

Minimal flavor violation in the minimal $U(1)_{B-L}$ model and resonant leptogenesisNobuchika Okada,¹ Yuta Orikasa,² and Toshifumi Yamada³¹*Department of Physics and Astronomy, University of Alabama, Tuscaloosa, Alabama 35487, USA*²*Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*³*Department of Physics, University of Tokyo, 7-3-1 Bunkyo, Tokyo 113-0033, Japan*

(Received 5 July 2012; published 2 October 2012)

We investigate the resonant leptogenesis scenario in the minimally $U(1)_{B-L}$ extended standard model with minimal flavor violation. In our model, the $U(1)_{B-L}$ gauge symmetry is broken at the TeV scale and standard model singlet neutrinos gain Majorana masses of order TeV. In addition, we introduce a flavor symmetry on the singlet neutrinos at a scale higher than TeV. The flavor symmetry is explicitly broken by the neutrino Dirac Yukawa coupling, which induces splittings in the singlet neutrino Majorana masses at lower scales through renormalization group evolutions. We call this setup *minimal flavor violation*. The mass splittings are proportional to the tiny Dirac Yukawa coupling, and hence they automatically enhance the CP asymmetry parameter necessary for the resonant leptogenesis mechanism. In this paper, we calculate the baryon number yield by solving the Boltzmann equations, including the effects of $U(1)_{B-L}$ gauge boson that also has TeV scale mass and causes washing-out of the singlet neutrinos in the course of thermal leptogenesis. The Dirac Yukawa coupling for neutrinos is fixed in terms of neutrino oscillation data and an arbitrary 3×3 complex-valued orthogonal matrix. We show that the right amount of baryon number asymmetry can be achieved through thermal leptogenesis in the context of the minimal flavor violation with singlet neutrinos and $U(1)_{B-L}$ gauge boson at the TeV scale. These particles can be discovered at the LHC in the near future.

DOI: [10.1103/PhysRevD.86.076003](https://doi.org/10.1103/PhysRevD.86.076003)

PACS numbers: 11.30.Fs, 12.60.Cn, 14.60.St

I. INTRODUCTION

The baryon number asymmetry of the Universe has been measured to be [1]

$$\frac{n_B}{s} = (0.88 \pm 0.02) \times 10^{-10}. \quad (1)$$

Since the electroweak baryogenesis scenario based on the standard model (SM) is incapable of explaining this amount, we need an extension of SM. Leptogenesis [2] is an appealing scenario, as it explains the tiny neutrino masses and the baryon number asymmetry simultaneously with heavy singlet neutrinos [3]. What is problematic in the leptogenesis scenario is that it usually cannot be tested experimentally; the efficiency of leptogenesis is proportional to CP violation which appears in interference terms between tree and one-loop diagrams involving singlet neutrinos. Therefore, sufficiently large neutrino Yukawa coupling is required in order that the efficiency be high enough to produce the observed amount of baryon asymmetry. This, in turn, implies that the singlet neutrinos must be heavy enough to explain the tiny neutrino masses, and hence they are beyond the energy scale of collider experiments.

However, if singlet neutrinos have nearly degenerate Majorana masses, resonance in loop diagrams enhances the efficiency of leptogenesis, making it possible to produce the right amount of baryon asymmetry even with small neutrino Yukawa couplings, which is called *resonant leptogenesis* [4]. With the resonant leptogenesis mechanism, models with TeV scale singlet neutrinos can explain both the baryon number asymmetry and the light neutrino

masses. Still, collider experiments cannot test such models because the singlet neutrinos couple to SM particles only through the small Yukawa coupling, and hence they cannot be produced at colliders. If there is a new gauge interaction that is broken at TeV scale and the singlet neutrinos are charged under it, we may be able to produce the TeV scale singlet neutrinos at colliders and even measure the amount of CP violation through their decays [5]. Another problem of models with TeV scale singlet neutrinos is its naturalness. Since successful resonant leptogenesis requires highly degenerate Majorana masses, we have to explain the origin of the degeneracy from some new symmetry or mechanism.

In this paper, we propose a model that solves these problems and show that it can explain both the light neutrino masses and the baryon number asymmetry without losing its experimental accessibility and without requiring unnatural fine-tuning. In addition to the SM content, our model has $U(1)_{B-L}$ gauge symmetry and contains three singlet neutrinos which are necessary for its anomaly cancellation. This gauge symmetry is broken at the TeV scale so that $U(1)_{B-L}$ gauge boson and the singlet neutrinos obtain masses of order TeV [6]. We further introduce a flavor symmetry on the singlet neutrinos at an energy scale above TeV, which is broken explicitly by the neutrino Dirac Yukawa coupling.

Because of the flavor symmetry of the singlet neutrinos, the differences in their Majorana masses arise solely from the neutrino Dirac Yukawa coupling through renormalization group (RG) evolutions, which we call *minimal flavor*

violation for singlet neutrinos. The ratios of the mass differences over the mass scale are therefore of the same order as the neutrino Dirac Yukawa coupling. It turns out that such mass difference is of an appropriate order to produce the right amount of the baryon number asymmetry with resonant enhancement, and no fine-tuning is required in our setup. Moreover, since the differences in the Majorana masses are related to the neutrino Dirac Yukawa coupling, our model is more restrictive than the original resonant leptogenesis scenario, where the mass difference is a free parameter.

Two of the authors have discussed in Ref. [7] resonant leptogenesis in the presence of $U(1)_{B-L}$ gauge symmetry that is broken at TeV scale. In that paper, the tiny difference in the Majorana masses of two singlet neutrinos is put in by hand, whereas in our paper it arises from RG evolutions involving the neutrino Dirac Yukawa coupling.

Resonant leptogenesis with a flavor symmetry of singlet neutrinos at a high scale has been first proposed in Ref. [8]. However the models in that paper does not contain $U(1)_{B-L}$ gauge symmetry that is broken at TeV scale because its motivation lies in lowering the reheating temperature of the Universe rather than observing singlet neutrinos by collider experiments. We expect that the washing-out of singlet neutrinos via $U(1)_{B-L}$ gauge interaction in the course of thermal leptogenesis reduces the resultant baryon number asymmetry, making it difficult to produce the right amount of baryon asymmetry. This is particularly so when the mass of $U(1)_{B-L}$ gauge boson is larger than twice the Majorana masses, because in this case on-shell $U(1)_{B-L}$ gauge boson can mediate the interactions between singlet neutrinos and SM particles. From the point of view of collider experiments, however, this case is favorable because the singlet neutrinos can be produced by the decay of $U(1)_{B-L}$ gauge boson. We will see that even with the $U(1)_{B-L}$ gauge interaction at the TeV scale, we can obtain enough baryon asymmetry through resonant leptogenesis with a flavor symmetry of singlet neutrinos at a high scale.

Another feature of our study is that we adopt Casas-Ibarra parametrization [9] for the neutrino Dirac Yukawa coupling. When the charged lepton Yukawa coupling is neglected, the processes of leptogenesis depend only on $(Y_\nu^\dagger)_{ai}(Y_\nu)_{ib}$, where Y_ν denotes the neutrino Dirac Yukawa coupling, i the flavor index of lepton doublets and a, b that of singlet neutrinos. In Casas-Ibarra parametrization, this quantity is parametrized in terms of the light neutrino mass eigenvalues and an arbitrary complex-valued *orthogonal* matrix, independently of the neutrino flavor-mixing matrix. This parametrization thus makes it clear whether the right amount of baryon number asymmetry can be produced in a given setup.

