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Light stops from Seiberg duality

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If low-energy supersymmetry is realized in nature, a seemingly contrived hierarchy in the squark mass spectrum appears to be required. We show that composite supersymmetric theories at the bottom of the conformal window can automatically yield the spectrum that is suggested by experimental data and naturalness. With a nontuned choice of parameters, the only superpartners below 1 TeV will be the partners of the Higgs, the electroweak gauge bosons, the left-handed top and bottom, and the right-handed top, which are precisely the particles needed to make weak scale supersymmetry breaking natural. In the model considered here, these correspond to composite (or partially composite) degrees of freedom via the Seiberg duality, while the other minimal supersymmetric standard model fields, with their heavier superpartners, are elementary. The key observation is that at or near the edge of the conformal window, soft supersymmetry breaking scalar and gaugino masses are transmitted only to fundamental particles at leading order. With the potential that arises from the duality, a Higgs with a 125 GeV mass, with nearly standard model production rates, is naturally accommodated without tuning. The lightest ordinary superpartner is either the lightest stop or the lightest neutralino. If it is the stop, it is natural for it to be almost degenerate with the top, in which case it decays to top by emitting a very soft gravitino, making it quite difficult to find this mode at the LHC and more challenging to find supersymmetry in general, yielding a simple realization of the stealth supersymmetry idea. We analyze four benchmark spectra in detail.

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I. INTRODUCTION

Supersymmetry potentially provides a complete theory of electroweak symmetry breaking, eliminating the hierarchy problem for the Higgs mass. But in a way supersymmetry is too efficient in suppressing the Higgs mass: the natural mass for a supersymmetry (SUSY) Higgs is often below 100 GeV so that large radiative corrections become essential. The simplest versions of the supersymmetric extension of the Standard Model (SM) are now being severely challenged: the Higgs sector must be fine tuned at the subpercent level in order to push the Higgs mass sufficiently far above the Z mass, and the nonobservation of missing energy events at the LHC [1,2] puts impressive bounds on squark and gluino masses. In popular versions of the minimal supersymmetric standard model (MSSM) with degenerate squarks, these masses are now constrained to be above 1 TeV. Minimizing fine-tuning in light of this data requires that the stop squark is lighter than the first and second generation squarks [3–6], leading to yet another hierarchy within SUSY models. The aim of this paper is to present a model where both the squark mass hierarchy and the little hierarchy are solved naturally via compositeness.

Compositeness is an intriguing idea for electroweak symmetry breaking: strong dynamics could either directly break electroweak symmetry or produce a composite Higgs boson without a hierarchy problem. Flavor poses the biggest challenges for such models, but compositeness might actually explain the much greater mass of the top quark: if the t and Higgs are composite while other quarks are not, then their Yukawa coupling is generically order one, while the other Yukawa couplings must be generated by higher-dimension operators. A fully composite SM (like that proposed by Abbott and Farhi [7]) is not expected to yield weakly interacting Ws and Zs. However we have learned from warped extra-dimensional models (which may be duals of approximately conformal 4D theories), like the Randall-Sundrum (RS) model [8], that large anomalous dimensions can save the composite Higgs scenario at the price of having both an elementary and a composite sector present, and having the t quark only partially composite (along with the W and Z). Even for these models, some fine tuning is nonetheless required to make the composite Higgs much lighter than the composite W'.

Since the problems of SUSY and of compositeness are complementary, it seems natural to try to combine the two to produce one complete, natural model of electroweak symmetry breaking (EWSB) at the TeV scale. In general this might seem artificial but existing Seiberg dualities automatically feature both. We will see in the models we consider that not only do we get the best features of both models, but also that supersymmetry breaking decouples at leading order from the IR composite states, somewhat analogously to what happens with UV supersymmetry

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breaking in RS-type models, leading to a natural hierarchy in the superpartner spectrum that readily accommodates current constraints.

Other ideas that have been explored include Refs. [9–12], in which strong SUSY dynamics trigger electroweak symmetric breaking by producing a composite Higgs that obtains a vacuum expectation value (VEV). Generically if the model reduces to the MSSM when the strong SUSY scale is taken to be much larger than the electroweak scale then the problems of the MSSM are reproduced. If, however, the model reduces to the next-to-minimal supersymmetric standard model (NMSSM) when the strong SUSY scale is taken to be large, then the Higgs mass can be much larger [9–12] than in the MSSM or even the NMSSM. This is because the cubic coupling between the composite singlet and composite Higgs doublets, which tends to increase the Higgs mass, can be much larger than in the NMSSM since its Landau pole merely signals the existence of the strong SUSY dynamics that generated the cubic coupling of the composites in the first place. This will be true for our model as well (as in Ref. [11]) and allows for sufficiently heavy Higgses.

Following the SUSY compositeness idea further, one must account for SUSY breaking. In this paper we show that (perhaps unexpectedly) the composite superpartners can remain light while the elementary superpartners can be heavy. One elegant idea for addressing SUSY breaking is to have strong SUSY dynamics break SUSY as well [12,13]. Such models are referred to as single-sector models. In this case however, the composites of the strong SUSY sector have large SUSY breaking masses. Avoiding fine-tuning calls for a light stop, $\tilde{t}$, so in these single sector models the t quark must be elementary, and thus the Higgs should be elementary as well in order to get a large t quark mass. Thus if we want a relatively light composite $\tilde{t}$ as well as a composite t and Higgs, SUSY breaking must come from outside the strong sector that produces composites. In this case, the leading contributions to the composite soft masses are calculable [14–18] when the Seiberg dual is weakly coupled in the infrared.

Generically the results are discouraging [14] since the squared soft masses of the mesons and dual quarks add to zero, so at least some of the composites will be tachyonic. However, at the boundary of the conformal window, the leading contributions to soft masses vanish. This suggests an interesting hierarchy of soft breaking masses: the composites (like the Higgsinos, $\tilde{t}_L$, $\tilde{b}_L$, and $\tilde{t}_R$) are much lighter than the other superpartners. If the W and Z are also partially composite, then their superpartners can also be lighter than the elementary superpartners. Note that these are exactly the particles that are needed to cancel the quadratic divergence in the Higgs mass. In this case the (approximately conformal) strong dynamics shields the composites from large supersymmetry breaking. On the other hand the little hierarchy problem of composite Higgses is resolved here via supersymmetry: the Higgs is a dual quark of Seiberg duality that can be much lighter than the compositeness scale because of supersymmetry. Moreover, because of the form of the potential that arises from Seiberg duality, the typical mass is on the order of the Higgs VEV without the usual MSSM suppression by a gauge coupling. Therefore in this model it is possible to accommodate a Higgs mass of 125 GeV without any tuning, while the production and decay rates of the Higgs will be close to SM values. In fact, the recently presented hints for a 125 GeV Higgs from ATLAS [19] and CMS [20] might even be further evidence that a viable supersymmetric theory should incorporate a low-scale cutoff, such as the compositeness scale presented here. The resulting spectrum is reminiscent of the ''more minimal supersymmetric standard model'' idea of Cohen *et al*. [21]. It can also be viewed as an explicit four-dimensional implementation of the warped extra- dimensional supersymmetric models of Refs. [22–24].

In this paper we analyze such models, which arise as dual composite gauge theories at the edge of the conformal window. These models have (partially) composite Higgs, t, W and Z and can address three problems at once: the hierarchy of Yukawa couplings, the little hierarchy of the Higgs mass, and the hierarchy of the squark soft masses. The same composite states that are needed for a dynamical Yukawa coupling are the ones needed to protect the Higgs mass. Such an unconventional superpartner spectrum has important consequences for SUSY searches at the LHC.

A limiting case is a nearly degenerate t-$\tilde{t}$ sector [25] that could be naturally produced by compositeness. In this case all superpartners decay via the next-to-lightest superpartner (NLSP) $\tilde{t}$ and not much missing energy. Models with new approximately degenerate superpartners that end the decay chains of Standard Model superpartners have been termed stealth SUSY models [26] precisely because of this lack of missing energy signatures. In generic stealth SUSY models, the approximate degeneracy is caused by a suppression of the coupling of the new states to the SUSY breaking sector. In the composite models we are discussing here, the suppression arises precisely because the states are composites of the strong SUSY dynamics, and the almost conformal strong dynamics screens SUSY breaking: the anomalous dimensions of the supersymmetry breaking terms suppress them up to possible threshold corrections. The threshold corrections are determined by holomorphy and also vanish in the conformal window.

Interestingly, the recently proposed [11] minimal composite supersymmetric standard model (MCSSM) has just these composite degrees of freedom and sits on the edge of the conformal window, so it provides a benchmark model for exploring this scenario and we focus on that model in this paper. Through most of the paper we will assume a low-scale mediation scenario, for which the prime example is gauge mediation. Many of the problems of gauge mediation simply do not arise here, since we can break electroweak symmetry in the SUSY limit there is no $B\mu$ problem, while singlet soft breaking terms are easily obtained since the singlet is a composite. We also consider one example of a possible high-scale supersymmetry breaking model as well.

Some of the key ideas here can be understood in analogy to the RS picture, where composites are localized near an IR brane, while the elementary fields are on the UV brane. The insensitivity of composites to SUSY breaking is simply captured by a small overlap of the IR localized composites with the UV localized SUSY breaking [22]. The other main ingredient is partial compositeness, which solves the major problems of fully composite theories. This is another very familiar feature of realistic RS models, corresponding to (almost) flat wave functions [27] for the W and Z.

The paper is organized as follows. First we discuss how external SUSY breaking feeds through to the composites of Seiberg duality. In Sec. III we review the MCSSM [11], which is the simplest model with composite Higgses, t's and partially composite Ws and Zs. In Sec. IV we estimate the sizes of realistic parameters for the MCSSM, discuss the electroweak symmetry breaking potential, and present the mass matrices for the light sparticles. In Sec. V we discuss the phenomenology by focusing on four benchmark spectra. Two of them have $\tilde{t}$ NLSP's, and one of these two has the lightest $\tilde{t}$ almost degenerate with the t as in Ref. [26] and therefore can be kinematically accessible to the current LHC run while nonetheless avoiding detection so far, while the other has a somewhat heavier $\tilde{t}$ below 300 GeV. The other two spectra are more conventional with neutralino NLSP's, one of which corresponds to a gauge mediated spectrum. All four of the spectra have $\tan\beta \sim 1$, and $\tan\beta$ can even be smaller than 1.

