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Dominant spin-orbit effects in radiative decays $\Upsilon(3S) \to \gamma + \chi_{bJ}(1P)$

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We show that there are two reasons why the partial width for the transition $\Gamma_1(\Upsilon(3S) \to \gamma + \chi_{b1}(1P))$ is suppressed. First, the spin-averaged matrix element $\overline{I(3S|r|1P_J)}$ is small, being equal to 0.023 GeV^{-1} in our relativistic calculations. Secondly, the spin-orbit splittings produce relatively large contributions, giving $I(3S|r|1P_2) = 0.066$ GeV^{-1}, while due to a large cancellation the matrix element $I(3S|r|1P_1) = -0.020$ GeV^{-1} is small and negative; at the same time the magnitude of $I(3S|r|1P_0) = -0.063$ GeV^{-1} is relatively large. These matrix elements give rise to the following partial widths: $\Gamma_2(\Upsilon(3S) \to \gamma + \chi_{b2}(1P)) = 212$ eV, $\Gamma_0(\Upsilon(3S) \to \gamma + \chi_{b0}(1P)) = 54$ eV, which are in good agreement with the CLEO and *BABAR* data, and also to $\Gamma_1(\Upsilon(3S) \to \gamma + \chi_{b1}(1P)) = 13$ eV, which satisfies the *BABAR* limit, $\Gamma_1(\mathrm{exp}) < 22$ eV.

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I. INTRODUCTION

In recent years, several new bottomonium states were discovered due to studies of radiative decays [1–4]. In [1] CLEO has observed the $\Upsilon(1D)$ in the four-photon decay cascade, $\Upsilon(3S) \to \gamma + \chi_b(2P)$, $\chi_b(2P) \to \gamma + \Upsilon(1D)$, $\Upsilon(1D) \to \gamma + \chi_b(1P)$, $\chi_b(1P) \to \gamma + \Upsilon(1S)$, and later this state was observed by *BABAR* in another four-photon cascade [2]. In 2008 a new state, $\eta_b(1P)$, was discovered by *BABAR*, first in radiative decay $\Upsilon(3S) \to \gamma + \eta_b(1S)$ [3] and then in $\Upsilon(2S) \to \gamma + \eta_b(1S)$ [4]; later $\eta_b(1S)$ was confirmed by CLEO [5]. Moreover, new or more precise data on different radiative transitions, like $\Upsilon(3S) \to \gamma + \chi_b(n^3P_J)$ $(n = 1, 2)$, $\chi_b(1P, 2P) \to \gamma + \Upsilon(1S)$, and $\chi_b(2P) \to \gamma + \Upsilon(2S)$, were presented in Refs. [6–9].

This new experimental information is of a special importance for the theory to provide a better understanding of the role of relativistic and spin-dependent effects in bottomonium, and may be used as a test of different models and approximations. There are a large number of papers devoted to radiative decays in bottomonium [10–14], and a comparison of different results was already presented in Refs. [12–14], where the predicted partial widths are shown to be rather close to each other for most radiative E1 transitions and to agree with the existing experimental data. The only exception is the radiative decays $\Upsilon(3S) \to \gamma + \chi_b(1P_J)$ $(J = 0, 1, 2)$, which are discussed in detail in Ref. [14]. Their partial widths are defined by the matrix element (m.e.) $I(3S|r|1P_J) \equiv \langle \Upsilon(3S)|r|1^3P_J\rangle$ $(J = 0, 1, 2)$ and below we shall also use the spin-averaged m.e., denoted as $\overline{I(3S|r|1P)}$.

These m.e. strongly differ in the nonrelativistic (NR) and relativistic cases, even within the same model. The predicted transition rate $\Gamma_1(\Upsilon(3S) \to \gamma + \chi_{b1}(1P))$ varies in a wide range, (3–110) eV [14] and is in many cases larger than the experimental width: $\Gamma_1 = (33 \pm 10)$ eV from the CLEO data [8]; a smaller value $\Gamma_1 = (10^{+8}_{-6})$ eV was measured by *BABAR* [9]. Moreover, even in the models which predict a small partial width Γ_1, their other two rates, $\Gamma_J(J = 0, 2)$, do not agree with the experimental values [15]. Therefore, the ratio of the transition rates, $r_{1,0} = \frac{\Gamma_1(\Upsilon(3S)\to\gamma+\chi_{b1}(1P))}{\Gamma_0(\Upsilon(3S)\to\gamma+\chi_{b0}(1P))}$, must be considered an important characteristic, which is small in experiments: $r_{1,0} \sim 0.5$ from the CLEO [8] and $r_{1,0} \sim 0.2$ from the *BABAR* data [9].

