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New physics in $\bar{b} \to \bar{s}$ transitions and the $B^0_{d,s} \to V_1 V_2$ angular analysis

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We suppose that there is new physics (NP) in $\bar{b} \to \bar{s}$ transitions and examine its effect on the angular distribution of $B^0_q \to V_1 V_2$ ($q = d, s$), where $V_{1,2}$ are vector mesons. We find that, in the presence of such NP, the formulas relating the parameters of the untagged, time-integrated angular distribution to certain observables (polarization fractions, CP-violating triple-product asymmetries, CP-conserving interference term) must be modified from their standard model forms. This modification is due in part to a nonzero B^0_q-$\bar{B}^0_q$ width difference, which is significant only for B^0_s decays. We reanalyze the $B^0_s \to \phi\phi$ data to see the effect of these modifications. As $\Delta\Gamma_s/2\Gamma_s \sim 10\%$, there are $O(10\%)$ changes in the derived observables. These are not large but may be important given that one is looking for signals of NP. In addition, if the NP contributes to the $\bar{b} \to \bar{s}$ decay, the measurement of the untagged time-dependent angular distribution provides enough information to extract all the NP parameters.

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I. INTRODUCTION

As recently as a year ago, there were several hints of physics beyond the standard model (SM) in $\bar{b} \to \bar{s}$ transitions. For example, the CDF [1] and D0 [2] Collaborations measured the CP asymmetry in $B^0_s \to J/\psi\phi$, and found a hint for indirect (mixing-induced) CP violation. This is counter to the expectation of the SM, which predicts this CP violation to be $\simeq 0$. In general, this result was interpreted as evidence for a nonzero value of the weak phase of B^0_s-$\bar{B}^0_s$ mixing ($2\beta_s$), and the contributions of various new physics (NP) models to the B_s mixing phase were explored [3–9]. It was also pointed out that NP in the decay $\bar{b} \to \bar{s}c\bar{c}$ could also play an important role [10]. In addition, the SM predicts that the measured indirect CP asymmetry in $\bar{b} \to \bar{s}s\bar{s}$ penguin decays should generally be equal to that found in charmonium decays such as $B^0_d \to J/\psi K_S$. However, it was found that these two quantities were not identical for several decays [11]. As a third example, the CDF Collaboration reported the measurement of $B(B^0_s \to \mu^+\mu^-) = (1.8^{+1.1}_{-0.9}) \times 10^{-8}$ [12]. This is larger than the SM prediction for this branching ratio, which is $B(B^0_s \to \mu^+\mu^-) = (3.35 \pm 0.32) \times 10^{-9}$ [13]. There were a number of other effects—in all cases, the disagreement with the SM was not large, $\leq 2\sigma$. Still, it was intriguing that all appear in $\bar{b} \to \bar{s}$ transitions.

In addition, the D0 Collaboration reported an anomalously large CP-violating like-sign dimuon charge asymmetry in the B system. In Ref. [14], the asymmetry was found to be

$$A^b_{\rm sl} = -(9.57 \pm 2.51 \pm 1.46) \times 10^{-3}, \tag{1}$$

which is a 3.2σ deviation from the SM prediction, $A^{b,{\rm SM}}_{\rm sl} = (-2.3^{+0.5}_{-0.6}) \times 10^{-4}$ [15]. In fact, the updated measurement [16] exhibits an even larger discrepancy:

$$A^b_{\rm sl} = -(7.87 \pm 1.72 \pm 0.93) \times 10^{-3}, \tag{2}$$

a 3.9σ deviation. This suggests NP in B^0_d-$\bar{B}^0_d$ and/or B^0_s-$\bar{B}^0_s$ mixing.

This was quantified in Ref. [17]. Here, NP only in B^0_q-$\bar{B}^0_q$ ($q = d, s$) mixing was considered (i.e., in M^q_{12}); NP in the decay was excluded. Three different NP scenarios were examined. In all cases a fit was performed to all data that is affected by NP in B^0_q-$\bar{B}^0_q$ mixing. This includes ${\rm BR}(B \to \tau\nu)$, whose measured value disagrees with the SM fit prediction [18], and possibly points to NP in B^0_d-$\bar{B}^0_d$ mixing. The details of the conclusions depend on the NP scenario, but a NP contribution to B^0_q-$\bar{B}^0_q$ mixing of up to 40% is possible. We therefore see that, at this time, NP in $\bar{b} \to \bar{s}$ transitions was entirely conceivable.

However, with recent measurements, many of the NP hints have largely disappeared, or at least been reduced. First, LHCb has measured the indirect CP asymmetry in $B^0_s \to J/\psi\phi$ and finds $\phi^{c\bar{c}s}_s \simeq 0$, in agreement with the SM[1] [19]. Specifically, they measure $\phi^{c\bar{c}s}_s = (-0.06 \pm 5.77({\rm stat}) \pm 1.54({\rm syst}))^\circ$. Still, the errors are large enough that NP cannot be excluded. Second, with the latest indirect CP asymmetry data, the Heavy Flavor Averaging Group [20] finds that the B^0_d-$\bar{B}^0_d$ mixing phase $\sin 2\beta$ is measured to be (i) 0.68 ± 0.02 in charmonium decays, and (ii) 0.64 ± 0.03 in $\bar{b} \to \bar{s}s\bar{s}$ penguin decays. These numbers are quite similar, so that no real discrepancy can be claimed. On the other hand, several of the $\bar{b} \to \bar{s}s\bar{s}$ decays

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[1] $\beta_s \equiv \arg[-(V^*_{tb}V_{ts})/(V^*_{cb}V_{cs})]$. For the measured B^0_s-$\bar{B}^0_s$ mixing phase, it is common to use the symbol $\phi^{c\bar{c}s}_s$, which is equal to $-2\beta_s$ in the SM.

have additional contributions with a different weak phase, and so the Heavy Flavor Averaging Group urges that the *naive average* in (ii) be used with extreme caution. Third, the recent LHCb update does not confirm the CDF $B_s^0 \to \mu^+\mu^-$ result [21]. They improve the present upper bound to $B(B_s^0 \to \mu^+\mu^-) \leq 4.5(3.8) \times 10^{-9}$ at 95% C.L. (90% C.L.), in agreement with the SM. Most of the other effects have similarly gone away, or are simply not large enough to be compelling.

On the other hand, the D0 measurement of an anomalously large $A_{\rm sl}^b$ is still present. However, there have been direct measurements of the semileptonic charge asymmetry in B_s^0 [22,23] and B_d^0 [24] decays. While these results show no significant deviation from the SM predictions, the errors are large enough that they are also not in contradiction with the D0 measurement.

Reference [25] presents an update of the analysis of Ref. [17], including the latest LHCb results. It is found that the SM is still disfavored, by 2.4σ. However, in contrast to Ref. [17], the problem cannot now be rectified by NP in $M_{12}^{d,s}$ alone. But if one also allows NP contributions to $\Gamma_{12}^{d,s}$ (i.e., the width differences), the data can be accommodated.

The bottom line is that, at present, the status of $\bar{b} \to \bar{s}$ NP is uncertain. The effect of such NP cannot be very large, but a smaller effect is still possible. In this paper we make the assumption that NP *is* present in $\bar{b} \to \bar{s}$ transitions. However, in addition to taking into account its effect on B_s mixing, which is what is conventionally done, we also consider its effect on $\bar{b} \to \bar{s}$ decays. The main aim is to examine the effect of $\bar{b} \to \bar{s}$ NP on the angular distribution of $B_q^0 \to V_1 V_2$ ($q = d, s$), where $V_{1,2}$ are vector mesons. In particular, we consider final states that are self-conjugate, so that both B_q^0 and $\bar{B}_q^0$ can decay to $V_1 V_2$, generating indirect (mixing-induced) CP-violating effects.

There are three classes of B_q^0 decays that can be affected by $\bar{b} \to \bar{s}$ NP:

(1) B_s^0 decays with $\bar{b} \to \bar{s}$,
(2) B_d^0 decays with $\bar{b} \to \bar{s}$,
(3) B_s^0 decays with $\bar{b} \to \bar{d}$.

Our analysis is completely general and can be applied to any of these classes. However, we also focus specifically on $B_s^0 \to \phi\phi$. There are two reasons. First, this is a pure $\bar{b} \to \bar{s}$ penguin decay, and so there can well be NP contributions to any of the loop-level penguin decay amplitudes.[2] Second, the untagged angular distribution of the decay has already been measured by the CDF [27] and LHCb [28] Collaborations, and so their results can be (re) interpreted in the context of $\bar{b} \to \bar{s}$ NP contributions.

The result of this analysis—and this is the main point of the paper—is as follows. The parameters of the untagged, time-integrated angular distribution can be measured experimentally. Certain observables can be derived from these parameters. However, in the presence of NP, the formulas that relate the observables and parameters are modified compared to their SM forms. There are six terms ($i = 1$–6) in the angular distribution, and we correspondingly find six observables for which the relation between the experimental data and theoretical parameters must be modified. For $i = 1$–3 they are the polarization fractions, for $i = 4, 6$ they are the CP-violating triple-product asymmetries, and $i = 5$ corresponds to a CP-conserving observable. The modifications for the polarization fractions are particularly striking. Here there are corrections to the SM formulas that are proportional to the width difference in the B_q^0-$\bar{B}_q^0$ system. Now, the width difference $\Delta\Gamma$ is sizeable only for B_s^0 decays.[3] Thus, the formulas' modifications due to NP are important only for class-(1) and class-(3) decays, which include $B_s^0 \to \phi\phi$. $\Delta\Gamma_s/2\Gamma_s \sim 10\%$, so that the modifications lead to $O(10\%)$ changes in the derived observables. These are not large but may be important given that one is looking for signals of NP.

Another result is that, if the untagged, time-dependent angular distribution can be measured, 12 observables can be obtained. If the NP contributes to the $\bar{b} \to \bar{s}$ decay, there are fewer than 12 unknown NP parameters. Thus, all of these parameters can be extracted from the angular distribution. This may allow the identification of the NP.

We begin in Sec. II by presenting the most general $B_{d,s}^0 \to V_1 V_2$ angular distribution, allowing for NP in the mixing and/or the decay. We consider the angular distribution for several different scenarios: at $t = 0$ (Sec. II B 1), time dependent (Sec. II B 2), untagged time dependent (Sec. II C), untagged time integrated (Sec. II D). In Sec. III we examine the untagged time-dependent and time-integrated distributions for $B_s^0 \to \phi\phi$ within the SM. The study of $B_s^0 \to \phi\phi$ is extended to the SM + NP in Sec. IV. We discuss observables such as the polarization fractions, CP-violating triple-product asymmetries, and the CP-conserving interference term, and note the changes in the formulas used for their extraction necessitated by the inclusion of $\bar{b} \to \bar{s}$ NP. We also show that all the unknown NP parameters in the $\bar{b} \to \bar{s}$ decay can be determined from the measurement of the untagged, time-dependent angular distribution. In Sec. V we present a numerical reanalysis of the $B_s^0 \to \phi\phi$ data allowing for the possibility of $\bar{b} \to \bar{s}$ NP contributions in the decay. We conclude in Sec. VI.

II. $B \to V_1 V_2$ ANGULAR DISTRIBUTION

A. Generalities

The most general Lorentz-covariant amplitude for the decay $B(p) \to V_1(k_1, \varepsilon_1) + V_2(k_2, \varepsilon_2)$ is given by [31,32]

[2] $B_s^0 \to \phi\phi$ and $B_s^0 \to J/\psi\phi$ were examined in Ref. [26], but only NP in B_s^0-$\bar{B}_s^0$ mixing was considered, not NP in the decay.

