of the bubble will be considered in the next chapter when we study the evolution of the bubble after its materialization.

In our case the shell trajectory follows from the action (see Refs. [59,60])

$$S = \int dy\{2\pi\epsilon\bar{a}_0^4[\chi - \cos(\chi)\sin(\chi)] - 4\pi\sigma\bar{a}_0^3\sin^2(\chi)\sqrt{1-\chi'^2}\}, \tag{11}$$

where we have denoted the coordinate radius of the shell as χ, and we have written the static ($a = a_0 = Cte$) version of the metric Eq. (1) as

$$ds^2 = \bar{a}_0^2(dy^2 - d\chi^2 - \sin^2(\chi)d\Omega^2), \tag{12}$$

with $\frac{r}{R} = \sin(\chi)$, $\bar{a}_0 = Ra_0$, $dt = \bar{a}_0 dy$ and prime means derivatives respect to y.

In the action (11), ϵ and σ denote, respectively, the latent heat and the surface energy density (surface tension) of the shell.

The action (11) describes the classical trajectory of the shell after the tunneling. This trajectory emanates from a classical turning point, where the canonical momentum

$$P = \frac{\partial S}{\partial \chi'} = 4\pi\sigma\bar{a}_0^3\chi'\frac{\sin^2(\chi)}{\sqrt{1-\chi'^2}}, \tag{13}$$

vanishes [59]. In order to consider tunneling, we evolve this solution back to the turning point, and then try to shrink the bubble to zero size along a complex y contour, see Refs. [57,59]. For each solution, the semiclassical tunneling rate is determined by the imaginary part of its action, see Ref. [59]:

$$\Gamma \approx e^{-2\mathrm{Im}[S]}. \tag{14}$$

From the action (11) we found the equation of motion

$$\frac{\sin^2(\chi)}{\sqrt{1-\chi'^2}} = \frac{\epsilon\bar{a}_0}{2\sigma}[\chi - \cos(\chi)\sin(\chi)]. \tag{15}$$

The action (11) can be put in a useful form by using Eq. (15), and changing variables to χ:

$$S = \int d\chi\frac{4\pi}{3}\epsilon a_0^4\sin^2(\chi)\sqrt{\left(\frac{3[\chi - \cos(\chi)\sin(\chi)]}{2\sin^2(\chi)}\right)^2 - \bar{r}_0^2}, \tag{16}$$

where $\bar{r}_0 = \frac{r_0}{R}$ and $r_0 = \frac{3\sigma}{\epsilon a_0}$ is the radio of nucleation of the bubble when the space is flat ($R \to \infty$) and static (i.e., when the space is Minkowsky).

The nucleation radius $\bar{\chi}$ (i.e., the coordinate radius of the bubble at the classical turning point), is a solution to the condition $P = 0$. Then from Eq. (13) we obtain

$$\frac{\bar{\chi} - \cos(\bar{\chi})\sin(\bar{\chi})}{\sin^2(\bar{\chi})} = \frac{2\sigma}{\epsilon\bar{a}_0}. \tag{17}$$

The action (11) has an imaginary part coming from the part of the trajectory $0 < \chi < \bar{\chi}$, when the bubble is tunneling:

$$\mathrm{Im}[S] = \frac{4\pi}{3}\epsilon a_0^4\int_0^{\bar{\chi}} d\chi\sin^2(\chi) \times\sqrt{\bar{r}_0^2 - \left(\frac{3[\chi - \cos(\chi)\sin(\chi)]}{2\sin^2(\chi)}\right)^2}. \tag{18}$$

Expanding (18) at first nonzero contribution in $\beta = (r_0/R)^2$ we find

$$\mathrm{Im}[S] = \frac{27\sigma^4\pi}{4\epsilon^3}\left[1 - \frac{1}{2}\beta^2\right]. \tag{19}$$

This result is in agreement with the expansion obtained in Ref. [61]. Then, the nucleation rate is

$$\Gamma \approx e^{-2\mathrm{Im}S} \approx \exp\left[-\frac{27\sigma^4\pi}{2\epsilon^3}\left(1 - \frac{9\sigma^2}{2\epsilon^3a_0^2R^2}\right)\right]. \tag{20}$$

We can note that the probability of the bubble nucleation is enhanced by the effect of the curvature of the closed static universe background.