In Sec. II, we describe the model which contains $U(1)_{B-L}$ gauge symmetry and a flavor symmetry of singlet neutrinos. In Sec. III, we discuss its RG evolutions, which

are essential in our model as they give rise to the small mass splittings of Majorana masses necessary for resonant leptogenesis. In Sec. IV, we review on the Boltzmann equations that calculate the baryon number yield from leptogenesis. In Sec. V, we calculate the baryon number yield for various values of singlet neutrino masses and $U(1)_{B-L}$ gauge boson mass, and study whether the observed amount of the baryon asymmetry can be produced. Section VI is devoted to the conclusions and discussions.

II. SETUP

We consider the minimally $U(1)_{B-L}$ extended standard model, where the gauge symmetry is $U(1)_Y \times U(1)_{B-L} \times SU(2)_L \times SU(3)_c$ and right-handed singlet neutrinos, N_a ($a = 1, 2, 3$), and a scalar field, Φ , are added to the SM matter content. Φ obtains the vacuum expectation value (VEV) and breaks the $U(1)_{B-L}$ gauge symmetry through the Higgs mechanism, giving rise to the $U(1)_{B-L}$ gauge boson mass and the Majorana mass term for the singlet neutrinos. We denote this VEV by v_{B-L} . We are especially interested in the case where v_{B-L} is at the TeV scale, and hence the $U(1)_{B-L}$ gauge boson and the singlet neutrinos are observable at collider experiments. In this case, the components of Y_ν are smaller than about 10^{-5} . The field content is summarized in Table I. We assume that the physical scalar particle that originates from Φ after the $U(1)_{B-L}$ symmetry breaking is heavier than the singlet neutrinos, and we may neglect it in the discussion of thermal leptogenesis.

We take the combination of the $U(1)_Y$ and $U(1)_{B-L}$ gauge bosons such that the mixing of their kinetic terms are zero throughout the RG evolutions. Then the covariant derivative, D_μ , can be written as

$$D_\mu = \partial_\mu - i\sqrt{\frac{3}{5}}g_1 Y B_\mu - i(\tilde{g}Y + g_{B-L}Y_{B-L})B'_\mu - ig_2 \frac{\sigma^\alpha}{2} W_\mu^\alpha - ig_3 T^\alpha G_\mu^\alpha, \quad (2)$$

where the coupling $\tilde{g}Y$ reflects the mixing of the $U(1)_Y$ and $U(1)_{B-L}$ gauge bosons. \tilde{g}, g_{B-L} are defined so that we have $\tilde{g} = 0$ at the $U(1)_{B-L}$ breaking scale. Since Φ has $B-L$ charge 2, its VEV induces the mass of the $U(1)_{B-L}$ gauge boson as

TABLE I. Field content in the minimally $U(1)_{B-L}$ extended standard model.

Field	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
L_i	2	-1/2	-1
N_a	1	0	-1
H	2	-1/2	0
Φ	1	0	+2

$$M_{Z'} = 2\sqrt{2}g_{B-L}v_{B-L}. \quad (3)$$

The Dirac and Majorana Yukawa couplings for the neutrinos are given by

$$\mathcal{L} \supset -(Y_\nu)_{ia} H L_i^\dagger N_a - \text{H.c.} - \frac{1}{2} (Y_M)_{ab} \Phi N_a N_b - \text{H.c.} \quad (4)$$

Note that Y_M is a symmetric matrix with complex values. The term $\frac{1}{2} (Y_M)_{ab} \Phi N_a N_b$ becomes the Majorana mass term,

$$\frac{1}{2} v_{B-L} (Y_M)_{ab} N_a N_b, \quad (5)$$

when Φ obtains VEV.

The light neutrino mass matrix takes the form:

$$(m_\nu)_{ij} = (Y_\nu)_{ia} (Y_M^{-1})_{ab} (Y_\nu^T)_{bj} \frac{v^2}{v_{B-L}}. \quad (6)$$

We move to the flavor basis where the charged leptons and the singlet neutrinos are in their mass eigenstates. On this basis, we have

$$m_\nu^\dagger m_\nu = U_{MNS} \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U_{MNS}^\dagger, \quad (7)$$

$$Y_M = \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix}, \quad (8)$$

where $(m_i)^2$'s denote the eigenvalues of the light neutrino mass squared, U_{MNS} denotes the neutrino flavor-mixing matrix, and y_i 's denote the eigenvalues of Y_M . We see from (6)–(8) that Y_ν on this basis can generally be written as

$$(Y_\nu)_{ia} = U_{MNS}^* \begin{pmatrix} (m_1^2)^{1/4} & 0 & 0 \\ 0 & (m_2^2)^{1/4} & 0 \\ 0 & 0 & (m_3^2)^{1/4} \end{pmatrix} \times O \begin{pmatrix} \sqrt{y_1} & 0 & 0 \\ 0 & \sqrt{y_2} & 0 \\ 0 & 0 & \sqrt{y_3} \end{pmatrix} \frac{\sqrt{v_{B-L}}}{v}, \quad (9)$$

where O denotes an arbitrary *orthogonal* matrix with *complex* values, which appears in Casas-Ibarra parametrization [9]. We hereafter call O *Yukawa kernel matrix*.

Let us introduce a symmetry of two generations of singlet neutrinos above an energy scale Λ , which is assumed higher than the TeV scale. Then we may write

$$Y_M[\Lambda] \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{pmatrix} \quad (10)$$

in the basis where Y_M is diagonal. This is the key assumption that realizes the resonant leptogenesis without fine-tuning of the parameters. Below the scale Λ , Y_ν explicitly breaks the flavor symmetry and comes into effect through RG evolutions. The difference between the first two components of Y_M at lower scales is proportional to $Y_\nu^\dagger Y_\nu$, and hence the singlet neutrinos obtain Majorana masses whose difference between the first two components is proportional to $Y_\nu^\dagger Y_\nu$ when Φ gains VEV. This mass difference is essential for successful resonant leptogenesis.

Our flavor symmetry is inspired by a class of A_4 symmetric models where the three flavors of the singlet neutrinos transform as 3 representation of A_4 group (see, e.g., Table 1 of Ref. [10] for a list of such models), although we do not specify the details of the flavor symmetry. If the singlet neutrinos are in 3 representation and the $B-L$ breaking field Φ is in 1 representation of A_4 group, the Majorana Yukawa coupling will be common for the three flavors. In realistic A_4 models, especially when one takes into account the observed value of the neutrino mixing angle θ_{13} [11], A_4 symmetry should be spontaneously broken below some scale. In our model, the spontaneous breaking of a flavor symmetry is effectively described by the Dirac Yukawa coupling that explicitly violates the flavor symmetry and that comes into play below the scale Λ .

III. RG EVOLUTIONS

In the following discussion, we set the $U(1)_{B-L}$ breaking scale, M_0 , at $|y|v_{B-L}$, where y is of the order of the eigenvalues of Y_M . The results do not change with $O(1)$ variation of the $U(1)_{B-L}$ breaking scale. We further make an approximation that the $U(1)_{B-L}$ gauge boson mass, $M_{Z'} = 2\sqrt{2}g_{B-L}v_{B-L}$, is of the same order as M_0 .