[3] There are many theoretical methods that rely on a sizeable $\Delta\Gamma_s$. Two recent examples are given by Refs. [29,30].

$$M = a\varepsilon_1^* \cdot \varepsilon_2^* + \frac{b}{m_B^2}(p \cdot \varepsilon_1^*)(p \cdot \varepsilon_2^*) + i\frac{c}{m_B^2}\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon_1^{*\rho}\varepsilon_2^{*\sigma}, \tag{3}$$

where $q \equiv k_1 - k_2$. The quantities a, b, and c are complex and contain in general both CP-conserving strong phases and CP-violating weak phases. In $B \to V_1V_2$ decays, the final state can have total spin 0, 1, or 2, which correspond to the V_1 and V_2 having relative orbital angular momentum $l = 0$ (s wave), $l = 1$ (p wave), or $l = 2$ (d wave), respectively. The a and b terms correspond to combinations of the parity-even s- and d-wave amplitudes, while the c term corresponds to the parity-odd p-wave amplitude.

In order to obtain the angular distribution for $B \to V_1V_2$, one uses the linear polarization basis. Here, one decomposes the decay amplitude into components in which the polarizations of the final-state vector mesons are either longitudinal (A_0), or transverse to their directions of motion and parallel ($A_\|$) or perpendicular ($A_\perp$) to one another. The transversity amplitudes A_h ($h = 0, \|, \perp$) are related to a, b, and c of Eq. (3) via [32]

$$A_\| = \sqrt{2}a, \qquad A_0 = -ax - \frac{m_1m_2}{m_B^2}b(x^2 - 1),$$
$$A_\perp = 2\sqrt{2}\frac{m_1m_2}{m_B^2}c\sqrt{x^2 - 1}, \tag{4}$$

where $x = k_1 \cdot k_2/(m_1m_2)$ (m_1 and m_2 are the masses of V_1 and V_2, respectively).

The amplitude for $\bar{B}(p) \to \bar{V}_1(k_1, \varepsilon_1) + \bar{V}_2(k_2, \varepsilon_2)$ can be obtained by operating on Eq. (3) with CP. This yields

$$\bar{M} = \bar{a}\varepsilon_1^* \cdot \varepsilon_2^* + \frac{\bar{b}}{m_B^2}(p \cdot \varepsilon_1^*)(p \cdot \varepsilon_2^*) - i\frac{\bar{c}}{m_B^2}\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon_1^{*\rho}\varepsilon_2^{*\sigma}, \tag{5}$$

in which $\bar{a}$, $\bar{b}$, and $\bar{c}$ are equal to a, b, and c, respectively, except that the weak phases are of opposite sign. The above equation can be obtained from Eq. (3) by changing $a \to \bar{a}$, $b \to \bar{b}$, and $c \to -\bar{c}$. Similarly, one defines $\bar{A}_0$, $\bar{A}_\|$, and $\bar{A}_\perp$, which are equal to A_0, $A_\|$, and $A_\perp$, respectively, but with weak phases of opposite sign.

B. $B^0_{d,s} \to V_1V_2$

As mentioned in the Introduction, we are interested in the decays $B_q^0 \to V_1V_2$ ($q = d, s$), in which both B_q^0 and $\bar{B}_q^0$ can decay to V_1V_2. Because of B_q^0-$\bar{B}_q^0$ mixing, the amplitude is time dependent. Assuming that $V_{1,2}$ both decay into two pseudoscalars, i.e., $V_1 \to P_1P_1'$, $V_2 \to P_2P_2'$, the angular distribution is given in terms of the vector $\vec{\omega} = (\cos\theta_1, \cos\theta_2, \Phi)$ [33,34]:

$$\frac{d^4\Gamma(t)}{dtd\vec{\omega}} = \frac{9}{32\pi}\sum_{i=1}^{6} K_i(t)f_i(\vec{\omega}). \tag{6}$$

Here, θ_1 (θ_2) is the angle between the directions of motion of the P_1 (P_2) in the V_1 (V_2) rest frame and the V_1 (V_2) in the B rest frame, and Φ is the angle between the normals to the planes defined by P_1P_1' and P_2P_2' in the B rest frame. The angular-dependent terms are given by

$$\begin{aligned}
f_1(\vec{\omega}) &= 4\cos^2\theta_1\cos^2\theta_2,\\
f_2(\vec{\omega}) &= 2\sin^2\theta_1\sin^2\theta_2\cos^2\Phi,\\
f_3(\vec{\omega}) &= 2\sin^2\theta_1\sin^2\theta_2\sin^2\Phi,\\
f_4(\vec{\omega}) &= -2\sin^2\theta_1\sin^2\theta_2\sin2\Phi,\\
f_5(\vec{\omega}) &= \sqrt{2}\sin2\theta_1\sin2\theta_2\cos\Phi,\\
f_6(\vec{\omega}) &= -\sqrt{2}\sin2\theta_1\sin2\theta_2\sin\Phi.
\end{aligned} \tag{7}$$

1. $t = 0$

At $t = 0$, the K_i are

$$K_1 = |A_0|^2, \quad K_2 = |A_\||^2, \quad K_3 = |A_\perp|^2,$$
$$K_4 = \mathrm{Im}(A_\perp A_\|^*), \quad K_5 = \mathrm{Re}(A_\|A_0^*), \quad K_6 = \mathrm{Im}(A_\perp A_0^*). \tag{8}$$

The angular distribution for the CP-conjugate decay $\bar{B}_q^0 \to V_1V_2$ is the same as that given above with the replacements $K_i \to \bar{K}_i$ and $A_h \to \bar{A}_h$.

The quantities K_4 and K_6 are particularly interesting. They are related to the $\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon_1^{*\rho}\varepsilon_2^{*\sigma}$ term of Eq. (3), which is proportional to $\vec{q} \cdot (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$ in the rest frame of the B. This is the triple product (TP). The TP is odd under both parity and time reversal and thus constitutes a potential signal of CP violation. However, here one has to be a bit careful. As noted above, the A_h possess both weak (CP-odd) and strong (CP-even) phases. Thus, K_4 and/or K_6 can be nonzero even if the weak phases vanish. In order to obtain a true signal of CP violation, one has to compare the B and $\bar{B}$ decays. Now, $\bar{K}_4$ is the same as K_4, except that (i) the weak phases change sign, and (ii) there is an overall relative minus sign due to the presence of $\bar{A}_\perp/A_\perp$, and similarly for $\bar{K}_6$ and K_6. This implies that the true (CP-violating) TP's are given by the untagged observables $K_4 + \bar{K}_4$ and $K_6 + \bar{K}_6$. There are also fake (CP-conserving) TP's, due only to strong phases of the A_h's, given by $K_4 - \bar{K}_4$ and $K_6 - \bar{K}_6$.

2. Time dependence

In order to calculate the $K_i(t)$, one proceeds as follows. Because of B_q^0-$\bar{B}_q^0$ mixing, the time evolution of the states $|B_q^0(t)\rangle$ and $|\bar{B}_q^0(t)\rangle$ can be described by the relations [35]

$$\begin{aligned}
|B_q^0(t)\rangle &= g_+(t)|B_q^0\rangle + \frac{q}{p}g_-(t)|\bar{B}_q^0\rangle,\\
|\bar{B}_q^0(t)\rangle &= \frac{p}{q}g_-(t)|B_q^0\rangle + g_+(t)|\bar{B}_q^0\rangle,
\end{aligned} \tag{9}$$

where $q/p = e^{-i\phi_q}$. Here, ϕ_q is the phase of B_q^0-$\bar{B}_q^0$ mixing. In the SM, we have $\phi_d = 2\beta = (42.8 \pm 1.6)^\circ$ from charmonium decays [20]. Also, assuming no NP in the decay, the LHCb Collaboration measures $\phi_s = (-0.06 \pm 5.77(\text{stat}) \pm 1.54(\text{syst}))^\circ$ in $B_s^0 \to J/\psi\phi$ [19]. Although this agrees with the SM prediction of $\phi_s \simeq 0$, the errors are still large enough that NP in the decay and/or mixing cannot be excluded.

In the above, we have

$$g_+(t) = \frac{1}{2}(e^{-(iM_L+\Gamma_L/2)t} + e^{-(iM_H+\Gamma_H/2)t}),$$
$$g_-(t) = \frac{1}{2}(e^{-(iM_L+\Gamma_L/2)t} - e^{-(iM_H+\Gamma_H/2)t}), \tag{10}$$

where L and H indicate the light and heavy states, respectively. The average mass and width, and the mass and width differences of the B-meson eigenstates are defined by

$$m = \frac{M_H + M_L}{2}, \qquad \Gamma = \frac{\Gamma_L + \Gamma_H}{2},$$
$$\Delta m = M_H - M_L, \qquad \Delta\Gamma = \Gamma_L - \Gamma_H. \tag{11}$$

Δm is positive by definition. For B_d^0 mesons, $\Gamma_L \simeq \Gamma_H$, so that $\Delta\Gamma_d = 0$. However, for B_s^0 mesons, $\Delta\Gamma_s$ is reasonably large: $|\Delta\Gamma_s| = 0.116 \pm 0.018(\text{stat}) \pm 0.006(\text{syst})\,\text{ps}^{-1}$ [19]. In our convention the SM prediction for $\Delta\Gamma_s$ is positive, and it has been recently confirmed experimentally that $\Delta\Gamma_s > 0$ [36].

The time dependence of the transversity amplitudes A_h is due to B_q^0-$\bar{B}_q^0$ mixing. For the decay to a final state f we have

$$A_h(t) = \langle f|B_q^0(t)\rangle_h = [g_+(t)A_h + \eta_h q/p g_-(t)\bar{A}_h],$$
$$\bar{A}_h(t) = \langle f|\bar{B}_q^0(t)\rangle_h = [p/q g_-(t)A_h + \eta_h g_+(t)\bar{A}_h], \tag{12}$$

where $A_h = \langle f|B_q^0\rangle_h$, $\bar{A}_h = \langle f|\bar{B}_q^0\rangle_h$, and $\eta_{0,\|} = 1$, $\eta_\perp = -1$. In calculating the $K_i(t)$, the following relations are useful:

$$|g_\pm(t)|^2 = \frac{1}{2}e^{-\Gamma t}(\cosh(\Delta\Gamma/2)t \pm \cos\Delta m t),$$
$$g_+^*(t)g_-(t) = \frac{1}{2}e^{-\Gamma t}(-\sinh(\Delta\Gamma/2)t + i\sin\Delta m t). \tag{13}$$