II. SOFT BREAKING TERMS FOR COMPOSITES

Before we present the concrete composite SUSY model that solves both the little hierarchy problem and predicts light $\tilde{t}$'s, we first address the question of the magnitudes of the soft breaking terms in composite SUSY models. Since these will essentially determine the characteristics of the spectrum, this is the critical feature of this class of models. We will assume that the strong dynamics can be captured via Seiberg duality, and ask the question of how UV soft breaking terms for the elementary ("electric") degrees of freedom get transmitted to the composite ("magnetic") degrees of freedom. We apply the method of analytic continuation into superspace [15–17] to find the mapping of soft breaking terms under duality. We start with the Lagrangian for the electric quarks Q, $\bar{Q}$ of an electric $SU(N)$ gauge theory with F flavors of these quarks.

We want to compare the soft mass for some spectator "elementary" degrees of freedom that do not have strong interactions with the soft masses of the composites in the IR. From the RS picture, we expect that composites (localized in the IR) will be insensitive to soft SUSY breaking in the UV, while the elementary fields should be sensitive. Indeed the soft breaking masses for the elementary fields undergo a perturbative RG running between the UV and IR scales characterized by small perturbative anomalous dimensions,

$$m_{\rm el}^2(\mu) = m_{\rm UV}^2\left(\frac{\mu}{\Lambda}\right)^{\mathcal{O}(\alpha)}, \qquad (2.1)$$

up to perturbative threshold corrections. The composite fields can in principle have both nonperturbative finite terms and nonperturbative anomalous dimensions,

$$m_{\rm comp}^2(\mu) = m_{\rm IR}^2 + m_{\rm UV}^2\left(\frac{\mu}{\Lambda}\right)^{\gamma}. \qquad (2.2)$$

This equation is schematic, when γ is a function of μ, then the renormalization group equation solution has the form of an exponential of an integral of $\gamma(\mu)$. Unlike the running term, the interpretation of the finite threshold term $m_{\rm IR}^2$ is not immediately obvious in the RS picture. When the dual theory is weakly coupled in the IR, the finite term, m_{IR}^2, can be calculated [14,16] using holomorphy. Meanwhile, the existence of a well-behaved Seiberg dual requires that the anomalous dimension, $\gamma(\mu)$ is positive, and for a weakly coupled dual, $\gamma(\mu) \sim \mathcal{O}(1)$ at or just below the strong coupling scale.[1] A large positive anomalous dimension rapidly drives the second term in (2.2) to zero. This analysis extends into the conformal window as well [17], where it further can be shown [18] that $m_{\rm IR} = 0$. This simply means that the fixed point is attractive, and these soft mass terms are irrelevant and vanish at the fixed point. As a consequence, one expects $m_{\rm IR}$ to also vanish just at the boundary between the conformal window and the free magnetic phase. At the bottom end of the conformal window $\gamma(\mu)$ is still $\mathcal{O}(1)$ but is perturbative at the top of the window, which means that at the top of the window the approach to the fixed point can be very slow, and in this case one exits the RGE long before the fixed point is approached. In the free magnetic phase similar conclusions hold, but with $m_{\rm IR} \neq 0$ in general, as one can see from the low-energy effective Kähler potential [16]. As we shall see, the two approaches agree at the bottom edge of the conformal window.

Next we will explicitly show the calculation for $m_{\rm IR}^2$ in the weakly coupled, "free-magnetic" phase. We will also include a small supersymmetric mass (matrix) μ_f for the electric quarks, much smaller than the dynamical scale of the theory. One of these will correspond to the term triggering electroweak symmetry breaking, which in this

[1] It has been recently pointed out in Ref. [28] that in SUSY QCD for $F \leq 3/2N$ the eventual approach in the deep IR is only logarithmic due to the appearance of accidental symmetries. For the applications considered here the only relevant issue is that there is a sufficiently large region with power-law running to ensure the suppression of the soft breaking terms.

model will happen even in the absence of supersymmetry breaking, but via the composite dynamics. Thus one needs to assume that the relevant μ_f is related to the magnitude of the Higgs VEV v, and this parameter is what sets the electroweak scale. Although we do not explain this choice of parameter, we expect that in a more complete model of supersymmetry breaking this can be related to the soft supersymmetry breaking scale as well.

The effects of the soft SUSY breaking terms for the elementary fields are incorporated into the Lagrangian by using the real and chiral spurions Z and U with nonzero θ components [14–17]:

$$\mathcal{L} = \int d^4\theta(Q^\dagger Z e^V Q + \bar{Q}^\dagger Z e^V \bar{Q}) + \int d^2\theta(U W^\alpha W_\alpha + \mu_f \bar{Q} Q) + \text{H.c.} \tag{2.3}$$

To introduce a soft squark mass $m_{\rm UV}$, a gaugino mass m_λ, and a soft breaking B term (with $m_{\rm UV}^2 \sim m_\lambda^2 \sim B$) we Taylor expand the spurions in superspace coordinates:

$$Z = 1 - \theta^2 B - \bar{\theta}^2 B - \theta^2\bar{\theta}^2(m_{\rm UV}^2 - |B|^2), \tag{2.4}$$

$$U = \frac{1}{2g^2} - i\frac{\theta_{YM}}{16\pi^2} + \theta^2\frac{m_\lambda}{g^2}, \tag{2.5}$$

where we have also included the CP violating parameter $\theta_{\rm YM}$ (not to be confused with the superspace coordinate). The spurion U is related to the holomorphic strong scale Λ_h which acts as a chiral superfield spurion that is also an RG invariant:

$$\Lambda_h = \mu e^{-16\pi^2 U(\mu)/b}, \tag{2.6}$$

where b is the one-loop β-function coefficient $b = 3N - F$ and μ is the RG scale. In the model presented in the next section we will choose $N = 4$ and $F = 6$.

We can also include these spurions in the composite description since the structure of the low-energy theory is constrained by symmetries including an anomalous axial $U(1)$ symmetry. In other words Z and U are also spurions of the anomalous axial $U(1)$. Under axial transformations, where the rotation parameter is promoted to a chiral superfield A, we have

$$Q \to e^A Q, \qquad \bar{Q} \to e^A \bar{Q}, \tag{2.7}$$

$$Z \to e^{-A-A^\dagger}, \qquad \Lambda_h \to e^{2F/bA}\Lambda_h. \tag{2.8}$$

It is convenient to introduce a redundant scale that is invariant under axial transformations

$$\Lambda^2 = \Lambda_h^\dagger Z^{2F/b}\Lambda_h, \tag{2.9}$$

which is also a SUSY breaking spurion

$$\log\frac{\Lambda}{\mu} = \frac{-8\pi^2}{bg^2} + \frac{-8\pi^2 m_\lambda}{bg^2}(\theta^2 + \bar{\theta}^2) - \frac{F}{b} m_{UV}^2 \theta^2\bar{\theta}^2. \tag{2.10}$$

This Λ is the invariant scale that can be used for dimensional analysis once the anomalous $U(1)$ charge is fixed.

In the composite theory, "magnetic" states transform under the dual gauge $SU(F - N)$ gauge group, and include the meson M and dual quarks q, $\bar{q}$. Due to the operator mapping

$$Q\,\bar{Q} \leftrightarrow M, \quad Q^N \leftrightarrow q^{F-N}, \quad \bar{Q}^N \leftrightarrow \bar{q}^{F-N}, \tag{2.11}$$

we have the following axial transformations for the composite states:

$$q \to e^{AN/(F-N)} q \tag{2.12}$$

$$\bar{q} \to e^{AN/(F-N)}\bar{q} \tag{2.13}$$

$$M \to e^{2A} M. \tag{2.14}$$

Since the dual composite theory is in the weakly coupled phase we can write an approximately canonical Kähler potential. Requiring SUSY and axial invariance and using dimensional analysis we find the dual Lagrangian

$$\mathcal{L} = \int d^4\theta\left[\frac{M^\dagger Z^2 M}{\Lambda^2} + \frac{q^\dagger Z^{N/(F-N)} e^{\tilde{V}} q}{\Lambda^{(4N-2F)/(F-N)}} + \frac{\bar{q}^\dagger Z^{N/(F-N)} e^{\tilde{V}}\bar{q}}{\Lambda^{(4N-2F)/(F-N)}}\right] + \int d^2\theta\left[U\tilde{W}^\alpha\tilde{W}_\alpha + \frac{yMq\bar{q}}{\Lambda_h^{b/(F-N)}} + \mu_f M\right] + \text{H.c.} \tag{2.15}$$

We can read off the soft masses near the infrared fixed point [16,17] for the composites from the Kähler term by Taylor expanding in superspace:

$$m_M^2 = 2\frac{3N-2F}{b} m_{\rm UV}^2, \quad m_q^2 = -\frac{3N-2F}{b} m_{\rm UV}^2. \tag{2.16}$$

Generically these results spell trouble for composite models: some of the dual quark or meson soft breaking masses should be tachyonic, and this would apply for the entire multiplet. However, for the case when $F = 3N/2$, that is at the lower end of the conformal window these leading calculable terms vanish. This is exactly the right region for the model considered later in this paper ($F = 4$, $N = 6$). In this case the leading terms will come from the second term in (2.2) corresponding to the fact that we do not run all the way to $\mu = 0$ but stop at a scale given by (2.2) $\mu^2 \sim m_{\rm UV}^2\mu/\Lambda$, so that the corrections are $\mathcal{O}(m_{\rm UV}^4/\Lambda^2)$ which can also be seen as the effects of higher-order terms in the Kähler potential suppressed by additional powers of Λ [16]. The perturbative dual-gauge group corrections are included in this estimate. In addition to power corrections, there are also perturbative corrections from SM gauge interactions that could dominate when Λ is very large.

