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Wess-Zumino-Witten action and photons from the chiral magnetic effect

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We revisit the chiral magnetic effect (CME) using the chiral Lagrangian. We demonstrate that the electric-current formula of the CME is derived immediately from the contact part of the Wess-Zumino-Witten action. This implies that the CME could be, if observed, a signature for the local parity violation, but a direct evidence for neither quark deconfinement nor chiral restoration. We also discuss the reverse chiral magnetic Primakoff effect, i.e., the real photon production through the vertex associated with the CME, which is kinematically possible for space-time inhomogeneous configurations of magnetic fields and the strong θ angle. We make a qualitative estimate for the photon yield to find that it is comparable to the thermal photon.

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The vacuum structure in quantum chromodynamics (QCD) has been an important subject investigated in theory for a long time. It has been well-known that gauge configurations with topologically nontrivial winding such as the instanton, the magnetic monopole, etc., should play a crucial role in the spontaneous breaking of chiral symmetry [1], color confinement [2], the mass of η' meson [3], and the strong θ angle [4].

Among others the problem of the strong θ angle is still posing a theoretical challenge. There is no consensus on the unnatural smallness of θ and thus the absence of $\mathcal{P}$ and $\mathcal{CP}$ violation in the strong interaction. Recently, more and more researchers in the field of the relativistic heavy-ion collision are getting interested in the possibility of fluctuating θ in a transient state of QCD matter and searching for a signature to detect the local $\mathcal{P}$ violation (LPV) experimentally [5,6].

In this context the discovery of the chiral magnetic effect (CME) [7,8] has triggered constructive discussions and lots of works have been devoted to the interplay between the topological effects and the external magnetic field $\boldsymbol{B}$ [9–11], while the strong-$\boldsymbol{B}$ effect itself on nuclear or QCD matter has been Ref. [12] and are still [13] attracting theoretical interest. (See Ref. [14] for earlier works related to the CME.) If θ temporarily takes a nonzero value in hot and dense QCD matter, its time derivative induces an excess of either left-handed or right-handed quarks. Due to the alignment of the spin and the momentum directions of left-handed and right-handed quarks, $\boldsymbol{B}$ would generate a net electric current parallel to $\boldsymbol{B}$, which may be in principle probed by the fluctuations of $\mathcal{P}$-odd observables in the heavy-ion collision [6].

It should be an urgent problem of paramount importance, we believe, to sort out the proper physics interpretation of the CME and the LPV in general since the LPV is under intensive investigations in ongoing experiments at present. It is also under active discussions whether the chiral magnetic wave should account for the discrepancy between the elliptic flows of positively and negatively charged hadrons [15]. It is often said that the CME could be a signature for quark deconfinement and chiral symmetry restoration, as stated also by one of the present authors in Ref. [8]. This was conjectured because the intuitive explanation for the CME seemed to require almost massless u and d quarks. One should be, however, careful of the physics interpretation of anomalous phenomena which sometimes look counterintuitive. The first half of our discussions are devoted to considerations on the implication of the CME in terms of the chiral Lagrangian. We conclude that the CME is insensitive to whether the fundamental degrees of freedom are quarks or hadrons, so that it could be seen without deconfinement. Chiral symmetry restoration is, on the other hand, necessary to realize the hadronic LPV in the same manner as in the case of the disoriented chiral condensate (DCC) [16].

