Coulomb gauge Yang-Mills theory at finite temperatures: Glueballs versus quasi-gluons

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We consider a variational approach to the finite-temperature Yang-Mills theory in the Coulomb gauge. The partition function is computed in the ensemble of glueballs and quasigluons which emerge as eigenstates of the Coulomb gauge Hamiltonian. We compute the energy density and pressure and compare with results of lattice computations for both SU(2) and SU(3). The emergence of a phase transition is discussed.

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I. INTRODUCTION

In recent years there has been an expansion in studies aimed at the determination of the patterns of the QCD phase transitions [1,2]. These are crucial for an understanding of the mechanism of confinement and dynamical chiral symmetry breaking. At high temperature and/or density, due to the asymptotic freedom, it is expected that the weak interaction between quarks and gluons determine the properties of the quark-gluon plasma [3–7]. Lattice simulations at finite temperature are a good tool to investigate these phase transitions [8–12], while phenomenological models also enable studies of the high density regime where the restoration of chiral symmetry is expected [13–17].

The present paper investigates the thermal properties of a phenomenological model motivated by the canonical approach to QCD in the physical, Coulomb gauge quantization. We compute the partition function in the ensemble of glueballs and quasigluons. There are numerous studies of QCD thermal properties in covariant gauges that include for example Dyson-Schwinger based models [18–21] or approaches based on the renormalization group flow [22], or direct models of the equation of state [23–27]. The few approaches that exist in physical gauges are rather loosely related to the underlying QCD interactions [28–32]. Recently, there has been also an attempt to explore the dynamical breaking of chiral symmetry in a self-consistent calculation at finite density [33].

The advantages of physical gauges for phenomenology and for developing physical intuition are clear, and we summarize them here. The degrees of freedom of the pure Yang-Mills (YM) theory are transverse gluons, and thermal excitations connect color-singlet states of an arbitrary number of gluons. Transverse gluons are expected to be effective only at high temperatures, while at low temperatures it is more effective to compute the partition function in terms of the ground state glueballs [34,35]. The underlying interactions in Coulomb gauge are dominated by the instantaneous Coulomb potential acting between color charges. In the non-Abelian theory, the potential not only couples charges but it also depends on the gluon distribution of the state in which it is calculated. At zero temperature, in the vacuum state this distribution is such that the Coulomb potential becomes confining, i.e., proportional to the distance R between the external color charges, $V(R) = \sigma_C R$ [36,37]. Using various approximate, variational models for the ground state YM wave functional, it has been possible to obtain a potential that is confining [38] or almost confining, i.e., $V(R) \to R^{1-\epsilon}$ with $\epsilon \approx O(10\%)$ [39–41]. The Coulomb string tension σ_c is larger than the string tension computed from the temporal Wilson loop. This is because the Coulomb potential represents the energy of a static quark-antiquark pair submersed in the QCD vacuum, while the Wilson loop measures the energy of the exact quark-antiquark state in which the gluon distribution is squeezed by closed vortex lines. Since the Coulomb potential is an instantaneous observable, one might expect that it remains confining even in the high-temperature limit [37]: At high temperatures the integration over transverse fields becomes even less restricted than in the vacuum, and, according to the Gribov-Zwanziger confinement scenario [42,43], Coulomb confinement originates from large field configurations near the Gribov horizon.

Recently the variational approaches to Yang-Mills theory in Coulomb gauge [38,40,41] have been extended to full QCD [44] and to finite temperatures [1,2], assuming a quasiparticle picture for the gluons. In the present paper we study the thermodynamic properties not only of a system of quasigluons but also include glueballs, which are the physical constituents of the Yang-Mills ensemble in the confining phase. In the following, we investigate the finite-temperature properties of Coulomb gauge Yang-Mills theory with a focus on the aspects of thermodynamical properties and their behavior around the critical temperature of the phase transition. In particular, we compute the energy density and pressure in the ensembles of glueballs and quasigluons and compare with SU(2) and SU(3) lattice results. In Sec. II we present the general setting for the

finite-temperature, canonical Coulomb gauge problem as well as general properties of the thermal average when the glueballs basis is used. In Sec. III we present the numerical results. The summary and outlook are given in Sec. IV. Finally, details of the important expressions are presented in the Appendixes.

II. HAMILTONIAN APPROACH AT FINITE TEMPERATURES

The Coulomb gauge Yang-Mills Hamiltonian is obtained after gauge fixing and elimination of the Gauss's law constraint on the longitudinal component of the electric field:

$$H_{\mathrm{YM}} = \frac{1}{2}\int \mathrm{d}^3x (J^{-1}[A]\Pi J[A]\Pi + B^2) + H_{\mathrm{C}} \equiv H_K + H_B + H_{\mathrm{C}}, \tag{1}$$

$$H_{\mathrm{C}} = \frac{g^2}{2}\int \mathrm{d}^3x \mathrm{d}^3y J^{-1}[A]\rho^a(x) J[A] F_A^{ab}(x,y)\rho^b(y). \tag{2}$$

Here $\Pi^a(x) = -\mathrm{i}\delta/\delta A^a(x)$ is the canonical momentum (electric field) operator, and

$$J[A] = \mathrm{Det}(-D\nabla) \tag{3}$$

is the Faddeev-Popov determinant with

$$D = \nabla + g\hat{A}, \qquad \hat{A}^{ab} = \hat{T}_c A^c, \qquad (\hat{T}_c)^{ab} = f^{acb}, \tag{4}$$

being the covariant derivative in the adjoint representation. Furthermore,

$$\rho^a(x) = -f^{abc}A^b \cdot \Pi^c \tag{5}$$

is the color charge density of the gluons and

$$F_A^{ab}(x,y) = \langle x,a|(-D\nabla)^{-1}(-\nabla^2)(-D\nabla)^{-1}|y,b\rangle \tag{6}$$

is the gluon field dependent Coulomb kernel. The vacuum expectation value of this kernel plays the role of an instantaneous potential between color charges. At zero temperature and at large distances $\langle F_A^{ab}(x,y)\rangle$ is well approximated by a linear rising potential. The gauge fixed Hamiltonian in Eq. (1) is highly nonlocal due to Coulomb kernel $F_A(x,y)$, Eq. (6), and the Faddeev-Popov determinant $J[A]$, Eq. (3). In addition, the latter also occurs in the functional integration measure of the scalar product of the Coulomb gauge wave functionals

$$\langle\psi_1|O|\psi_2\rangle = \int DAJ[A]\psi_1^*[A]O\psi_2[A]. \tag{7}$$

In Ref. [40] the Yang-Mills Schrödinger equation was solved by the variational principle using the following ansatz for the vacuum wave functional:

$$\langle A|0\rangle = \frac{1}{\sqrt{J[A]}}\langle A|\tilde{0}\rangle, \qquad \langle A|\tilde{0}\rangle = \mathcal{N}\exp\left(-\frac{1}{2}\int\frac{\mathrm{d}^3k}{(2\pi)^3}A(-k)\omega(k)A(k)\right). \tag{8}$$

The preexponential factor removes the Faddeev-Popov determinant from the scalar product Eq. (7). The kernel $\omega(k)$ was determined by minimizing the vacuum energy $\langle H_{\mathrm{YM}}\rangle$, which yields an $\omega(k)$ that can be well fitted by Gribov's formula

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}, \tag{9}$$

and which is in satisfactory agreement with the lattice data [45], for $M \approx 880$ MeV.