The matching of the gaugino masses follows simply from the invariance of Λ, implying $m_\lambda/(bg^2) = m_{\tilde{\lambda}}/(\tilde{b}\tilde{g}^2)$ in the holomorphic basis. After the rescaling by couplings to get into the canonical basis one obtains the well-known answer

$$m_{\tilde{\lambda}} = -\frac{3N-2F}{3N-F} m_\lambda, \tag{2.17}$$

thus the leading contribution of the composite gaugino mass also vanishes at the boundary of the conformal window.

To get the soft terms that come from the superpotential couplings we must rescale the fields to get canonical Kähler terms. Since we need terms only of order θ^2 we can write

$$Z = \xi^\dagger \xi, \qquad \xi = 1 - \theta^2 B, \tag{2.18}$$

and then rescale chiral fields only via the holomorphic quantities ξ, Λ_h. We then find the superpotential terms in the canonical basis:

$$\int d^2\theta (yMq\bar{q} + \mu_f \Lambda_h M \xi^{\frac{2(2F-3N)}{(3N-F)}} + \text{H.c.}). \tag{2.19}$$

Since the cubic superpotential is independent of the supersymmetry breaking spurions, we find that the A-term vanishes in the IR limit for any F:

$$A = \mathcal{O}\left(\frac{m_{\text{UV}}^2}{\Lambda}\right). \tag{2.20}$$

We also find a SUSY breaking scalar tadpole for the meson

$$T = \mu_f \Lambda \left(-\frac{16\pi^2 m_\lambda}{bg^2} - \frac{2(2F-3N)}{3N-F} B\right). \tag{2.21}$$

While the second term vanishes for $F = 3/2N$ the first one does not: this is not surprising since this is the effect of an explicit breaking of the conformality on the elementary side. The expected magnitude for T will then be of order

$$T \sim \mu_f \Lambda \times m_{\text{UV}}, \tag{2.22}$$

where m_{UV} represents the characteristic magnitude of the gaugino mass m_λ that appears on the right-hand side of Eq. (2.17). Thus we find that the IR limit of all soft breaking parameters for composites vanish at the edge of the conformal window, except for the scalar tadpole, which is related to the explicit breaking term and the elementary SUSY breaking terms. For phenomenological reasons that will be explicit in the next section we parameterize the superpotential term linear in the meson field in (2.19) as yf^2M, where f must be chosen to be of order of the weak scale, and y is the dynamical Yukawa coupling that runs down to $\mathcal{O}(1)$ at the electroweak scale, which is the right size to give the correct t mass. In terms of the duality mapping given above, we see that by definition $yf^2 \equiv \mu_f \Lambda$, so we find that the magnitude of the scalar tadpole is of order

$$T \sim f^2 m_{\text{UV}}. \tag{2.23}$$

Thus we find that at the edge of the conformal window one has a hierarchy of the soft breaking terms, which, writing the soft scale for the elementary fields as $m_{\text{el}} \sim m_{\text{UV}}$, takes the form

$$A, \qquad m_{\tilde{q},\tilde{g}} \sim \frac{m_{\text{el}}^2}{\Lambda} \ll m_{\text{el}}$$
$$T \sim \mu_f \Lambda \times m_{\text{el}} \equiv f^2 m_{\text{el}} \ll m_{\text{el}}^3. \tag{2.24}$$

As a check of the duality mapping, note that the scale-matching relation between the electric and dual magnetic theories is defined in the frame where the dual quarks are canonically normalized, and the meson is mapped to $Q\bar{Q}$. In this frame the dual quarks carry anomalous charge 1, and the scale matching relation is [29]

$$\Lambda_h^b \tilde{\Lambda}_h^{\tilde{b}} = (-1)^N \Lambda_M^F, \tag{2.25}$$

where Λ_M can be expressed in terms of Λ_h and ξ by matching the anomalous charge as:

$$\Lambda_M = \Lambda_h \xi^{\frac{3(2N-F)}{3N-F}}. \tag{2.26}$$

By rescaling the terms in (2.15) to move to a frame with canonically normalized dual quarks we find that as expected Λ_M is also the parameter appearing in the dual superpotential in this frame: $Mq\bar{q}/\Lambda_M$, as predicted in Ref. [29].

III. MCSSM: THE MODEL FOR A COMPOSITE THIRD GENERATION

A concrete model (that we refer to as the minimal composite supersymmetric standard model or MCSSM) of supersymmetric composite Higgs and t quarks (and partially composite W and Z) was recently proposed in Ref. [11]. The main idea is that an asymptotically free gauge group becomes strongly interacting and the IR theory will contain composite gauge bosons, mesons and dual quarks, some of which are to be identified with the W, Z, t, and Higgs of the MSSM. To get a realistic theory, the composite W and Z need to be mixed with elementary W and Z gauge bosons that couple to the elementary quarks and leptons. The electric theory of the simplest such model is given by (corresponding to $N = 4$, $F = 6$)

	$SU(4)$	$SU(6)_1$	$SU(6)_2$	$U(1)_V$	$U(1)_R$
Q	$\square$	$\overline{\square}$	**1**	1	$\frac{1}{3}$
$\bar{Q}$	$\overline{\square}$	**1**	$\overline{\square}$	−1	$\frac{1}{3}$

, (3.1)

where the $SU(4)$ is the strong gauge group and the other groups are the global symmetries, some of which are weakly gauged. In particular, the elementary gauge symmetries $SU(3) \times SU(2)_{\text{el}} \times U(1)$ are embedded into these global symmetries. We will also allow small tree-level masses for the electric quarks.

The IR behavior of this strongly coupled theory is given by the Seiberg dual [29]

$$\begin{array}{cccccc} & SU(2)_{\rm mag} & SU(6)_1 & SU(6)_2 & U(1)_V & U(1)_R \\ q & \square & \square & \mathbf{1} & 2 & \frac{2}{3} \\ \bar{q} & \overline{\square} & \mathbf{1} & \square & -2 & \frac{2}{3} \\ M & \mathbf{1} & \overline{\square} & \overline{\square} & 0 & \frac{2}{3} \end{array} \tag{3.2}$$

with the additional dynamical superpotential term

$$W_{\rm dyn} = y\bar{q}Mq. \tag{3.3}$$

The SM gauge groups are embedded in the global symmetry as

$$\begin{aligned} SU(6)_1 &\supset SU(3)_c \times SU(2)_{\rm el} \times U(1)_Y \\ SU(6)_2 &\supset SU(3)_X \times SU(2)_{\rm el} \times U(1)_Y, \end{aligned} \tag{3.4}$$

where $SU(3)_X$ is a global $SU(3)$ which will be broken by (elementary) Yukawa couplings. The $SU(2)_{\rm mag} \times SU(2)_{\rm el}$ will eventually be broken to the diagonal subgroup which will be identified with the SM $SU(2)_L$. The embedding is chosen so that the dual quarks contain the left-handed third-generation quark doublet, two Higgses $H_{u,d}$, and two bifundamentals $\mathcal{H}$, $\bar{\mathcal{H}}$ that will be responsible for breaking the $SU(2)_{\rm mag} \times SU(2)_{\rm el}$ to the diagonal and generating the partially composite W and Z. Fields are embedded into the dual quarks as

$$q = Q_3, \mathcal{H}, H_d, \qquad \bar{q} = X, \bar{\mathcal{H}}, H_u. \tag{3.5}$$

From the q, $\bar{q}$ charge assignments it follows that the meson M contains the right-handed t, the two singlets S and P, two additional Higgses $\Phi_{u,d}$ transforming under the elementary $SU(2)_{\rm el}$, a second right-handed up-type quark U and some exotics V, E, R, G:

$$M = \begin{pmatrix} V & U & \bar{t} \\ E & G + P & \phi_u \\ R & \phi_d & S \end{pmatrix}, \tag{3.6}$$

where the quantum numbers under $SU(3)_c \times SU(2)_{\rm el}$ for the meson fields are as follows: V represents three $(\bar{3}, 1)$'s, U is a $(\bar{3}, 2)$, E represents three (1,2)'s, G is a (1,3), ϕ_d and ϕ_u are (1,2)'s, P and S are singlets, and R represents three singlets. The hypercharge assignments for the electric quarks, the dual quarks, and the mesons are then

$$\begin{array}{ccccccc} & \mathcal{Q}_1 & \mathcal{Q}_2 & \mathcal{Q}_3 & \mathcal{Q}_4 & \mathcal{Q}_5 & \mathcal{Q}_6 \\ Y & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & -\frac{1}{2}, \end{array}$$
$$\begin{array}{cccccccccccccc} & Q_3 & \mathcal{H}, \bar{\mathcal{H}} & H_u & H_d & X & V & U & \bar{t} & E & \phi_u & R & \phi_d & G, P, S \\ Y & \frac{1}{6} & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{6} & 0 & -\frac{1}{6} & -\frac{2}{3} & \frac{1}{6} & -\frac{1}{2} & \frac{2}{3} & \frac{1}{2} & 0 \end{array}. \tag{3.7}$$

With these quantum numbers the most general gauge invariant renormalizable electric superpotential is given by

$$W_{\rm tree} = \mu_{\mathcal{F}}(\mathcal{Q}_4\bar{\mathcal{Q}}_4 + \mathcal{Q}_5\bar{\mathcal{Q}}_5) + \mu_f \mathcal{Q}_6\bar{\mathcal{Q}}_6. \tag{3.8}$$

These will get mapped into tadpoles for the singlets P and S on the magnetic side. The P tadpole will be responsible for the breaking of the $SU(2)_{\rm mag} \times SU(2)_{\rm el}$ to the diagonal, while the S tadpole will be responsible for electroweak symmetry breaking. Note that the embedding of the SM gauge symmetries into the global symmetries together with the superpotential (3.8) imply that there are no accidental global symmetries appearing in the IR. This can be seen from the fact that the only gauge singlet composites are the S, P components of the meson, but these are precisely the fields for which a tree-level superpotential has been added. The absence of accidental global symmetries implies that there is no danger of the logarithmic IR running associated with accidental global symmetries described in Ref. [28].