In the last half of our discussions, as an application of the chiral Lagrangian, we address the real photon production through the process that we call the reverse chiral magnetic Primakoff effect. The typical process in the ordinary Primakoff effect is the π^0 (or some neutral meson generally) production from a single photon picking up another photon from the external electromagnetic field [17]. In the relativistic heavy-ion collision the neutral pseudoscalar field $\theta(x)$ can couple to a photon in $\boldsymbol{B}$ leading to a single photon emission, i.e., $\theta + B \to \gamma$, which can be viewed as a reverse process of the Primakoff effect. Such a mechanism for the photon production can be traced back to the old idea to detect the axion via the Primakoff effect [18], and is similar to the recent idea on a novel source of photons from the conformal anomaly [19]. In short, a crucial difference between our idea and that in Ref. [19] lies in the neutral meson involved in the process: the σ meson (which turns to a hydrodynamic mode) in the conformal anomaly case and θ or η_0 in our case of the CME vertex (see also Ref. [20] for the diphoton emission from the σ meson). From this point of view, it would be very natural to think of photons as a signature of the CME; instead of the axion [18], CME requires a *background* $\theta(x)$, which may cause the same process of the single photon production as the axion detection.

The interesting point in our arguments for the photon production is that the real photon emission is attributed to exactly the same vertex as to describe the electric-current generation in the CME. As long as $\boldsymbol{B}$ and θ are spatially homogeneous, as often assumed for simplicity, the real particle production is prohibited kinematically, but once $\boldsymbol{B}$ and θ are space-time dependent (and they are indeed so in the heavy-ion collision), the energy-momentum conservation is satisfied, so that the real photon can come out.

Here, one might have wondered how the physical constant θ can be lifted up in hot and dense matter and treated as if it were a particle. In other words, what is the origin of the chiral chemical potential μ_5 in the hadronic environment? This is an important question and related to the physical mechanism that causes the LPV. At an extremely high energy the color glass condensate and the Glasma initial condition [21] may be the most relevant and their characteristic scale is then given by the saturation scale Q_s. In this case the role of θ in the pure Yang-Mills dynamics is more nontrivial [22] than full QCD with dynamical quarks where θ can be regarded as the $\mathrm{U}(1)_\mathrm{A}$ rotation angle. In the hadronic phase at low energy, the chiral Lagrangian provides us with a clear picture, which consists of three parts:

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_\chi + \mathcal{L}_{\mathrm{WZW}} + \mathcal{L}_{\mathrm{P}}, \tag{1}$$

where the first one is the usual chiral Lagrangian that is given by [23–25]

$$\begin{aligned}\mathcal{L}_\chi = &\frac{f_\pi^2}{4}\,\mathrm{tr}[D_\mu U^\dagger D^\mu U + 2\chi(MU^\dagger + UM)] \\ &- \frac{N_\mathrm{f}\chi_{\mathrm{top}}}{2}\left[\theta - \frac{i}{2}\,\mathrm{tr}(\ln U - \ln U^\dagger)\right]^2,\end{aligned} \tag{2}$$

in the lowest order including the topological terms that break $\mathrm{U}(1)_\mathrm{A}$ symmetry. Here, χ_{top} represents the pure topological susceptibility, the covariant derivative involves the vector and the axial-vector fields as $D_\mu U \equiv \partial_\mu U - ir_\mu U + iUl_\mu + \frac{i}{2}(\partial_\mu\theta + 2\,\mathrm{tr}(a_\mu))U$ with $r_\mu \equiv v_\mu + a_\mu$ and $l_\mu = v_\mu - a_\mu$, and $\chi \equiv -\langle\bar{q}q\rangle/f_\pi^2$ from the Gell-Mann–Oakes–Renner relation. It is obvious that, as discussed in Ref. [23], the θ-dependence is to be absorbed in the phase of U if the current quark mass matrix M has a zero component. Then, one can understand that θ and the phase of U or η_0/f_{η_0} are simply identifiable apart from the mass terms proportional to χ and M. This means that, if the system has the DCC in the isosinglet channel $\eta_0(x)$ and if $\chi M \simeq 0$ at high enough T, we can interpret this $\eta_0(x)$ as an effective $\theta(x)$ in a transient state (this reinterpretation exactly corresponds to the normalization condition for U in Ref. [25]). We note that in the whole argument this is the only place where (partial) chiral symmetry restoration is required in the hadronic picture of the CME. Hence, in the hadronic phase, the DCC of η_0 is the source for $\mu_5(x)$. Its strength and distribution could be in principle figured out in numerical simulations as in Ref. [26].