The expressions for the time-dependent functions $K_i(t)$ are given by

$$\begin{aligned}
K_1(t) = |A_0(t)|^2 &= \frac{1}{2}e^{-\Gamma t}[(|A_0|^2 + |\bar{A}_0|^2)\cosh(\Delta\Gamma/2)t + (|A_0|^2 - |\bar{A}_0|^2)\cos\Delta m t \\
&\quad - 2\,\text{Re}(A_0^*\bar{A}_0)(\cos\phi_q \sinh(\Delta\Gamma/2)t - \sin\phi_q \sin\Delta m t) - 2\,\text{Im}(A_0^*\bar{A}_0)(\cos\phi_q \sin\Delta m t + \sin\phi_q \sinh(\Delta\Gamma/2)t)], \\
K_2(t) = |A_\|(t)|^2 &= \frac{1}{2}e^{-\Gamma t}[(|A_\||^2 + |\bar{A}_\||^2)\cosh(\Delta\Gamma/2)t + (|A_\||^2 - |\bar{A}_\||^2)\cos\Delta m t \\
&\quad - 2\,\text{Re}(A_\|^*\bar{A}_\|)(\cos\phi_q \sinh(\Delta\Gamma/2)t - \sin\phi_q \sin\Delta m t) - 2\,\text{Im}(A_\|^*\bar{A}_\|)(\cos\phi_q \sin\Delta m t + \sin\phi_q \sinh(\Delta\Gamma/2)t)], \\
K_3(t) = |A_\perp(t)|^2 &= \frac{1}{2}e^{-\Gamma t}[(|A_\perp|^2 + |\bar{A}_\perp|^2)\cosh(\Delta\Gamma/2)t + (|A_\perp|^2 - |\bar{A}_\perp|^2)\cos\Delta m t \\
&\quad + 2\,\text{Re}(A_\perp^*\bar{A}_\perp)(\cos\phi_q \sinh(\Delta\Gamma/2)t - \sin\phi_q \sin\Delta m t) + 2\,\text{Im}(A_\perp^*\bar{A}_\perp)(\cos\phi_q \sin\Delta m t + \sin\phi_q \sinh(\Delta\Gamma/2)t)], \\
K_4(t) = \text{Im}(A_\perp(t)A_\|^*(t)) &= \frac{1}{2}e^{-\Gamma t}[(\text{Im}(A_\perp A_\|^*) - \text{Im}(\bar{A}_\perp\bar{A}_\|^*))\cosh(\Delta\Gamma/2)t + (\text{Im}(A_\perp A_\|^*) + \text{Im}(\bar{A}_\perp\bar{A}_\|^*))\cos\Delta m t \\
&\quad + (\text{Im}(A_\perp\bar{A}_\|^*) - \text{Im}(\bar{A}_\perp A_\|^*))(-\sinh(\Delta\Gamma/2)t\cos\phi_q + \sin\Delta m t \sin\phi_q) + (\text{Re}(A_\perp\bar{A}_\|^*) \\
&\quad + \text{Re}(\bar{A}_\perp A_\|^*))(-\sinh(\Delta\Gamma/2)t\sin\phi_q - \sin\Delta m t\cos\phi_q)], \\
K_5(t) = \text{Re}(A_\|(t)A_0^*(t)) &= \frac{1}{2}e^{-\Gamma t}[(\text{Re}(A_\| A_0^*) + \text{Re}(\bar{A}_\|\bar{A}_0^*))\cosh(\Delta\Gamma/2)t + (\text{Re}(A_\| A_0^*) - \text{Re}(\bar{A}_\|\bar{A}_0^*))\cos\Delta m t \\
&\quad + (\text{Re}(A_\|\bar{A}_0^*) + \text{Re}(\bar{A}_\| A_0^*))(-\sinh(\Delta\Gamma/2)t\cos\phi_q + \sin\Delta m t\sin\phi_q) + (\text{Im}(A_\|\bar{A}_0^*) \\
&\quad - \text{Im}(\bar{A}_\| A_0^*))(\sinh(\Delta\Gamma/2)t\sin\phi_q + \sin\Delta m t\cos\phi_q)], \\
K_6(t) = \text{Im}(A_\perp(t)A_0^*(t)) &= \frac{1}{2}e^{-\Gamma t}[(\text{Im}(A_\perp A_0^*) - \text{Im}(\bar{A}_\perp\bar{A}_0^*))\cosh(\Delta\Gamma/2)t + (\text{Im}(A_\perp A_0^*) + \text{Im}(\bar{A}_\perp\bar{A}_0^*))\cos\Delta m t \\
&\quad + (\text{Im}(A_\perp\bar{A}_0^*) - \text{Im}(\bar{A}_\perp A_0^*))(-\sinh(\Delta\Gamma/2)t\cos\phi_q + \sin\Delta m t\sin\phi_q) + (\text{Re}(A_\perp\bar{A}_0^*) \\
&\quad + \text{Re}(\bar{A}_\perp A_0^*))(-\sinh(\Delta\Gamma/2)t\sin\phi_q - \sin\Delta m t\cos\phi_q)].
\end{aligned} \tag{14}$$

The expressions for the time-dependent $\bar{K}_i(t)$'s can be obtained from the $K_i(t)$'s by changing the sign of the weak phases in both the decay ($A_h \leftrightarrow \eta_h \bar{A}_h$) and the mixing ($\phi_q \to -\phi_q$).

C. Untagged decays

In the previous subsections, we presented the angular distribution for the case in which the initial decay meson is tagged, so that one can distinguish the B_q^0 and $\bar{B}_q^0$ decays. In practice, however, tagging is difficult. Thus, as a first step, experiments will examine the untagged decay, and this is considered here.

The untagged time-dependent angular distribution is given by

$$\frac{d^4(\Gamma^{B_q} + \Gamma^{\bar{B}_q})}{dt d\vec{\omega}} = \frac{9}{32\pi} \sum_{i=1}^{6} (K_i(t) + \bar{K}_i(t)) f_i(\vec{\omega}), \quad (15)$$

where the untagged observables can be found using Eq. (14):

$$\begin{aligned}
K_1(t) + \bar{K}_1(t) &= e^{-\Gamma t}[(|A_0|^2 + |\bar{A}_0|^2)\cosh(\Delta\Gamma/2)t - 2(\mathrm{Re}(A_0^*\bar{A}_0)\cos\phi_q + \mathrm{Im}(A_0^*\bar{A}_0)\sin\phi_q)\sinh(\Delta\Gamma/2)t],\\
K_2(t) + \bar{K}_2(t) &= e^{-\Gamma t}[(|A_\parallel|^2 + |\bar{A}_\parallel|^2)\cosh(\Delta\Gamma/2)t - 2(\mathrm{Re}(A_\parallel^*\bar{A}_\parallel)\cos\phi_q + \mathrm{Im}(A_\parallel^*\bar{A}_\parallel)\sin\phi_q)\sinh(\Delta\Gamma/2)t],\\
K_3(t) + \bar{K}_3(t) &= e^{-\Gamma t}[(|A_\perp|^2 + |\bar{A}_\perp|^2)\cosh(\Delta\Gamma/2)t + 2(\mathrm{Re}(A_\perp^*\bar{A}_\perp)\cos\phi_q + \mathrm{Im}(A_\perp^*\bar{A}_\perp)\sin\phi_q)\sinh(\Delta\Gamma/2)t],\\
K_4(t) + \bar{K}_4(t) &= e^{-\Gamma t}[(\mathrm{Im}(A_\perp A_\parallel^*) - \mathrm{Im}(\bar{A}_\perp \bar{A}_\parallel^*))\cosh(\Delta\Gamma/2)t - ((\mathrm{Im}(A_\perp \bar{A}_\parallel^*) - \mathrm{Im}(\bar{A}_\perp A_\parallel^*))\cos\phi_q\\
&\quad + (\mathrm{Re}(A_\perp \bar{A}_\parallel^*) + \mathrm{Re}(\bar{A}_\perp A_\parallel^*))\sin\phi_q)\sinh(\Delta\Gamma/2)t],\\
K_5(t) + \bar{K}_5(t) &= e^{-\Gamma t}[(\mathrm{Re}(A_\parallel A_0^*) + \mathrm{Re}(\bar{A}_\parallel \bar{A}_0^*))\cosh(\Delta\Gamma/2)t - ((\mathrm{Re}(A_\parallel \bar{A}_0^*) + \mathrm{Re}(\bar{A}_\parallel A_0^*))\cos\phi_q\\
&\quad - (\mathrm{Im}(A_\parallel \bar{A}_0^*) - \mathrm{Im}(\bar{A}_\parallel A_0^*)]\sin\phi_q)\sinh(\Delta\Gamma/2)t],\\
K_6(t) + \bar{K}_6(t) &= e^{-\Gamma t}[(\mathrm{Im}(A_\perp A_0^*) - \mathrm{Im}(\bar{A}_\perp \bar{A}_0^*))\cosh(\Delta\Gamma/2)t - ((\mathrm{Im}(A_\perp \bar{A}_0^*) - \mathrm{Im}(\bar{A}_\perp A_0^*))\cos\phi_q\\
&\quad + (\mathrm{Re}(A_\perp \bar{A}_0^*) + \mathrm{Re}(\bar{A}_\perp A_0^*))\sin\phi_q)\sinh(\Delta\Gamma/2)t]. \qquad (16)
\end{aligned}$$

Note that the CP properties of all the terms are respected. For example, the $K_i(t) + \bar{K}_i(t)$ ($i = 1, 2, 3, 5$) are supposed to be CP even. But they contain terms proportional to $\sin\phi_q$, which is CP odd. This is accounted for because, in all cases, $\sin\phi_q$ is multiplied by a term involving the helicity amplitudes which is also CP odd. Similarly, $\cos\phi_q$ (CP even) is multiplied by a helicity-amplitude term that is also CP even. The upshot is that the $K_i(t) + \bar{K}_i(t)$ ($i = 1$, 2, 3, 5) are indeed CP even. And it is straightforward to verify that the $K_i(t) + \bar{K}_i(t)$ ($i = 4, 6$) are CP odd.

The key point here is the following. The individual K_i's and $\bar{K}_i$'s [Eq. (14)] depend on four functions of time: $e^{-\Gamma t}\cos\Delta mt$, $e^{-\Gamma t}\sin\Delta mt$, $e^{-\Gamma t}\cosh(\Delta\Gamma/2)t$, and $e^{-\Gamma t}\sinh(\Delta\Gamma/2)t$. However, in the expressions above, the dependence on the functions $e^{-\Gamma t}\cos\Delta mt$ and $e^{-\Gamma t}\sin\Delta mt$ cancels, so that the untagged observables depend only on $e^{-\Gamma t}\cosh(\Delta\Gamma/2)t$ and $e^{-\Gamma t}\sinh(\Delta\Gamma/2)t$. For B_d^0 mesons, $\Delta\Gamma = 0$, so that the untagged observables are equal to $e^{-\Gamma t} \times$ simple sums of functions of the A_i and $\bar{A}_i$. On the other hand, since $\Delta\Gamma \neq 0$ for B_s^0 mesons, the untagged observables are now complicated functions of the A_i and $\bar{A}_i$.

In addition, we have

$$\begin{aligned}
e^{-\Gamma t}\cosh(\Delta\Gamma/2)t &= \frac{1}{2}(e^{-\Gamma_L t} + e^{-\Gamma_H t}),\\
e^{-\Gamma t}\sinh(\Delta\Gamma/2)t &= \frac{1}{2}(e^{-\Gamma_L t} - e^{-\Gamma_H t}).
\end{aligned} \quad (17)$$

If the $e^{-\Gamma_L t/2}$ and $e^{-\Gamma_H t/2}$ terms can be distinguished experimentally, which is doable for B_s^0 decays, the untagged time-dependent angular distribution provides 12 observables, 2 for each $K_i(t) + \bar{K}_i(t)$. Thus, $B_s^0 \to V_1 V_2$ decays are particularly interesting.