The RG equations for the $U(1)$ gauge couplings, g_1 , g_{B-L} , \tilde{g} , are given as follows, where M_0 is the $U(1)_{B-L}$ breaking scale:

$$16\pi^2 \mu \frac{d}{d\mu} g_1 = \frac{53}{15} g_1^3 \quad \text{for } \mu < m_\nu, \quad (11)$$

$$16\pi^2 \mu \frac{d}{d\mu} g_1 = \frac{41}{10} g_1^3 \quad \text{for } \mu > m_\nu,$$

$$16\pi^2 \mu \frac{d}{d\mu} g_{B-L} = 12g_{B-L}^3 + 2 \times \frac{16}{3} g_{B-L}^2 \tilde{g} + \frac{41}{6} g_{B-L} \tilde{g}^2 \quad \text{for } \mu > M_0, \quad (12)$$

$$16\pi^2 \mu \frac{d}{d\mu} \tilde{g} = \frac{41}{6} \tilde{g} (\tilde{g}^2 + \frac{6}{5} g_1^2) + 2 \times \frac{16}{3} g_{B-L} (\tilde{g}^2 + \frac{3}{5} g_1^2) + 12g_{B-L}^2 \tilde{g} \quad \text{for } \mu > M_0. \quad (13)$$

The RG equations for g_2, g_3 are the same as in the standard model.

The RG equations for the Yukawa couplings Y_M , $Y_\nu^\dagger Y_\nu$ and the top Yukawa coupling, y_t , are given as

$$16\pi^2 \mu \frac{d}{d\mu} (Y_M)_{ab} = (Y_\nu^\dagger Y_\nu)_{ac} (Y_M)_{cb} + (Y_M)_{ac} (Y_\nu^\dagger Y_\nu)_{cb} \\ + (Y_M Y_M^* Y_M)_{ab} + \frac{1}{2} (Y_M)_{ab} \text{tr}[Y_M Y_M^\dagger] \\ - 6(g'_1)^2 (Y_M)_{ab} \quad \text{for } \mu > M_0, \quad (14)$$

$$16\pi^2 \mu \frac{d}{d\mu} (Y_\nu^\dagger Y_\nu)_{ab} = (Y_\nu^\dagger Y_\nu)_{ab} \left(6y_t^2 - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 \right. \\ \left. - 12g_{B-L}^2 - \frac{3}{2}\tilde{g}^2 - 6g_{B-L}\tilde{g} \right) \\ + \frac{1}{2} (Y_M Y_M^* Y_\nu^\dagger Y_\nu)_{ab} \\ + \frac{1}{2} (Y_\nu^\dagger Y_\nu Y_M Y_M^*)_{ab} \quad \text{for } \mu > M_0, \quad (15)$$

$$Y_\nu^\dagger Y_\nu[M_0] \simeq \begin{pmatrix} \sqrt{y^*} & 0 & 0 \\ 0 & \sqrt{y^*} & 0 \\ 0 & 0 & \sqrt{y_3^*} \end{pmatrix} O^\dagger \begin{pmatrix} \sqrt{m_1^2} & 0 & 0 \\ 0 & \sqrt{m_2^2} & 0 \\ 0 & 0 & \sqrt{m_3^2} \end{pmatrix} O \begin{pmatrix} \sqrt{y} & 0 & 0 \\ 0 & \sqrt{y} & 0 \\ 0 & 0 & \sqrt{y_3} \end{pmatrix} \frac{v_{B-L}}{v^2}, \quad (18)$$

$$\tilde{g}[M_0] = 0. \quad (19)$$

Note that $Y_\nu^\dagger Y_\nu$ is independent of the neutrino flavor-mixing matrix, U_{MNS} . In the first and second equations, \tilde{y} , \tilde{y}_3 and y , y_3 are related through RG evolutions from the scale Λ to M_0 . In order to obtain their relation, we solve Eq. (14) neglecting the term $Y_\nu^\dagger Y_\nu$.

We numerically solve the RG equations using the following best fit values (with 3σ ranges) from the Particle Data Group [12], which were derived assuming nonzero θ_{13} :

$$m_2^2 - m_1^2 = 7.58_{-0.59}^{+0.60} \times 10^{-5} \text{ eV}^2, \quad (20)$$

$$|m_3^2 - m_1^2| = 2.35_{-0.29}^{+0.32} \times 10^{-3} \text{ eV}^2. \quad (21)$$

Our analysis is done for both the normal hierarchy ($m_3^2 > m_1^2$) and the inverted hierarchy ($m_3^2 < m_1^2$) of the light neutrino masses. We fix the mass of the lightest neutrino at zero, i.e., we set $m_1^2 = 0$ for the normal hierarchy case and $m_3^2 = 0$ for the inverted hierarchy case. The Yukawa kernel matrix O can be any 3×3 orthogonal matrix. However, since the purpose of this paper is to show that successful leptogenesis can be realized in our framework, we here restrict our analysis to a specific choice of the matrix O that can be expressed as

$$16\pi^2 \mu \frac{d}{d\mu} y_t = \frac{9}{2}y_t^3 - y_t \left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 \right) \\ \text{for } m_t < \mu < M_0, \\ 16\pi^2 \mu \frac{d}{d\mu} y_t = \frac{9}{2}y_t^3 - y_t \left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 + \frac{17}{12}\tilde{g}^2 \right. \\ \left. + \frac{2}{3}g_{B-L}^2 + \frac{5}{3}\tilde{g}g_{B-L} \right) \quad \text{for } \mu > M_0. \quad (16)$$

In the RG equations above, we neglect the Yukawa couplings except the top Yukawa coupling. Note that, when the charged lepton Yukawa couplings are neglected, the RG evolution for Y_ν can be expressed in terms of that for $Y_\nu^\dagger Y_\nu$.

The boundary conditions for the RG equations are

$$Y_M[\Lambda] = \begin{pmatrix} \tilde{y} & 0 & 0 \\ 0 & \tilde{y} & 0 \\ 0 & 0 & \tilde{y}_3 \end{pmatrix}, \quad (17)$$

$$O = \begin{pmatrix} 0 & 0 & 1 \\ \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \end{pmatrix} \text{ for the normal hierarchy case,} \\ O = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ for the inverted hierarchy case,} \quad (22)$$

where α takes a *complex* value. With these assumptions, the upper-left 2×2 part of $Y_\nu^\dagger Y_\nu[M_0]$ takes the following simple form for the normal hierarchy case:

$$Y_\nu^\dagger Y_\nu[M_0]_{2 \times 2} \simeq \begin{pmatrix} \cos\alpha^* & -\sin\alpha^* \\ \sin\alpha^* & \cos\alpha^* \end{pmatrix} \begin{pmatrix} \sqrt{m_2^2} & 0 \\ 0 & \sqrt{m_3^2} \end{pmatrix} \\ \times \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \frac{|y|v_{B-L}}{v^2}. \quad (23)$$

For the inverted hierarchy case, $\sqrt{m_2^2}$ and $\sqrt{m_3^2}$ in the above formula are replaced with $\sqrt{m_1^2}$ and $\sqrt{m_2^2}$.

We calculate the mass spectrum for various values of α and substitute it into the Boltzmann equations that

calculate the baryon number yield. Since we are assuming a symmetry on only two generations of singlet neutrinos, it is natural that the other singlet neutrino has distinctively different Majorana mass. This singlet neutrino then does not contribute to resonant leptogenesis. We here neglect this singlet neutrino and make a two-flavor approximation, i.e., we only consider the two generations of singlet neutrinos, on solving the Boltzmann equations.

IV. BARYON NUMBER ASYMMETRY FROM LEPTOGENESIS

At high temperatures, the singlet neutrinos as well as the SM particles are in thermal equilibrium. (Remember that the $U(1)_{B-L}$ gauge boson mediates interactions between the singlet neutrinos and other particles). As the Universe cools down and the temperature becomes below the order of M_0 , the singlet neutrinos decouple from the thermal bath and decay into the Higgs doublet and the SM leptons. Because of the CP phases of Y_ν , the decays of the singlet neutrinos exhibit a CP asymmetry, which gives rise to nonzero lepton number density, or equivalently nonzero $B-L$ charge density. The lepton number density is converted into the baryon number asymmetry through the sphaleron process [13,14] when the temperature of the Universe is higher than the decoupling temperature of the sphaleron process $T_{\text{sph}} \sim 150$ GeV [15].