The present paper is devoted to study Yang-Mills theory at finite temperatures, which is defined by the density operator

$$\mathcal{D} = Z^{-1}\exp(-\beta H_{\mathrm{YM}}), \tag{10}$$

where $\beta = 1/T$ is the inverse temperature and

$$Z = \mathrm{Tr}\,\mathrm{e}^{-\beta H_{\mathrm{YM}}} \tag{11}$$

is the partition function. Exact calculation of the trace in the thermal averages

$$\langle O\rangle = \mathrm{Tr}(O\mathcal{D}) \tag{12}$$

is not possible for the YM theory and the way we proceed is, following Ref. [1], to replace $\mathcal{D}$ by a variational ansatz. This is achieved by first defining a suitable basis in the gluonic Fock space. It is chosen as follows. In the standard fashion we Fourier decompose the gauge field in terms of creation and annihilation operators

$$A_i^a(\mathbf{x}) = \int\frac{d^3q}{(2\pi)^3}\frac{1}{\sqrt{2\omega(q)}}[a_i^a(\boldsymbol{q}) + a_i^{a\dagger}(-\boldsymbol{q})]e^{-i\mathbf{q}\cdot\mathbf{x}}, \qquad \Pi_i^a(\mathbf{x}) = -i\int\frac{d^3q}{(2\pi)^3}\sqrt{\frac{\omega(q)}{2}}[a_i^a(\boldsymbol{q}) - a_i^{a\dagger}(-\boldsymbol{q})]e^{-i\mathbf{q}\cdot\mathbf{x}}, \tag{13}$$

where $a_i^b(\boldsymbol{q}) = \sum_\lambda \epsilon_i(\boldsymbol{q},\lambda)a(\boldsymbol{q},\lambda,b)$ (λ and b are the helicity and color indices, respectively). Choosing here $\omega(q)$ to be the kernel of the vacuum wave functional (8) the operators $a_i^a(q)$ annihilate this state, i.e.,

$$a_i^a(k)|\tilde{0}\rangle = 0. \tag{14}$$

Then a complete basis in the gluonic Fock space is given by

$$\{|\tilde{n}\rangle\} = \{|\tilde{0}\rangle, a_i^{a\dagger}(k)|\tilde{0}\rangle, a_i^{a\dagger}(k)a_j^{b\dagger}(q)|\tilde{0}\rangle, \ldots\}. \tag{15}$$

As shown in Ref. [1], however, this quasigluon basis, even when restricted to color singlet states, is not ideal for studies of thermal properties of the YM plasma. Because of confinement, energies of isolated gluons are large

(infinite) and subtle cancellation of infrared divergencies has to occur in color singlet states containing multiple gluons with low relative momenta. For the same reason gluons in these low momentum states are expected to strongly bind into color singlet, also known as glueball, states. Thus instead of working directly with the basis of Eq. (15) we choose to work with a basis of glueballs, which are constructed using creation operators defined by

$$G^\dagger(\alpha) = \frac{1}{\mathcal{V}} \sum_{(1,2)} \Psi^\alpha(1,2) a^\dagger(1) a^\dagger(2), \tag{16}$$

where $\mathcal{V} = (2\pi)^3 \delta(0)$ is the volume factor. Here the summation extends over single gluon helicities $(\lambda_{1,2})$ and color $(c_{1,2})$ and it also includes integration over individual gluon momenta $(p_{1,2})$. Strong interactions lead to effective, quasigluon mass in the range of 600–800 MeV. Thus bound states with a minimal number of gluons are expected to dominate the low energy spectrum. The basis of two-gluon glueballs in Eq. (16) is therefore expected to describe the lowest glueball spectrum and dominate thermodynamical properties at low temperatures. When acting on a vacuum state $G^\dagger$ creates a glueball state with quantum numbers $\alpha = (P, J^{PC})$, where P and J^{PC} are the total momentum, and the total spin, parity and charge conjugation of the glueball, respectively. The cutoff on relative momentum is implicit in the glueball wave function Ψ^α. The latter can be written explicitly as

$$\begin{aligned} \Psi^\alpha(1,2) &\equiv \Psi^\alpha_{\lambda_1\lambda_2;c_1c_2}(p_1, p_2) \\ &= (2\pi)^3 \delta^3(P - p_1 - p_2) \\ &\quad \times \frac{\delta^{c_1c_2}}{\sqrt{N_C^2 - 1}} \frac{\Psi^\alpha_{\lambda_1\lambda_2}(\boldsymbol{p}_2 - \boldsymbol{p}_1)}{\sqrt{2}}, \end{aligned} \tag{17}$$

and it is normalized by

$$\sum_{\lambda_1\lambda_2} \int [dp_1 dp_2]_P \Psi^\alpha_{\lambda_1\lambda_2}(p_1, p_2)|^2 = 1, \tag{18}$$

with the measure defined as

$$[dp_1 dp_2]_P = \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} (2\pi)^3 \delta(P - p_1 - p_2). \tag{19}$$

Bose symmetry implies that the wave function is symmetric under exchange of the quantum numbers of the two gluons,: $\Psi^\alpha_{\lambda_1\lambda_2}(\boldsymbol{p}_1, \boldsymbol{p}_2) = \Psi^\alpha_{\lambda_2\lambda_1}(\boldsymbol{p}_2, \boldsymbol{p}_1)$. In terms of this single glueball operator, multiple glueball states $|n_{\alpha_1} n_{\alpha_2} \ldots\rangle$ are given by

$$|n_{\alpha_1} n_{\alpha_2} \ldots\rangle = \prod_i \frac{(G^\dagger(\alpha_i))^{n_{\alpha_i}}}{\sqrt{n_{\alpha_i}!}} |\tilde{0}\rangle. \tag{20}$$

In the following we will ignore the Faddeev-Popov determinant. As will be shown below, the expression we obtain from variational principle is closely related to that for the spectrum of $H_{\rm YM}$ and at the end given by the eigenvalues of the Hamiltonian. Thus the Faddeev-Popov contributions to the formulas for the free energy can in principle be restored by comparing with those for $H_{\rm YM}$.

We introduce a variational ansatz for the thermal density operator by replacing $H_{\rm YM}$ in Eq. (10) by the following single particle operator:

$$h = \int \frac{d^3k}{(2\pi)^3} \Omega(\boldsymbol{k}) \sum_{i,b} a_i^{b\dagger}(\boldsymbol{k}) a_i^b(\boldsymbol{k}). \tag{21}$$

The optimal value of free energy is obtained by taking a variation with respect to Ω, which obviously has the meaning of the gluon energy.