The m.e. $I(3S|r|1P_J)$ may differ several times in NR and relativistic calculations, even within the same model or while different static potentials are used [11,16]. In Ref. [17] the suppression of this m.e. was shown to be quite strong in the NR limit for the power-law potentials $V(r) \sim r^\alpha$ with $-1 < \alpha < 2$. Since in bottomonium, even for $\Upsilon(3S)$, the relativistic corrections are not large, $\frac{\boldsymbol{p}^2}{m_b^2} \lesssim 0.1$, one may assume that this fact occurs because of the different asymptotics of the wave function (w.f.) of the Schrödinger and relativistic equations.

An interesting result was obtained in Ref. [16], where for the NR Hamiltonian the partial width $\Gamma_2 = \Gamma(\Upsilon(3S) \to \gamma + \chi_{b2}(1P))$ decreases ten times, if instead of the Cornell potential with $\alpha(\mathrm{static}) = \mathrm{constant}$, the Wisconsin potential which takes into account the asymptotic freedom behavior of the vector strong coupling, is used. This result reminds of the situation with the dielectron widths of $\Upsilon(nS)$ $(n = 1, 2, 3)$, where agreement with the experiment is reached only for the potential with the asymptotic freedom behavior of the strong coupling [18].

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However, even for this kind of potentials the spin-averaged m.e. $\overline{I(3S|r|1P)}$ appears to depend on the freezing (critical) value of the vector strong coupling used. In this paper we consider gluon-exchange (GE) potentials with two different values of $\alpha_{\rm crit}$.

It is also evident that since the m.e. $\overline{I(3S|r|1P_J)}$ is small, it may strongly depend on other small effects, in particular, on the spin-orbit interaction used. Here we show that due to the spin-orbit splittings the m.e. $I(3S|r|1P_J)$ acquire corrections of the same order as the value of the spin-averaged m.e. $\overline{I(3S|r|1P)}$, and a large cancellation takes place in the m.e. with $J = 1$. Here in our calculations we use the relativistic string Hamiltonian (RSH) [19], which was already tested in a number of papers, devoted to different bottomonium properties [20].

II. RADIATIVE DECAYS

Electric dipole transitions between an initial state $i = 3^3S_1$, and a final state $f = 1^3P_J$, are defined by the partial width [10–14],

$$\Gamma(i \xrightarrow{\rm E1} \gamma + f) = \frac{4}{3}\alpha e_Q^2 E_\gamma^3 (2J' + 1) S_{if}^{\rm E} |\mathcal{E}_{if}|^2, \tag{1}$$

where $J' = J_f$, $l' = l_f$, and the statistical factor $S_{if}^{\rm E} = S_{fi}^{\rm E}$ is given by

$$S_{if}^{\rm E} = \max(l, l') \begin{Bmatrix} J & 1 & J' \\ l' & s & l \end{Bmatrix}^2 \tag{2}$$

and for the transitions between the n^3S_1 and m^3P_J states with the same spin $S = 1$ this coefficient $S_{if}^{E} = 1/9$. In Eq. (1) $e_{\rm Q}$ is the electric charge of the heavy quark Q ($e_b = -1/3$); α is the fine structure constant; $E_\gamma = \frac{M_i^2 - M_f^2}{2M_i}$ is the photon energy, where M_i and M_f are the masses of the initial- and final-quarkonium states. The value $\mathcal{E}_{if}$ is the electric overlap integral,

$$\mathcal{E}_{if} = \int_0^\infty dr \varphi_{nl}(r) r \varphi_{n'l'}(r), \tag{3}$$

where $\varphi_{nl}(r)(\varphi_{n'l'}(r))$ is the radial w.f. of the initial (final) state. Our definition of the partial width does not include possible relativistic corrections to the leading order of the multiple E_1 expansion, which are discussed in Ref. [11], and relativity is taken into account in our calculations only via the relativistic radial w.f., calculated here with the use of the RSH.