The cancellation of SM gauge anomalies requires the presence of some spectator fields in the electric theory that only have SM gauge couplings. A simple choice for this anomaly cancellation is to include elementary fields that are conjugate to the representations of composite mesons V, U, R, $\phi_{u,d}$, G. Trilinear superpotential terms between these spectators and electric quarks will map to mass terms in the dual description, and the extra degrees of freedom will decouple, while the fields E, X will pair together to obtain a mass from the VEV of the bifundamental $\mathcal{H}$. The remaining standard model fields (first two generation quarks, right-handed bottom and all leptons) are assumed to be elementary fields transforming under $SU(3)_c \times SU(2)_{\rm el} \times U(1)_Y$. This charge assignment will be automatically anomaly free, and is capable of producing the usual flavor structure and Cabibbo-Kobayashi-Maskawa mixing matrix.

The relevant part of the superpotential (3.3) together with the singlet tadpoles from (3.8) can then be written as

$$\begin{aligned} W \supset\ & yP(\mathcal{H}\bar{\mathcal{H}} - \mathcal{F}^2) + yS(H_uH_d - f^2) \\ & + yQ_3H_u\bar{t} + yH_u\mathcal{H}\phi_u + yH_d\bar{\mathcal{H}}\phi_d. \end{aligned} \tag{3.9}$$

The first term is responsible for the breaking of $SU(2)_{\rm el} \times SU(2)_{\rm mag}$ to the diagonal group, the second term will trigger electroweak symmetry breaking, the third will give rise to the t Yukawa coupling and the last two terms give rise to a mixing of the Higgs with a heavy Higgs $\phi_{u,d}$. At this point the low-energy effective theory below the scale $\mathcal{F}$ (and assuming that $\mathcal{F} \gg f$) is that of the NMSSM with a composite Higgs, Q_3 and t. As explained above the rest of the SM particles are assumed to be elementary, that is made of fields that do not transform under the strongly coupled $SU(4)$. They simply carry the usual SM quantum numbers under $SU(2)_{\rm el} \times SU(3)_c \times U(1)_Y$.

At high energies there are three sets of Higgses: the composite $H_{u,d}$ from the dual quarks transforming under the composite $SU(2)_{\rm mag}$, the composite $\phi_{u,d}$ from the mesons transforming under the elementary $SU(2)_{\rm el}$, and a set of elementary Higgses $\phi'_{u,d}$ transforming under the elementary $SU(2)_{\rm el}$. These latter fields need to be present to remove $\phi_{u,d}$ from the spectrum via a trilinear superpotential term, which after duality maps into a mass term. The elementary Higgses $\phi'_{u,d}$ also have ordinary Yukawa couplings with the light elementary SM matter fields in addition to their mass with $\phi_{u,d}$, After integrating out $\phi_{u,d}$, $\phi'_{u,d}$ effective Yukawa couplings between the remaining light composite Higgses $H_{u,d}$ and the light SM fermions are generated. For more details see Ref. [11]. The resulting theory of the Higgses in the low-energy potential has the necessary Yukawa couplings and as we will now see it also has a viable and interesting potential.

IV. ELECTROWEAK SYMMETRY BREAKING, SOFT BREAKING PATTERNS AND MASS SPECTRUM

The Higgs potential relevant for electroweak symmetry breaking (assuming $\mathcal{F} \gg f$) is (including soft breaking terms)

$$V = y^2|H_u H_d - f^2|^2 + y^2|S|^2(|H_u|^2 + |H_d|^2) + m_S^2|S|^2 + m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + (ASH_uH_d + TS + \text{H.c.}) + \frac{g^2 + g'^2}{8}(|H_u|^2 - |H_d|^2)^2, \tag{4.1}$$

where m^2_{S,H_u,H_d}, A and T are soft supersymmetry breaking parameters, and the last term is the usual MSSM D-term. This is quite different from the usual MSSM potential, and the traditional source of fine-tuning related to the need of large $\tilde{t}$ loop corrections for the quartic are not produced. While the matter content of the Higgs sector is that of an NMSSM, the actual potential is quite different from what is traditionally used in a Z_3 symmetric NMSSM. Electroweak symmetry is broken in the supersymmetric limit, and a Higgs mass much bigger than in the MSSM is ensured since the quartic does not come from D-terms and thus the Higgs mass is not related to the Z-mass. Such Higgs sectors are natural in the context of composite "fat Higgs"-like models [9,10]: the NMSSM singlet S is simply one of the composite meson components. The NMSSM-like superpotential given in Eq. (3.9) is the one that appears most naturally in Seiberg duals. The electroweak symmetry breaking scale is determined by the magnitude of the S-tadpole f, which means that electroweak symmetry breaking in general is not dependent or related to supersymmetry breaking, but that f has to be of the order of the Higgs VEV v. For a completely natural model, one would hope for a deeper relation between f and v. This is similar to the usual μ-problem of the MSSM (without a corresponding $B\mu$ problem). The traditional way of solving this would be to assume that the electric theory has a global Peccei-Quinn-type symmetry that forbids the mass term for the electric quarks that eventually turn into the composite S, and that this PQ symmetry is only broken in the supersymmetry breaking sector. Coupling the electric quarks to the supersymmetry breaking sector can then give a PQ-violating superpotential term proportional to the supersymmetry breaking scale just like in the usual Giudice-Masiero mechanism. We will not try to build a complete model for the supersymmetry breaking sector in this paper.

We will use the usual parametrization of the Higgs fields:

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \tag{4.2}$$

$$\langle H_u^0 \rangle = \frac{v}{\sqrt{2}} \sin\beta, \qquad \langle H_d^0 \rangle = \frac{v}{\sqrt{2}} \cos\beta. \tag{4.3}$$

Since the interaction with the singlet provides a sizable quartic, it is not important to have a large $\tan\beta$, it actually can be close to one, or even less than one. Minimizing the potential with respect to the scalar S we find the scalar VEV

$$\langle S \rangle = -\frac{\sqrt{2}(\mathrm{A}v^2 \sin\beta\cos\beta + 2T)}{2M_S^2 + y^2v^2}. \tag{4.4}$$

A combination of the other two equations yield an expression that is analogous to the usual fine-tuning condition for the Higgs VEV:

$$\frac{y^2v^2}{2} = \frac{2(y^2f^2 - AS)}{\sin 2\beta} - 2y^2S^2 - m_{H_u}^2 - m_{H_d}^2. \tag{4.5}$$

Thus the fine-tuning can now be characterized by

$$\frac{y^2v^2}{2m_{H_u}^2}. \tag{4.6}$$

In most supersymmetric models, the $\tilde{t}$'s have to be sufficiently heavy to generate a large enough Higgs quartic (or equivalently, a large enough physical Higgs mass). On the other hand, heavy $\tilde{t}$'s also give a large contribution to $m_{H_u}^2$ leading to large tuning. In our models, one has a large tree-level quartic from compositeness, and the $\tilde{t}$'s are light, thus (4.6) can be of $\mathcal{O}(1)$ with composite $\tilde{t}$ masses in the 200–500 GeV range. Even so, since the gluino is elementary and thus in the few TeV range the two-loop corrections to the Higgs mass via gluino-$\tilde{t}$ loops can potentially be too large. The leading two-loop correction to $m_{H_u}^2$ due to the gluino loop is

$$\Delta m_{H_u}^2 \sim -\frac{2y_t^2\alpha_s^2}{\pi^3}|m_{\tilde{g}}|^2\log^2\left(\frac{\Lambda}{\mathrm{TeV}}\right). \tag{4.7}$$

Note that due to compositeness, the cutoff scale of the logarithm is small here. Even for low $\tan\beta$, one gets only about ten percent tuning for a gluino as heavy as 3 TeV.

We conclude that in principle, a gluino heavier than those that are usually considered natural would be allowed. However, a heavy gluino mass would also contribute to the $\tilde{t}$ masses, and in our models we assume light top squark masses. The leading log correction to the $\tilde{t}$ mass parameters is of the order

$$\Delta m_{\tilde{t}} \sim \frac{32}{3}\frac{\alpha_s}{4\pi}|M_3|^2\log(\frac{\Lambda}{\mathrm{TeV}}). \tag{4.8}$$

Even with this additional consideration on naturalness, since the logarithm is quite small (corresponding to the running between the duality scale and the TeV scale, $\log\frac{\Lambda}{\mathrm{TeV}} \sim 2$), one can naturally maintain a hierarchy between the gluino and the $\tilde{t}$ mass. However this hierarchy cannot be very large if we want to keep the top squark light. A gluino of about 1.5 TeV would be natural with a 400 GeV $\tilde{t}$ without much tuning. If one were to allow ten percent tuning the gluino mass could be raised to about 3 TeV. We will however not do that, and restrict the gluino mass to be below 1.5 TeV in order to protect the squark mass hierarchies obtained from the strong dynamics. Note, that the experimental lower bound on the gluino is around 700 GeV even if it only decays via third-generation squarks [30].

We now discuss the pattern of soft breaking terms and the magnitudes of the relevant parameters of the model. While we do not fully specify the mechanism of supersymmetry breaking mediation to the elementary (‘‘electric’’) fields here, we will usually assume some form of low-scale mediation mechanism, in order to have the gravitino be the lightest superpartner (LSP). The prime example of such models is gauge mediation. However, even if we assume gauge mediation applies, this is a non-standard application, since we are eventually ending up with the NMSSM. Naively one would think that gauge mediation can not be applied to an NMSSM-type theory, since the singlet will not obtain SUSY breaking terms. However, in this case gauge mediation is assumed to happen above the compositeness (‘‘duality’’) scale. Since the singlet is a composite (it is a component of the meson) a soft breaking term (suppressed as with all composites) will be induced for it. The mass for the fermionic partner of the singlet (the singlino) is model dependent. There can be a singlino mass from nonrenormalizable terms for the elementary fields $(\bar{Q}_6 Q_6)^2/\Lambda_{\mathrm{UV}}$ giving a singlino mass of order $m_{S_f} \sim \Lambda^2/\Lambda_{\mathrm{UV}}$. There will also be a singlino mass generated by the strong dynamics of order $\frac{f^4}{\Lambda^4}m_{\mathrm{el}}$ which is typically quite small. We will not be making a definite assumption on the size of the singlino mass, but explore spectra both with small and sizeable values for it.