A. The partition function

Computation of the partition function (11) with $H_{\rm YM}$ replaced by h (21) in the glueball basis (20)

$$\mathcal{Z} = \sum_{n_{\alpha_1} n_{\alpha_2} \ldots} \langle n_{\alpha_1} n_{\alpha_2} \ldots | e^{-\beta \sum_j \Omega_j a_j^\dagger a_j} | n_{\alpha_1} n_{\alpha_2} \ldots \rangle \tag{22}$$

is straightforward and yields

$$\mathcal{Z} = \exp\left[V \int \frac{d^3P}{(2\pi)^3} \sum_{J^{PC}} \ln(1 + n_\alpha(P)) \right], \tag{23}$$

where

$$n_\alpha(P) = \frac{e^{-\beta E_\alpha}}{1 - e^{-\beta E_\alpha}} \tag{24}$$

is the glueball thermal occupation number. Here the effective glueball energy E_α is given by

$$e^{-\beta E_\alpha(P)} = \sum_{\lambda_{1,2}} \int [dp_1 dp_2]_P |\Psi^\alpha_{\lambda_1\lambda_2}(p_1, p_2)|^2 e^{-\beta \Omega_{1,2}}, \tag{25}$$

where

$$\Omega_{1,2} \equiv \Omega(p_1) + \Omega(p_2). \tag{26}$$

Since the density operator is defined in the gluon basis but the thermal average is evaluated in the basis of glueballs the effective Boltzmann factor $\exp(-\beta E_\alpha)$ is determined by averaging gluon thermal distribution over the glueball wave function.

B. The internal energy

Using the Fourier decomposition (13) we can calculate thermal expectation values of the Yang-Mills Hamiltonian Eq. (1). After normal ordering, the Hamiltonian contains the vacuum contribution and one- and two-body gluon operators. Those are given explicitly in Appendix A. Since vacuum contribution is temperature independent it can be removed by defining the free energy with respect to that of the vacuum. The final expression for the thermal average of the Hamiltonian is then given by

$$\frac{\langle H_{\rm YM}\rangle}{\mathcal{V}} = \sum_{J^{PC}} \int \frac{d^3P}{(2\pi)^3} [\mathcal{E}_\alpha(P) + \mathcal{B}_\alpha(P) + \mathcal{C}_\alpha(P)] \times [1 + n_\alpha(P)], \tag{27}$$

where the three terms represent contributions from the one-body operators describing single gluon energies averaged over the glueball state ($\mathcal{E}$), the two-body magnetic contribution from the four-gluon vertex ($\mathcal{B}$), and the two-body Coulomb interaction ($\mathcal{C}$), respectively. The explicit formulas for the three terms are given in Appendix B.

C. The free energy

The glueball wave function is the solution of the Hamiltonian bound state problem and as discussed previously defines the basis over which thermal averages are computed. The variational estimate for the free energy F,

$$\mathcal{F} = \langle H_{\rm YM}\rangle - TS = \langle H_{\rm YM}\rangle - \frac{\ln \mathcal{Z}}{\beta} + \frac{\partial \ln \mathcal{Z}}{\partial \beta} \tag{28}$$

is in turn obtained by minimization with respect to the single gluon energy $\Omega(k)$. Before proceeding, however, we note that boost invariance requires the density matrix to depend only on the total momentum of the two-gluon state. Thus the factor $\exp(-\beta(\Omega(p_1) + \Omega(p_2)))$ which appears in matrix elements should be replaced by $\exp(-\beta\sqrt{P_\alpha^2 + M_\alpha^2})$, where M_α is a Lorentz scalar. From Eq. (25) it then follows immediately that

$$E_\alpha(P) = \sqrt{P_\alpha^2 + M_\alpha^2}, \tag{29}$$

and the internal energy given by Eq. (27) becomes

$$\frac{\langle H_{\rm YM}\rangle}{\mathcal{V}} = \sum_{J^{PC}} \int \frac{d^3P}{(2\pi)^3} n_\alpha(P)[\mathcal{E}^0_\alpha(P) + \mathcal{B}^0_\alpha(P) + \mathcal{C}^0_\alpha(P)], \tag{30}$$

where the subscript 0 indicates that the corresponding quantities are to be evaluated at $\beta = 0$ [cf. Eqs. (B5), (B7), and (B8)]. Finally the entropy reduces to [1]

$$\frac{S}{\mathcal{V}} = \sum_{J^{PC}} \int \frac{d^3P}{(2\pi)^3} [\ln(1 + n_\alpha(P)) + \beta E_\alpha(P) n_\alpha(P)]. \tag{31}$$

Minimization of $\mathcal{F}$ is now performed with respect to M_α and yields the following relation:

$$\mathcal{E}^0_\alpha(P) + \mathcal{B}^0_\alpha(P) + \mathcal{C}^0_\alpha(P) = E_\alpha(P), \tag{32}$$

which is immediately recognized as the zero-temperature eigenvalue equation for the Hamiltonian $H_{\rm YM}$ projected onto the glueball sector. If the Faddeev-Popov contributions were retained, the structure of Eq. (32) would remain unchanged but the individual terms would be modified. It thus follows that M_α is the glueball mass obtained from diagonalizing the Hamiltonian. With Eq. (32) we find from Eqs. (30) and (31) for the minimum of the free energy (28)

$$\frac{\mathcal{F}}{\mathcal{V}} = \frac{1}{\beta} \sum_{J^{PC}} \int \frac{d^3P}{(2\pi)^3} \ln(1 - e^{-\beta\sqrt{P_\alpha^2 + M_\alpha^2}}). \tag{33}$$

D. Scattering states

If the Coulomb interaction $F_A^{ab}(x, y)$ [cf. Eq. (1)] is replaced by its vacuum expectation value, which is expected to be strictly confining [36], quasigluon bound states saturate the spectrum of $H_{\rm YM}$. However, since the Coulomb kernel couples gluon Fock sectors with an arbitrary large number of gluons, reduction of the full Hamiltonian to the two-gluon subspace must break down above energies where excitations of multiple quasigluon states becomes relevant. At this point we qualitatively describe the spectrum in terms of single quasigluons, which are no longer confined but screened. Calculation of the expectation value of $H_{\rm YM}$ in the basis (15) of quasifree gluons was done in Ref. [1] and gives

$$\frac{\langle H_{\rm YM}\rangle}{\mathcal{V}} = 2(N_C^2 - 1) \int \frac{d^3q}{(2\pi)^3} n(q) e(q) + \int \frac{d^3q}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \times [b(p, q) + c(p, q)] n(p)[1 + n(q)]. \tag{34}$$

Here the three terms, given explicitly in Appendix B, represent contributions from the single gluon energy, the four-gluon magnetic term and the Coulomb interaction, respectively. The free energy is obtained from Eq. (28) with gluon entropy given by [1]

$$\frac{S}{\mathcal{V}} = 2(N_C^2 - 1) \int \frac{d^3q}{(2\pi)^3} [\ln[1 + n(q)] + \beta\Omega(q) n(q)]. \tag{35}$$

Minimizing the free energy with respect to the density matrix $\delta\mathcal{F}/\delta\Omega(k) = 0$ results in the following expression for the effective gluon energy $\Omega(k)$:

$$\Omega(k) = e(q) + \int \frac{d^3p}{(2\pi)^3} [b(q, p) + c(q, p)][1 + 2n(q)], \tag{36}$$

which, when substituted into the expression for free energy, yields

$$\frac{F}{\mathcal{V}} = -\frac{2(N_C^2 - 1)}{\beta} \int \frac{d^3q}{(2\pi)^3} \ln[1 + n(q)] + \frac{2(N_C^2 - 1)}{\beta} \times \int \frac{d^3q}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} 4n(p) n(q)[b(p, q) + c(p, q)]. \tag{37}$$

Here the first term represents the contribution from the free gas of quasigluons and the second one is the one-loop correction due to residual interactions. We emphasize that in order for the two-component glueball plus gluon model to be valid, the interaction in the Coulomb term in Eq. (37) (implicit in the term proportional to c) has to be screened. That is, $c(p, p)$ is assumed to be free from the infrared singularity at $p = q$ normally associated with

confinement. Scattering corrections, i.e., the one-loop term, are then expected to be weak and we ignore them in the numerical studies.