D. Time-integrated untagged distribution

As noted in the previous subsection, because $\Delta\Gamma \neq 0$ for B_s^0 mesons, B_s^0 decays can be treated without tagging. The time-integrated untagged angular distribution can be obtained by integrating the $K_i(t) + \bar{K}_i(t)$ observables over time:

$$\frac{d^3\langle\Gamma(B_s^0 \to f)\rangle}{d\vec{\omega}} = \frac{9}{32\pi} \sum_{i=1}^{6} \langle K_i \rangle f_i(\vec{\omega}), \quad (18)$$

where

$$\begin{aligned}
\langle\Gamma(B_s^0 \to f)\rangle &= \frac{1}{2}\int_0^\infty dt(\Gamma^{B_s} + \Gamma^{\bar{B}_s}),\\
\langle K_i \rangle &= \frac{1}{2}\int_0^\infty dt(K_i(t) + \bar{K}_i(t)).
\end{aligned} \quad (19)$$

One can obtain the $\langle K_i \rangle$'s from Eq. (16):

$$
\begin{aligned}
\langle K_1\rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)}[(|A_0|^2+|\bar{A}_0|^2)-2(\mathrm{Re}(A_0^*\bar{A}_0)\cos\phi_s+\mathrm{Im}(A_0^*\bar{A}_0)\sin\phi_s)y_s],\\
\langle K_2\rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)}[(|A_\|| ^2+|\bar{A}_\||^2)-2(\mathrm{Re}(A_\|^*\bar{A}_\|)\cos\phi_s+\mathrm{Im}(A_\|^*\bar{A}_\|)\sin\phi_s)y_s],\\
\langle K_3\rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)}[(|A_\perp|^2+|\bar{A}_\perp|^2)+2(\mathrm{Re}(A_\perp^*\bar{A}_\perp)\cos\phi_s+\mathrm{Im}(A_\perp^*\bar{A}_\perp)\sin\phi_s)y_s],\\
\langle K_4\rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)}[(\mathrm{Im}(A_\perp A_\|^*)-\mathrm{Im}(\bar{A}_\perp\bar{A}_\|^*))-((\mathrm{Im}(A_\perp\bar{A}_\|^*)-\mathrm{Im}(\bar{A}_\perp A_\|^*))\cos\phi_s+(\mathrm{Re}(A_\perp\bar{A}_\|^*)+\mathrm{Re}(\bar{A}_\perp A_\|^*))\sin\phi_s)y_s],\\
\langle K_5\rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)}[(\mathrm{Re}(A_\| A_0^*)+\mathrm{Re}(\bar{A}_\|\bar{A}_0^*))-((\mathrm{Re}(A_\|\bar{A}_0^*)+\mathrm{Re}(\bar{A}_\| A_0^*))\cos\phi_s-(\mathrm{Im}(A_\|\bar{A}_0^*)-\mathrm{Im}(\bar{A}_\| A_0^*))\sin\phi_s)y_s],\\
\langle K_6\rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)}[(\mathrm{Im}(A_\perp A_0^*)-\mathrm{Im}(\bar{A}_\perp\bar{A}_0^*))-((\mathrm{Im}(A_\perp\bar{A}_0^*)-\mathrm{Im}(\bar{A}_\perp A_0^*))\cos\phi_s+(\mathrm{Re}(A_\perp\bar{A}_0^*)+\mathrm{Re}(\bar{A}_\perp A_0^*))\sin\phi_s)y_s],
\end{aligned}
\tag{20}
$$

where $y_s \equiv \Delta\Gamma_s/2\Gamma_s$.

At this stage, one clearly sees the effect of a nonzero $\Delta\Gamma_s$ (or y_s). For B_d^0 decays, $\Delta\Gamma_d = 0$, so there are no terms proportional to $y_d \equiv \Delta\Gamma_d/2\Gamma_d$ in the $\langle K_i\rangle$. Indeed, the $\langle K_i\rangle$ take the same form as the $(K_i(t)+\bar{K}_i(t))|_{t=0}$ [Eq. (8)]. However, this does not hold for B_s^0 decays. Because of the nonzero y_s, the $\langle K_i\rangle$, which are time-integrated quantities, take a different form than they did at $t=0$. And this means that, if general $\bar{b}\to\bar{s}$ NP is considered, the formulas relating certain observables to the $\langle K_i\rangle$ must necessarily include terms proportional to y_s. As we will see, this holds specifically for the polarization fractions, *CP*-violating triple-product asymmetries, and the *CP*-conserving interference term.

E. Effective lifetime

In general, the expressions for $K_i(t)+\bar{K}_i(t)$ [Eq. (16)] and $\langle K_i\rangle$ [Eq. (20)] have the form

$$
\begin{aligned}
K_i(t)+\bar{K}_i(t) &= 2e^{-\Gamma t}[\mathcal{A}_i^{\rm ch}\cosh(\Delta\Gamma/2)t\\
&\quad + \mathcal{A}_i^{\rm sh}\sinh(\Delta\Gamma/2)t],\\
\langle K_i\rangle &= \frac{\tau_{B_s}}{(1-y_s^2)}[\mathcal{A}_i^{\rm ch}+\mathcal{A}_i^{\rm sh}y_s],
\end{aligned}
\tag{21}
$$

where the experimental observables (dependent on the K_i) are on the left-hand side, and the theoretical expressions (dependent on $\mathcal{A}_i^{\rm ch}$ and $\mathcal{A}_i^{\rm sh}$) are on the right-hand side. (We have implicitly assumed that $\Delta\Gamma \neq 0$, which implies a B_s^0 decay.) $\mathcal{A}_i^{\rm ch}$ and $\mathcal{A}_i^{\rm sh}$ can be related to the experimental observables via the effective lifetime [29]:

$$
\begin{aligned}
\tau_{B_s}^{{\rm eff},i} &= \frac{\int_0^\infty t(K_i(t)+\bar{K}_i(t))dt}{\int_0^\infty(K_i(t)+\bar{K}_i(t))dt}\\
&= \frac{\tau_{B_s}}{(1-y_s^2)}\frac{(1+2\mathcal{A}_{\Delta\Gamma}^i y_s+y_s^2)}{(1+\mathcal{A}_{\Delta\Gamma}^i y_s)},
\end{aligned}
\tag{22}
$$

where $\mathcal{A}_{\Delta\Gamma}^i \equiv \mathcal{A}_i^{\rm sh}/\mathcal{A}_i^{\rm ch}$.

Using Eqs. (21) and (22), one can relate the $\mathcal{A}_i^{\rm ch}$ to the $\langle K_i\rangle$:

$$
\mathcal{A}_i^{\rm ch} = \frac{\langle K_i\rangle}{\tau_{B_s}}\left(2-\frac{\tau_{B_s}^{{\rm eff},i}}{\tau_{B_s}}(1-y_s^2)\right). \tag{23}
$$

The $\mathcal{A}_i^{\rm sh}$ can be obtained from $\mathcal{A}_{\Delta\Gamma}^i$.

III. $B_s^0 \to \phi\phi$ − SM

The results of the previous section are completely general. In this section we focus on the angular distribution of the pure $\bar{b}\to\bar{s}$ penguin decay $B_s^0\to\phi\phi$ within the SM.

In the SM, the amplitude for $B_s^0\to\phi\phi$ can be written

$$
\begin{aligned}
\mathcal{A}(B_s^0\to\phi\phi) &= \lambda_t^{(s)}P_t' + \lambda_c^{(s)}P_c' + \lambda_u^{(s)}P_u'\\
&= \lambda_t^{(s)}P_{tc}' + \lambda_u^{(s)}P_{uc}',
\end{aligned}
\tag{24}
$$

where $\lambda_q^{(s)} \equiv V_{qb}^*V_{qs}$. (As this is a $\bar{b}\to\bar{s}$ transition, the diagrams are written with primes.) In the second line, we have used the unitarity of the Cabibbo-Kobayashi-Maskawa matrix $[\lambda_u^{(s)}+\lambda_c^{(s)}+\lambda_t^{(s)}=0]$ to eliminate the c-quark contribution: $P_{tc}' \equiv P_t' - P_c'$, $P_{uc}' \equiv P_u' - P_c'$.

Now, $|\lambda_t^{(s)}|$ and $|\lambda_u^{(s)}|$ are $O(\lambda^2)$ and $O(\lambda^4)$, respectively, where $\lambda = 0.23$ is the sine of the Cabibbo angle. This suggests that the $\lambda_u^{(s)}P_{uc}'$ term can be neglected compared to $\lambda_t^{(s)}P_{tc}'$. However, if one does this, one must be consistent and neglect *all* $O(\lambda^4)$ terms. In particular, $\mathrm{Im}(\lambda_t^{(s)})$ is $O(\lambda^4)$, and so it too can be neglected. But since $2\beta_s = -\arg((q/p)(\bar{\mathcal{A}}/\mathcal{A}))$, one also has $\beta_s = 0$ because $(q/p) = (\bar{\mathcal{A}}/\mathcal{A}) = 1$ in the limit where $\lambda_t^{(s)}$ is real. Thus, in the approximation of neglecting all quantities of $O(\lambda^4)$, there are no nonzero weak phases in $B_s^0\to\phi\phi$, either in the mixing or in the decay.

A. Untagged distribution

In the approximation of neglecting all weak phases in $B_s^0\to\phi\phi$, the untagged observables [Eq. (16)] are

$$
\begin{aligned}
(K_1(t) + \bar{K}_1(t))_{\rm SM} &= e^{-\Gamma t}[2|A_0|^2(\cosh(\Delta\Gamma/2)t \\
&\quad - \sinh(\Delta\Gamma/2)t)], \\
(K_2(t) + \bar{K}_2(t))_{\rm SM} &= e^{-\Gamma t}[2|A_\parallel|^2(\cosh(\Delta\Gamma/2)t \\
&\quad - \sinh(\Delta\Gamma/2)t)], \\
(K_3(t) + \bar{K}_3(t))_{\rm SM} &= e^{-\Gamma t}[2|A_\perp|^2(\cosh(\Delta\Gamma/2)t \\
&\quad + \sinh(\Delta\Gamma/2)t)], \\
(K_4(t) + \bar{K}_4(t))_{\rm SM} &= 0, \\
(K_5(t) + \bar{K}_5(t))_{\rm SM} &= e^{-\Gamma t}[2\,{\rm Re}(A_\parallel A_0^*)(\cosh(\Delta\Gamma/2)t \\
&\quad - \sinh(\Delta\Gamma/2)t)], \\
(K_6(t) + \bar{K}_6(t))_{\rm SM} &= 0.
\end{aligned}
\tag{25}
$$

We have $\mathcal{A}_i^{\rm sh} = \mp \mathcal{A}_i^{\rm ch}$ [Eq. (21)], where the minus sign is for $i = 1, 2, 5$, the plus sign for $i = 3$, and both quantities vanish when $i = 4, 6$. The effective lifetimes are then predicted to be

$$
\begin{aligned}
\tau_{B_s,{\rm SM}}^{{\rm eff},i} &= \frac{\tau_{B_s}}{(1+y_s)}, \quad i = 1, 2, 5, \\
\tau_{B_s,{\rm SM}}^{{\rm eff},i} &= \frac{\tau_{B_s}}{(1-y_s)}, \quad i = 3.
\end{aligned}
\tag{26}
$$

If the measurement of an effective lifetime differs from the SM prediction, this will be a sign for NP [29].

The SM untagged time-dependent angular distribution for $B_s^0 \to \phi\phi$ takes the form

$$
\frac{d^4(\Gamma^{B_s} + \Gamma^{\bar{B}_s})}{dt d\vec{\omega}} = \frac{9}{32\pi}[\mathcal{F}_L(q^2, \vec{\omega})\mathcal{K}_L(t) + \mathcal{F}_H(q^2, \vec{\omega})\mathcal{K}_H(t)],
\tag{27}
$$

where the angular and time-dependent terms are

$$
\begin{aligned}
\mathcal{F}_L(\vec{\omega}) &= [|A_0|^2 f_1(\vec{\omega}) + |A_\parallel|^2 f_2(\vec{\omega}) \\
&\quad + |A_0||A_\parallel|\cos(\delta_\parallel - \delta_0) f_5(\vec{\omega})], \\
\mathcal{F}_H(\vec{\omega}) &= |A_\perp|^2, \\
\mathcal{K}_L(t) &= 2e^{-\Gamma_L t} = 2e^{-\Gamma t}(\cosh(\Delta\Gamma/2)t - \sinh(\Delta\Gamma/2)t), \\
\mathcal{K}_H(t) &= 2e^{-\Gamma_H t} = 2e^{-\Gamma t}(\cosh(\Delta\Gamma/2)t + \sinh(\Delta\Gamma/2)t),
\end{aligned}
\tag{28}
$$

in which $(\delta_\parallel - \delta_0) = \arg(A_\parallel A_0^*)$.