IV. EVOLUTION OF THE BUBBLE

In this section we study the evolution of the bubble after the process of tunneling. During this study we are going to consider the gravitational backreaction of the bubble. We follow the approach used in Ref. [58] where it is assumed that the bubble wall separates space-time into two parts, described by different metrics and containing different kinds of matter. The bubble wall is a timelike, spherically symmetric hypersurface Σ, the interior of the bubble is described by a de Sitter space-time and the exterior by the static universe discussed in Sec. II. The Israel junction conditions [62] are implemented in order to join these two manifolds along their common boundary Σ. The evolution of the bubble wall is determined by implementing these conditions.

We will follow the scheme and notation of Fischler *et al.* [58]. Then, Latin and Greek indices denote 3-dimensional objects defined on the shell and 4-dimensional quantities, respectively. The projectors are $e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$ and the semicolon is shorthand for the covariant derivative. Unit as such that $8\pi G = 1$.

In particular, the exterior of the bubble is described by the metric Eq. (1) and the equations (2)–(5), previously discussed in Sec. II. At the end, the static solution for these equations will be assumed. The interior of the bubble will be described by the metric of the de Sitter space-time in its open foliation, see Ref. [32]

$$ds^2 = dT^2 - b^2(T)\left(\frac{dz^2}{1+z^2} + z^2 d\Omega_2\right), \tag{21}$$

where the scale factor satisfies

$$\left(\frac{db}{dT}\right)^2 = \left(\frac{V_T}{3}\right)b^2(T) + 1. \tag{22}$$

These two regions are separated by the bubble wall Σ, which will be assumed to be a thin-shell and spherically symmetric. Then, the intrinsic metric on the shell is [63]

$$ds^2|_\Sigma = d\tau^2 - B^2(\tau)d\Omega_2, \tag{23}$$

where τ is the shell proper time.

Now we proceed to impose the Israel conditions in order to joint the manifolds along their common boundary Σ. The first of Israel's conditions impose that the metric induced on the shell from the bulk 4-metrics on either side should match, and be equal to the 3-metric on the shell. Then by looking from the outside to the bubble-shell we can parametrize the coordinates $r = x(\tau)$ and $t = t(\tau)$, obtaining the following match conditions, see Ref. [58]

$$a(t)x = B(\tau), \qquad \left(\frac{dt}{d\tau}\right)^2 = 1 + \frac{a(t)^2}{1-(\frac{x}{R})^2}\left(\frac{dx}{d\tau}\right)^2, \tag{24}$$

where all the variables in these equations are thought as functions of τ. On the other hand, the angular coordinates of metrics (1) and (23) can be just identified in virtue of the spherical symmetry.

The second junction condition could be written as follows

$$[K_{ab}] - h_{ab}[K] = S_{ab}, \tag{25}$$

where K_{ab} is the extrinsic curvature of the surface Σ, and square brackets stand for discontinuities across the shell. Following [58], we assume that the surface energy-momentum tensor S_{ab} has a perfect fluid form given by $S_\tau{}^\tau \equiv \sigma$ and $S_\theta{}^\theta = S_\phi{}^\phi \equiv -\bar{P}$, where $\bar{P} = (\bar{\gamma} - 1)\sigma$. Also, because of the spherical symmetry and the form of the metric Eq. (23), the extrinsic curvature $K_a{}^b$ has only independent components $K_\tau{}^\tau$ and $K_\theta{}^\theta = K_\phi{}^\phi$. Then, from the second junction condition we obtain the following independent equations

$$-\frac{\sigma}{2} = [K^\theta_\theta], \tag{26}$$

$$\bar{P} = [K^\tau_\tau] + [K^\theta_\theta], \tag{27}$$

where σ and $\bar{P}$ are considered as purely functions of τ. Also, the junctions conditions imply a conservation law [63], which in this case take the following form

$$\frac{d\sigma}{d\tau} + \frac{2}{B}\frac{dB}{d\tau}(\sigma + P) + [T^n_\tau] = 0, \tag{28}$$

where

$$[T^n_\tau] = (e^\alpha_\tau T^\beta_\alpha n_\beta)_{\text{out}} - (e^\alpha_\tau T^\beta_\alpha n_\beta)_{\text{in}}, \tag{29}$$

and n_α is the outward normal vector to the surface Σ.