The RSH is simplified in the case of bottomonium, where in the Hamiltonian the string and self-energy corrections can be neglected because they are very small, ≤ 1 MeV. Then the original form of the RSH with the static potential

$$V_{\rm B}(r) = \sigma r - \frac{4}{3}\frac{\alpha_{\rm B}(r)}{r} \tag{4}$$

is

$$H = \frac{\boldsymbol{p}^2 + m_b^2}{\omega} + \omega + V_B(r). \tag{5}$$

Here m_b is the b-quark pole mass, while the value of ω is determined from the extremum condition $\frac{\partial H}{\partial \omega} = 0$, which gives $\omega = \sqrt{\boldsymbol{p}^2 + m_b^2}$, being equal to the kinetic energy of a b quark. Substituting this ω into Eq. (5) one arrives at the spinless Salpeter equation (SSE):

$$H_0 = 2\sqrt{\boldsymbol{p}^2 + m_b^2} + V_{\rm B}(r). \tag{6}$$

The kinetic term occurring in (6) is widely used in relativistic potential models [21–23], however, as compared to constituent potential models, the RSH has several important differences.

(1) By derivation, the mass of the b quark in the kinetic term cannot be chosen arbitrarily: it must be equal to the pole mass of a b quark, which takes into account perturbative in $\alpha_s(m_b)$ corrections. In two-loop approximations $m_b(\text{pole}) = \bar{m}_b(\bar{m}_b)(1 + 0.09 + 0.05)$ [24], where the second and third numbers come from the α_s and α_s^2 corrections, respectively. In our calculations $m_b(\text{pole}) = 4.83$ GeV is used, which corresponds to the conventional current mass $\bar{m}_b(\bar{m}_b) \simeq 4.24$ GeV.

(2) H_0, as well as the mass $M(nl)$, does not contain an overall additive (fitting) constant.

(3) The string tension $\sigma = 0.18\ \text{GeV}^2$, used in the RSH, cannot be considered a fitting parameter, because it is fixed by the slope of the Regge trajectories for light mesons.

(4) In the GE potential the asymptotic freedom behavior of the vector strong coupling $\alpha_{\rm B}(r)$ is taken into account, being expressed via the "vector" QCD constant $\Lambda_{\rm B}$, which is not a fitting parameter but defined by the conventional $\Lambda_{\overline{\rm MS}}$ according to the relation: $\Lambda_{\rm B}(n_f = 3) = 1.4753\Lambda_{\overline{\rm MS}}(n_f = 3)$ and $\Lambda_{\rm B}(n_f = 5) = 1.3656\Lambda_{\overline{\rm MS}}(n_f = 5)$ [25]. On the other hand, the value of $\Lambda_{\overline{\rm MS}}(n_f = 5)$ is fixed by the known value of $\alpha_s(M_Z)$ at the scale $M_Z = 91.19$ GeV. Here $\alpha_s(M_Z) = 0.1191$ is used, which in the two-loop approximation gives $\Lambda_{\overline{\rm MS}}(n_f = 5) = 240$ MeV and correspondingly, $\Lambda_{\rm B}(n_f = 5) \simeq 330$ MeV.

The relation between $\Lambda_{\rm B}(n_f)$ and $\Lambda_{\overline{\rm MS}}(n_f)$ used above, follows from the important relation between the strong coupling $\alpha_{\rm B}(q^2)$ in the vector space and $\alpha_{\overline{\rm MS}}(q^2)$ in the $\overline{\rm MS}$ renormalization scheme, which was derived in Ref. [25]:

$$\alpha_{\rm B}(q^2) = \alpha_{\overline{\rm MS}}(q^2)\left(1 + a_1 \frac{\alpha_{\overline{\rm MS}}(q^2)}{4\pi}\right), \tag{7}$$

where $a_1 = \frac{31}{3} - \frac{10}{9} n_f$. In order to establish the connection between $\Lambda_{\rm B}$ and $\Lambda_{\overline{\rm MS}}$ it is sufficient to consider the case of very large momentum squared q^2, where one can use the one-loop approximation for both strong couplings: $\alpha_B = \frac{4\pi}{\beta_0}\frac{1}{\log(q^2/\Lambda_B^2)}$ ($\beta_0 = 11 - \frac{2}{3}n_f$) and the analogous expression for $\alpha_{\overline{MS}}$. Then in the limit of large q^2 and introducing $\tilde{\alpha} = \frac{\alpha}{4\pi}$ one obtains the relation:

$$\tilde{\alpha}_B(q^2) = \tilde{\alpha}_{\overline{MS}}(q^2)(1 + a_1\tilde{\alpha}_{\overline{MS}}(q^2)) \approx \frac{\tilde{\alpha}_{\overline{MS}}}{1 - a_1\tilde{\alpha}_{\overline{MS}}}. \qquad (8)$$