III. NUMERICAL RESULTS

We present a thermodynamical study for the energy density and pressure of the SU(2) and SU(3) gauge theories separately for the quasigluon and glueball ensembles. In the glueball ensemble the degeneracy factor determines the high-temperature limiting value of the energy density but the behavior of the transition is not well known as well as the location of the critical temperature. In the thermodynamical quasigluon study, the phase transition, the critical temperature T_C, the behavior below and above the transition and the high-temperature limit are compared with SU(2) and SU(3)-lattice results [23–26], when two dispersion relations $\Omega(k)$ are used.

A. Glueball energy density and pressure

Energy density and pressure are computed from the free energy differentiating $\ln\mathcal{Z}$ with respect to T and $\mathcal{V}$:

$$\epsilon = \frac{T^2}{\mathcal{V}}\frac{\partial \ln\mathcal{Z}}{\partial T}, \tag{38}$$

$$p = T\frac{\partial \ln\mathcal{Z}}{\partial \mathcal{V}}, \tag{39}$$

with $\mathcal{Z} = \exp(-\beta\mathcal{F})$. The energy density in the pure gauge theory has been found to rise rapidly at T_C and approach the high-temperature ideal gas (Stefan-Boltzmann) limit from below [24]. In the high-temperature limit one finds in leading order perturbation theory for SU(N) [12]

$$\frac{\epsilon}{T^4} = (N^2-1)\frac{\pi^2}{15}\left[1 + \frac{\alpha_s 5N_C}{\pi} + O(\alpha_s^2)\right]. \tag{40}$$

In the following we denote energy density and pressure for the ensemble of glueballs by ϵ_1 and p_1 and for the ensemble of gluons by ϵ_2 and p_2, respectively. In particular, glueball energy density and pressure, respectively, are given by

$$\begin{aligned} p_1 &= T\sum_{J^{PC}}\int\frac{d^3P_\alpha}{(2\pi)^3}\ln\left(1+\frac{1}{e^{\beta\sqrt{P_\alpha^2+M_G^2}}-1}\right), \\ \epsilon_1 &= \sum_{J^{PC}}\int\frac{d^3P_\alpha}{(2\pi)^3}\frac{\sqrt{P_\alpha^2+M_G^2}}{e^{\beta\sqrt{P_\alpha^2+M_G^2}}-1}. \end{aligned} \tag{41}$$

In the case of glueballs we need to know the expected degeneracy. Since explicit digitalization of the Coulomb gauge YM Hamiltonian [34,35] reproduces lattice glueball spectrum [46] we use the latter to determine the number of states. In particular we will consider glueballs up to 2.5 GeV [46–50], i.e., with $J^{PC} = 0^{+-}$, 0^{-+}, 2^{++}. The numerical results for energy density and pressure are shown in Figs. 1 and 2. At high temperatures $T \gg M_G$

$$\begin{aligned} \frac{1}{\mathcal{N}}\frac{\epsilon_1}{T^4}\bigg|_{T\gg M_G} &= \frac{1}{2\pi^2}\int_0^\infty dx\frac{x^3e^{-x}}{1-e^{-x}} = \frac{\pi^2}{30}, \\ \frac{1}{\mathcal{N}}\frac{p_1}{T^4}\bigg|_{T\gg M_G} &= \frac{1}{2\pi^2}\int_0^\infty dx x^2\ln\left[\frac{1}{1-e^{-x}}\right] = \frac{\pi^2}{90}, \end{aligned} \tag{42}$$

where $\mathcal{N} = \sum_\alpha$ is the degeneracy factor. Inspecting the numerical results it is evident that because of the high degeneracy, glueballs contribute too much to thermodynamical quantities as compared with QCD expectations from lattice simulations. It implies that glueballs must evaporate below the critical temperature (see below), and a Hamiltonian model based on confined potential without mixing with open channels becomes inadequate at fairly low temperatures.

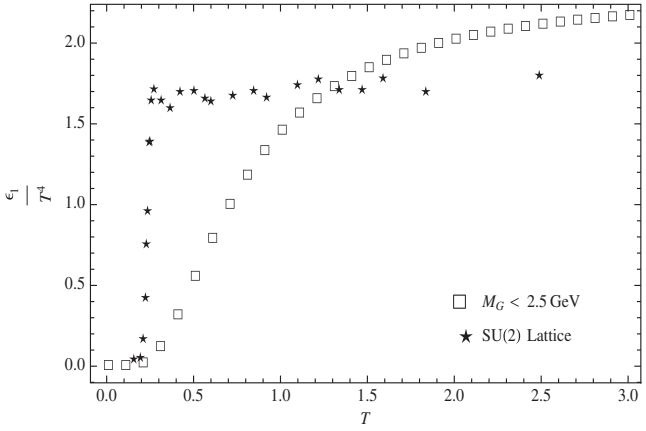

FIG. 1. Energy density versus temperature [GeV] in the case of glueballs for $J^{PC} = 0^{+-}$, 0^{-+}, 2^{++}.

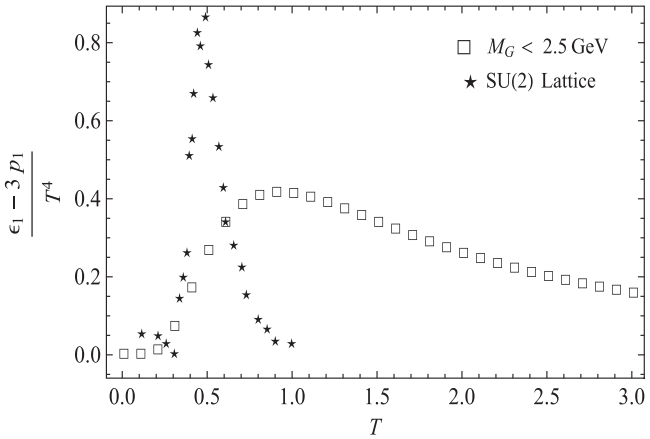

FIG. 2. The same as in Fig. 1 for the combination of energy density and pressure $(\epsilon - 3p)/T^4$.