Note that the usual $B\mu$ problem is simply not present, since the potential contains only trilinear and tadpole terms, both of which are induced as described in Sec. II. While the μ-problem is solved as usual in NMSSM-type models, an issue similar to the μ-problem is why the parameter f is close to the electroweak scale, which as we discussed before is likely to be addressed with a more complete model of SUSY breaking.

The message from the general discussion of Sec. II is that soft breaking terms for the composites are suppressed compared to those of the elementary fields, while the scalar tadpole T is unsuppressed. We choose parameters consistent with the hierarchies explained in the previous explained in the previous section of order

$$\begin{aligned} &m_{\mathrm{el}} \sim M_3 \sim \mathrm{few}\cdot\mathrm{TeV} \\ &\Lambda \sim 5\text{–}10\,\mathrm{TeV}\, m_{\mathrm{comp}} \sim \frac{m_{\mathrm{el}}^2}{\Lambda} \sim M_1 \sim M_2 \sim A \sim \mathrm{few}\cdot 100\,\mathrm{GeV} \\ &f \sim 100\,\mathrm{GeV}\, T \sim f^2 m_{\mathrm{el}} \sim \mathrm{few}\cdot 10^7\,\mathrm{GeV}^3 \\ &\mathcal{F} \sim \mathrm{few}\cdot\mathrm{TeV}\,\mu_{\mathrm{eff}} = y\langle S\rangle \sim A \end{aligned} \tag{4.9}$$

$$\tan\beta \sim \mathcal{O}(1). \tag{4.10}$$

Here m_{el} includes the soft breaking scalar masses of the first two generation squarks, the right handed sbottom, $\tilde{b}$ and all sleptons, while m_{comp} includes $m_{Q_{33}}$ and $m_{U_{33}}$. The soft terms include the dynamical noncalculable contributions of $\mathcal{O}(m_{\mathrm{el}}^2/\Lambda)$ and the additional radiative corrections $\propto \log\frac{\Lambda}{\mathrm{TeV}}$. The latter can be comparable to the dynamical terms as we discussed for the gluino loops. The effective $B\mu$ term is $A\langle S\rangle \sim \mu_{\mathrm{eff}}^2$. However, as stated previously, in this model electroweak symmetry is broken in the supersymmetric limit, so the magnitude of $B\mu$ is not very crucial. Note, that flavor constraints for such models with heavy first- and second-generation squarks and sleptons are largely satisfied if the scale of the heavy squark masses is around 5 TeV [31], and if the heavy squarks are close to degenerate, which would be the case if they get their masses from gauge mediation.

With this choice of parameters we can then go ahead and evaluate the full sparticle spectrum. We present the relevant expressions for the masses below, while in the next section we focus on four benchmark spectra.

Given all the soft SUSY breaking terms the spectrum calculation proceeds in a similar fashion to the MSSM and NMSSM. The $\tilde{t}$ mass matrix is

$$m_{\tilde{t}}^2 = \begin{pmatrix} m_{Q33}^2 + m_t^2 + \delta_u & v(As_\beta - \mu_{\mathrm{eff}} y_t c_\beta)/\sqrt{2} \\ v(As_\beta - \mu_{\mathrm{eff}} y_t c_\beta)/\sqrt{2} & m_{\bar{u}33}^2 + m_t^2 + \delta_{\bar{u}} \end{pmatrix}, \tag{4.11}$$

where the D-term contribution is as usual

$$\delta_f = -gT_f^3\langle D^3\rangle - g'Y_f\langle D'\rangle = (T_f^3 - Q_f s_W^2)\cos 2\beta M_Z^2. \tag{4.12}$$

Since the $\tilde{b}$ mass is constrained by the LHC to be above ~250–280 GeV [5], m_{Q33} should not be too small, since this sets the mass of the lighter $\tilde{b}$. The right-handed $\tilde{t}$ mass, $m_{\bar{u}33}$, can be somewhat smaller than m_{Q33}, and with A not too large one gets a spectrum with the right–handed $\tilde{t}$ as the lightest sfermion, a somewhat heavier left-handed $\tilde{t}$ and left-handed $\tilde{b}$, while the elementary fields are quite a bit heavier.

The explicit form of the $\tilde{b}$ mass matrix is

$$m_{\tilde{b}}^2 = \begin{pmatrix} m_{Q33}^2 + m_b^2 + \delta_d & v(A_{d33}c_\beta - \mu_{\rm eff} y_b s_\beta)/\sqrt{2} \\ v(A_{d33}c_\beta - \mu_{\rm eff} y_b s_\beta)/\sqrt{2} & m_{\bar{d}33}^2 + m_b^2 + \delta_{\bar{d}} \end{pmatrix}, \tag{4.13}$$

where the right-handed $\tilde{b}$ is elementary, so its soft breaking mass is expected to be large $m_{\bar{d}33}^2 \sim m_{\rm el}$, while $m_{Q33}^2 \sim m_{\rm comp}$ is suppressed.

Due to the extra $SU(2)$ group we have an additional set of charginos and neutralinos, and the singlet S also contributes to the neutralino mass matrix. The chargino mass matrix is

$$\begin{pmatrix} \tilde{W}_{2,\rm el}^- & \tilde{\phi}_d^- & \tilde{H}_d^- & \tilde{W}_{2,\rm mag}^- \end{pmatrix} \times \begin{pmatrix} M_2 & \frac{g_{\rm el}}{\sqrt{2}}\mathcal{F} & 0 & 0 \\ \frac{g_{\rm el}}{\sqrt{2}}\mathcal{F} & y\langle P\rangle & 0 & \frac{g_{\rm mag}}{\sqrt{2}}\mathcal{F} \\ 0 & 0 & \mu_{\rm eff} & \frac{g_{\rm mag}}{\sqrt{2}} s_\beta v \\ 0 & \frac{g_{\rm mag}}{\sqrt{2}}\mathcal{F} & \frac{g_{\rm mag}}{\sqrt{2}} c_\beta v & M_{2,\rm mag} \end{pmatrix} \begin{pmatrix} \tilde{W}_{2,\rm el}^+ \\ \tilde{\phi}_u^+ \\ \tilde{H}_u^+ \\ \tilde{W}_{2,\rm mag}^+ \end{pmatrix}, \tag{4.14}$$

where we have also added the elementary winos for the $SU(2)_{\rm el}$ group and the Higgsinos from $\phi_{u,d}$, and also $\mathcal{F} = \langle\mathcal{H}\rangle$ is the bifundamental VEV that breaks the composite $SU(2)_{\rm mag}$ and the elementary $SU(2)_{\rm el}$ down to the diagonal $SU(2)_L$ subgroup, while g_c and g_e represent the two gauge couplings giving rise to the SM couplings via the mixing

$$s_\theta = \frac{g_{\rm el}}{\sqrt{g_{el}^2 + g_{\rm mag}^2}}, \qquad c_\theta = \frac{g_{\rm mag}}{\sqrt{g_{el}^2 + g_{\rm mag}^2}},$$
$$g_2 = g_{\rm mag} s_\theta = g_{\rm el} c_\theta, \tag{4.15}$$

and $\mu_{\rm eff} = y\langle S\rangle$. In the limit $g_{\rm mag} \gg g_{\rm el}$, $\mathcal{F} \gg M_2, M_{2,\rm mag}$ this can be approximately diagonalized, and the heavy combination of gauginos (corresponding mostly to the composite charginos and the $\mathcal{H}$'s) can be integrated out with only small corrections to the mass spectrum that results from the ordinary MSSM mass matrix of the form

$$\begin{pmatrix} \tilde{H}_d^- & \tilde{W}_{2,L}^- \end{pmatrix} \begin{pmatrix} \mu_{\rm eff} & \frac{g_2}{\sqrt{2}} s_\beta v \\ \frac{g_2}{\sqrt{2}} c_\beta v & M_2 \end{pmatrix} \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{W}_{2,L}^+ \end{pmatrix} \tag{4.16}$$

with the elementary gaugino mass playing approximately the role of the MSSM M_2 parameter. In some regions of parameter space the extra mixing can change the chargino spectrum but we will not consider that case here. Thus to leading order it is the elementary gaugino that will be lighter, due to the large coupling of the composite gaugino. When gauginos are light, it is as usual only because of the suppression by the small SM gauge couplings.

Similarly the neutralino mass matrix reduces to the NMSSM form after integrating out the heavy neutral fermions corresponding to the composite neutral gauginos:

$$\begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W & 0 \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z c_W s_\beta & 0 \\ -M_Z c_\beta s_W & M_Z s_\beta s_W & 0 & -\mu_{\rm eff} & -yvs_\beta \\ M_Z c_\beta c_W & -M_Z c_W s_\beta & -\mu_{\rm eff} & 0 & -yvc_\beta \\ 0 & 0 & -yvs_\beta & -yvc_\beta & M_{Sf} \end{pmatrix}, \tag{4.17}$$

where $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, where again $M_{1,2}$ are approximately given by the elementary gaugino masses, and we have also included a soft breaking Majorana mass for the singlino. All other fields either correspond to elementary fields with large SUSY breaking terms, or are vector-like and also assumed to have large masses. This way we obtain the particle spectrum we will be investigating in the next section: the lightest $\tilde{t}$ within a few hundred GeV of the top mass, heavier $\tilde{t}$ and lighter $\tilde{b}$ below 500 GeV, neutralinos and charginos and the full scalar Higgs sector below a TeV, while all other particles are above one TeV.

V. PHENOMENOLOGY OF A LIGHT COMPOSITE STOP

Finally we discuss the phenomenology of composite supersymmetric models with light $\tilde{t}$'s. We restrict our analysis to regions of parameter space for which the lighter $\tilde{t}$ (which is mostly the right handed $\tilde{t}$) is within a few hundred GeV of the top. We examine four different spectra in order to display a variety of phenomenological possibilities.

The NLSP will be either the $\tilde{t}$ or the lightest neutralino, N_1. The first two spectra have $\tilde{t}$ NLSP. The two spectra are distinguished by the degree of degeneracy of the t and right-handed $\tilde{t}$. In the first, the $\tilde{t}$ NLSP is nearly degenerate with the t, generating a stealth stop spectrum, while the $\tilde{t}$ is a bit heavier for the second parameter set. The third spectrum has a neutralino NLSP and supersymmetry breaking arises from standard gauge mediation. The last spectrum has a neutralino (N)LSP, but the dominant contribution to the soft mass parameters is assumed to be the radiative contributions, and not the power suppressed corrections. In other words, this model assumes a relatively high compositeness scale.