The evolution of the shell is completely determined by Eq. (26) and (28). Following [58] we write these matching conditions in terms of the outside coordinates.

The extrinsic curvature could be written as

$$K_{ab} = n_{\alpha;\beta}e^\alpha_a e^\beta_b. \tag{30}$$

The projectors of the static side are

$$u^\alpha \equiv e^\alpha_\tau = \left(\frac{dt}{d\tau}, \frac{dx}{d\tau}, 0, 0\right), \tag{31}$$

$$e^\alpha_\theta = (0, 0, 1, 0), \qquad e^\alpha_\phi = (0, 0, 0, 1). \tag{32}$$

We can note that u^α is the 4-velocity of the bubble-shell. Then we obtain

$$n_\alpha = \frac{a}{\sqrt{1-(\frac{x}{R})^2}}(-\dot{x}, \dot{t}, 0, 0), \tag{33}$$

where dots mean differentiation with respect to τ and we have used the following conditions $u^\alpha n_\alpha = 0$ and $n^\alpha n_\alpha = -1$, in order to determinate n_α.

Then K^θ_θ on the static side becomes

$$K^\theta_{\theta(\text{out})} = \left(\frac{ax\dot{x}a_{,t} + (1 - x^2/R^2)\dot{t}}{B\sqrt{1-(\frac{x}{R})^2}}\right). \tag{34}$$

Repeating the above calculation for K^θ_θ on the inside we obtain

$$K^{\theta}_{\theta(\mathrm{in})} = \left(\frac{zb\frac{db}{dT}\dot{z} + (1+z^2)\dot{T}}{B\sqrt{1+z^2}}\right). \tag{35}$$

By using Eq. (34) and (35) we can obtain the explicit form of the junction condition Eq. (26). Nevertheless, it is most convenient to write this condition as follows, see Refs. [58,63],

$$\sqrt{\dot{B}^2 - \Delta_{\mathrm{out}}} - \sqrt{\dot{B}^2 - \Delta_{\mathrm{in}}} = -\frac{\sigma B}{2}. \tag{36}$$

Where we have defined

$$\Delta_{\mathrm{out}} = -1 + \left(\frac{A}{3a^{3\gamma}} + \frac{V_F}{3}\right)B^2, \tag{37}$$

$$\Delta_{\mathrm{in}} = -1 + \frac{V_T}{3}B^2. \tag{38}$$

Now we proceed to write the equations for the evolution of the bubble in outside coordinates. In order to do that we rewrite Eq. (36), by using Eqs. (37) and (38), obtaining

$$\dot{B}^2 = B^2C^2 - 1, \tag{39}$$

where

$$C^2 = \frac{V_T}{3} + \left(\frac{\sigma}{4} + \frac{1}{\sigma}\left[\frac{V_F - V_T}{3} + \frac{A}{3a^{3\gamma}}\right]\right)^2. \tag{40}$$

In the outside coordinates we parametrize $x(t)$ as the curve for the bubble evolution (the bubble radius in these coordinates). Since x and t are dependent variables on the shell, this is legitimate. We write $B = ax$, then by using $\dot{B} = a_{,t}x\dot{t} + a\dot{x}$ and

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{a^2}{(1-x^2/R^2)}(\frac{dx}{dt})^2}}, \tag{41}$$

obtained from Eq. (24), we can express Eq. (39) as follows

$$\frac{dx}{dt} = \pm\sqrt{\frac{(R^2 - x^2)(a_0^2C^2x^2 - 1)}{x^2a_0^2(a_0^2C^2R^2 - 1)}}. \tag{42}$$

The evolution of σ is determinate by Eq. (28) which could be converted to outside coordinates by using Eq. (41) obtaining

$$\frac{d\sigma}{dt} = -2\left(\frac{\bar{\gamma}\sigma}{x}\right)\frac{dx}{dt} + \frac{a_0\gamma\rho_0}{\sqrt{-(\frac{dx}{dt})^2a_0^2 + 1 - \frac{x^2}{R^2}}}\frac{dx}{dt}. \tag{43}$$