B. Gluon energy density and pressure

The gluon energy density and pressure, respectively, are given by

$$p_2 = 2[N_C^2 - 1]T \int \frac{d^3q}{(2\pi)^3} \ln\left(1 + \frac{1}{e^{\beta\Omega(q)} - 1}\right),$$
$$\epsilon_2 = 2[N_C^2 - 1] \int \frac{d^3q}{(2\pi)^3} \frac{\Omega(q)}{e^{\beta\Omega(q)} - 1}, \tag{43}$$

which, assuming $\Omega(q) \longrightarrow q$ for large moments, in the high-temperature limit reduce to

$$\frac{1}{2(N_C^2 - 1)} \frac{\epsilon_2}{T^4}\bigg|_{T\to\infty} = \frac{\pi^2}{30},$$
$$\frac{1}{2(N_C^2 - 1)} \frac{p_2}{T^4}\bigg|_{T\to\infty} = \frac{\pi^2}{90}, \tag{44}$$

which, of course, yields the correct Stefan-Boltzmann limit given by Eq. (40) with $\alpha_s = 0$.

For the gluon dispersion relation we use the Gribov formula

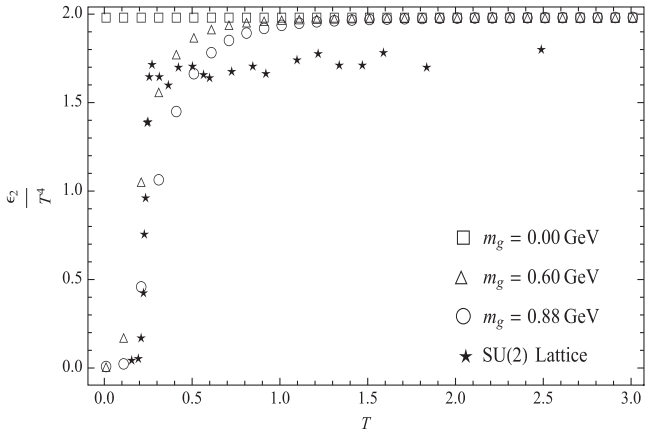

FIG. 3. Energy density versus temperature [GeV] for the $N_C = 2$ gluon ensemble with a Gribov dispersion relation.

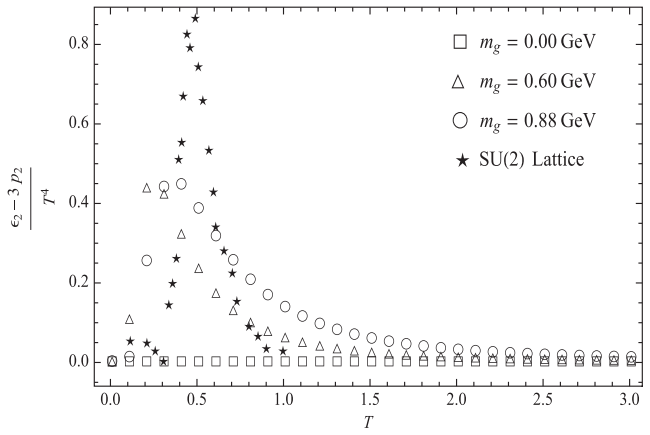

FIG. 4. The same as in Fig. 3 for the combination of energy density and pressure $(\epsilon - 3p)/T^4$.

$$\Omega(k) = \sqrt{k^2 + \frac{m_g^4}{k^2}}, \tag{45}$$

and choose the Gribov mass m_g in the range from zero (perturbative gluons) to 880 MeV. The latter value is found on the lattice [45]. In Figs. 3–5 we summarize the results for $N_C = 2$ and $N_C = 3$, respectively. Of course, in this calculation the transition temperature is set by the Gribov mass m_g. To calculate the transition temperature T_C we can calculate the derivative of the energy density and look for the location of the peak in the specific heat. For $m_g = 0.880$ GeV we find $T_C \approx 220$ MeV, which is in a reasonable agreement with lattice results [23,24]. For the lattice value $m_g = 880$ MeV the quasigluon ensemble reproduces

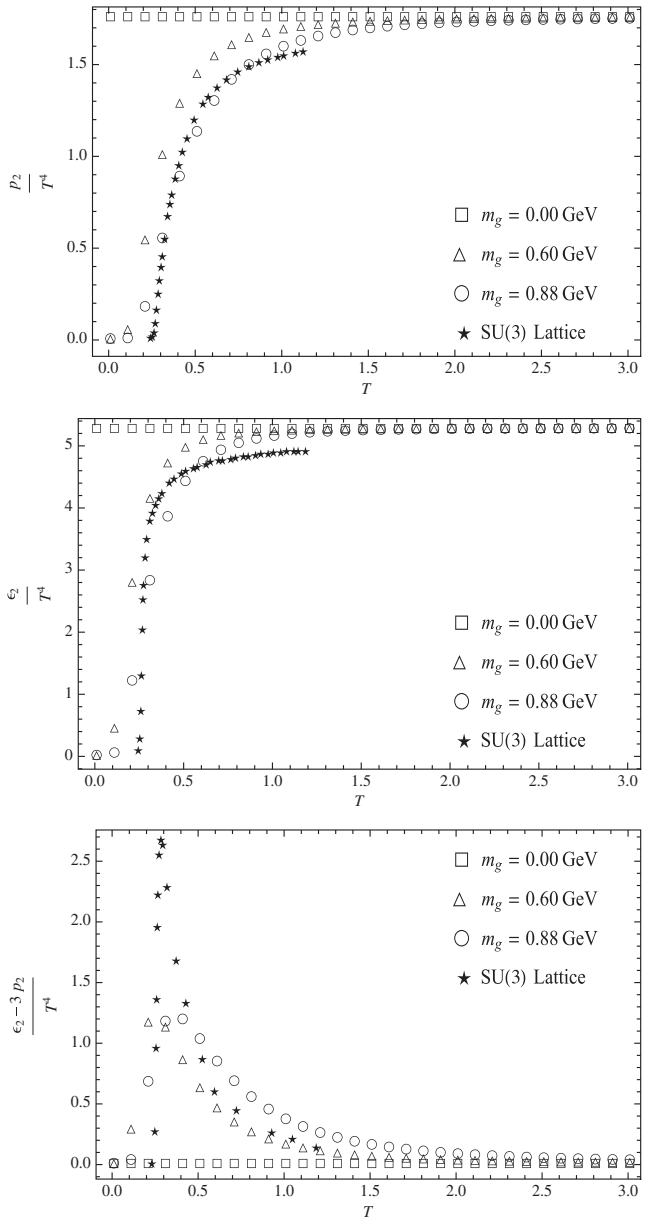

FIG. 5. The same as in Figs. 3 and 4 for $N_C = 3$.

the lattice energy density reasonably well up to the transition temperature but substantial deviations occur above the phase transition. This should come as no surprise. In a self-consistent treatment of the finite-temperature quasigluon ensemble in the variational approach in Coulomb gauge [2] one finds that at the deconfinement phase transition the gluon dispersion relation switches from the Gribov formula (45) in the confining phase to the massive dispersion relation

$$\Omega(k) = \sqrt{k^2 + m^2}, \tag{46}$$

where the gluon mass m is temperature dependent and growths linearly in T for large T. Its minimal value can be as low as 200 MeV. We have also computed energy and pressure for the massive dispersion relation (46), with m in a range from zero (perturbative) to 880 MeV. As expected, the Gribov formula reproduces the critical temperature and the overall shape more accurately than the massive dispersion relation.