When the NLSP is the $\tilde{t}$, it will decay to t plus gravitino. If it is the N_1, then the $\tilde{t}$ will decay (depending on kinematics) either to $t + N_1$ or bottom plus chargino ($b + C^-$), while the N_1 will decay to photon plus gravitino or Higgs$/Z+$ gravitino. Alternatively the N_1 may be the LSP itself, with higher-scale SUSY breaking and heavier gravitino. In either case there will be missing energy signals from neutralino production.

For the spectra where the $\tilde{t}$ is lighter than the N_1, we assume a low-scale for supersymmetry breaking $\sqrt{F} \leq 10^{10}$ GeV, implying an LSP gravitino mass of a few GeV or less. As long as the mediation scale $M_{\rm SUSY}$ is well above the duality scale $\Lambda \sim$ 5–10 TeV the assumption that supersymmetry breaking must be fed through the duality applies.

The viability of a t-$\tilde{t}$ sector with a $\tilde{t}$ NLSP decaying via the gravitino has recently been investigated in detail by Kats and Shih in Ref. [25] using Tevatron and first-year LHC data (35 pb^{-1}). They found that using searches based on these data sets that the data on the lightest $\tilde{t}$ mass decaying to t plus gravitino sets a bound of about $\tilde{t}$ mass of about 150 GeV, and that bounds of about 180 GeV are expected using 3 fb^{-1} data. If the lightest $\tilde{t}$ mass is almost degenerate with the top, then there will not be much missing energy in the decays leading to the stealth supersymmetry scenario mentioned in Ref. [26]. The most recent papers [3–6] on light third-generation bounds from 1 fb^{-1} of LHC data have also considered the possibility of the lightest $\tilde{t}$ decaying to top plus gravitino. They have found (in agreement with Ref. [25]) that currently there is no bound [5] over 200 GeV for such a $\tilde{t}$.

These most recent analyses [3–6] have also examined bounds on the heavier $\tilde{t}$ and the left-handed $\tilde{b}$. These are assumed to decay to neutralinos/charginos, and for decays of this type the currents bounds are found to be around 270 GeV. We take it as an indication that left-handed $\tilde{t}$'s and $\tilde{b}$'s of order 300 GeV are experimentally viable, even though in some of the spectra presented here the leading decays of the heavier $\tilde{t}$, $\tilde{b}$ will actually involve the lighter $\tilde{t}$.

We now discuss our choice of input parameters that correspond to these spectra. When minimizing the Higgs potential (4.1), we impose the EWSB vacuum with the correct value of v and a fixed $\tan\beta$, with an appropriate choice of the scalar tadpole f. This fixes the values of the Higgs soft breaking terms $m^2_{H_{u,d}}$, which will not be treated as inputs. We do however check that these terms have the correct magnitudes presented in the previous section. The other relevant input parameters to fix are the composite soft breaking masses $m_{Q_{33}}$, $m_{u_{33}}$, $M_{1,2}$ and M_S. As discussed before, a Majorana mass for the singlet fermion M_{S_f} may also be present, and in the second spectrum we add a term that raises the neutralino mass. In all other spectra this term is set to zero. The A-terms for the SH_uH_d and the t Yukawa interaction originate from the same dynamical term and are thus assumed to be equal. Finally we need to assign the soft breaking scalar tadpole T. All other soft breaking masses are assumed to be above one TeV, ensuring that the rest of the sparticle spectrum is essentially decoupled due to them being elementary degrees of freedom. Elementary Higgses responsible for generating the Yukawa couplings for the elementary fields are assumed to be heavy and integrated out for the purposes of this paper, but it could be interesting to investigate a theory with the elementary Higgses included as light fields as well.

The input parameters for the four benchmark spectra are given in Table I. Minimizing (4.1) and imposing the correct electroweak symmetry breaking VEV's fixes $\mu_{\rm eff}$, $m^2_{H_u}$, $m^2_{H_d}$; the corresponding values are given in Table II. The first two spectra we examine have $\tilde{t}$ NLSP's, while the second two have neutralino NLSP/LSP's. The singlino mass is set to zero in all but the second spectrum, where it is used to raise the lightest neutralino mass above the $\tilde{t}$ mass. The first spectrum has the lightest $\tilde{t}$ almost degenerate with the t, and is thus more "stealthy," while the second one has heavier $\tilde{t}$'s with it still being the NLSP. The third spectrum implements minimal gauge mediation to the electric degrees of freedom: the ratio of gaugino masses here is given by the coupling constant squares (with the gluino at 1 TeV), and the other soft breaking masses for the composites taken equal. The fourth spectrum was chosen such that the soft breaking Higgs masses are rather small so this scenario could correspond to a high duality scale with radiatively generated $\tilde{t}$ and $\tilde{b}$ masses. While we are assuming some form of low-scale supersymmetry breaking in all but one of the spectra, only the third one corresponds to minimal gauge mediation. In the minimal case the gaugino mass ratios are determined by the SM gauge couplings, and the upper bound on the gluino mass implies a fairly light

TABLE I. Input parameters for the four sample spectra. In spectrum 1, the $\tilde{t}$ is the NLSP and very degenerate with the top, generating a stealth stop spectrum. In spectrum 2, the $\tilde{t}$ is the NLSP but is a bit heavier. Spectrum 3 has a neutralino NLSP and is generated through a gauge mediated spectrum. Spectrum 4 has a neutralino (N)LSP, and the compositeness scale is assumed high enough that radiative corrections to soft composite superpartners dominate.

Parameter	Spectrum 1	Spectrum 2	Spectrum 3	Spectrum 4
$\tan\beta$	0.85	1.3	1.0	0.97
A	300 GeV	540 GeV	350 GeV	400 GeV
T	4×10^7 GeV^3	4×10^7 GeV^3	3.35×10^7 GeV^3	6×10^6 GeV^3
mQ_{33}	500 GeV	500 GeV	350 GeV	400 GeV
mU_{33}	250 GeV	350 GeV	350 GeV	400 GeV
M_1	600 GeV	700 GeV	85 GeV	600 GeV
M_2	800 GeV	800 GeV	282 GeV	1200 GeV
mS	400 GeV	350 GeV	350 GeV	100 GeV
M_{Sf}	0 GeV	−350 GeV	0 GeV	0 GeV
f	100 GeV	100 GeV	293 GeV	100 GeV

TABLE II. Output parameters for the four benchmark spectra.

Parameter	Spectrum 1	Spectrum 2	Spectrum 3	Spectrum 4
μ_{eff}	−416 GeV	−639 GeV	−422 GeV	−342 GeV
$m^2_{H_u}$	$-(176\ \text{GeV})^2$	$-(244\ \text{GeV})^2$	$(350\ \text{GeV})^2$	$(40.3\ \text{GeV})^2$
$m^2_{H_d}$	$-(218\ \text{GeV})^2$	$(207\ \text{GeV})^2$	$(350\ \text{GeV})^2$	$-(46.6\ \text{GeV})^2$

bino below 100 GeV and thus a neutralino LSP (unless the a large contribution to the singlino mass is present). The cases with heavier gaugino masses (and $\tilde{t}$ NLSP's) can be thought of as cases corresponding to a general gauge mediated spectrum [32] to the electric degrees of freedom.

We have chosen the parameters of all four spectra such that the lightest Higgs mass is around 125 GeV. This is not a necessity dictated by the model, and one can easily obtain spectra with heavier Higgses. We also made sure that for these points we are sufficiently close to the decoupling limit, such that Higgs production and decay rates are not too far from the corresponding SM values. Note that choosing the input parameters given above does not involve any extensive tuning: no automated scans had to be performed for finding these points.

In order to calculate the spectrum and widths we have modified the NMSSMTools [33,34] package, which deals with the Z_3 symmetric NMSSM. The modified package (MCSSMTools) [35] handles the minimal composite supersymmetric standard model considered here, where a linear superpotential term, tadpole soft breaking term, and a singlino mass are also allowed.

The mass spectra are presented graphically in Fig. 1 (benchmark spectra 1 and 2 with $\tilde{t}$ NLSP's) and Fig. 2 (benchmark spectra 3 and 4 with neutralino NLSP/LSP's). The numerical values for the masses for spectra 1 and 2 are presented in Table III, while the leading decay modes are in Table IV. The physical masses for spectra 3 and 4 are in Table V, with decay modes in Table VI. The spectrum and decay chains can be interactively visualized online at Ref. [36]. Table VII contains the couplings of the lightest Higgs relative to their SM values. One can see that we are close to the decoupling limit in each case: gluon couplings are within 65–83% of the SM values, while the photon coupling varies between 85–102% of the SM size for the same Higgs mass.

A. Spectrum 1: stealth stop NLSP

The first spectrum corresponds to a stealthy $\tilde{t}$ scenario [26], with the $\tilde{t}_1$ almost degenerate with the t. The largest LHC SUSY production process in this scenario is $pp \to \tilde{t}_1\tilde{t}_1^*$ production, which is about 12% of the $t\bar{t}$ production cross section [37] at the 7 TeV LHC. For a small enough gravitino mass, the $\tilde{t}_1$ decays promptly with very little missing transverse energy. The $\tilde{t}_1$ can only be uncovered by a precise measurement of the $t\bar{t}$ cross section or a shape analysis of the invariant mass distribution of $t\bar{t}$ pairs.