Then from Eq. (8) in the one-loop approximation it follows that $\log(\frac{q^2}{\Lambda_B^2}) = \log(\frac{q^2}{\Lambda_{\overline{MS}}^2}) - \frac{a_1}{\beta_0}$ and the solution of this equation just gives the relation,

$$\Lambda_B = \Lambda_{\overline{MS}} \exp\left(\frac{a_1}{2\beta_0}\right). \qquad (9)$$

The values of the factor $\exp(\frac{a_1}{2\beta_0})$ grow for a smaller n_f, being equal to 1.3656 for $n_f = 5$ and 1.4753 for $n_f = 3$. Thus our scheme of calculations appears to be very restrictive in the case of bottomonium and only small variations of the fundamental parameters are admissible. However, some uncertainty comes from the value of the freezing constant, $\alpha_B(r \to \infty) \equiv \alpha_{crit}$, which properties are discussed in Ref. [26]. Here we use the vector coupling in the range $0.49 \le \alpha_{crit} \le 0.60$. Then for a given multiplet nl the centroid mass $M_{cog}(nl)$ coincides with the eigenvalue $M(nl)$ of the SSE:

$$\left[2\sqrt{\boldsymbol{p}^2 + m_b^2} + V_B(r)\right]\varphi_{nl} = M(nl)\varphi_{nl}. \qquad (10)$$

For this relativistic equation the NR limit and the so-called *einbein* approximation may also be used and in both approximations a good description of the bottomonium spectrum is obtained, even for the higher states [20]. For most radiative decays (in bottomonium) the m.e. like $I(mS|r|nP_J)$ and $I(nP_J|r|mS)$ differ only by 10%–20% between the NR and relativistic cases, with the exception of the transitions $\Upsilon(3S) \to \gamma + \chi_{bJ}(1P)$. In this case our calculations give $\overline{I(3S|r|1P)} = 0.007\ \text{GeV}^{-1}$ in the NR case, being ~3 times smaller than $\overline{I(3S|r|1P)} = 0.023\ \text{GeV}^{-1}$ for the SSE (here $\alpha_{crit} = 0.49$ was used). Notice that for a stronger GE potential with $\alpha_{crit} = 0.60$ these spin-averaged m.e. appear to be larger: $\overline{I(3S|r|1P)} = 0.011\ \text{GeV}^{-1}$ in the NR case and $0.036\ \text{GeV}^{-1}$ for the SSE.

Since the same static potential is used for the SSE as in the NR case, such a difference between the m.e. may be explained by two factors: the different asymptotic behavior of the w.f. of the SSE and Schrödinger equations, and also a smaller value of the w.f. at the origin for the Schrödinger equation as compared to that for the SSE. However, it is known that the w.f. $R_{nS}(r)$, as well as the derivative $R'_{1P}(r)$ for the $1P$ state, diverge near the origin for the SSE (these divergences are discussed in details in Ref. [16]) and the calculated values of the w.f. (or its derivative) at the origin are obtained with the use of a regularization procedure. This regularization introduces a theoretical error, which is estimated to be $\lesssim 10\%$.

TABLE I. The m.e. $I(3S|r|1P_J)$ (in GeV^{-1}) in the relativistic and NR cases.

Transition	NR	RA[a]	NR[b]	SSE
	[13]	[13]	this paper	this paper
$\langle 3S\|r\|1P_2\rangle$	0.016	0.063	0.047	0.066
$\langle 3S\|r\|1P_1\rangle$	0.011	0.063	−0.033	−0.020
$\langle 3S\|r\|1P_0\rangle$	0.004	0.063	−0.073	−0.063

[a]Given numbers refer to the variant RA [13], where a scalar linear potential is used.
[b]Given numbers refer to the NR limit of the SSE Eq. (10) with the same potential $V_B(r)$ and $\alpha_{crit} = 0.49$.

In Table I we give the m.e. $I(3S|r|1P_J)$, calculated here for the SSE and in the NR limit, together with their values from the second paper of Ref. [13] (Table 4.16) for the NR and the relativistic variant (RA), where a scalar confining potential, as in our calculations, is used.