IV. SUMMARY AND OUTLOOK

We studied the Coulomb gauge Yang-Mills theory at finite temperatures using a variational approach. The partition function has been computed in the ensemble of glueballs and quasigluons. Working with both ensembles we present the possibility of a phase transition since gluons with low relative momenta are expected to strongly bind into color singlet, glueball states. The thermodynamical limits for the energy density and pressure are different in each ensemble. This is expected since the partition function of the glueballs depends on the states J^{PC} included and their degeneracy, while the partition function of the (quasi) gluons depends on the number of colors N_C. In the present work we have considered the glueball states up to $M_G = 2.5$ GeV, i.e., $J^{PC} = 0^{+-}, 0^{-+}, 2^{++}$, and showed that the thermodynamical limit is rapidly overshot, indicating the possibility that the glueballs may evaporate at some finite temperature. Furthermore, the more glueballs one adds, the higher the degeneracy factor and the thermodynamical limit will be overshot faster, indicating that the glueballs evaporate at quite low (below T_C) temperatures. A more realistic description of the deconfinement phase transition would assume a two-component picture, in which glueballs coexist with gluons. Well below the deconfinement phase transition the finite-temperature Yang-Mills ensemble would dominantly consist of glueballs, which dissociate at the deconfinement phase transition into pairs of gluons.

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APPENDIX A: YM HAMILTONIAN CONTRIBUTIONS

The Hamiltonian thermal average is computed in the ensemble of glueballs and quasigluons which immediately rules out contributions from an odd number of particle creation and annihilation operators. The relevant terms of the YM Hamiltonian are

$$H_K \to \frac{1}{4}\int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\sum_{a,i}\sqrt{\omega(q)\omega(q')}(2\pi)^3\delta(\boldsymbol{q}+\boldsymbol{q}') \times [a_i^a(\boldsymbol{q})a_i^{a\dagger}(-\boldsymbol{q}') + a_i^{a\dagger}(-\boldsymbol{q})a_i^a(\boldsymbol{q}')], \tag{A1}$$

$$H_B \to \frac{1}{4}\int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\sum_a\sum_{ijklm}\epsilon_{ijk}\epsilon_{ilm}\frac{q_j(-q_l')}{\sqrt{\omega(q)\omega(q')}}(2\pi)^3\delta(\boldsymbol{q}+\boldsymbol{q}')[a_k^a(\boldsymbol{q})a_m^{a\dagger}(-\boldsymbol{q}') + a_k^{a\dagger}(-\boldsymbol{q})a_m^a(\boldsymbol{q}')]$$
$$+ \frac{g^2}{32}\int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3}\frac{d^3q_4}{(2\pi)^3}\sum_{abcde}\sum_{ijklm}\epsilon_{ijk}\epsilon_{ilm}\epsilon^{abc}\epsilon^{ade}\frac{(2\pi)^3\delta(\boldsymbol{q}_1+\boldsymbol{q}_2+\boldsymbol{q}_3+\boldsymbol{q}_4)}{\sqrt{\omega(q_1)\omega(q_2)\omega(q_3)\omega(q_4)}}$$
$$\times [a_j^b(\boldsymbol{q}_1)a_k^c(\boldsymbol{q}_2)a_l^{d\dagger}(-\boldsymbol{q}_3)a_m^{e\dagger}(-\boldsymbol{q}_4) + a_j^b(\boldsymbol{q}_1)a_k^{c\dagger}(-\boldsymbol{q}_2)a_l^d(\boldsymbol{q}_3)a_m^{e\dagger}(-\boldsymbol{q}_4) + a_j^b(\boldsymbol{q}_1)a_k^{c\dagger}(-\boldsymbol{q}_2)a_l^{d\dagger}(-\boldsymbol{q}_3)a_m^e(\boldsymbol{q}_4)$$
$$+ a_j^{b\dagger}(-\boldsymbol{q}_1)a_k^c(\boldsymbol{q}_2)a_l^d(\boldsymbol{q}_3)a_m^{e\dagger}(-\boldsymbol{q}_4) + a_j^{b\dagger}(-\boldsymbol{q}_1)a_k^c(\boldsymbol{q}_2)a_l^{d\dagger}(-\boldsymbol{q}_3)a_m^e(\boldsymbol{q}_4) + a_j^{b\dagger}(-\boldsymbol{q}_1)a_k^{c\dagger}(-\boldsymbol{q}_2)a_l^d(\boldsymbol{q}_3)a_m^e(\boldsymbol{q}_4)], \tag{A2}$$

$$H_C \to \frac{g^2}{8}\int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3}\frac{d^3q_4}{(2\pi)^3}\sum_{abcde}\sum_{ij}\epsilon^{abc}\epsilon^{ade}(2\pi)^3\delta(\boldsymbol{q}_1+\boldsymbol{q}_2+\boldsymbol{q}_3+\boldsymbol{q}_4)F(-\boldsymbol{q}_3-\boldsymbol{q}_4)$$
$$\times\sqrt{\frac{\omega(q_2)\omega(q_4)}{\omega(q_1)\omega(q_3)}}[a_i^b(\boldsymbol{q}_1)a_i^c(\boldsymbol{q}_2)a_j^{d\dagger}(-\boldsymbol{q}_3)a_j^{e\dagger}(-\boldsymbol{q}_4) - a_i^b(\boldsymbol{q}_1)a_i^{c\dagger}(-\boldsymbol{q}_2)a_j^d(\boldsymbol{q}_3)a_j^{e\dagger}(-\boldsymbol{q}_4)$$
$$+ a_i^b(\boldsymbol{q}_1)a_i^{c\dagger}(-\boldsymbol{q}_2)a_j^{d\dagger}(-\boldsymbol{q}_3)a_j^e(\boldsymbol{q}_4) + a_i^{b\dagger}(-\boldsymbol{q}_1)a_i^c(\boldsymbol{q}_2)a_j^d(\boldsymbol{q}_3)a_j^{e\dagger}(-\boldsymbol{q}_4) - a_i^{b\dagger}(-\boldsymbol{q}_1)a_i^c(\boldsymbol{q}_2)a_j^{d\dagger}(-\boldsymbol{q}_3)a_j^e(\boldsymbol{q}_4)$$
$$+ a_i^{b\dagger}(-\boldsymbol{q}_1)a_i^{c\dagger}(-\boldsymbol{q}_2)a_j^d(\boldsymbol{q}_3)a_j^e(\boldsymbol{q}_4)]. \tag{A3}$$