To estimate the resultant baryon number asymmetry, we solve the Boltzmann equations governing the number densities of the particles concerned [16]. We normalize the temperature, T , of the radiation-dominated Universe as

$$z \equiv y\nu_{B-L}/T,$$

where $y\nu_{B-L}$ approximately equals to the mass of the two singlet neutrinos that are relevant to leptogenesis. We respectively denote the yields, n/s (n : number density of a particle species, s : entropy density), of the singlet neutrinos N_a ($a = 1, 2$) and the $B-L$ charge by Y_{N_a} , Y_{B-L} .

The CP asymmetry parameter ϵ_a for the decay of singlet neutrino N_a is defined as

$$\epsilon_a \equiv \frac{\sum_j [\Gamma(N_a \rightarrow L_j H^\dagger) - \Gamma(N_a \rightarrow L_j^\dagger H)]}{\sum_j [\Gamma(N_a \rightarrow L_j H^\dagger) + \Gamma(N_a \rightarrow L_j^\dagger H)]}. \quad (24)$$

Nonzero ϵ_a comes from the interference of the diagrams in Fig. 1. It is generally given by

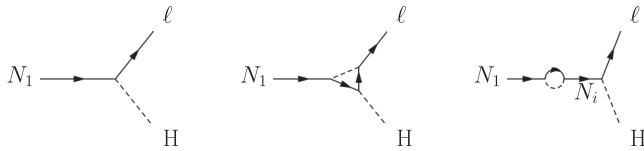


FIG. 1. Decay of a singlet neutrino at tree and one-loop levels.

$$\epsilon_a = -\sum_{b \neq a} \frac{M_a}{M_b} \frac{\Gamma_b}{M_b} \left(\frac{V_b}{2} + S_b \right) \frac{\text{Im}[(Y_\nu^\dagger Y_\nu)_{ab}^2]}{(Y_\nu^\dagger Y_\nu)_{aa} (Y_\nu^\dagger Y_\nu)_{bb}}, \quad (25)$$

where M_a , Γ_a denote the Majorana mass and the decay width of the singlet neutrino N_a and we have $\Gamma_a = M_a (Y_\nu^\dagger Y_\nu)_{aa} / 8\pi$. V_b reflects the interference of the left and the middle diagrams in Fig. 1 and S_b that of the left and the right diagrams. They are given by

$$V_b = 2 \frac{M_b^2}{M_a^2} \left[\left(1 + \frac{M_b^2}{M_a^2} \right) \log \left(1 + \frac{M_a^2}{M_b^2} \right) - 1 \right], \quad (26)$$

$$S_b = \frac{M_b^2 (M_b^2 - M_a^2)}{(M_b^2 - M_a^2)^2 + M_a^2 \Gamma_b^2}. \quad (27)$$

In our model, the mass difference of the relevant two singlet neutrinos, $M_2^2 - M_1^2$, comes from the RG evolutions from the scale Λ down to M_0 . Therefore it is estimated as

$$M_2^2 - M_1^2 \propto Y_\nu^\dagger Y_\nu (y\nu_{B-L})^2. \quad (28)$$

Since we are assuming $y\nu_{B-L} \sim \text{TeV}$, it follows from Eq. (9) that the components of Y_ν is at least as small as $(\Delta m_{\text{atm}}^2 / \text{TeV})^{1/4} \sim 10^{-4}$. Then S_1, S_2 are enhanced by the factor of $(1/Y_\nu^\dagger Y_\nu) \sim 10^8$, whereas V_1, V_2 remain $O(1)$. In Eq. (25), this enhancement cancels the suppression factor $Y_\nu^\dagger Y_\nu$ that comes from the term Γ_b/M_b , and we have $O(1)$ values of ϵ_1, ϵ_2 . This is the heart of resonant leptogenesis naturally realized with the minimal flavor violation.

The Boltzmann equations that govern the yields of the singlet neutrinos N_1, N_2 and the $B-L$ charge are written as

$$\frac{dY_{N_1}}{dz} = -\frac{z}{sH(y\nu_{B-L})} \left[\left(\frac{Y_{N_1}}{Y_{N_1}^{\text{eq}}} - 1 \right) \gamma_{D_1} + \left(\left(\frac{Y_{N_1}}{Y_{N_1}^{\text{eq}}} \right)^2 - 1 \right) \gamma_{Z'} \right], \quad (29)$$

$$\frac{dY_{N_2}}{dz} = -\frac{z}{sH(y\nu_{B-L})} \left[\left(\frac{Y_{N_2}}{Y_{N_2}^{\text{eq}}} - 1 \right) \gamma_{D_2} + \left(\left(\frac{Y_{N_2}}{Y_{N_2}^{\text{eq}}} \right)^2 - 1 \right) \gamma_{Z'} \right], \quad (30)$$

$$\frac{dY_{B-L}}{dz} = -\frac{z}{sH(y\nu_{B-L})} \left[\sum_{a=1}^2 \left(\frac{1}{2} \frac{Y_{B-L}}{Y_L^{\text{eq}}} - \epsilon_a \left(\frac{Y_{N_a}}{Y_{N_a}^{\text{eq}}} - 1 \right) \right) \gamma_{D_a} \right]. \quad (31)$$

$Y_{N_a}^{\text{eq}}, Y_L^{\text{eq}}$ respectively denote the yields of the singlet neutrino N_a and the lepton doublets L_i s, and $H(y\nu_{B-L})$ denotes the Hubble constant at the temperature $T = y\nu_{B-L}$. γ_{D_a} is the space-time density of the decay events of N_a , and $\gamma_{Z'}$ the space-time density of the processes mediated by the $U(1)_{B-L}$ gauge boson: $N_a N_a \leftrightarrow f\bar{f}$, where

f indicates a standard model fermion. We neglect other terms that give negligibly small contributions to the yields, in accordance with Ref. [7]. The general form of the Boltzmann equation and expressions of γ 's are given in Ref. [17].

At small z , $U(1)_{B-L}$ gauge interaction keeps the singlet neutrinos in thermal equilibrium. Below the sphaleron decoupling temperature, the sphaleron process ends and the $B-L$ charge density is no longer converted to the baryon number density. Noting these, we numerically solved the Boltzmann equations from $z = 0.001$ to $z = y\nu_{B-L}/T_{\text{sph}}$ with the initial conditions:

$$Y_{N_a}(z = 0.001) = Y_{N_a}^{\text{eq}}(z = 0.001), \quad (32)$$

$$Y_{B-L}(z = 0.001) = 0, \quad (33)$$

where we simply assumed that there is initially no $B-L$ charge density. The resultant $B-L$ charge density is converted to the baryon number density through the sphaleron process [14]. The yield of the baryon number, n_B/s , is related to Y_{B-L} by the following formula:

$$\frac{n_B}{s} = \frac{28}{79} Y_{B-L}(z = y\nu_{B-L}/T_{\text{sph}}). \quad (34)$$

V. NUMERICAL ANALYSIS ON RESONANT LEPTOGENESIS

Solving the RG equations in Sec. III and the Boltzmann equations in Sec. IV, we calculate the baryon number density for various values of the parameters, and compare it with the observed value Eq. (1). The parameters are

- (i) The $U(1)_{B-L}$ gauge boson mass (at the scale M_0): $M_{Z'} = 2\sqrt{2}g_{B-L}\nu_{B-L}$
- (ii) The $U(1)_{B-L}$ gauge coupling constant (at the scale M_0): g_{B-L}
- (iii) Scale above which the flavor symmetry of two singlet neutrinos appears: Λ
- (iv) Approximate value of the mass of the singlet neutrinos (at the scale M_0): $y\nu_{B-L}$
- (v) The parameter of the Yukawa kernel matrix O : $\alpha(\in \mathbb{C})$