Comparison of the m.e. presented in Table I shows the following:

(1) In Ref. [13] for the relativistic variant RA the m.e. $I(3S|r|1P_J)$ is $\lesssim 4$ times larger than in the NR case; a similar result is obtained here for the spin-averaged m.e., where $\overline{I(3S|r|1P)} = 0.023\ \text{GeV}^{-1}$ for the SSE and has an ≈ 3 times smaller value $0.007\ \text{GeV}^{-1}$ in its NR limit.
From Table I one can see that in the NR case there are large differences between our m.e. and those from Ref. [13]. We suppose that it happens because of different parameters being used in the GE potential and also different b-quark masses taken. At the same time large difference in the values of the m.e. in the NR and relativistic cases, which takes place for any models, may occur because of the different asymptotic behavior of their w.f.: in the NR case the w.f. behaves as the Airy function, while in our case the asymptotic behavior of the relativistic w.f. is close to exponential.

(2) Corrections $\delta I_{so}(J) = I(3S|r|1P_J) - \overline{I(3S|r|1P)}$, due to the spin-orbit potential, have a relatively large value, e.g., $\delta I_{so}(J = 2) = 0.043\ \text{GeV}^{-1}$, being almost two times larger than $\overline{I(3S|r|1P)}$ in the spin-averaged case [see Eq. (13) below].

(3) In Ref. [13] the splittings between the m.e. $I(3S|r|1P_J)$ with different J are much smaller than in our calculations.

(4) In the spin-orbit potential we take the strong coupling $\alpha_{so}(\mu) = 0.38$, which is close to the value $\alpha_{so}(\mu(2P))$ used for the $\chi_{bJ}(2P)$ states (this value was extracted in Ref. [23] from the experimental masses of the members of the $\chi_{bJ}(2P)$ multiplet). Our calculations here show that the nondiagonal

TABLE II. The partial widths $\Gamma(\Upsilon(3S) \to \gamma + \chi_b(1P_J))$ (in eV)

Transition	E_γ (MeV)	RA [13]	NR This paper	SSE This paper	Experiment CLEO [6]	Experiment *BABAR* [8]
$\Gamma_2(\Upsilon(3S) \to \gamma + \chi_b(1P_2))$	433.5	195	108	213	157 ± 30	216 ± 25
$\Gamma_1(\Upsilon(3S) \to \gamma + \chi_b(1P_1))$	452.1	134	36	13	33 ± 10	<22
$\Gamma_0(\Upsilon(3S) \to \gamma + \chi_b(1P_0))$	483.9	54	72	54	61 ± 23	55 ± 10

m.e., like $\langle nP|V_{so}|mP\rangle$ ($n \neq m$, $n = 1, 2, 3$), are of the same order or have even larger values than the diagonal m.e. $\langle 2P|V_{so}(r)|2P\rangle$.

The calculated $E1$ transition rates are presented in Table II together with their values from Ref. [13]; they correspond to the m.e. from Table I.

In the relativistic case our transition rates appear to be very close to those from the *BABAR* data [9]. Even in the NR case, due to large spin-orbit corrections, the calculated partial widths do not contradict the CLEO data [8].

We make some remarks on the contribution δI_{so} to the m.e. $I(3S|r|1P_J)$ from the spin-orbit potential, $\hat{V}_{so}(r) = \boldsymbol{L} \cdot \boldsymbol{S} V_{so}(r)$, for which the splittings $a_{so}(nP|1P) = \langle nP|V_{so}|1P\rangle$, ($n = 2, 3$) are taken as for the one-gluon exchange interaction, i.e., neglecting the second order corrections in $\alpha_s(\mu)$ (it may be shown that the second order corrections are negative and small, ~ -0.7 MeV). In this approximation we find

$$a_{so}(nP, 1P) = \frac{1}{2\omega_b^2}\{4\alpha_{so}\langle r^{-3}\rangle_{nP,1P} - \sigma\langle r^{-1}\rangle_{nP,1P}\}, \qquad (11)$$

where we take $\alpha_{so} = 0.38$, which provided a good description of the fine-structure splittings for the $\chi_b(2P_J)$ multiplet. To determine the corrections to the w.f. of the $\chi_{bJ}(1P)$ states, the potential $\hat{V}_{so}$ is considered as a perturbation and the following mass differences between the centroid masses are used:

$$\begin{aligned} M_{cog}(2P) - M_{cog}(1P) &= 360 \text{ MeV}, \\ M_{cog}(3P) - M_{cog}(1P) &= 640 \text{ MeV}. \end{aligned} \qquad (12)$$

Notice that the correction from the $3P$ state is not small, while the value of the centroid mass $M_{cog}(3P)$, $M(\chi_b(3P)) \simeq 10.54$ GeV, is taken from the recent ATLAS experiment [27].