Thus, if the $e^{-\Gamma_L t/2}$ and $e^{-\Gamma_H t/2}$ terms in the time-dependent angular distribution [see Eq. (17)] can be distinguished experimentally, the $|A_h|$ and $\cos(\delta_\parallel - \delta_0)$ can be measured. However, as we will see in the next subsection, these observables can be obtained from time-integrated measurements.

B. Untagged time-integrated distribution

In the SM, the observables in the time-integrated untagged distribution are

$$
\begin{aligned}
\langle K_1 \rangle &= \frac{\tau_{B_s}}{1+y_s}|A_0|^2, \qquad \langle K_2 \rangle = \frac{\tau_{B_s}}{1+y_s}|A_\parallel|^2, \\
\langle K_3 \rangle &= \frac{\tau_{B_s}}{1-y_s}|A_\perp|^2, \qquad \langle K_4 \rangle = 0, \\
\langle K_5 \rangle &= \frac{\tau_{B_s}}{1+y_s}|A_0||A_\parallel|\cos(\delta_\parallel - \delta_0), \quad \langle K_6 \rangle = 0.
\end{aligned}
\tag{29}
$$

We have $y_s = 0.088 \pm 0.014$ and $\tau_{B_s}^{-1} = (0.6580 \pm 0.0085)\ {\rm ps}^{-1}$ [19,29]. With this knowledge, the $|A_h|$ and $\cos(\delta_\parallel - \delta_0)$ can be extracted from the above measurements. This is what CDF and LHCb have presented [27,28].

C. Polarization fractions

With no weak phases in the decay, we have $A_h = \bar{A}_h$, and the $|A_h|^2$ can be measured in the untagged time-integrated distribution [Eq. (29)]. The polarization fractions are given by

$$
\begin{aligned}
f_0 &= \frac{|A_0|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2}, \quad f_\parallel = \frac{|A_\parallel|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2}, \\
f_\perp &= \frac{|A_\perp|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2},
\end{aligned}
\tag{30}
$$

with total polarization $f_{\rm tot} = f_0 + f_\parallel + f_\perp = 1$.

Now, in the presence of NP the distribution changes, and so the experimental measurements have to be reinterpreted. We address this issue in the next section.

IV. $B_s^0 \to \phi\phi$ − SM + NP

In this section, we consider NP contributions to $B_s^0 \to \phi\phi$, in the mixing and/or in the decay.

A. Polarization fractions

The polarization fractions can be written as

$$
\begin{aligned}
f_0 &= \frac{|A_0|^2 + |\bar{A}_0|^2}{|A_0|^2 + |\bar{A}_0|^2 + |A_\parallel|^2 + |\bar{A}_\parallel|^2 + |A_\perp|^2 + |\bar{A}_\perp|^2} \\
&= \frac{\mathcal{A}_1^{\rm ch}}{\sum_{i=1,2,3}\mathcal{A}_i^{\rm ch}}, \\
f_\parallel &= \frac{|A_\parallel|^2 + |\bar{A}_\parallel|^2}{|A_0|^2 + |\bar{A}_0|^2 + |A_\parallel|^2 + |\bar{A}_\parallel|^2 + |A_\perp|^2 + |\bar{A}_\perp|^2} \\
&= \frac{\mathcal{A}_2^{\rm ch}}{\sum_{i=1,2,3}\mathcal{A}_i^{\rm ch}}, \\
f_\perp &= \frac{|A_\perp|^2 + |\bar{A}_\perp|^2}{|A_0|^2 + |\bar{A}_0|^2 + |A_\parallel|^2 + |\bar{A}_\parallel|^2 + |A_\perp|^2 + |\bar{A}_\perp|^2} \\
&= \frac{\mathcal{A}_3^{\rm ch}}{\sum_{i=1,2,3}\mathcal{A}_i^{\rm ch}}.
\end{aligned}
\tag{31}
$$

In the above, the f_h are written in terms of the $|A_h|^2$ and $|\bar{A}_h|^2$. However, as noted above, what is measured experimentally in the time-integrated untagged distribution are the $\langle K_i \rangle$. It is therefore necessary to express the f_h in terms of the $\langle K_i \rangle$. This is done as follows. Using Eq. (23), one can write

$$\mathcal{A}_i^{\rm ch} = \frac{\langle K_i \rangle}{\tau_{B_s}}(1 + \eta_i y_s) Y_i, \quad i = 1, 2, 3, \tag{32}$$

where the quantity Y_i is related to $\tau_{B_s}^{{\rm eff},i}$ or $\mathcal{A}_{\Delta\Gamma}^i$:

$$Y_i = \frac{1}{(1+\eta_i y_s)}\left(2 - \frac{\tau_{B_s}^{{\rm eff},i}}{\tau_{B_s}}(1 - y_s^2)\right) = \frac{(1-\eta_i y_s)}{(1+\mathcal{A}_{\Delta\Gamma}^i y_s)}, \tag{33}$$

with $\eta_{1,2} = 1$, and $\eta_3 = -1$. From Eq. (20) we have

$$\mathcal{A}_{\Delta\Gamma}^i = \mathcal{A}_i^{\rm sh}/\mathcal{A}_i^{\rm ch} = \begin{cases} -2({\rm Re}(A_0^*\bar{A}_0)\cos\phi_s + {\rm Im}(A_0^*\bar{A}_0)\sin\phi_s)/(|A_0|^2 + |\bar{A}_0|^2), & i = 1, \\ -2({\rm Re}(A_\parallel^*\bar{A}_\parallel)\cos\phi_s + {\rm Im}(A_\parallel^*\bar{A}_\parallel)\sin\phi_s)/(|A_\parallel|^2 + |\bar{A}_\parallel|^2), & i = 2, \\ 2({\rm Re}(A_\perp^*\bar{A}_\perp)\cos\phi_s + {\rm Im}(A_\perp^*\bar{A}_\perp)\sin\phi_s)/(|A_\perp|^2 + |\bar{A}_\perp|^2), & i = 3. \end{cases} \tag{34}$$

In the SM, the weak phases of the A_h vanish and $\phi_s = 0$, so that $\mathcal{A}_{\Delta\Gamma}^i = \pm 1$ (the minus sign is for $i = 1, 2$, and the plus sign is for $i = 3$). This implies that $Y_{1,2,3} = 1$, so that the polarization fractions are

$$\begin{aligned} f_0^{\rm SM} &= \frac{\langle K_1\rangle(1+y_s)}{\langle K_1\rangle(1+y_s) + \langle K_2\rangle(1+y_s) + \langle K_3\rangle(1-y_s)}, \\ f_\parallel^{\rm SM} &= \frac{\langle K_2\rangle(1+y_s)}{\langle K_1\rangle(1+y_s) + \langle K_2\rangle(1+y_s) + \langle K_3\rangle(1-y_s)}, \\ f_\perp^{\rm SM} &= \frac{\langle K_3\rangle(1-y_s)}{\langle K_1\rangle(1+y_s) + \langle K_2\rangle(1+y_s) + \langle K_3\rangle(1-y_s)}. \end{aligned} \tag{35}$$

Note that these are consistent with Eq. (29). However, if there is NP in the mixing and/or the decay, we have $Y_{1,2,3} \neq 1$, so that the polarization fractions take the form

$$\begin{aligned} f_0 &= \frac{\langle K_1\rangle(1+y_s)Y_1}{\langle K_1\rangle(1+y_s)Y_1 + \langle K_2\rangle(1+y_s)Y_2 + \langle K_3\rangle(1-y_s)Y_3}, \\ f_\parallel &= \frac{\langle K_2\rangle(1+y_s)Y_2}{\langle K_1\rangle(1+y_s)Y_1 + \langle K_2\rangle(1+y_s)Y_2 + \langle K_3\rangle(1-y_s)Y_3}, \\ f_\perp &= \frac{\langle K_3\rangle(1-y_s)Y_3}{\langle K_1\rangle(1+y_s)Y_1 + \langle K_2\rangle(1+y_s)Y_2 + \langle K_3\rangle(1-y_s)Y_3}. \end{aligned} \tag{36}$$

The f_h are expressed completely in terms of measured quantities. The $\langle K_i \rangle$'s are obtained from the untagged angular distribution, and one can calculate the Y_i using the measured effective lifetimes. If the effective lifetimes have not been measured, then $\mathcal{A}_{\Delta\Gamma}^i$ can be varied within a certain range to get a range for the Y_i.

Thus, to obtain the correct polarization fractions in the presence of NP, Eq. (36), which includes factors of Y_i, must be used. This is one of the main points of the paper. However, experiments have used Eq. (35), so they have effectively excluded NP. If this possibility is allowed, the analysis must be redone and we discuss this in Sec. V.

The difference between Eqs. (35) and (36) is related to the difference $Y_i - 1$. One can see from Eq. (33) that $Y_i - 1 \to 0$ in the limit that $y_s \to 0$. This indicates that $f_h - f_h^{\rm SM} = O(y_s)$. Since $y_s = 0.088 \pm 0.014$, this corresponds to a correction to the polarization fractions of $O(10\%)$. This is not large, but it may be important given that the measurements hope to identify the presence of NP.

B. Other observables

In Sec. II B, we noted that the angular distribution of the decay $B_q^0 \to V_1 V_2$ $(q = d,\ s)$ is proportional to $\sum_{i=1}^6 K_i(t) f_i(\vec{\omega})$, where $\vec{\omega} = (\cos\theta_1, \cos\theta_2, \Phi)$ [Eq. (6)]. In the previous subsection, we discussed polarization fractions, observables that are dependent on $\langle K_i \rangle$, $i = 1, 2, 3$. We now turn to $i = 4, 6$.

In the present case, K_4 and K_6 are related to the triple products in $B_s^0 \to \phi\phi$. The expressions for the untagged observables in $B_{d,s}^0 \to V_1 V_2$ are given in Eq. (16). For convenience, $K_i(t) + \bar{K}_i(t)$ $(i = 4, 6)$ are repeated below:

$$\begin{aligned} K_4(t) + \bar{K}_4(t) = e^{-\Gamma_s t}[&({\rm Im}(A_\perp A_\parallel^*) - {\rm Im}(\bar{A}_\perp \bar{A}_\parallel^*)) \\ &\times \cosh(\Delta\Gamma_s/2)t - (({\rm Im}(A_\perp \bar{A}_\parallel^*) \\ &- {\rm Im}(\bar{A}_\perp A_\parallel^*))\cos\phi_s + ({\rm Re}(A_\perp \bar{A}_\parallel^*) \\ &+ {\rm Re}(\bar{A}_\perp A_\parallel^*))\sin\phi_s)\sinh(\Delta\Gamma_s/2)t], \\ K_6(t) + \bar{K}_6(t) = e^{-\Gamma_s t}[&({\rm Im}(A_\perp A_0^*) - {\rm Im}(\bar{A}_\perp \bar{A}_0^*)) \\ &\times \cosh(\Delta\Gamma_s/2)t - (({\rm Im}(A_\perp \bar{A}_0^*) \\ &- {\rm Im}(\bar{A}_\perp A_0^*))\cos\phi_s + ({\rm Re}(A_\perp \bar{A}_0^*) \\ &+ {\rm Re}(\bar{A}_\perp A_0^*))\sin\phi_s)\sinh(\Delta\Gamma_s/2)t]. \end{aligned} \tag{37}$$

Now, as discussed earlier, in the SM the weak phases in $B_s^0 \to \phi\phi$, both in the mixing and in the decay, are all approximately zero, so that $K_4(t) + \bar{K}_4(t)$ and $K_6(t) + \bar{K}_6(t)$ vanish. Thus, if one finds evidence for a nonzero TP, this is a clear sign of NP.