We set the $U(1)_{B-L}$ breaking scale M_0 at $y\nu_{B-L}$, and the sphaleron decoupling temperature T_{sph} at 150 GeV [15]. We adopt the following values:

$$\Lambda = 10^5 \text{ GeV}, \quad \frac{g_{B-L}^2}{4\pi} = 0.005,$$

while substituting various values into $y\nu_{B-L}$ and $M_{Z'}$. We assume the normal hierarchy of the light neutrino masses. With these parameters, we make contour plots of the baryon number yield on the plane of real and complex parts of α that parametrizes the neutrino Dirac Yukawa matrix Y_ν . The baryon number density depends on α only

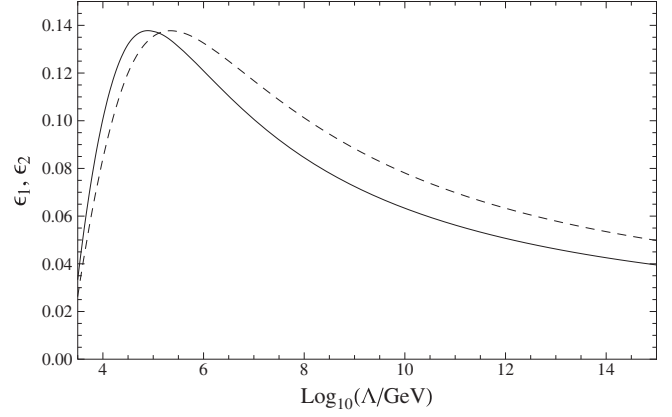


FIG. 2. CP asymmetry parameters, ϵ_1 (solid) and ϵ_2 (dashed), at the scale M_0 as functions of the singlet neutrino symmetry scale, Λ .

through the form of $Y_\nu^\dagger Y_\nu$, (17). Since $Y_\nu^\dagger Y_\nu$ is invariant under the transformation: $\text{Re}[\alpha] \rightarrow \text{Re}[\alpha] + \pi$, we restrict the range of α to $0 \leq \text{Re}[\alpha] \leq \pi$.

We also make contour plots of the baryon number yield for $\Lambda = 10^{15}$ GeV, for $\frac{g_{B-L}^2}{4\pi} = 0.02$ and for the inverted hierarchy of the light neutrino masses, to study the impacts of these parameters on the result.

In Fig. 2, we show how the CP asymmetry parameters, ϵ_1 , ϵ_2 , evaluated at the scale M_0 depend on Λ . Here we set $g_{B-L}^2/4\pi = 0.005$, $y\nu_{B-L} = 2$ TeV, $M_{Z'} = 5$ TeV and $\alpha = 2 + i$. We see that above the scale of 10^4 GeV, the CP asymmetry parameters of order 0.1 are automatically realized by the RG evolutions. Also, they only weakly depend on the scale of Λ , which means that the baryon number yield is not affected by the scale above which the flavor symmetry of singlet neutrinos appears.

On the left and right of Figs. 3–6, we show contour plots of the baryon number yield, n_B/s , on the plane of $(\text{Re}[\alpha], \text{Im}[\alpha])$ for $(M_{Z'}, y\nu_{B-L}) = (3, 1)$, $(3, 2)$, $(4, 1.5)$, $(4, 3)$, $(5, 2)$, $(5, 3)$ TeV. Thick solid (red) contours denote the regions where the resultant baryon number yield fits with the observed value Eq. (1). Cases with $(M_{Z'}, y\nu_{B-L}) = (3, 1)$, $(4, 1.5)$, $(5, 2)$ TeV are particularly interesting because the singlet neutrinos can be produced by the decay of $U(1)_{B-L}$ gauge boson if it is produced as a resonant state at colliders.

Figure 6 is the same contour plot as the left of Fig. 5, but we here take $\Lambda = 10^{15}$ GeV. Figure 7 is the same contour plot as the left of Fig. 5, but we here take $g_{B-L}^2/4\pi = 0.02$. Figure 8 is the same contour plot as the left of Fig. 5, but we here assume the inverted hierarchy of the light neutrino masses.

We notice from the left panels of Figs. 4 and 5 that a sufficiently large amount of the baryon number yield can be produced in benchmark points with $(M_{Z'}, y\nu_{B-L}) = (4, 1.5)$ TeV and $(5, 2)$ TeV, and $g_{B-L}^2/4\pi = 0.005$, which are of interest for collider studies. Moreover, the observed

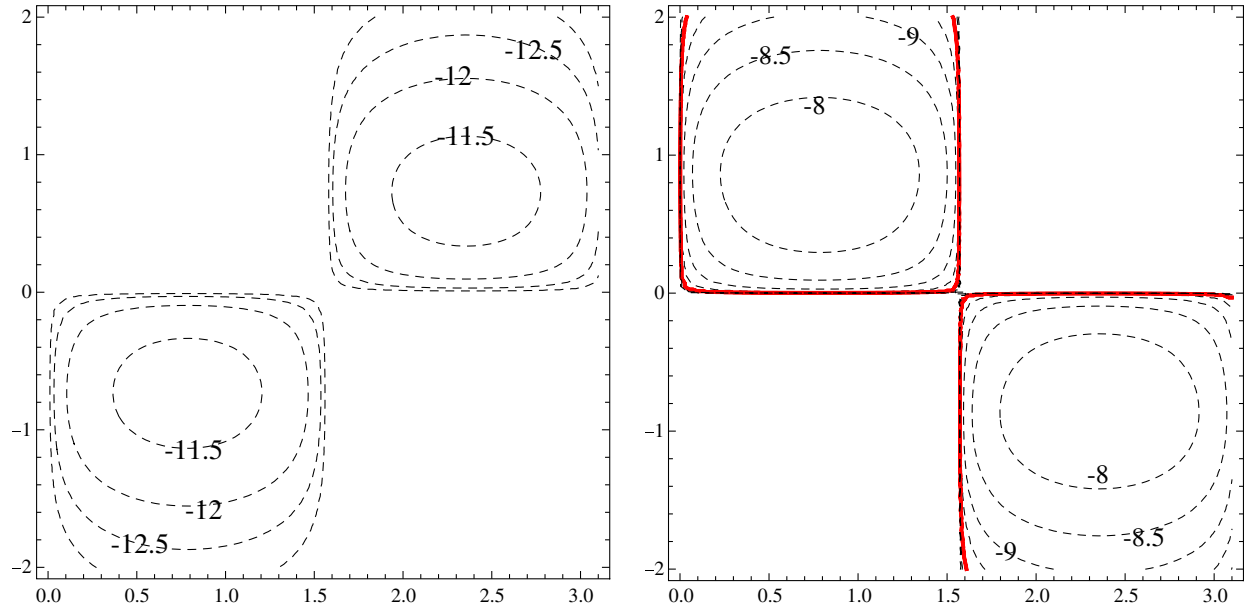


FIG. 3 (color online). Contour plots of the baryon number yield, n_B/s , as a function of the complex parameter α that parametrizes the neutrino Dirac Yukawa matrix Y_ν . We take $g_{B-L}^2/4\pi = 0.005$ and $\Lambda = 10^5$ GeV, and assume the normal hierarchy of the light neutrino masses. *Left*: $(M_{Z_l}, yv_{B-L}) = (3, 1)$ TeV, *Right*: $(M_{Z_l}, yv_{B-L}) = (3, 2)$ TeV. The numbers on the contours denote the corresponding values of $\log_{10}(n_B/s)$. The thick red line denote the region where the baryon number yield fits with the observed value Eq. (1).

baryon number yield, Eq. (1), is obtained for moderate values of $(\text{Re}[\alpha], \text{Im}[\alpha])$, which implies that no fine-tuning is required for the neutrino Dirac Yukawa coupling in order to reproduce the observed value.