The anomalous processes such as $\pi^0 \to \gamma\gamma$ and $\gamma\pi^0 \to \pi^+\pi^-$ are described by the Wess-Zumino-Witten (WZW) part that can be written in a concise way in the two-flavor case [25] as

$$\begin{aligned}\mathcal{L}_{\mathrm{WZW}} = &-\frac{N_\mathrm{c}}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}[\mathrm{tr}\{U^\dagger\hat{r}_\mu U\hat{l}_\nu - \hat{r}_\mu\hat{l}_\nu \\ &+ i\Sigma_\mu(U^\dagger\hat{r}_\nu U + \hat{l}_\nu)\}\mathrm{tr}(v_{\rho\sigma}) \\ &+ \frac{2}{3}\,\mathrm{tr}(\Sigma_\mu\Sigma_\nu\Sigma_\rho)\mathrm{tr}(v_\sigma)],\end{aligned} \tag{3}$$

with $v_{\mu\nu} \equiv \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu]$, and $\Sigma_\mu \equiv U^\dagger\partial_\mu U$. A hat symbol indicates the traceless part, i.e., $\hat{r}_\mu \equiv r_\mu - \frac{1}{2}\mathrm{tr}(r_\mu)$ and $\hat{l}_\mu \equiv l_\mu - \frac{1}{2}\mathrm{tr}(l_\mu)$. There is one more part that has no dynamics of chiral field U and thus is called the contact part:

$$\begin{aligned}\mathcal{L}_\mathrm{P} = &\frac{N_\mathrm{c}}{8N_\mathrm{f}\pi^2}\epsilon^{\mu\nu\rho\sigma}\bigg\{\mathrm{tr}\bigg[v_\mu\bigg(\partial_\nu v_\rho - \frac{2i}{3}v_\nu v_\rho\bigg)\bigg]\partial_\sigma\theta \\ &+ \mathrm{tr}(a_\mu D_\nu^v a_\rho)\bigg(\frac{4}{3}\,\mathrm{tr}(a_\sigma) + \partial_\sigma\theta\bigg) \\ &- \frac{2}{3N_\mathrm{f}}\,\mathrm{tr}(a_\mu)\,\mathrm{tr}(\partial_\nu a_\rho)\partial_\sigma\theta\bigg\},\end{aligned} \tag{4}$$

where $D_\mu^v a_\nu \equiv \partial_\mu a_\nu - iv_\mu a_\nu - ia_\mu v_\nu$.

Now that we have the chiral effective Lagrangian that should encompass the anomalous processes, it is straightforward to read the current in the presence of space-time dependent $\theta(x)$ and the electromagnetic field A_μ. To this end, in the two-flavor case, the vector and the axial-vector fields are set to be

$$v_\mu = eQA_\mu, \qquad a_\mu = 0, \tag{5}$$

with the electric-charge matrix, $Q = \mathrm{diag}(2/3, -1/3) = 1/6 + \tau_3$.

Let us first simplify $\mathcal{L}_{\mathrm{WZW}}$ and $\mathcal{L}_\mathrm{P}$, respectively, which are of our central interest. It should be mentioned that the quadratic terms of A_μ vanish due to the antisymmetric tensor, $\epsilon^{\mu\nu\rho\sigma}$. Then, the first term in Eq. (3) vanishes and the rest takes the following form:

$$\begin{aligned}\mathcal{L}_{\mathrm{WZW}} = &-\frac{N_\mathrm{c}\mathrm{tr}(Q)}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\{ie^2\,\mathrm{tr}[(\Sigma_\mu + \tilde{\Sigma}_\mu)\tau_3]A_\nu\partial_\rho A_\sigma \\ &- \frac{2e}{3}\,\mathrm{tr}(\Sigma_\mu\Sigma_\nu\Sigma_\rho)A_\sigma\},\end{aligned} \tag{6}$$

where we defined $\tilde{\Sigma}_\mu = (\partial_\mu U)U^\dagger$. Similarly the contact term can become as simple as

$$\mathcal{L}_\mathrm{P} = \frac{N_\mathrm{c}e^2\,\mathrm{tr}(Q^2)}{8N_\mathrm{f}\pi^2}\epsilon^{\mu\nu\rho\sigma}A_\mu(\partial_\nu A_\rho)\partial_\sigma\theta. \tag{7}$$