Suppose first that there is NP, with a nonzero weak phase, only in the mixing. In this case, the first two terms in each of $K_i(t) + \bar{K}_i(t)$ $(i = 4, 6)$ are zero, but the third is nonzero. This is a particularly interesting situation, as it corresponds to a TP generated through mixing. It arises only because $\Delta\Gamma_s$ is nonzero; mixing-induced TP's cannot be produced in B_d^0 decays. And, although $\Delta\Gamma_s \neq 0$, it is still not large, so that the associated TP is also rather small.

The second possibility is that there is NP, with a nonzero weak phase, only in the decay. In this case, the first two terms in each of $K_i(t) + \bar{K}_i(t)$ ($i = 4, 6$), proportional to $\cosh(\Delta\Gamma_s/2)t$ and $\cos\phi_s = 1$, are nonzero, but the third is zero. And of course one can have NP in both the mixing and the decay. If a TP is seen, its source can be determined through its time dependence.

Both $K_i(t) + \bar{K}_i(t)$ ($i = 4, 6$) are CP violating, so they correspond to true TP's. They can be nonzero only if there are two interfering amplitudes with a relative weak phase. If there is NP in the mixing, the amplitudes are $A(B_s^0 \to \phi\phi)$ and $A(B_s^0 \to \bar{B}_s^0 \to \phi\phi)$; if there is NP in the decay, the amplitudes are $A(B_s^0 \to \phi\phi)_{\rm SM}$ and $A(B_s^0 \to \phi\phi)_{\rm NP}$. In addition, in order to produce a TP, the two interfering amplitudes must be kinematically different [32]. For the case of NP in the decay, this is satisfied straightforwardly. But for NP in the mixing, how are $B_s^0 \to \phi\phi$ and $B_s^0 \to \bar{B}_s^0 \to \phi\phi$ kinematically different? The point is that mixing-induced TP's are generated due to a nonzero $\Delta\Gamma_s$. That is, although $B_s^0 \to \phi\phi$ is a penguin decay, $\bar{B}_s^0 \to \phi\phi$ occurs via a mechanism that contributes to $\Delta\Gamma_s$. For example, one possibility is the $B_s^0 \to \bar{B}_s^0$ transition via the intermediate states $D_s^{*+}D_s^{*-}$ [37], with the $\bar{B}_s^0$ decaying to $\phi\phi$. The B_s^0 and $\bar{B}_s^0$ decays are clearly kinematically different.

We now turn to the measurement of TP's. Here we focus on the time-integrated untagged observables, $\langle K_i\rangle$. We have $\langle K_i\rangle \propto \mathcal{A}_i^{\rm ch} + \mathcal{A}_i^{\rm sh} y_s$ [Eq. (21)]. Specifically, the $\langle K_{4,6}\rangle$ are given in Eq. (20):

$$\begin{aligned}\langle K_4\rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)}[(\mathrm{Im}(A_\perp A_\parallel^*) - \mathrm{Im}(\bar{A}_\perp \bar{A}_\parallel^*)) \\ &\quad - ((\mathrm{Im}(A_\perp \bar{A}_\parallel^*) - \mathrm{Im}(\bar{A}_\perp A_\parallel^*))\cos\phi_s \\ &\quad + (\mathrm{Re}(A_\perp \bar{A}_\parallel^*) + \mathrm{Re}(\bar{A}_\perp A_\parallel^*))\sin\phi_s)y_s], \\ \langle K_6\rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)}[(\mathrm{Im}(A_\perp A_0^*) - \mathrm{Im}(\bar{A}_\perp \bar{A}_0^*)) \\ &\quad - ((\mathrm{Im}(A_\perp \bar{A}_0^*) - \mathrm{Im}(\bar{A}_\perp A_0^*))\cos\phi_s \\ &\quad + (\mathrm{Re}(A_\perp \bar{A}_0^*) + \mathrm{Re}(\bar{A}_\perp A_0^*))\sin\phi_s)y_s].\end{aligned} \tag{38}$$

The TP's in the untagged distribution can be measured by constructing asymmetries involving the angular variables. We start by integrating Eq. (18) over θ_1 and θ_2 to obtain the differential rate:

$$\begin{aligned}\frac{d\langle\Gamma(B_q^0 \to V_1V_2)\rangle}{d\Phi} &= \frac{1}{2\pi}[\langle K_1\rangle + 2\langle K_2\rangle\cos^2\Phi \\ &\quad + 2\langle K_3\rangle\sin^2\Phi - 2\langle K_4\rangle\sin 2\Phi].\end{aligned} \tag{39}$$

Note that the time-integrated untagged decay rate can be obtained by integrating out the azimuthal angle Φ:

$$\langle\Gamma(B_q^0 \to V_1V_2)\rangle = [\langle K_1\rangle + \langle K_2\rangle + \langle K_3\rangle]. \tag{40}$$

Following Ref. [26] we can define asymmetries to measure the TP's. We begin with $i = 4$, for which $f_4(\vec{\omega}) = -2\sin^2\theta_1\sin^2\theta_2\sin 2\Phi$. We define $u \equiv \sin 2\Phi$. The TP asymmetry between the number of decays involving positive and negative values of u is given by [26,32]

$$\begin{aligned}\mathcal{A}_u &= \frac{1}{2}\left[\frac{\langle\Gamma(B_s^0 \to \phi\phi), u>0\rangle - \langle\Gamma(B_s^0 \to \phi\phi), u<0\rangle}{\langle\Gamma(B_s^0 \to \phi\phi), u>0\rangle + \langle\Gamma(B_s^0 \to \phi\phi), u<0\rangle}\right] \\ &= -\frac{2}{\pi}[\mathcal{A}_T^{(2)}]_{\rm exp}, \\ [\mathcal{A}_T^{(2)}]_{\rm exp} &= \frac{\langle K_4\rangle}{\langle\Gamma(B_s^0 \to \phi\phi)\rangle}.\end{aligned} \tag{41}$$

As noted above, if $\mathcal{A}_u \neq 0$ is found, this would clearly indicate NP. However, we would like to know the relation between $[\mathcal{A}_T^{(2)}]_{\rm exp}$ and the theoretical expression for the TP in Eq. (40). The measured TP $[\mathcal{A}_T^{(2)}]_{\rm exp}$ is related to $[\mathcal{A}_T^{(2)}]_{\rm theo}$ via

$$[\mathcal{A}_T^{(2)}]_{\rm exp} = [\mathcal{A}_T^{(2)}]_{\rm theo}\frac{\tau_{B_s}}{\langle\Gamma(B_s^0 \to \phi\phi)\rangle}\frac{(1 + A_{\Delta\Gamma}^{(4)}y_s)}{(1-y_s^2)}, \tag{42}$$

where $A_{\Delta\Gamma}^{(4)} = \mathcal{A}_4^{\rm sh}/\mathcal{A}_4^{\rm ch}$ and

$$[\mathcal{A}_T^{(2)}]_{\rm theo} = \mathcal{A}_4^{\rm ch} = \frac{1}{2}(\mathrm{Im}(A_\perp A_\parallel^*) - \mathrm{Im}(\bar{A}_\perp \bar{A}_\parallel^*)) \tag{43}$$

If we define the dimensionless theoretical TP as

$$\mathcal{TP}_2 \equiv [\mathcal{A}_T^{(2)}]_{\rm theo}\frac{\tau_{B_s}}{\langle\Gamma(B_s^0 \to \phi\phi)\rangle}, \tag{44}$$

Eq. (42) details the corrections to the naive relation $[\mathcal{A}_T^{(2)}]_{\rm exp} = \mathcal{TP}_2$ due to a nonzero (NP) $\mathcal{A}_4^{\rm sh}$. (In the SM, $[\mathcal{A}_T^{(2)}]_{\rm theo} = 0$, so the relation is trivial.)

For $i = 6$ we have $f_6(\vec{\omega}) = -\sqrt{2}\sin 2\theta_1\sin 2\theta_2\sin\Phi$. We define $v \equiv \mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\Phi$, which has the following associated TP asymmetry [26]:

$$\begin{aligned}\mathcal{A}_v &= \frac{1}{2}\left[\frac{\langle\Gamma(B_s^0 \to \phi\phi), v>0\rangle - \langle\Gamma(B_s^0 \to \phi\phi), v<0\rangle}{\langle\Gamma(B_s^0 \to \phi\phi), v>0\rangle + \langle\Gamma(B_s^0 \to \phi\phi), v<0\rangle}\right] \\ &= -\frac{\sqrt{2}}{\pi}[\mathcal{A}_T^{(1)}]_{\rm exp}, \\ [\mathcal{A}_T^{(1)}]_{\rm exp} &= \frac{\langle K_6\rangle}{\langle\Gamma(B_s^0 \to \phi\phi)\rangle}.\end{aligned} \tag{45}$$

Then

$$[\mathcal{A}_T^{(1)}]_{\rm exp} = [\mathcal{A}_T^{(1)}]_{\rm theo}\frac{\tau_{B_s}}{\langle\Gamma(B_s^0 \to \phi\phi)\rangle}\frac{(1 + A_{\Delta\Gamma}^{(6)}y_s)}{(1-y_s^2)}, \tag{46}$$

where $A_{\Delta\Gamma}^{(6)} = \mathcal{A}_6^{\rm sh}/\mathcal{A}_6^{\rm ch}$ and

$$[\mathcal{A}_T^{(1)}]_{\rm theo} = \mathcal{A}_6^{\rm ch} = \frac{1}{2}(\mathrm{Im}(A_\perp A_0^*) - \mathrm{Im}(\bar{A}_\perp \bar{A}_0^*)). \tag{47}$$

We can again define the dimensionless theoretical TP as

$$\mathcal{T}\mathcal{P}_1 \equiv [\mathcal{A}_T^{(1)}]_{\rm theo} \frac{\tau_{B_s}}{\langle\Gamma(B_s^0 \to \phi\phi)\rangle}. \quad (48)$$

Equation (46) gives the corrections to the naive relation $[\mathcal{A}_T^{(1)}]_{\rm exp} = \mathcal{T}\mathcal{P}_1$.