From Figs. 3–5, we see that the heavier the singlet neutrinos are, the more baryon number yield we have. This is simply due to the fact that leptogenesis ends at

the sphaleron decoupling temperature, which is assumed to be $T_{\text{sph}} \approx 150$ GeV. If the singlet neutrino mass is smaller, less singlet neutrinos decouple from the thermal bath and do out-of-equilibrium decay before the radiation of the Universe cools down to the sphaleron decoupling temperature. We also notice that we have more baryon number yield when $M_{Z_l} < 2yv_{B-L}$ than otherwise. This is because

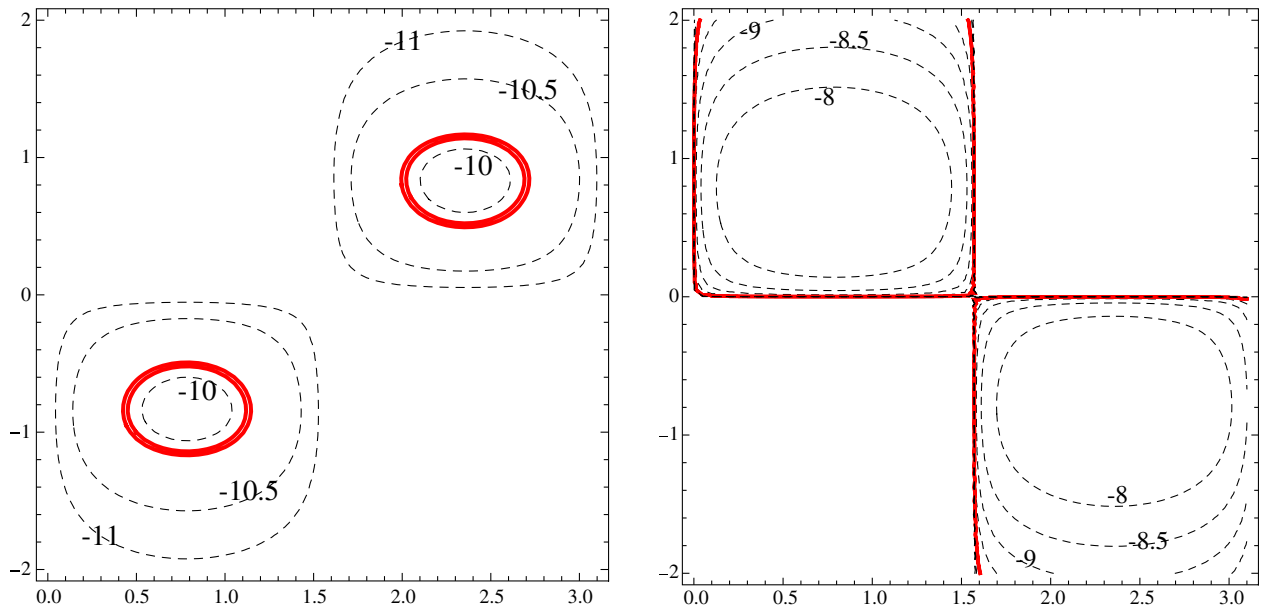


FIG. 4 (color online). Same as Fig. 3, but we here have *Left*: $(M_{Z_l}, yv_{B-L}) = (4, 1.5)$ TeV, *Right*: $(M_{Z_l}, yv_{B-L}) = (4, 3)$ TeV.

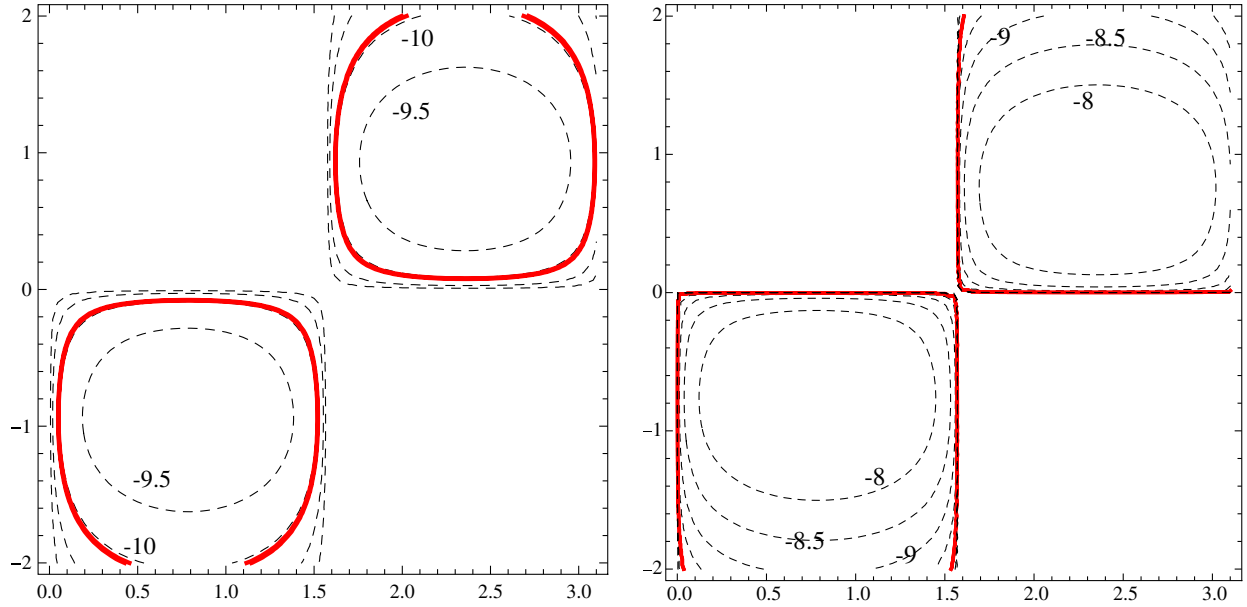


FIG. 5 (color online). Same as Fig. 3, but we here have *Left*: $(M_{Z'}, y\nu_{B-L}) = (5, 2)$ TeV, *Right*: $(M_{Z'}, y\nu_{B-L}) = (5, 3)$ TeV.

in the latter case, an on-shell $U(1)_{B-L}$ gauge boson mediates the processes: $N_a N_a \leftrightarrow Z' \leftrightarrow f \bar{f}$ (N_a : singlet neutrino, f : standard model fermion), and singlet neutrinos are washed out more efficiently.

From Figs. 5 and 6, we confirm that the baryon number yield hardly depends on the scale Λ , above which the flavor symmetry of the singlet neutrinos appears. This is in accord with Fig. 2, which shows that the CP asymmetry parameters vary only by factor 2 with the change of Λ .

Comparing Fig. 7 with Fig. 5, we see that the baryon number yield decreases for larger g_{B-L} , which reflects the fact that the washing-out of singlet neutrinos is more efficient when the $U(1)_{B-L}$ gauge coupling is stronger.