Now, we are ready to confirm that we can reproduce the electric current corresponding to the CME in the hadronic phase. We shall next compute the electric current by taking the differentiation of the effective action with respect to the gauge field coupled to it, that is

$$j^\mu(x) = \frac{\delta}{\delta A_\mu(x)} \int d^4x \mathcal{L}_{\rm eff}. \quad (8)$$

The current from the usual chiral Lagrangian $\mathcal{L}_\chi$ at the lowest order results in

$$j^\mu_\chi = -i\frac{ef_\pi^2}{4}\,\mathrm{tr}[(\Sigma^\mu - \tilde{\Sigma}^\mu)\tau^3] \simeq e(\pi^- i\partial^\mu\pi^+ - \pi^+ i\partial^\mu\pi^-) + \cdots, \quad (9)$$

which represents the electric current carried by the flow of charged pions, $\pi^\pm$, which is clear from the expanded expression. There appears no term involving $\partial_\mu\theta$ in this part. More nontrivial and interesting is the current associated with the WZW terms, leading to

$$j^\mu_{\rm WZW} = -\frac{N_{\rm c}\,\mathrm{tr}(Q)}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\bigg\{2ie^2\,\mathrm{tr}[(\Sigma_\nu + \tilde{\Sigma}_\nu)\tau_3]\partial_\rho A_\sigma + e^2\,\mathrm{tr}[\partial_\rho(\Sigma_\nu + \tilde{\Sigma}_\nu)\tau_3]A_\sigma - \frac{2e}{3}\,\mathrm{tr}(\Sigma_\nu\Sigma_\rho\Sigma_\sigma)\bigg\}. \quad (10)$$

The physical meaning of this current will be transparent in the expanded form using $U \sim 1 + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}/f_\pi + \cdots$. Then we find that the first term in Eq. (10) is written as,

$$j^\mu_{\rm WZW} = \frac{N_{\rm c}\,\mathrm{tr}(Q)e^2}{8\pi^2 f_\pi}\epsilon^{\mu\nu\rho\sigma}(\partial_\nu\pi^0)F_{\rho\sigma}. \quad (11)$$

The second term in Eq. (10) is vanishing and the last term represents a topological current purely from the entanglement of all π^0 and $\pi^\pm$. The physics implication of Eq. (11) has been discussed with the π^0-domain wall [9] and the pion profile in the Skyrmion [27]. Finally we can reproduce the CME current from the contact interaction as

$$j^\mu_{\rm P} = \frac{N_{\rm c}e^2\,\mathrm{tr}(Q^2)}{4N_{\rm f}\pi^2}\epsilon^{\mu\nu\rho\sigma}(\partial_\nu A_\rho)\partial_\sigma\theta. \quad (12)$$

We can rewrite the above expression in a more familiar form using $\mu_5 = \partial_0\theta/(2N_{\rm f})$ and $B^i = \epsilon^{ijk}\partial_j A_k$ to reach

$$\boldsymbol{j}_{\rm P} = \frac{N_{\rm c}e^2\,\mathrm{tr}(Q^2)}{2\pi^2}\mu_5\boldsymbol{B}. \quad (13)$$

It should be noted that $\epsilon_{0123} = +1$ in our convention.

This derivation of the CME is quite suggestive on its own and worth several remarks.