Finally, we turn to $i = 5$, which corresponds to a CP-conserving observable. From Eq. (20),

$$\begin{aligned}\langle K_5\rangle = \frac{\tau_{B_s}}{2(1-y_s^2)}[&(\mathrm{Re}(A_\parallel A_0^*) + \mathrm{Re}(\bar{A}_\parallel \bar{A}_0^*)) \\ &- ((\mathrm{Re}(A_\parallel \bar{A}_0^*) + \mathrm{Re}(\bar{A}_\parallel A_0^*))\cos\phi_s \\ &- (\mathrm{Im}(A_\parallel \bar{A}_0^*) - \mathrm{Im}(\bar{A}_\parallel A_0^*))\sin\phi_s)y_s]. \quad (49)\end{aligned}$$

We have $f_5(\vec{\omega}) = \sqrt{2}\sin2\theta_1 \sin2\theta_2 \cos\Phi$, so we define $w \equiv \mathrm{sign}(\cos\theta_1 \cos\theta_2)\cos\Phi$. The associated asymmetry is

$$\begin{aligned}\mathcal{A}_w &= \frac{1}{2}\left[\frac{\langle\Gamma(B_s^0\to\phi\phi), w>0\rangle - \langle\Gamma(B_s^0\to\phi\phi), w<0\rangle}{\langle\Gamma(B_s^0\to\phi\phi), w>0\rangle + \langle\Gamma(B_s^0\to\phi\phi), w<0\rangle}\right] \\ &= \frac{\sqrt{2}}{\pi}[A^{(5)}]_{\rm exp}, \\ [A^{(5)}]_{\rm exp} &= \langle K_5\rangle / \langle\Gamma(B_s^0\to\phi\phi)\rangle. \quad (50)\end{aligned}$$

We have

$$\begin{aligned}[A^{(5)}]_{\rm exp} &= [A^{(5)}]_{\rm theo}\frac{\tau_{B_s}}{\langle\Gamma(B_s^0\to\phi\phi)\rangle}\frac{(1+A_{\Delta\Gamma}^{(5)}y_s)}{(1-y_s^2)} \\ &= [A^{(5)}]_{\rm theo}\frac{\tau_{B_s}}{\langle\Gamma(B_s^0\to\phi\phi)\rangle}\left(2 - \frac{\tau_{B_s}^{\rm eff,5}}{\tau_{B_s}}(1-y_s^2)\right), \quad (51)\end{aligned}$$

where $A_{\Delta\Gamma}^{(5)} = \mathcal{A}_5^{\rm sh}/\mathcal{A}_5^{\rm ch}$, the effective lifetime $\tau_{B_s}^{\rm eff,5}$ is defined in Eq. (22), and

$$[A^{(5)}]_{\rm theo} = \mathcal{A}_5^{\rm ch} = \frac{1}{2}(\mathrm{Re}(A_\parallel A_0^*) + \mathrm{Re}(\bar{A}_\parallel \bar{A}_0^*)). \quad (52)$$

C. NP parameters

Twelve observables can be measured from the time-dependent untagged angular distribution (Sec. II C). With these, one can identify if NP is present in the mixing and/or the decay. However, we will also want to identify its properties. To be specific, if there is NP in the decay amplitude, it will be important to measure the various NP parameters. With this in mind, the question is, how many theoretical unknowns are there in the most general SM + NP$B_s^0 \to \phi\phi$ amplitude? If there are fewer than 12, then we can extract all the unknowns.

In writing the SM + NP$B_s^0 \to \phi\phi$ amplitude, we have the following points:

(i) The SM weak phases are $\simeq 0$.

(ii) Assuming that the NP amplitudes satisfy $|\mathcal{A}_h^{\rm NP}| < |\mathcal{A}_h^{\rm SM}|$, the NP strong phases are negligible [38]. This means that if there are many NP amplitudes, they can all be combined into a single term with an effective magnitude and weak phase.

(iii) In the heavy-quark limit, we have $\mathcal{A}_\perp^{\rm SM} = -\mathcal{A}_\parallel^{\rm SM}$ [39].

Taking these points into account, the most general SM + NP$B_s^0 \to \phi\phi$ helicity amplitude can then be written

$$\mathcal{A}_h = |\mathcal{A}_h^{\rm SM}|e^{i\delta_h^{\rm SM}} + |\mathcal{A}_h^{\rm NP}|e^{i\phi_h}. \quad (53)$$

There are a total of 11 unknown theoretical parameters—5 magnitudes (2 SM, 3 NP), 2 SM strong phases, 3 NP weak phases, and the mixing phase ϕ_s. In principle, these can all be extracted from the 12 observables.

However, note that Eq. (53) includes a different NP weak phase ϕ_h for each helicity amplitude. But in many NP models the weak phases are helicity independent. In this case there is only one NP weak phase ϕ, and the number of theoretical unknowns is reduced to 9. This is a model-dependent result, but it is still very general.

Finally, if the NP is purely left handed or right handed, then $\mathcal{A}_\perp^{\rm NP} = \mp\mathcal{A}_\parallel^{\rm NP}$ [40], which further reduces the number of theoretical unknowns by 1.

In all cases, assuming the time-dependent untagged angular distribution can be measured, there are more observables than unknowns, and so we will be able to extract all the NP parameters in the decay. In this way, we may be able to identify the type of NP that is present.

V. NUMERICAL ANALYSIS

Recently, the CDF and LHCb Collaborations have reported measurements for the polarization amplitudes, the strong-phase difference between $A_\parallel$ and A_0, and the triple-product asymmetries in $B_s^0 \to \phi\phi$. The LHCb results [28] are summarized in Table I. The values are in good agreement with those reported by the CDF Collaboration [27].

The experiments have measured the $\langle K_i\rangle$ and constructed the polarization fractions assuming the SM. As discussed previously, if one allows for the possibility of NP in $\bar{b} \to \bar{s}$ transitions, this analysis must be modified. This is done below.

We denote the measured value of y_s as y_{s0}. From Eq. (29) we have

$$\begin{aligned}\langle K_1\rangle &= \frac{\tau_{B_s}}{1+y_s}|A_0|^2 = \frac{\tau_{B_s}}{1+y_{s0}}|A_0|^2_{y_s=y_{s0}}, \quad (54)\\ \langle K_2\rangle &= \frac{\tau_{B_s}}{1+y_s}|A_\parallel|^2 = \frac{\tau_{B_s}}{1+y_{s0}}|A_\parallel|^2_{y_s=y_{s0}}, \\ \langle K_3\rangle &= \frac{\tau_{B_s}}{1-y_s}|A_\perp|^2 = \frac{\tau_{B_s}}{1-y_{s0}}|A_\perp|^2_{y_s=y_{s0}}.\end{aligned}$$

The experimental measurements in Table I are then

TABLE I. Measured polarization amplitudes, strong-phase difference, and triple-product asymmetries in $B_s^0 \to \phi\phi$ [28]. The sum of the $|A_h|^2_{\rm exp}$ terms is constrained to unity.

Observable	Measurement
$\lvert A_0\rvert^2_{\rm exp}$	$0.365 \pm 0.022({\rm stat}) \pm 0.012({\rm syst})$
$\lvert A_\perp\rvert^2_{\rm exp}$	$0.291 \pm 0.024({\rm stat}) \pm 0.010({\rm syst})$
$\lvert A_\parallel\rvert^2_{\rm exp}$	$0.344 \pm 0.024({\rm stat}) \pm 0.014({\rm syst})$
$\cos(\delta_\parallel - \delta_0)$	$-0.844 \pm 0.068({\rm stat}) \pm 0.029({\rm syst})$
$\mathcal{A}_u$	$-0.055 \pm 0.036({\rm stat}) \pm 0.018({\rm syst})$
$\mathcal{A}_v$	$0.010 \pm 0.036({\rm stat}) \pm 0.018({\rm syst})$

$$
\begin{aligned}
|A_0|^2_{\rm exp} &= \frac{|A_0|^2_{y_s=y_{s0}}}{|A_0|^2_{y_s=y_{s0}} + |A_\parallel|^2_{y_s=y_{s0}} + |A_\perp|^2_{y_s=y_{s0}}}, \\
|A_\parallel|^2_{\rm exp} &= \frac{|A_\parallel|^2_{y_s=y_{s0}}}{|A_0|^2_{y_s=y_{s0}} + |A_\parallel|^2_{y_s=y_{s0}} + |A_\perp|^2_{y_s=y_{s0}}}, \\
|A_\perp|^2_{\rm exp} &= \frac{|A_\perp|^2_{y_s=y_{s0}}}{|A_0|^2_{y_s=y_{s0}} + |A_\parallel|^2_{y_s=y_{s0}} + |A_\perp|^2_{y_s=y_{s0}}}.
\end{aligned}
\tag{55}
$$

One can now calculate the polarization fractions in the SM as a function of y_s. Inputting the expressions for the $\langle K_i\rangle$ from Eq. (54) into Eq. (35), and using Eq. (55), we obtain

$$
\begin{aligned}
f_0^{\rm SM} &= \frac{|A_0|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}}}{|A_0|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}} + |A_\parallel|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}} + |A_\perp|^2_{\rm exp}\frac{1-y_s}{1-y_{s0}}}, \\
f_\parallel^{\rm SM} &= \frac{|A_\parallel|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}}}{|A_0|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}} + |A_\parallel|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}} + |A_\perp|^2_{\rm exp}\frac{1-y_s}{1-y_{s0}}}, \\
f_\perp^{\rm SM} &= \frac{|A_\perp|^2_{\rm exp}\frac{1-y_s}{1-y_{s0}}}{|A_0|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}} + |A_\parallel|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}} + |A_\perp|^2_{\rm exp}\frac{1-y_s}{1-y_{s0}}}.
\end{aligned}
\tag{56}
$$

Hence the $|A_i|^2_{\rm exp}$ in Table I are just the $f_i^{\rm SM}$ defined in Eq. (56) with $y_s = y_{s0}$.

The true polarization fractions can then be obtained by inputting the expressions for the $\langle K_i\rangle$ from Eq. (54) into Eq. (36), and using Eq. (55):

$$
\begin{aligned}
f_0 &= \frac{|A_0|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}}Y_1}{|A_0|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}}Y_1 + |A_\parallel|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}}Y_2 + |A_\perp|^2_{\rm exp}\frac{1-y_s}{1-y_{s0}}Y_3}, \\
f_\parallel &= \frac{|A_\parallel|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}}Y_2}{|A_0|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}}Y_1 + |A_\parallel|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}}Y_2 + |A_\perp|^2_{\rm exp}\frac{1-y_s}{1-y_{s0}}Y_3}, \\
f_\perp &= \frac{|A_\perp|^2_{\rm exp}\frac{1-y_s}{1-y_{s0}}Y_3}{|A_0|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}}Y_1 + |A_\parallel|^2_{\rm exp}\frac{1+y_s}{1+y_{s0}}Y_2 + |A_\perp|^2_{\rm exp}\frac{1-y_s}{1-y_{s0}}Y_3}.
\end{aligned}
\tag{57}
$$

In Fig. 1 we plot the dependence of the polarization fractions f_0, $f_\parallel$, and $f_\perp$ as a function of y_s. This figure is read as follows. In all plots the horizontal region represents the experimental result, in which $[|A|^2_{h=0,\parallel,\perp}]_{\rm exp}$ is allowed to vary by $\pm 1\sigma$ (see Table I). Also, the vertical bands correspond to y_s, with $\pm 1\sigma$ (green) or $\pm 3\sigma$ (yellow) errors. In the SM we have $Y_i = 1$, corresponding to ($A^1_{\Delta\Gamma} = -1$, $A^2_{\Delta\Gamma} = -1$, $A^3_{\Delta\Gamma} = 1$) [Eq. (34)]. In order to illustrate the effect of NP, we take ($A^1_{\Delta\Gamma} = 1$, $A^2_{\Delta\Gamma} = -1$, $A^3_{\Delta\Gamma} = -1$) (red line) or ($A^1_{\Delta\Gamma} = -1$, $A^2_{\Delta\Gamma} = 1$, $A^2_{\Delta\Gamma} = 1$) (blue line). For these values of $A^i_{\Delta\Gamma}$, we have $Y_i \neq 1$. Consider first f_0. In the SM the experimental measurement implies $0.33 \leq f_0 \leq 0.40$. However, with NP, the value of f_0 can lie outside this range—for example, on the red line it can be as small as 0.29. The behavior is similar for $f_\parallel$ and $f_\perp$. This shows explicitly that, in the presence of NP, the $B_s^0 \to \phi\phi$ polarization fractions can be changed from their SM values by $O(10\%)$ for the current value of y_s.