For the SSE, the splittings $a_{so}(2P, 1P) = 12$ MeV and $a_{so}(3P, 1P) = 10.2$ MeV, were calculated and in the NR limit their values are $\sim$10% smaller. Then the nondiagonal m.e. $I(3S|r|1P_J)$ with the "spin-orbit" corrections can be presented (in GeV^{-1}) as

$$\begin{aligned} I(3S|r|1P_J) &= \overline{I(3S|r|1P)} + \delta I_{so}(J), \\ \delta I_{so}(J) &= 0.033\xi_J\overline{I(3S|r|2P)} + 0.016\xi_J\overline{I(3S|r|3P)}, \end{aligned} \qquad (13)$$

where $\xi_J = -2, -1, +1$ for $J = 0, 1, 2$ and $\overline{I(3S|r|1P)} = 0.023$ GeV^{-1} for the SSE (relativistic case) and 0.007 GeV^{-1} in the NR limit. To obtain the m.e. presented in Table I, we use also the spin-averaged nondiagonal m.e.: $\overline{I(3S|r|2P)} = -2.54$ GeV^{-1} and $\overline{I(3S|r|3P)} = 2.64$ GeV^{-1}.

In our calculations [see Eq. (13)] we have neglected possible contributions from nondiagonal m.e. of higher P-wave states. These contributions are expected to be small for two reasons. First, while the magnitudes of the m.e. $I(3S|r|2P) = -2.54$ GeV^{-1} and $I(3S|r|3P) = 2.64$ GeV^{-1} are large, the m.e. $I(3S|r|4P) = 0.34$ GeV^{-1} is $\sim$8 times smaller. Secondly, the $4P$ and higher states lie above the open beauty threshold and therefore their w.f. have larger sizes. Due to this fact their w.f. at the origin and the m.e. like $\langle r^{-3}\rangle$ are becoming significantly smaller, although their exact values may be calculated only within a multichannel approach. Indirectly, experimental data on dielectron widths of $\Upsilon(nS)$ with $n = 4, 5, 6$ (they are proportional to the squared w.f. at the origin) support this statement, being two to three times smaller than the dielectron width of $\Upsilon(3S)$.

We would like also to underline that the choice of the strong coupling in the spin-orbit potential is also important and here we have used the value close to that from the fine-structure analysis in Ref. [23]; the different choice of α_{so} may give rise to different splittings $\delta I_{so}(J)$ in Eq. (13).

III. CONCLUSIONS

For the $E1$ radiative transitions, $\Upsilon(3S) \to \gamma + \chi_b(1P_J)$ ($J = 0, 1, 2$), the spin-averaged m.e. $\overline{I(3S|r|1P_J)}$ are shown to be small, as it was predicted in a number of studies before.

However, due to spin-orbit effects the w.f. of the 1^3P_1 state is mixed with the $2P$, $3P$ states, for which the m.e. $\overline{I(3S|r|2P)}$ and $\overline{I(3S|r|3P)}$ are large and have different signs. Such a mixing is important, although the spin-orbit splittings themselves are not large and their typical values are $\sim$10–12 MeV. Due to this mixing, a strong cancellation takes place in the m.e. $I(3S|r|1P_1)$, which gives rise to a suppression of the transition rate for the radiative decay $\Upsilon(3S) \to \gamma + \chi_b(1^3P_1)$.

The following partial widths are predicted: $\Gamma_J(\Upsilon(3S) \to \gamma + \chi_b(1^3P_J)) = 213$ eV, 13 eV, and 54 eV for $J = 2, 1, 0$, which are in good agreement with the *BABAR* data, $\Gamma_2(\text{exp}) = 216 \pm 25$ eV and $\Gamma_0(\text{exp}) = 55 \pm 10$ eV [9].