The next largest SUSY production process are $pp \to \tilde{b}_1\tilde{b}_1^*$ and $pp \to \tilde{t}_2\tilde{t}_2^*$ which are about few $\times$ 10 fb, of the order of 0.1% of the $t\bar{t}$ production cross section [37] at the 7 TeV LHC. The experimental bounds on $\tilde{t}_2$, $\tilde{b}_1$ are around 270 GeV if decaying to N_1, C_1, while the bound from decays to light gravitinos/binos can be as high as 350 GeV [5]. The $\tilde{b}_1$ decays to $\tilde{t}_1 W$, giving rise to $t\bar{t}WW$ final states and, in principle, missing energy. The N_1 decays to $t\tilde{t}_1^*$, and the off-shell $\tilde{t}$'s will further decay to off-shell t's. The final state for a pair production of $\tilde{t}_2$ will

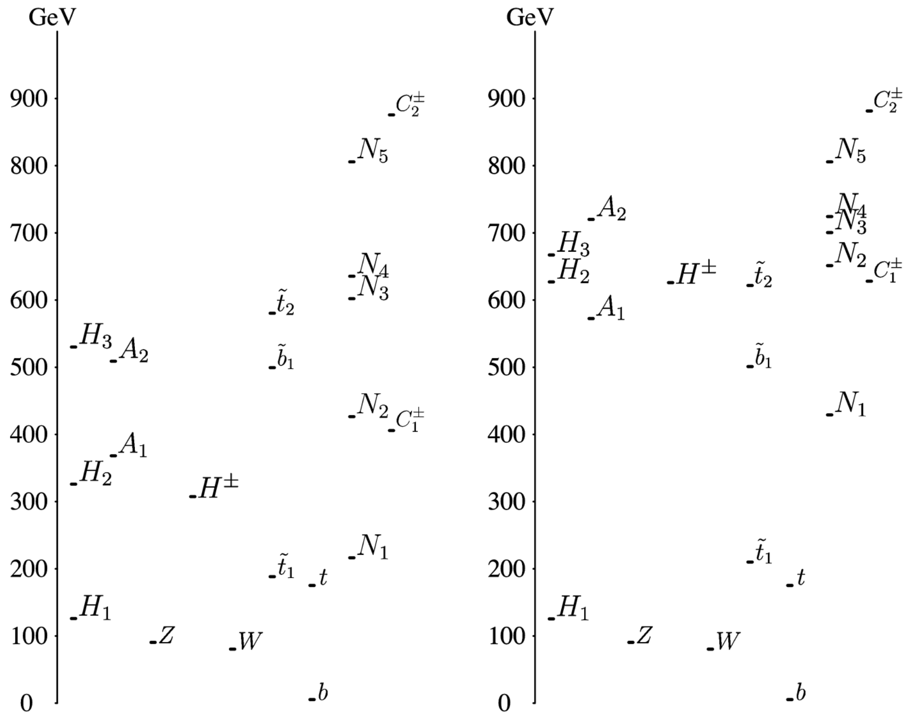

FIG. 1. Light superpartners and Higgs particles for benchmark spectra 1 and 2 with a $\tilde{t}$ NLSP.

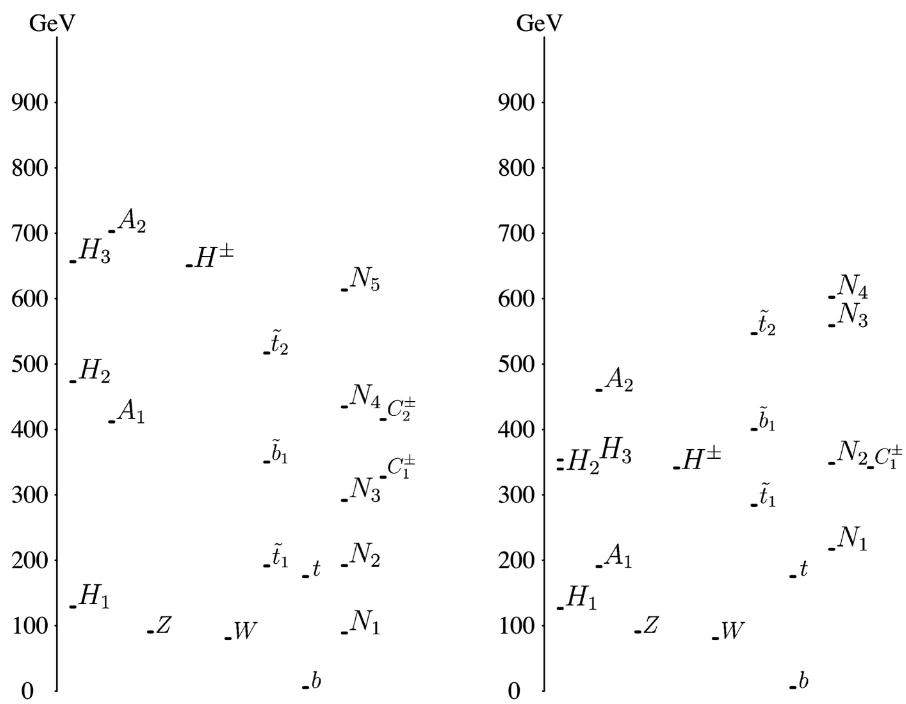

FIG. 2. Light superpartners and Higgs particles for benchmark spectra 3 and 4.

then contain *ttbb* plus the decay products of two off-shell W's. The $\tilde{b}$ decays would be the best channels for looking for this spectrum. However all of these events will have very little missing transverse energy. In the rest frame the gravitino will carry only a little energy. Even though the lightest $\tilde{t}$ will be boosted, boost factors of order a few will generically not bring the missing energy above the standard cuts. As seen in Table IV, almost all superpartner decay chains end in the NLSP, the $\tilde{t}_1$, which decays to t and a soft gravitino. The $\tilde{t}$ lifetime is [38]

TABLE III. Light superpartners and Higgs particles for benchmark spectra 1 and 2 with a $\tilde{t}$ NLSP. All other superpartners are above 1 TeV.

H_1	125 GeV	$\tilde{b}_1$	499 GeV
$\tilde{t}_1$	188 GeV	A_2	509 GeV
N_1	216 GeV	H_3	530 GeV
$H^\pm$	307 GeV	$\tilde{t}_2$	580 GeV
H_2	326 GeV	N_3	602 GeV
A_1	368 GeV	N_4	635 GeV
C_1	406 GeV	N_5	805 GeV
N_2	426 GeV	C_2	876 GeV
H_1	125 GeV	C_1	628 GeV
$\tilde{t}_1$	210 GeV	N_2	651 GeV
N_1	429 GeV	H_3	667 GeV
$\tilde{b}_1$	501 GeV	N_3	700 GeV
A_1	572 GeV	A_2	720 GeV
$\tilde{t}_2$	621 GeV	N_4	724 GeV
$H^\pm$	626 GeV	N_5	806 GeV
H_2	627 GeV	C_2	881 GeV

TABLE IV. Branching fractions for benchmark spectra 1 and 2 with a $\tilde{t}$ NLSP.

$\tilde{t}_2 \to t + LSP$	100%
$C_1 \to \tilde{t}_1 + b^\dagger$	84%
$C_1 \to N_1 + W^\pm$	16%
$\tilde{b}_1 \to \tilde{t}_1 + W^-$	97%
$\tilde{b}_1 \to \tilde{t}_1 + H^-$	3%
$\tilde{t}_2 \to \tilde{t}_1 + Z$	51%
$\tilde{t}_2 \to t + N_1$	27%
$\tilde{t}_2 \to b + C_1^+$	11%
$\tilde{t}_2 \to \tilde{t}_1 + H_1$	10%
$\tilde{t}_1 \to t + LSP$	100%
$N_1 \to t + \tilde{t}^*$	50%
$N_1 \to \bar{t} + \tilde{t}$	50%
$\tilde{b}_1 \to \tilde{t}_1 + W^-$	100%
$\tilde{t}_2 \to \tilde{t}_1 + Z$	78%
$\tilde{t}_2 \to \tilde{b}_1 + W^+$	14%
$\tilde{t}_2 \to \tilde{t}_1 + H_1$	8%

$$\Gamma = \frac{m_{\tilde{t}}^5}{16\pi F^2}\left(1 - \frac{m_t^2}{m_{\tilde{t}}^2}\right)^4. \qquad (5.1)$$

For $m_{\tilde{t}} < 200$ GeV, a prompt decay requires $\sqrt{F}$ less than 50 TeV, in which case there is no easy way to find a SUSY signal [25] from this mode. For bigger values of F there will be displaced vertices involving t quarks.

B. Spectrum 2: stop NLSP with heavier N_1

The phenomenology with the second set of input parameters is fairly similar with a slightly heavier $\tilde{t}$. The main difference is that we no longer have a stealth spectrum, N_1 is quite a bit heavier, and more of the spectrum is pushed above a TeV. Because of the heavier N_1 mass, it can now decay on shell to $t + \tilde{t}$, giving rise to events with $t\bar{t}t\bar{t}$ in addition to the $t\bar{t}WW$ states from the $\tilde{b}$ decays. Since the $\tilde{t}_1$ is still quite light, the amount of missing energy in these

TABLE V. Benchmark spectra 3 and 4.

N_1	88 GeV	C_2	415 GeV
H_1	128 GeV	N_4	434 GeV
$\tilde{t}_1$	191 GeV	H_2	473 GeV
N_2	192 GeV	$\tilde{t}_2$	517 GeV
N_3	291 GeV	N_5	613 GeV
C_1	327 GeV	$H^\pm$	650 GeV
$\tilde{b}_1$	350 GeV	H_3	657 GeV
A_1	412 GeV	A_2	702 GeV
H_1	126 GeV	N_2	348 GeV
A_1	190 GeV	H_3	353 GeV
N_1	217 GeV	$\tilde{b}_1$	400 GeV
$\tilde{t}_1$	284 GeV	A_2	460 GeV
H_2	339 GeV	$\tilde{t}_2$	546 GeV
$H^\pm$	341 GeV	N_3	559 GeV
C_1	341 GeV	N_4	602 GeV

TABLE VI. Branching fractions for benchmark spectra 3 and 4.