The relation between $\mathcal{A}_u$ and $[\mathcal{A}_T^{(2)}]_{\rm theo}$ is given in Eqs. (41) and (42); that between $\mathcal{A}_v$ and $[\mathcal{A}_T^{(1)}]_{\rm theo}$ is given in Eqs. (45) and (46). These can be rewritten as

$$
\begin{aligned}
[\mathcal{A}_T^{(2)}]_{\rm theo}\frac{\tau_{B_s}}{\langle\Gamma(B_s^0\to\phi\phi)\rangle} &= -\frac{\pi}{2}\mathcal{A}_u\frac{(1-y_s^2)}{(1+A^{(4)}_{\Delta\Gamma}y_s)}, \\
[\mathcal{A}_T^{(1)}]_{\rm theo}\frac{\tau_{B_s}}{\langle\Gamma(B_s^0\to\phi\phi)\rangle} &= -\frac{\pi}{\sqrt{2}}\mathcal{A}_v\frac{(1-y_s^2)}{(1+A^{(6)}_{\Delta\Gamma}y_s)}.
\end{aligned}
\tag{58}
$$

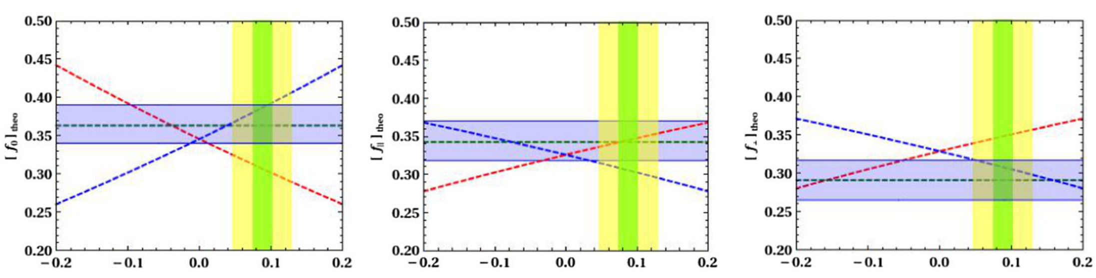

FIG. 1 (color online). The dependence of the theoretical polarization fractions f_0, $f_\parallel$, and $f_\perp$ on the decay width parameter y_s for different values of $\mathcal{A}^{1,2,3}_{\Delta\Gamma}$. The red line (dashed, pointing down, left to right) corresponds to ($A^1_{\Delta\Gamma} = 1$, $A^2_{\Delta\Gamma} = -1$, $A^3_{\Delta\Gamma} = -1$), while the blue line (dashed, pointing up, left to right) has ($A^1_{\Delta\Gamma} = -1$, $A^2_{\Delta\Gamma} = 1$, $A^2_{\Delta\Gamma} = 1$). In all plots the experimental result $[|A|^2_{h=0,\parallel,\perp}]_{\rm exp}$ (horizontal region) is allowed to vary by $\pm 1\sigma$ (see Table I). The vertical bands correspond to y_s, with $\pm 1\sigma$ green (inner) or $\pm 3\sigma$ yellow (outer) errors.

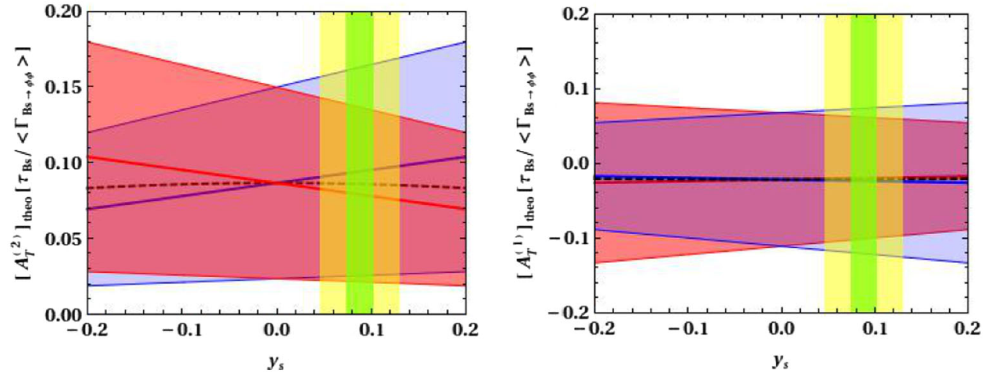

FIG. 2 (color online). The dependence of the theoretical TP's $[A_T^{(2)}]_{\rm theo}\tau_{B_s}/\langle\Gamma(B_s^0\to\phi\phi)\rangle$ (left) and $[A_T^{(1)}]_{\rm theo}\tau_{B_s}/\langle\Gamma(B_s^0\to\phi\phi)\rangle$ (right) on y_s for different values of $A_{\Delta\Gamma}^{4(6)}$. In the red (wider on left-hand side) and blue (narrower on left-hand side) regions, we take $A_{\Delta\Gamma}^{4(6)}=\pm 1$, respectively. Also, $\mathcal{A}_u$ (left) and $\mathcal{A}_v$ (right) are allowed to vary by $\pm 1\sigma$ (see Table I). The dashed black lines correspond to the central values of $\mathcal{A}_u$ (left) and $\mathcal{A}_v$ (right) with $A_{\Delta\Gamma}^{4(6)}=0$. The vertical bands correspond to y_s, with $\pm 1\sigma$ green (inner) or $\pm 3\sigma$ yellow (outer) errors.

In Fig. 2 we plot the dependence of the theoretical TP's $[A_T^{(2)}]_{\rm theo}\tau_{B_s}/\langle\Gamma(B_s^0\to\phi\phi)\rangle$ and $[A_T^{(1)}]_{\rm theo}\tau_{B_s}/\langle\Gamma(B_s^0\to\phi\phi)\rangle$ as a function of y_s. The dashed black lines correspond to the central values of $\mathcal{A}_u$ (left) and $\mathcal{A}_v$ (right) with $A_{\Delta\Gamma}^{4(6)}=0$. The vertical bands correspond to y_s, with $\pm 1\sigma$ (green) or $\pm 3\sigma$ (yellow) errors. In the red and blue regions, we take $A_{\Delta\Gamma}^{4(6)}=\pm 1$, respectively, and allow $\mathcal{A}_u$ (left) and $\mathcal{A}_v$ (right) to vary by $\pm 1\sigma$ (see Table I). It is clear from these figures that, in the presence of NP, the values of the theoretical TP's can differ significantly from the measured asymmetries. (This is not surprising since the TP's vanish in the SM.)

Finally, for $i=5$, we have estimated the measured value of the CP-conserving observable as follows:

$$\begin{aligned}[A^{(5)}]_{\rm exp} &= |A_0|_{\rm exp}|A_\parallel|_{\rm exp}\cos(\delta_\parallel-\delta_0)\\ &= -0.299\pm 0.030.\end{aligned} \tag{59}$$

The relation between $[A^{(5)}]_{\rm exp}$ and $[A^{(5)}]_{\rm theo}$ is given by [see Eq. (51)]

$$[A^{(5)}]_{\rm theo}\frac{\tau_{B_s}}{\langle\Gamma(B_s^0\to\phi\phi)\rangle}=[A^{(5)}]_{\rm exp}\frac{(1-y_s^2)}{(1+A_{\Delta\Gamma}^{(5)}y_s)}. \tag{60}$$

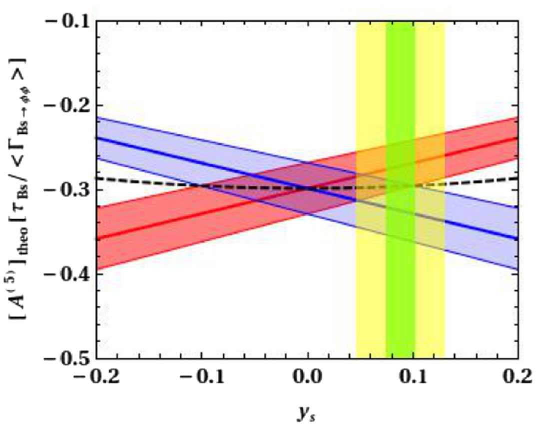

FIG. 3 (color online). The dependence of $[A_T^{(5)}]_{\rm theo}\tau_{B_s}/\langle\Gamma(B_s^0\to\phi\phi)\rangle$ on y_s for different values of $A_{\Delta\Gamma}^5$. In the red (lower on left-hand side) and blue (upper on left-hand side) regions, we take $A_{\Delta\Gamma}^5=\pm 1$, respectively. Also, $[A^{(5)}]_{\rm exp}$ is allowed to vary by $\pm 1\sigma$ [see Eq. (59)]. The dashed black line corresponds to the central value of $[A^{(5)}]_{\rm exp}$ with $A_{\Delta\Gamma}^5=0$. The vertical bands correspond to y_s, with $\pm 1\sigma$ green (inner) or $\pm 3\sigma$ yellow (outer) errors.

In Fig. 3 we plot the dependence of $[A^{(5)}]_{\rm theo}\tau_{B_s}/\langle\Gamma(B_s^0\to\phi\phi)\rangle$ as a function of y_s. As before, the value of this quantity can differ from Eq. (59) by as much as $O(10\%)$ for the current value of y_s.

VI. CONCLUSIONS

The main goal of studying the B system is to find evidence for physics beyond the standard model. One possibility is new physics in $\bar{b}\to\bar{s}$ transitions. At present its status is uncertain. It seems unlikely that the effect of such NP can be very large, but a smaller effect is still possible. In this paper, we consider $\bar{b}\to\bar{s}$ NP. However, in contrast to what is usually done, i.e., considering only NP in B_s^0-$\bar{B}_s^0$ mixing, here we also allow NP in the decay. In particular, we examine the effect of such NP on the angular distribution of $B_q^0\to V_1V_2$ ($q=d,s$), where $V_{1,2}$ are vector mesons.

Our principal result is the following. The parameters of the untagged, time-integrated angular distribution can be measured experimentally, and certain observables can be derived from these parameters. However, in the presence of NP, the formulas that relate the parameters to the observables must be modified from their SM forms. We find six observables for which the relation between the experimental data and theoretical parameters must be modified, corresponding to the six terms ($i=1$–6) in the angular distribution. For $i=1$–3 they are the polarization fractions, for

$i = 4,6$ they are the CP-violating triple-product asymmetries, and $i = 5$ corresponds to a CP-conserving observable. The modifications for the polarization fractions are most interesting. These are due in part to the nonzero width difference in the B_q^0-$\bar{B}_q^0$ system, and so are important only for B_s^0 decays. In particular, there can be important effects on the pure $\bar{b} \to \bar{s}$ penguin decay $B_s^0 \to \phi\phi$.

In light of this, we reanalyze the $B_s^0 \to \phi\phi$ data to see the effect of these modifications. $\Delta\Gamma_s/2\Gamma_s \sim 10\%$, so that the modifications of the formulas lead to $O(10\%)$ changes in the polarization fractions. These are not large, but may be important given that one is looking for signals of NP.

Finally, if the NP contributes to the $\bar{b} \to \bar{s}$ decay, we show that the measurement of the untagged time-dependent angular distribution provides enough information—12 observables—to extract all the NP parameters.

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