The positive energy condition $\sigma > 0$ together with Eq. (36) impose the following restriction to σ

$$0 < \sigma \leq 2\sqrt{\frac{V_F - V_T}{3} + \frac{\rho_0}{3}}. \tag{44}$$

Also, from the definition of x and Eq. (42) we obtain the following restriction for x

$$\frac{1}{a_0C} \leq x \leq R. \tag{45}$$

We solved the Eqs. (42) and (43) numerically by considering different kinds and combinations of the matter

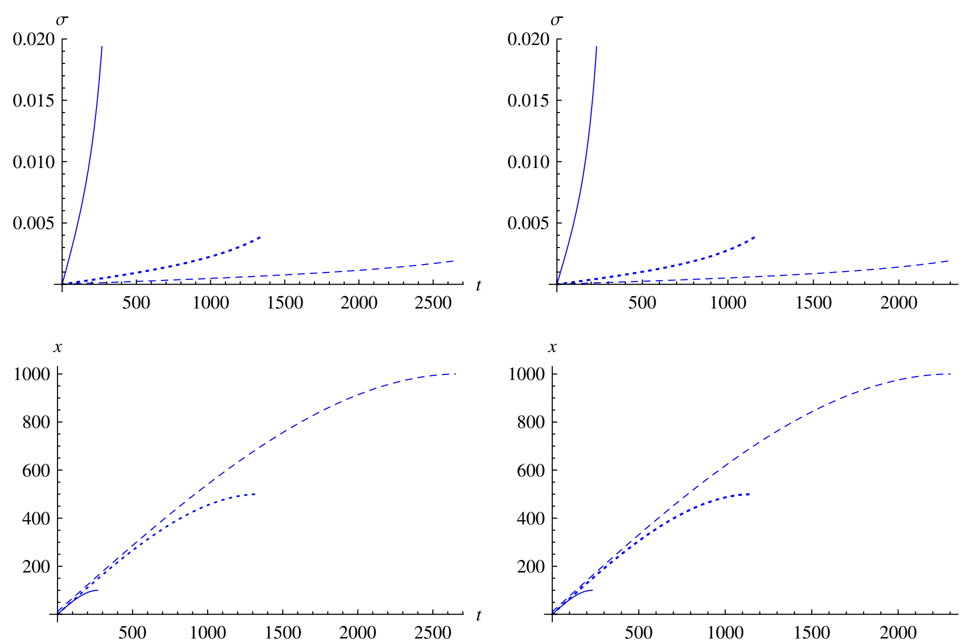

FIG. 4 (color online). Time evolution of the bubble in the outside coordinates $x(t)$, and time evolution of the surface energy density $\sigma(t)$. The left panel is for a static universe dominated by dust and the bubble wall containing dust. The right panel is the same situation but with radiations instead of dust. In all these graphics we have considered dashed line for $R = 1000$, dotted line for $R = 500$, and continuous line for $R = 100$.

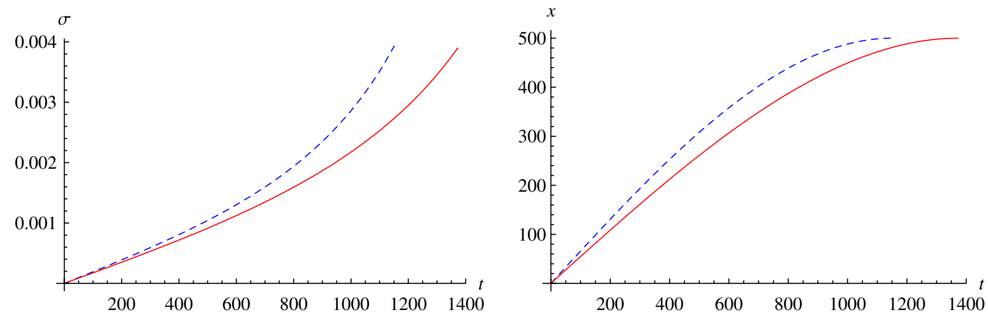

FIG. 5 (color online). Time evolution of the bubble in the outside coordinates $x(t)$, and time evolution of the surface energy density $\sigma(t)$, for a background with $R = 500$. Dashed line corresponds to a static universe dominated by dust and bubble wall containing radiation. Continuous line corresponds to a static universe dominated by radiation and a bubble wall containing dust.

content of the background and the bubble wall. From these solutions we found that once the bubble has materialized in the background of an ES universe, it grows, filling completely the background space.