First, it is known that the contact term $\mathcal{L}_{\rm P}$ is not renormalization-group invariant [25]. This means that $\mathcal{L}_{\rm P}$ and, thus, $j_{\rm P}$ are scale dependent like the running coupling constant. It is often said that $j_{\rm P}$ is an exact result from the quantum anomaly, but it may be a little misleading. The functional form itself could be protected (though there is no rigourous proof) but $\boldsymbol{B}$ and μ_5 in Eq. (13) should be renormalized ones. Indeed it has been pointed out that interaction vertices in the (axial) vector channels result in the dielectric correction to $\boldsymbol{B}$ [28]. The knowledge on the chiral Lagrangian strongly supports the results of Ref. [28].

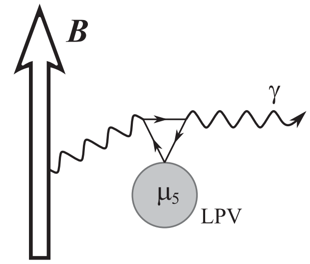

FIG. 1. Schematic figure for the single photon production as a consequence of the axial anomaly and the external magnetic field. The angular distribution of the emitted photons is proportional to $(q_z^2 + q_x^2)/(q_x^2 + q_y^2 + q_z^3)$ where q_y is in the direction parallel to $\boldsymbol{B}$ and q_z and q_x perpendicular to $\boldsymbol{B}$.

Second, to find Eq. (13), we do not need quark degrees of freedom explicitly but only hadronic variables. This is naturally so because the idea of the WZW action is to capture the anomalous effects from the ultraviolet regime in terms of infrared variables. It is clear from the above derivation, therefore, that the CME occurs without massless quarks in the quark-gluon plasma. (See also Ref. [29] for another derivations of the CME without referring to quarks explicitly.) Then, a conceptual confusion might arise: what really flows that contributes to an electric current in the hadronic phase? One may have thought that it is $\pi^\pm$, but such a current is rather given by j^μ_χ, and the CME current $j^\mu_{\rm P}$ originates from the contact part that is decoupled from U. The same question is applied to Eq. (11) if the system has a π^0 condensation.

In a sense these currents associated with the $\theta(x)$ or $\pi^0(x)$ backgrounds are reminiscent of the Josephson current in superconductivity. Suppose that we have a π^0 condensate, then such a coherent state behaves like a macroscopic wave function of π^0 field. Then, a microscopic current inside of the wave function π^0 could be a macroscopic current in the whole system since the wave function spreads over the whole system. In the case of the CME, $\theta(x)$ or $\eta_0(x)$ plays the same role as $\pi^0(x)$. In this way, strictly speaking, it is a high-momentum component of quarks and antiquarks in the wave function of π^0 or η_0 that really flow to make a current, though these quarks do not have to get deconfined.

This sort of confusing interpretation of the CME current arises from the assumption that $\theta(x)$ and $\boldsymbol{B}(x)$ are spatially homogeneous. Once this assumption is relaxed, as we discuss in what follows, an interesting new possibility opens, which may be more relevant to experiments.

From now on, let us revisit Eq. (7) from a different point of view. If we literally interpret Eq. (7) as usual in the quantum field theory, it should describe a vertex of the processes involving two photons and the θ field such as $\theta \to \gamma\gamma$ and $\theta + B \to \gamma$ in the magnetic field. The latter process can be viewed as the reverse of the Primakoff effect involving the $\theta(x)$ background instead of neutral mesons. It is a very intriguing question how many photons can be produced from this reverse Primakoff effect. For this purpose we shall decompose the vector potential into the background $\bar{A}_\mu$ (corresponding to B) and the fluctuation $\mathcal{A}_\mu$ (corresponding to photon). Then, Eq. (7) turns into

$$\mathcal{L}_\mathrm{P} = \frac{N_\mathrm{c} e^2 \,\mathrm{tr}(Q^2)}{8N_\mathrm{f}\pi^2} \epsilon^{\mu\nu\rho\sigma}[\mathcal{A}_\mu(\partial_\nu \mathcal{A}_\rho) + \mathcal{A}_\mu \bar{F}_{\nu\rho}]\partial_\sigma \theta, \tag{14}$$

where the first term represents the two-photon production process $\theta \to \gamma\gamma$ similar to $\pi^0 \to \gamma\gamma$, and the second represents the reverse Primakoff effect ($\theta + B \to \gamma$) involving the background field strength $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$ (see Fig. 1 for a schematic illustration). Here we are interested only in the situation where the background field is so strong that we can neglect the contribution from the first term.