Also for $J = 1$ the calculated partial width $\Gamma_1 = 13$ eV satisfies the upper limit, $\Gamma_1 < 22$ eV, obtained in the *BABAR* experiment. More precise measurements of the transition rate for $\Upsilon(3S) \to \gamma + \chi_{b1}(1P)$ could give additional restrictions on the spin-orbit effects in radiative decays.

We predict the following ratio of the partial widths: $r_{1,0} = \frac{\Gamma_1}{\Gamma_0} = 0.24$, which should be considered as an important feature of the transition rates where spin-orbit dynamics dominate.

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[1] G. Bonvicini *et al.* (CLEO Collaboration), Phys. Rev. D **70**, 032001 (2004).
[2] P. del Amo Sanchez *et al.* (*BABAR* Collaboration), Phys. Rev. D **82**, 111102 (2010).
[3] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **101**, 071801 (2008).
[4] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **103**, 161801 (2009).
[5] G. Bonvicini *et al.* (CLEO Collaboration), Phys. Rev. D **81**, 031104(R) (2010).
[6] M. Artuso *et al.* (CLEO Collaboration), Phys. Rev. Lett. **94**, 032001 (2005).
[7] D. M. Asner *et al.* (CLEO Collaboration), Phys. Rev. D **78**, 091103 (2008).
[8] M. Kornicer *et al.* (CLEO Collaboration), Phys. Rev. D **83**, 054003 (2011); arXiv:1012.0589.
[9] J. P. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. D **84**, 072002 (2011).
[10] W. Kwong and J. L. Rosner, Phys. Rev. D **38**, 279 (1988).
[11] N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011); N. Brambilla, P. Pietrulewicz, and A. Vairo, Phys. Rev. D **85**, 09 4005 (2012).
[12] E. Eichten, S. Godfrey, H. Mahlke, and J. L. Rosner, Rev. Mod. Phys. **80**, 1161 (2008).
[13] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **67**, 014027 (2003); N. Brambilla *et al.*, arXiv:hep-ph/0412158.
[14] J. L. Rosner, arXiv:1107.1273, and references therein.
[15] H. Grotch, D. A. Owen, and K. J. Sebastian, Phys. Rev. D **30**, 1924 (1984); S. N. Gupta, S. F. Radford, and W. W. Repko, Phys. Rev. D **30**, 2424 (1984).
[16] S. Jacobs, M. G. Olsson, and C. Suchyta, Phys. Rev. D **33**, 3338 (1986).
[17] A. K. Grant and J. L. Rosner, Phys. Rev. D **46**, 3862 (1992).
[18] A. M. Badalian, B. L. G. Bakker, and I. V. Danilkin, Phys. At. Nucl. **73**, 138 (2010); arXiv:0903.3643.
[19] A. Yu. Dubin, A. B. Kaidalov, and Yu. A. Simonov, Phys. At. Nucl. **56**, 1745 (1993); arXiv:hep-ph/9311344; Phys. Lett. B **323**, 41 (1994); Yu. A. Simonov, arXiv:hep-ph/9911237.
[20] A. M. Badalian, B. L. G. Bakker, and I. V. Danilkin, Phys. Rev. D **81**, 071502 (2010); **81**, 099902(E) (2010); **79**, 037505 (2009); A. M. Badalian, A. I. Veselov, and B. L. G. Bakker, Phys. Rev. D **70**, 016007 (2004).
[21] D. P. Stanley and D. Robson, Phys. Rev. D **21**, 3180 (1980); W. Lucha, F. F. Schoberl, and D. Gromes, Phys. Rep. **200**, 127 (1991), and references therein.
[22] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[23] A. M. Badalian and B. L. G. Bakker, Phys. Rev. D **62**, 094031 (2000).
[24] K. Nakamura (Particle Data Group) J. Phys. G **37**, 075021 (2010).
[25] M. Peter, Phys. Rev. Lett. **78**, 602 (1997); arXiv:hep-ph/9610209; Nucl. Phys. **B501**, 471 (1997); arXiv:hep-ph/9702245.
[26] Yu. A. Simonov, arXiv:1011.5386; A. M. Badalian and A. I. Veselov, Phys. At. Nucl. **68**, 582 (2005); A. M. Badalian and D. S. Kuzmenko, Phys. Rev. D **65**, 016004 (2001).
[27] G. Aad *et al.* (ATLAS Collaboration), Phys. Rev. Lett. **108**, 152001 (2012).