To compute the thermal averages we need to write products of particle operators in normal ordered form. Computation of thermal averages in the gluon basis (15) was given in Ref. [1] while in the case of the glueball ensemble the relevant matrix elements to compute are

$$\langle H_K\rangle \to \left\langle \int \frac{d^3q}{(2\pi)^3}\left(\mathcal{V}[N_C^2-1]\frac{\omega(q)}{2}+\sum_{i,b}\frac{\omega(q)}{2}a_i^{b\dagger}(\boldsymbol{q})a_i^b(\boldsymbol{q})\right)\right\rangle, \tag{A4}$$

$$\begin{aligned}\langle H_B\rangle \to \Bigg\langle & \frac{1}{4}\int\frac{d^3q}{(2\pi)^3}\sum_{ijklm}\epsilon_{ijk}\epsilon_{ilm}\frac{q_jq_l}{\omega(q)}(\mathcal{V}[N_C^2-1]t_{km}(q)+\sum_b[a_k^{b\dagger}(\boldsymbol{q})a_m^b(\boldsymbol{q})+(k\leftrightarrow m)])\\
&+\frac{g^2}{32}\int\frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\sum_{ijklm}\epsilon_{ijk}\epsilon_{ilm}\Bigg(\mathcal{V}[N_C^2-1]N_C\frac{t_{jl}(q')t_{km}(q)-t_{kl}(q')t_{jm}(q)}{\omega(q)\omega(q')}+\sum_{abcde}\epsilon^{abc}\epsilon^{ade}\\
&\times\left[2\frac{t_{kl}(q')}{\omega(q)\omega(q')}\delta^{cd}a_m^{e\dagger}(\boldsymbol{q})a_j^b(\boldsymbol{q})+(k\leftrightarrow j,b\leftrightarrow c)+(m\leftrightarrow l,d\leftrightarrow e)+(j\leftrightarrow k,b\leftrightarrow c;l\leftrightarrow m,d\leftrightarrow e)\right]\Bigg)\\
&+\frac{g^2}{32}\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3}\frac{d^3q_4}{(2\pi)^3}\sum_{ijklm}\sum_{abcde}\epsilon_{ijk}\epsilon_{ilm}\epsilon^{abc}\epsilon^{ade}\frac{1}{\sqrt{\omega(q_1)\omega(q_2)\omega(q_3)\omega(q_4)}}\\
&\times[(2\pi)^3\delta^3(\boldsymbol{q}_1+\boldsymbol{q}_3-\boldsymbol{q}_2-\boldsymbol{q}_4)a_k^{c\dagger}(\boldsymbol{q}_2)a_m^{e\dagger}(\boldsymbol{q}_4)a_j^b(\boldsymbol{q}_1)a_l^d(\boldsymbol{q}_3)+(3\leftrightarrow 4)+(1\leftrightarrow 2)+(1\leftrightarrow 2;3\leftrightarrow 4)]\rangle,\end{aligned} \tag{A5}$$

$$\begin{aligned}\langle H_C\rangle \to \Bigg\langle & \frac{g^2}{8}\int\frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\sum_{ij}\Bigg(\mathcal{V}[N_C^2-1]N_CF(\boldsymbol{q}-\boldsymbol{q}')\left[-1+\frac{\omega(q)}{\omega q'}\right]t_{ij}(q)t_{ij}(q')\\
&+\sum_{abcde}\epsilon^{abc}\epsilon^{ade}2t_{ij}(q')F(q-q')\frac{\omega(q)}{\omega(q')}[\delta^{bd}a_j^{e\dagger}(\boldsymbol{q})a_i^c(\boldsymbol{q})+(b\leftrightarrow c,d\leftrightarrow e)]\Bigg)\\
&+\frac{g^2}{8}\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3}\frac{d^3q_4}{(2\pi)^3}\sum_{ij}\sum_{abcde}\epsilon^{abc}\epsilon^{ade}\sqrt{\frac{\omega(q_2)\omega(q_4)}{\omega(q_1)\omega(q_3)}}\\
&\times[-(2\pi)^3\delta^3(\boldsymbol{q}_1+\boldsymbol{q}_3-\boldsymbol{q}_2-\boldsymbol{q}_4)F(-\boldsymbol{q}_3+\boldsymbol{q}_4)a_k^{c\dagger}(\boldsymbol{q}_2)a_m^{e\dagger}(\boldsymbol{q}_4)a_j^b(\boldsymbol{q}_1)a_l^d(\boldsymbol{q}_3)\\
&+(3\leftrightarrow 4)+(1\leftrightarrow 2)-(1\leftrightarrow 2;3\leftrightarrow 4)]\rangle,\end{aligned} \tag{A6}$$

with further details given below.

APPENDIX B: HAMILTONIAN THERMAL AVERAGE

1. $\langle H_{\mathrm{YM}}\rangle$ in a basis of glueballs

Computation of thermal averages of the Hamiltonian in the glueball ensemble involves expectation values of one-body and two-body operators. These are given below:

$$\begin{aligned}\left\langle\int\frac{d^3q}{(2\pi)^3}f(\boldsymbol{q})a_i^{b\dagger}(\boldsymbol{q})a_j^c(\boldsymbol{q})\right\rangle &= \mathcal{V}\sum_{J^{PC}}\sum_{\lambda_1,\lambda_2,\lambda_l}\int\frac{d^3P_\alpha}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}[dp_1dp_2]_P[dp_1'dp_2']_P\frac{\delta^{bc}}{N_C^2-1}\frac{e^{-\beta(\Omega(p_1')+\Omega(p_2'))}}{1-e^{-\beta E_\alpha}}\\
&\times[(2\pi)^3\delta(p_2'-p_2)(2\pi)^3\delta(q-p_1')(2\pi)^3\delta(q-p_1)f(q)\Psi^\alpha_{\lambda_1,\lambda_l}(p_1,p_2)H^{ij}_{\lambda_1\lambda_2}(q,q)\\
&\times\Psi^\alpha_{\lambda_2,\lambda_l}(p_1',p_2')+(p_1'\leftrightarrow p_2')]\\
&= \mathcal{V}\sum_{J^{PC}}\sum_{\lambda_1,\lambda_2,\lambda_l}\int\frac{d^3P_\alpha}{(2\pi)^3}[dp_1'dp_2']_P\frac{\delta^{bc}}{N_C^2-1}\frac{e^{-\beta(\Omega(p_1')+\Omega(p_2'))}}{1-e^{-\beta E_\alpha}}\\
&\times[f(p_1')\Psi^\alpha_{\lambda_1,\lambda_l}(p_1',p_2')H^{ij}_{\lambda_1\lambda_2}(p_1',p_1')\Psi^\alpha_{\lambda_2,\lambda_l}(p_1',p_2')+(p_1'\leftrightarrow p_2')],\end{aligned} \tag{B1}$$

where $H^{ij}_{\lambda_1\lambda_2}(q,q) = H^{ij}_{\lambda_1\lambda_2}(q) = \epsilon^*_i(q,\lambda_1)\epsilon_j(q,\lambda_2)$ is the one-body vertex factor and $1/1 - e^{-\beta E_\alpha} = [1 + n_\alpha(P)]$. In a similar way the two-body operator thermal average is given by