Even when $|eB| \sim \Lambda_\mathrm{QCD}$ in the heavy-ion collision, we can still utilize the perturbative expansion in terms of the electromagnetic coupling constant. In the leading order, from the Lehmann-Symanzik-Zimmermann reduction formula, the S-matrix element for the single-photon production with the momentum $q = (|\boldsymbol{q}|, \boldsymbol{q})$ and the polarization $\varepsilon^{(i)}(\boldsymbol{q})$ is deduced immediately from the vertex (14),

$$\begin{aligned} i\mathcal{M}(i;\boldsymbol{q}) &= \langle \varepsilon^{(i)}(\boldsymbol{q})|\Omega\rangle \\ &= i\frac{N_\mathrm{c} e^2 \,\mathrm{tr}(Q^2)}{8N_\mathrm{f}\pi^2\sqrt{(2\pi)^3 2q_0}} \epsilon^{\mu\nu\rho\sigma}\varepsilon^{(i)\mu}(\boldsymbol{q}) \\ &\quad \times \int d^4x e^{-iq\cdot x} \bar{F}_{\nu\rho}(x)\partial_\sigma \theta(x), \end{aligned} \tag{15}$$

where $q_0 = |\boldsymbol{q}|$. This expression becomes very simple when the background field has only the magnetic field in the y direction, i.e., $B = \bar{F}_{zx}$ and the rest is just vanishing. Thus, we have

$$\begin{aligned} &\epsilon^{\mu\nu\rho\sigma}\varepsilon^{(i)\mu}(\boldsymbol{q}) \int d^4x e^{-iq\cdot x} \bar{F}_{\nu\rho}(x)\partial_\sigma \theta(x) \\ &\quad = -2\varepsilon^{(i)y}(\boldsymbol{q}) \int d^4x e^{-iq\cdot x} B(x)\partial_0 \theta(x), \end{aligned} \tag{16}$$

and replacing $\partial_0\theta$ by the chiral chemical potential μ_5 as $\mu_5 = \partial_0\theta/(2N_\mathrm{f})$ and using $\sum_i \varepsilon^{(i)j}(\boldsymbol{q})\varepsilon^{(i)k}(\boldsymbol{q}) = \delta^{jk} - q^j q^k/\boldsymbol{q}^2$ with $\boldsymbol{q}^2 = q_x^2 + q_y^2 + q_z^2$, we can finally arrive at

$$\begin{aligned} q_0 \frac{dN_\gamma}{d^3q} &= q_0 \sum_i |\mathcal{M}(i;\boldsymbol{q})|^2 \\ &= \frac{1 - (q_y)^2/\boldsymbol{q}^2}{2(2\pi)^3}\bigg(\frac{N_\mathrm{c} e^2 \,\mathrm{tr}(Q^2)}{2\pi^2} \\ &\quad \times \int d^4x e^{-iq\cdot x} B(x)\mu_5(x)\bigg)^2 \\ &= \frac{q_z^2 + q_x^2}{2(2\pi)^3 \boldsymbol{q}^2} \cdot \frac{25\alpha_e \zeta(\boldsymbol{q})}{9\pi^3}, \end{aligned} \tag{17}$$

where we used $N_\mathrm{c} = 3$ and $\mathrm{tr}(Q^2) = 5/9$ for the two-flavor case in the last line and $\alpha_e \equiv e^2/(4\pi) \simeq 1/137$ is the fine structure constant. In the above, we defined

$$\zeta(\boldsymbol{q}) \equiv \bigg| \int d^4x e^{-iq\cdot x} eB(x)\mu_5(x) \bigg|^2. \tag{18}$$

It is quite interesting to see that the final expression is proportional to the momenta $q_z^2 + q_x^2$ which are perpendicular to the $\boldsymbol{B}$ direction. This could be another source for the elliptic flow v_2 of the direct photon in a similar mechanism as pointed out in Ref. [19].