In order to find the numerical solutions we chose the following values for the free parameters of the model, in units where $8\pi G = 1$:

$$a_0 = 1, \tag{46}$$

$$V_T = 0.1 V_F, \tag{47}$$

$$\sigma_{\rm init} = 10^{-6}. \tag{48}$$

The other parameters are fixed by the conditions discussed in Sec. II.

Some of the numerical solutions are shown in Figs. 4 and 5) where the evolution of the bubble, as seen by the outside observer, is illustrated. In these numerical solutions we have considered three different curvature radii ($R = 1000$, $R = 500$, $R = 100$) and various matter contents combinations for the background and the bubble wall. From these examples we can note that the bubble of the new face grows to fill the background space, where the shell coordinate asymptotically tends to the curvature radius R.

V. CONCLUSIONS

In this paper we explore an alternative scheme for an emergent universe scenario, where the universe is initially in a truly static state. This state is supported by a scalar field which is located in a false vacuum. The universe begins to evolve when, by quantum tunneling, the scalar field decays into a state of true vacuum.

In particular, in this work we study the process of tunneling of a scalar field from the false vacuum to the true vacuum and the consequent creation and evolution of a bubble of true vacuum in the background of Einstein static universe. The motivation in doing this is because we are interested in the study of new ways of leaving the static period and beginning the inflationary regime in the context of emergent universe models.

In the first part of the paper, we study an Einstein static universe dominated by two fluids, one is a standard perfect fluid and the other is a scalar field located in a false vacuum. The requisites for obtaining a static universe under these conditions are discussed. As was shown by Eddington [64], this static solution is unstable to homogeneous perturbations, furthermore it is always neutrally stable against small inhomogeneous vector and tensor perturbations and neutrally stable against adiabatic scalar density inhomogeneities with high enough sound speed [42,43,65,66]. This situation has implications for the EU scenario, see discussion bellow.

In the second part of the paper, we study the tunneling process of the scalar field from the false vacuum to the true vacuum and the consequent creation of a bubble of true vacuum in the background of Einstein static universe. Following the formalism presented in Ref. [59] we found the semiclassical tunneling rate for the nucleation of the bubble in this curved space. We conclude that the probability for the bubble nucleation is enhanced by the effect of the curvature of the closed static universe background.

In the third part of the paper, we study the evolution of the bubble after its materialization. By following the formalism developed by Israel [62] we found that once the bubble has materialized in the background of an ES universe, it grows filling completely the background space. In particular, we use the approach of Fischler *et al.* [58] to find the equations which govern the evolution of the bubble in the background of the ES universe. These equations are solved numerically, some of these solutions, concerning several types of matter combinations for the background and the bubble wall, are shown in Figs. 4 and 5.

In summary we have found that this new mechanism for an emergent universe is plausible and could be an interesting alternative to the realization of the emergent universe scenario.

We have postponed for future work the study of this mechanism applied to emergent universe based on alternative theories to general relativity (GR), like

Jordan-Brans-Dicke [67], which present stable past eternal static regime [19,20]. It is interesting to explore this possibility because emergent universe models based on GR suffer from instabilities, associated with the instability of the Einstein static universe. This instability is possible to cure by going away from GR, for example, by consider a Jordan Brans Dicke theory at the classical level, where it has been found that contrary to general relativity, a static universe could be stable, see Refs. [19,20]. Another possibility is considering nonperturbative quantum corrections of the Einstein field equations, either coming from a semi-classical state in the framework of loop quantum gravity [13,17] or braneworld cosmology with a timelike extra dimension [16,18]. In addition to this, consideration of the Starobinsky model, exotic matter [14,15] or the so-called two measures field theories [21–24] also can provide a stable initial state for the emergent universe scenario.

On the other hand, in the context of GR, the instability of the ES could be overcome by considering a static universe filled with a noninteracting mixture of isotropic radiation and a ghost scalar field [68] or by considering a negative cosmological constant with a universe dominated by a exotic fluid satisfies $P = (\gamma - 1)\rho$ with $0 < \gamma < 2/3$, see Ref. [69]. In this case it is important that the exotic matter source should not be a perfect fluid. It could be, for example, an assembly of randomly oriented domain walls [70].

We are interested in applying the scheme of emergent universe by tunneling developed here to models which present stable past eternal static regimes in the near future.

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