$$\begin{aligned}
&\left\langle \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3}\frac{d^3q_4}{(2\pi)^3} f(q_1,q_2,q_3,q_4)(2\pi)^3\delta(q_3+q_4-q_1-q_2)a_i^{c_1\dagger}(q_1)a_j^{c_2\dagger}(q_2)a_r^{c_3}(q_3)a_s^{c_4}(q_4)\right\rangle \\
&\quad = \mathcal{V}\int \frac{d^3P_\alpha}{(2\pi)^3}[dp_1dp_2]_P[dp_1'dp_2']_P \sum_{J^{PC}}\sum_{\lambda_1\lambda_2\lambda_3\lambda_4} \frac{e^{-\beta(\Omega(p_1')+\Omega(p_2'))}}{1-e^{-\beta E_\alpha}}\frac{\delta^{c_3c_4}\delta^{c_1c_2}}{N_C^2-1} \\
&\qquad \times \{f(p_1,p_2,p_1',p_2')\Psi^\alpha_{\lambda_1\lambda_2}(p_1,p_2)H^{ijrs}_{\lambda_1\lambda_2\lambda_3\lambda_4}(p_1,p_2,p_1',p_2')\Psi^\alpha_{\lambda_3\lambda_4}(p_1',p_2') + (p_1' \leftrightarrow p_2')\},
\end{aligned} \tag{B2}$$

where the vertex factor for the two-body operator is given by

$$H^{ijrs}_{\lambda_1\lambda_2\lambda_3\lambda_4}(p_1,p_2,p_1',p_2') = \epsilon^*_i(p_1,\lambda_1)\epsilon^*_j(p_2,\lambda_2)\epsilon_r(p_1',\lambda_3)\epsilon_s(p_2',\lambda_4). \tag{B3}$$

Calculation of the thermal average of the Hamiltonian is further simplified by the following relations involving vertex factors:

$$\begin{aligned}
&\sum_{jk} t_{jj}(\boldsymbol{q}')H^{kk}_{\lambda_1\lambda_2}(\boldsymbol{q}) = 2\delta_{\lambda_1\lambda_2}, \qquad \sum_{jk} t_{kj}(\boldsymbol{q}')H^{kj}_{\lambda_1\lambda_2}(\boldsymbol{q}) = \sum_j \frac{1+(\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{q}}')^2}{2}H^{jj}_{\lambda_1\lambda_2}(\boldsymbol{q},\boldsymbol{q}) = \frac{1+(\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{q}}')^2}{2}\delta_{\lambda_1\lambda_2} = \frac{1+x^2}{2}\delta_{\lambda_1\lambda_2}, \\
&\sum_{ij} H^{ijij}_{\lambda_1\lambda_2\lambda_3\lambda_4}(\boldsymbol{P}_\alpha - \boldsymbol{q}', \boldsymbol{q}, \boldsymbol{P}_\alpha - \boldsymbol{q}, \boldsymbol{q}') \to \sum_{ij} H^{ii}_{\lambda_1\lambda_3}(\boldsymbol{P}_\alpha - \boldsymbol{q}, \boldsymbol{P}_\alpha - \boldsymbol{q})H^{jj}_{\lambda_2\lambda_4}(\boldsymbol{q},\boldsymbol{q}) = \delta_{\lambda_1\lambda_3}\delta_{\lambda_2\lambda_4}, \\
&\sum_{jk} H^{kkjj}_{\lambda_1\lambda_2\lambda_3\lambda_4}(\boldsymbol{P}_\alpha - \boldsymbol{q}', \boldsymbol{q}', \boldsymbol{q}, \boldsymbol{P}_\alpha - \boldsymbol{q}) \approx \delta_{\lambda_1\lambda_2}\delta_{\lambda_3\lambda_4}.
\end{aligned} \tag{B4}$$

This finally leads to the expression in Eq. (27) with

$$\mathcal{E}_\alpha(P) = \sum_{\lambda_1,\lambda_2}\int [dp_1dp_2]_P |\Psi^\alpha_{\lambda_1,\lambda_2}(p_1,p_2)|^2[e(p_1)+e(p_2)]e^{-\beta(\Omega(p_1)+\Omega(p_2))}, \tag{B5}$$

with the single gluon energies $e(p)$ given by a sum of kinetic and self-energy terms, $e(p) = \omega(p)/2 + p^2/2\omega(p) + \Sigma_B(p) + \Sigma_C(p)$:

$$\Sigma_B(p) = \frac{g^2N_C}{8}\int \frac{d^3q}{(2\pi)^3}\frac{3-x^2}{\omega(p)\omega(q)}, \qquad \Sigma_C(p) = \frac{g^2N_C}{4}\int\frac{d^3q}{(2\pi)^3}(1+x^2)F(p-q)\frac{\omega(p)}{\omega(q)}. \tag{B6}$$

The explicit form of the two-body magnetic (B) contribution from the four-gluon vertex and the two-body Coulomb interaction (C), respectively, are

$$\begin{aligned}
\mathcal{B}_\alpha(P) = \frac{g^2N_C}{8}\sum_{\lambda_i}\int &\frac{[dp_1dp_2]_P[dp_1'dp_2']_P}{\sqrt{\omega(p_1)\omega(p_2)\omega(p_1')\omega(p_2')}}e^{-\beta(\Omega(p_1')+\Omega(p_2'))} \\
&\times [\Psi^\alpha_{\lambda_1\lambda_2}(p_1,p_2)\Psi^\alpha_{\lambda_1'\lambda_2'}(p_1',p_2')(\delta_{\lambda_1\lambda_2}\delta_{\lambda_1'\lambda_2'} - \delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}) + (p_1' \leftrightarrow p_2')],
\end{aligned} \tag{B7}$$

$$\begin{aligned}
\mathcal{C}_\alpha(P) = -\frac{g^2N_C}{4}\sum_{\lambda_i}&\int [dp_1dp_2]_P[dp_1'dp_2']_P e^{-\beta(\Omega(p_1')+\Omega(p_2'))} \\
&\times\left[F(p_1-p_1')\Psi^\alpha_{\lambda_1\lambda_2}(p_1,p_2)\Psi^\alpha_{\lambda_1\lambda_2}(p_1',p_2')\left(\sqrt{\frac{\omega(p_1)\omega(p_2')}{\omega(p_1')\omega(p_2)}} + \sqrt{\frac{\omega(p_1)\omega(p_2)}{\omega(p_1')\omega(p_2')}}\right) + (p_1' \leftrightarrow p_2')\right].
\end{aligned} \tag{B8}$$

2. $\langle H_{\rm YM}\rangle$ in a basis of quasigluons

The calculation of the thermal average $\langle H_{\rm YM}\rangle$ in the basis (15) of quasigluons was carried out in Ref. [1] with the result

$$\frac{\langle H_{\rm YM}\rangle}{\mathcal{V}} = 2(N_C^2-1)\int\frac{d^3q}{(2\pi)^3}n(q)e(q) + \int\frac{d^3q}{(2\pi)^3}\frac{d^3p}{(2\pi)^3} \times [b(p,q)+c(p,q)]n(p)[1+n(q)], \tag{B9}$$

with

$$e(q) = \frac{\omega(q)}{2} + \frac{q^2}{2\omega(q)}, \tag{B10}$$

$$b(p,q) = \frac{g^2N_C}{8}\frac{3-\hat{p}\cdot\hat{q}}{\omega(p)\omega(q)}, \tag{B11}$$

$$c(p,q) = \frac{g^2N_C}{4}[1+(\hat{p}\cdot\hat{q})^2]F(p-q)\frac{\omega(p)}{\omega(q)}. \tag{B12}$$

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