Because there is no reliable model to predict $\mu_5(x)$, it is difficult to calculate $\zeta(\boldsymbol{q})$ as a function of the momentum. For a first attempt, therefore, let us make a qualitative order estimate. The strength of the magnetic field is as large as Λ_QCD^2 or even bigger at initial time. A natural scale for μ_5 is also given by Λ_QCD, or if the origin of the LPV is the color flux-tube structure in the Glasma [21], the typical scale is the saturation momentum $Q_s \sim 2$ GeV for the RHIC energy. The space-time integration picks up the volume factor $\sim\tau_0^2 A_\perp$ with τ_0 being the lifetime of the magnetic field, i.e., $\tau_0 \simeq 0.01 \sim 0.1$ fm/c, and $A_\perp$ the transverse area $\sim$150 fm^2 for the Au-Au collision. Then, $\zeta \simeq 0.1$–10^3, where the smallest estimate is for $\tau_0 = 0.01$ fm/c and $\mu_5 \sim \Lambda_\mathrm{QCD}$ and the largest one is for $\tau_0 = 0.1$ fm/c and $\mu_5 \sim Q_s$. Then, the photon yield is expected to be $q_0(dN_\gamma/d^3q) \simeq (10^{-7} \sim 10^{-3})$ GeV^{-2}. This is of a detectable level of the photon yield as compared to the conventional photon production from the thermal medium [30]. If the backreactions to sustain $\boldsymbol{B}$ work efficiently, the relevant time scale τ_0 may be replaced by the lifetime of the plasma. Then, the photon contribution from the reverse Primakoff effect would be enhanced and appreciable even at the LHC energy.

We also remark about a hard scale such as Q_s in the above estimate. We postulated that the interaction vertex (14) makes sense also in the ultraviolet regime since the CME current (13) is kept unchanged through renormalization, which extends the validity of Eq. (14) to ultraviolet scales. It would be a nontrivial question whether or how the anomaly matching between the ultraviolet and infrared degrees of freedom could be realized, including a formalism based on the vector dominance [11], which is beyond our current scope.

One may think that not only the polarization but also $\zeta(\boldsymbol{q})$ has strong asymmetry because of the presence of $\boldsymbol{B}$. The typical domain size of the LPV should be, however, much smaller than the impact factor $b \sim$ a few fm at least, and thus the asymmetry effect turns out only negligible. In reality, depending on the spatial position, there is not only B_y, but B_x and B_z and also the electric fields E_x, E_y, and E_z. We are now performing full numerical calculations including all those fields and the LPV based on the Glasma flux-tube picture, which will be reported in a future publication.

In summary, we have formulated the CME in terms of the chiral Lagrangian with the WZW terms, which provides us with the physics picture to understand the CME in the hadronic phase. We derived the current of the CME correctly from the contact term that is not renormalization group invariant. We established how the CME could be realized through $\eta_0(x)$ as a result of the DCC in the iso-singlet channel. Then, the key observation in view of the chiral Lagrangian is that the vertex responsible for the CME also describes the single photon production. We have given the expression for the photon yield to find that its angular distribution is asymmetric with the direction perpendicular to $\boldsymbol{B}$ more preferred. We made a qualitative estimate for the yield and found it comparable to the thermal photon contribution. Unlike the thermal photon the p_t distribution should reflect the domain size of the LPV. Electromagnetic probes as a signature for the LPV (see Ref. [31] for the dilepton production) deserve further investigations and we believe that this work should shed light on future developments in this direction.

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