$\tilde{t}_1 \to N_1^+ b + W^+$	100%
$\tilde{b}_1 \to N_3 + b$	80%
$\tilde{b}_1 \to \tilde{t}_1 + W^-$	95%
$\tilde{b}_1 \to N_3 + b$	80%
$\tilde{b}_1 \to \tilde{t}_1 + W^-$	95%
$\tilde{b}_1 \to N_3 + b$	4%
$\tilde{b}_1 \to N_1 + b$	1%
$\tilde{t}_2 \to \tilde{t}_1 + b$	42%
$\tilde{t}_2 \to \tilde{b}_1 + W^+$	31%
$\tilde{t}_2 \to N_2 + t$	10%
$\tilde{t}_2 \to C_2^+ + b$	8%
$\tilde{t}_2 \to N_1 + t$	4%
$\tilde{t}_2 \to C_1^+ + b$	3%
$\tilde{t}_2 \to N_3 + t$	2%
$\tilde{t}_1 \to N_1 + c$	99%
$\tilde{t}_1 \to N_1 + u$	1%
$\tilde{b}_1 \to \tilde{t}_1 + W^-$	100%
$\tilde{t}_2 \to \tilde{t}_1 + Z$	28%
$\tilde{t}_2 \to C_1^+ + b$	24%
$\tilde{t}_2 \to \tilde{b}_1 + W^+$	20%
$\tilde{t}_2 \to N_2 + t$	15%
$\tilde{t}_2 \to N_2 + t$	14%

TABLE VII. Ratio of Higgs couplings to SM Higgs couplings for the same mass for the four benchmark spectra to various SM fields.

SM fields	Spectrum 1	Spectrum 2	Spectrum 3	Spectrum 4
$\gamma\gamma$	1.02	1.02	0.95	0.85
gluons	0.65	0.83	0.82	0.73
WW, ZZ	0.89	0.96	0.89	0.74
$u\bar{u}$	0.72	1.0	0.89	0.72
$d\bar{d}$	1.01	0.91	0.89	0.77

decays will still be limited. The $\tilde{t}_2$ will mainly decay to $Z + \tilde{t}_1$ giving rise to $t\bar{t}ZZ$ final states.

C. Spectrum 3: minimal gauge mediation

The third and fourth spectra both have neutralino (N) LSP's, thus the traditional missing energy signals of supersymmetry are expected. However, due to the heavy gluino and first two generations squarks, the rates are strongly reduced from those of the constrained MSSM. These spectra fall in the class of models considered in Ref. [5].

The third set of input parameters in particular represent a minimal gauge-mediated spectrum to the electric degrees of freedom. All the soft scalar masses are set equal to 350 GeV. Thus fixing $m^2_{H_u} = m^2_{H_d} = (350\ \text{GeV})^2$ means that f is no longer really an input parameter but is an output of fixing the right EWSB vacuum. Since we are considering gauge mediation, the expectation is that the LSP is again the gravitino, and the NLSP N_1 decays to photon plus gravitino. The lightest $\tilde{t}$ decays to $t^* N_1$, while the heavier $\tilde{t}$ has again many possible decay channels including $\tilde{t}_1 Z$, $\tilde{b}W$, $N_{1,2,3}t$, $C_{1,2}b$, while the sbottom again decays to $\tilde{t}W$. Depending on the N_1 lifetime, the final states will again either be j + MET, jt + MET, and $j + W/Z$ + MET, or the same final states with additional photons. This spectrum will also produce some longer SUSY cascades involving the same final states.

D. Spectrum 4: high duality scale

The fourth spectrum was chosen such that it can correspond to a higher duality scale, where the squark masses are mainly radiatively induced from the elementary gluino (and not coming from power-suppressed terms), while the other composite soft masses are small. In this case Higgs naturalness is especially good, since the Higgs soft breaking terms needed are around $(50\ \text{GeV})^2$. Third-generation squarks are in the 300–500 GeV range. The lightest $\tilde{t}$ decays via $\tilde{t}_1 \to N_1 c$, while the second $\tilde{t}$ has many possible decay modes to final states $\tilde{t}_1 Z$, $C_1^+ b$, $\tilde{b}W$ and $N_{1,2}t$. The sbottom decay is $\tilde{b}_1 \to \tilde{t}_1 W$. The characteristic final states will be j + MET, jt + MET, or jW/Z + MET events. This yields fairly traditional SUSY signals at reduced rate and no leptons (except from W and Z's).

VI. CONCLUSIONS

We have seen that by combining supersymmetry, which makes the theory calculable but also the Higgs too light and/or fine-tuned, with compositeness, which requires strong coupling and allows for a heavier Higgs with large dynamical Yukawa couplings to other composites, we can address three hierarchies: the hierarchy in Yukawa couplings, the little hierarchy problem, and the apparent hierarchy in squark soft masses. The strong dynamics determines which particles have significant coupling to the composite Higgs and can force the composite superpartners that are thus required for naturalness to be much lighter than the elementary superpartners.

In the model presented here Seiberg duality provides the crucial ingredient for resolving these hierarchies. The lessons could apply more generally but with Seiberg duality, we can explicitly determine the hierarchies in the spectrum of composite superpartners. The models we presented produce a composite Higgs, t and LH b along with partially composite W and Z. The low-energy dynamics is that of the NMSSM with a composite singlet, where the singlet couplings equal the t Yukawa coupling. This ensures that the Higgs can be sufficiently heavy. The flavor problem is addressed via the large dynamical top Yukawa, and the little hierarchy via the NMSSM-type singlet coupling that determines the effective μ-parameter and is related to the top Yukawa. The strong dynamics at the edge or just inside the conformal window will strongly suppress the soft breaking terms for the composites. This gives the necessary hierarchy among the squark masses, that will strongly reduce the SUSY production rates at the LHC and allow for a natural SUSY EWSB sector.

We have presented four distinct mass spectra corresponding to explicit implementations of this model. Two of them have the $\tilde{t}$ as the NLSP (with gravitino LSP's), while the other two have the N_1 as the (N)LSP. One of the spectra with a $\tilde{t}$ NLSP correspond to an explicit implementation of a stealthy stop, where most of the SUSY events would not contain much missing energy.

Although conventional supersymmetric models are being challenged by experiments and naturalness at this point, this model raises the hope that models with more subtle composite dynamics could in fact be the correct theory of nature.

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[1] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Rev. Lett. **107**, 221804 (2011).
[2] G. Aad, Phys. Lett. B **710**, 67 (2012).
[3] Y. Kats, P. Meade, M. Reece, and D. Shih, J. High Energy Phys. 02 (2012) 115.
[4] C. Brust, A. Katz, S. Lawrence, and R. Sundrum, J. High Energy Phys. 03 (2012) 103.
[5] M. Papucci, J. T. Ruderman, and A. Weiler, arXiv:1110.6926.
[6] R. Essig, E. Izaguirre, J. Kaplan, and J. G. Wacker, J. High Energy Phys. 01 (2012) 074.
[7] L. F. Abbott and E. Farhi, Phys. Lett. **101B**, 69 (1981).
[8] L. Randall and R. Sundrum, arXiv:9905221.
[9] M. A. Luty, J. Terning, and A. K. Grant, Phys. Rev. D **63**, 075001 (2001).
[10] R. Harnik, G. D. Kribs, D. T. Larson, and H. Murayama, Phys. Rev. D **70**, 015002 (2004).
[11] C. Csaki, Y. Shirman, and J. Terning, Phys. Rev. D **84**, 095011 (2011).
[12] N. Craig, D. Stolarski, and J. Thaler, J. High Energy Phys. 11 (2011) 145.
[13] N. Arkani-Hamed, M. A. Luty, and J. Terning, Phys. Rev. D **58**, 015004 (1998); M. A. Luty and J. Terning, Phys. Rev. D **62**, 075006 (2000); J. Terning and M. A. Luty, arXiv:9903393.
[14] H.-C. Cheng and Y. Shadmi, Nucl. Phys. **B531**, 125 (1998).
[15] N. Arkani-Hamed, G. F. Giudice, M. A. Luty, and R. Rattazzi, Phys. Rev. D **58**, 115005 (1998).
[16] N. Arkani-Hamed and R. Rattazzi, Phys. Lett. B **454**, 290 (1999).
[17] M. A. Luty and R. Rattazzi, J. High Energy Phys. 11 (1999) 001.
[18] S. Abel, M. Buican, and Z. Komargodski, Phys. Rev. D **84**, 045005 (2011).
[19] ATLAS Collaboration, Report No. ATLAS-CONF-2011-163.
[20] CMS Collaboration, Report No. CMS-PAS-HIG-11-032.
[21] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys. Lett. B **388**, 588 (1996).
[22] T. Gherghetta and A. Pomarol, Phys. Rev. D **67**, 085018 (2003).
[23] R. Sundrum, J. High Energy Phys. 01 (2011) 062.
[24] S. Abel and T. Gherghetta, J. High Energy Phys. 12 (2010) 091.
[25] Y. Kats and D. Shih, J. High Energy Phys. 08 (2011) 049.
[26] J. Fan, M. Reece, and J. T. Ruderman, J. High Energy Phys. 11 (2011) 012.
[27] A. Pomarol, Phys. Lett. B **486**, 153 (2000); H. Davoudiasl, J. L. Hewett, and T. G. Rizzo, Phys. Lett. B **473**, 43 (2000).
[28] M. Buican, arXiv:1206.3033.
[29] N. Seiberg, Nucl. Phys. **B435**, 129 (1995).
[30] M. Reece (private communication).
[31] G. F. Giudice, M. Nardecchia, and A. Romanino, Nucl. Phys. **B813**, 156 (2009).
[32] P. Meade, N. Seiberg, and D. Shih, Prog. Theor. Phys. Suppl. **177**, 143 (2009).
[33] NMSSMTools, http://www.th.u-psud.fr/NMHDECAY/nmssmtools.html.
[34] U. Ellwanger and C. Hugonie, Comput. Phys. Commun. **175**, 290 (2006); U. Ellwanger, J. F. Gunion, and C. Hugonie, J. High Energy Phys. 02 (2005) 066; U. Ellwanger and C. Hugonie, Comput. Phys. Commun. **177**, 399 (2007); A. Djouadi, M. Drees, U. Ellwanger, R. Godbole, C. Hugonie, S. F. King, S. Lehti, S. Moretti *et al.*, J. High Energy Phys. 07 (2008) 002.
[35] MCSSMTools, https://github.com/jterning/MCSSMTools.
[36] http://bit.ly/mcspect.
[37] W. Beenakker, S. Brensing, M. Kramer, A. Kulesza, E. Laenen, and I. Niessen, J. High Energy Phys. 08 (2010) 098.
[38] U. Sarid and S. D. Thomas, Phys. Rev. Lett. **85**, 1178 (2000).