Finally, from Figs. 5 and 8, we see that the baryon number yield drastically decreases with the inverted hierarchy of light neutrino masses. This is because the neutrino Dirac Yukawa coupling squared, Eq. (18), cannot have

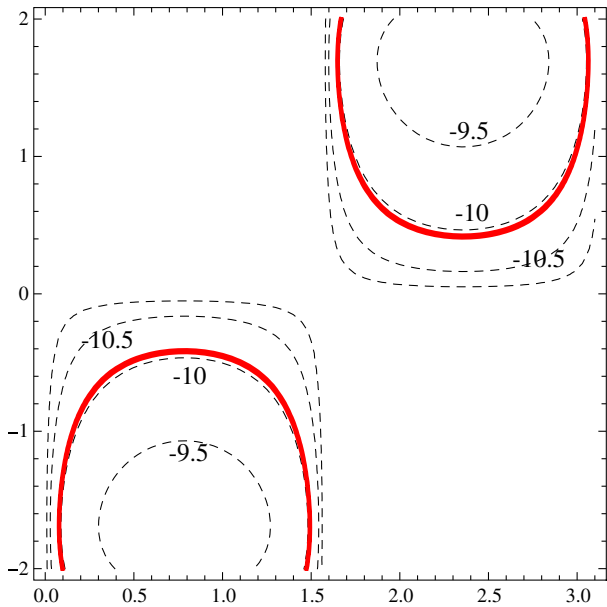


FIG. 6 (color online). Same as the left of Fig. 5, but we here take $\Lambda = 10^{15}$ GeV.

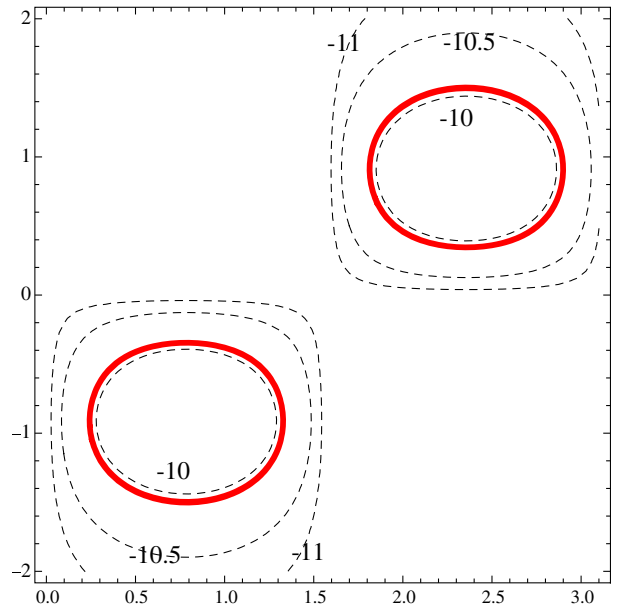


FIG. 7 (color online). Same as the left of Fig. 5, but we here take $g_{B-L}^2/4\pi = 0.02$.

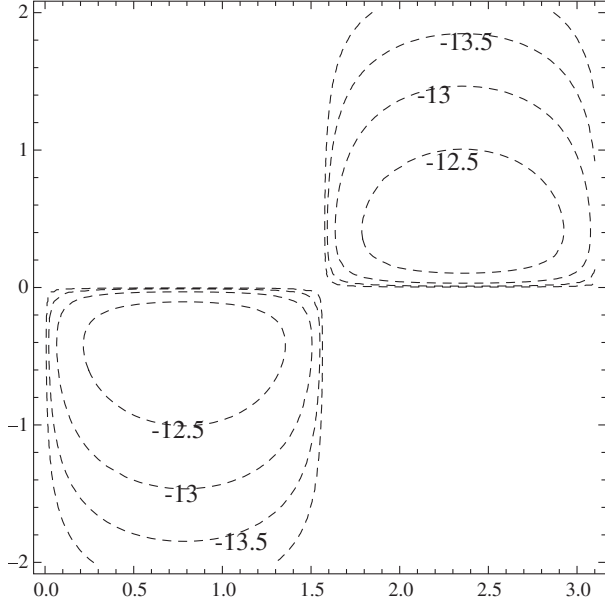


FIG. 8. Same as the left of Fig. 5, but we here assume the inverted hierarchy of the light neutrino masses.

large CP -violating phase when two of the mass eigenvalues for the light neutrinos are nearly degenerate, as in the inverted hierarchy case.

VI. CONCLUSIONS AND DISCUSSIONS

We have studied the resonant leptogenesis scenario in the model where SM is extended with $U(1)_{B-L}$ gauge symmetry which is broken by the Higgs mechanism at the TeV scale and with TeV scale singlet neutrinos. We further assume a flavor symmetry on two generations of the singlet neutrinos which is explicitly broken by the neutrino Dirac Yukawa coupling at a scale above TeV. Renormalization group evolutions generate a splitting of the singlet neutrino Majorana masses that is proportional to the neutrino Dirac Yukawa coupling. Hence, the CP asymmetry parameters for the singlet neutrino decay process are automatically enhanced to order 0.1 by the resonance, even though the neutrino Dirac Yukawa coupling is much smaller. With the above setup, we have calculated the baryon number yield by solving the Boltzmann equations, taking into account the effects of the $U(1)_{B-L}$ gauge boson that can wash out the singlet neutrinos. We parametrize the neutrino Dirac Yukawa coupling by the light neutrino mass eigenvalues obtained from the neutrino oscillation data and an arbitrary 2×2 complex-valued orthogonal matrix. We have derived the baryon number yield for various values of the singlet neutrino mass and $U(1)_{B-L}$ gauge boson mass, and compared it with the observed value, Eq. (1). We have shown that a sufficiently large amount of baryon number yield is generated with several TeV masses for the singlet neutrinos and $U(1)_{B-L}$ gauge boson. The most notable result is that the right amount of baryon number yield

can be obtained for $(M_{Z'}, yv_{B-L}) = (4, 1.5)$ TeV and $(5, 2)$ TeV with $g_{B-L}^2/4\pi = 0.005$, in spite of the fact that an on-shell $U(1)_{B-L}$ gauge boson washes out singlet neutrinos in the course of thermal leptogenesis.

Let us discuss the implications of the above result on collider physics. At the LHC with $\sqrt{s} = 14$ TeV, the cross section for a $U(1)_{B-L}$ gauge boson that eventually decays into $\mu^+ \mu^-$ is given by

$$\begin{aligned} \sigma(pp \rightarrow Z' \rightarrow \mu^+ \mu^-) &= 0.200 \text{ fb} \quad \text{for } M_{Z'} = 4 \text{ TeV} \\ \text{and } g_{B-L}^2/4\pi &= 0.005, \end{aligned} \quad (35)$$

$$\begin{aligned} \sigma(pp \rightarrow Z' \rightarrow \mu^+ \mu^-) &= 0.0271 \text{ fb} \quad \text{for } M_{Z'} = 5 \text{ TeV} \\ \text{and } g_{B-L}^2/4\pi &= 0.005, \end{aligned} \quad (36)$$

where we have integrated over the muon invariant mass from $M_{Z'} - 1$ TeV to $M_{Z'} + 1$ TeV. SM background is negligibly small in these regions. We therefore expect to discover the $U(1)_{B-L}$ gauge boson with the integrated luminosity below 100 fb^{-1} in these cases. The production cross section for the two generations of singlet neutrinos is given by ($i = 1, 2$)

$$\begin{aligned} \sigma(pp \rightarrow Z' \rightarrow N_i \bar{N}_i) &= 0.138 \text{ fb} \\ \text{for } (M_{Z'}, yv_{B-L}) &= (4, 1.5) \text{ TeV} \\ \text{and } g_{B-L}^2/4\pi &= 0.005, \end{aligned} \quad (37)$$

$$\begin{aligned} \sigma(pp \rightarrow Z' \rightarrow N_i \bar{N}_i) &= 0.0181 \text{ fb} \\ \text{for } (M_{Z'}, yv_{B-L}) &= (5, 2) \text{ TeV} \\ \text{and } g_{B-L}^2/4\pi &= 0.005. \end{aligned} \quad (38)$$

A singlet neutrino decays into a light neutrino and the Higgs boson or the longitudinal component of Z boson, or into a charged lepton and the longitudinal component of W boson. By combining several decay modes, it is possible to identify singlet neutrinos [18] with the integrated luminosity of several hundred fb^{-1} even in the case of $(M_{Z'}, yv_{B-L}) = (5, 2)$ TeV.

ACKNOWLEDGMENTS

This work is supported in part by the D.O.E. Grant No. DE-FG02-10ER41714 (N.O.), Grant-in-Aid for Scientific research on Innovative Areas, Grant No. 23104009 (Y.O.), and the grant of the Japan Society for the Promotion of Science, Grant No. 23-3599 (T.Y.).

APPENDIX

The number density n_ψ of a particle ψ (a right-handed neutrino in our case) with mass m_ψ in the early universe is evaluated by solving the Boltzmann equation of the form [16],

$$\begin{aligned} \frac{dY_\psi}{dz} = & -\frac{z}{sH(m_\psi)} \\ & \times \sum_{a,i,j,\dots} \left[\frac{Y_\psi Y_a \dots}{Y_\psi^{\text{eq}} Y_a^{\text{eq}} \dots} \gamma^{\text{eq}}(\psi + a + \dots \rightarrow i + j + \dots) \right. \\ & \left. - \frac{Y_i Y_j \dots}{Y_i^{\text{eq}} Y_j^{\text{eq}} \dots} \gamma^{\text{eq}}(i + j + \dots \rightarrow \psi + a + \dots) \right], \end{aligned} \quad (\text{A1})$$

where $Y_\psi = n_\psi/s$ is the ratio of n_ψ and the entropy density s , $z = \frac{m_\psi}{T}$, and $H(m_\psi)$ is the Hubble parameter at a temperature $T = m_\psi$. The right-hand side of Eq. (A1) describes the interactions that change number of ψ , and γ^{eq} is the space-time density of scatterings in thermal equilibrium. For a dilute gas we take into account decays, two-particle scatterings and the corresponding back reactions. One finds, for a decay the particle ψ ,

$$\gamma_D = \gamma^{\text{eq}}(\psi \rightarrow i + j + \dots) = n_\psi^{\text{eq}} \frac{K_1(z)}{K_2(z)} \tilde{\Gamma}_{rs}, \quad (\text{A2})$$

where K_1 and K_2 are the modified Bessel functions, and $\tilde{\Gamma}_{rs}$ is the decay width. For two body scattering, one has

$$\begin{aligned} \gamma(\psi + a \leftrightarrow i + j + \dots) \\ = \frac{T}{64\pi^4} \int_{(m_\psi + m_a)^2}^{\infty} ds \hat{\sigma}(s) \sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right), \end{aligned} \quad (\text{A3})$$

where s is the squared center-of-mass energy and the reduced cross section $\hat{\sigma}$ for the process $\psi + a \leftrightarrow i + j + \dots$ is related to the usual total cross section $\sigma(s)$ by

$$\hat{\sigma}(s) = \frac{8}{s} [(p_\psi \cdot p_a)^2 - m_\psi^2 m_a^2] \sigma(s). \quad (\text{A4})$$

The total reduced cross section for the process $f + \bar{f} \rightarrow N + N$ mediated by the Z' boson (f denotes the SM fermions) is given by

$$\hat{\sigma}_{Z'}(s) = \frac{104\pi}{3} \alpha_{B-L}^2 \frac{\sqrt{x}}{(x-y)^2 + yc} (x-4)^{\frac{3}{2}}, \quad (\text{A5})$$

where $y = \frac{m_{Z'}^2}{m_N^2}$, and $c = (\tilde{\Gamma}_{Z'}/M_1)^2$ with the decay width of Z' boson

$$\tilde{\Gamma}_{Z'} = \frac{\alpha_{B-L} m_{Z'}}{6} \left[3 \left(1 - \frac{4}{y} \right)^{\frac{3}{2}} \theta(y-4) + 13 \right]. \quad (\text{A6})$$

-
- [1] G. Hinshaw *et al.* (WMAP Collaboration), *Astrophys. J. Suppl. Ser.* **180**, 225 (2009); E. Komatsu *et al.*, *Astrophys. J. Suppl. Ser.* **192**, 18 (2011).
- [2] M. Fukugita and T. Yanagida, *Phys. Lett. B* **174**, 45 (1986).
- [3] P. Minkowski, *Phys. Lett.* **67B**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity*, edited by P. van Nieuwenhuizen *et al.* (North Holland, Amsterdam, 1979), p. 315; S. L. Glashow, in *Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons*, edited by M. Lévy *et al.* (Plenum Press, New York, 1980), p. 687; R. N. Mohapatra and G. Senjanović, *Phys. Rev. Lett.* **44**, 912 (1980).
- [4] M. Flanz, E. A. Paschos, U. Sarkar, and J. Weiss, *Phys. Lett. B* **389**, 693 (1996); A. Pilaftsis, *Phys. Rev. D* **56**, 5431 (1997); A. Pilaftsis and T. E. J. Underwood, *Nucl. Phys.* **B692**, 303 (2004).
- [5] Recent studies, see, for example, K. Huitu, J. Maalampi, A. Pietila, and M. Raidal, *Nucl. Phys.* **B487**, 27 (1997); K. Huitu, S. Khalil, H. Okada, and S. K. Rai, *Phys. Rev. Lett.* **101**, 181802 (2008); S. Blanchet, Z. Chacko, S. S. Granor, and R. N. Mohapatra, *Phys. Rev. D* **82**, 076008 (2010); L. Basso, A. Belyaev, S. Moretti, G. M. Pruna, and C. H. Shepherd-Themistocleous, *Eur. Phys. J. C* **71**, 1613 (2011);
- [6] S. Khalil, *J. Phys. G* **35**, 055001 (2008); M. Abbas and S. Khalil, *J. High Energy Phys.* **04** (2008) 56; S. Khalil and A. Masiero, *Phys. Lett. B* **665**, 374 (2008).
- [7] S. Iso, N. Okada, and Y. Orikasa, *Phys. Rev. D* **83**, 093011 (2011).
- [8] R. G. Felipe, F. R. Joaquim, and B. M. Nobre, *Phys. Rev. D* **70**, 085009 (2004).
- [9] J. A. Casas and A. Ibarra, *Nucl. Phys.* **B618**, 171 (2001).
- [10] J. Barry and W. Rodejohann, *Phys. Rev. D* **81**, 093002 (2010); **81**, 119901(E) (2010).
- [11] F. P. An *et al.* (Daya Bay Collaboration), *Phys. Rev. Lett.* **108**, 171803 (2012); J. K. Ahn *et al.* (RENO Collaboration), *Phys. Rev. Lett.* **108**, 191802 (2012).
- [12] K. Nakamura *et al.* (Particle Data Group), *J. Phys. G* **37**, 075021 (2010).
- [13] N. S. Manton, *Phys. Rev. D* **28**, 2019 (1983); F. R. Klinkhamer and N. S. Manton, *Phys. Rev. D* **30**, 2212 (1984).
- [14] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, *Phys. Lett.* **155B**, 36 (1985).
- [15] Y. Burnier, M. Laine, and M. Shaposhnikov, *J. Cosmol. Astropart. Phys.* **02** (2006) 007.
- [16] E. Kolb and M. Turner, *The Early Universe* (Westview Press, New York, 1994).
- [17] See, for example, M. Plumacher, *Z. Phys. C* **74**, 549 (1997).
- [18] L. Basso, Ph.D. thesis, University of Southampton [arXiv:1106.4462].