Casimir scaling in gauge theories with a gap: Deformed QCD as a toy model

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We study a Casimir-like behavior in a "deformed QCD." We demonstrate that for the system defined on a manifold of size $\mathbb{L}$, the difference $\Delta E \equiv E - E_{\rm Mink}$ between the energies of a system in a nontrivial background and Minkowski space-time geometry exhibits the Casimir-like scaling $\Delta E \sim \mathbb{L}^{-1}$, despite the presence of a mass gap in the system, in contrast with naive expectation $\Delta E \sim \exp(-m\mathbb{L})$, which would normally originate from any physical massive propagating degrees of freedom consequent to conventional dispersion relations. The Casimir-like behavior in our system comes instead from a nondispersive ("contact") term which is not related to any physical propagating degrees of freedom, such that the naive argument is simply not applicable. These ideas can be explicitly tested in weakly coupled deformed QCD. We comment on profound consequences for cosmology of this effect if it persists in real strongly coupled QCD.

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I. INTRODUCTION AND MOTIVATION

The main motivation for the present studies is a recent suggestion on the dynamical dark energy (DE) model, which is entirely rooted in the strongly coupled QCD, without any new fields and/or coupling constants [1–4]. The key element of the proposal [1–4] is based on a paradigm that the relevant energy which enters the Einstein equations is, in fact, the difference $\Delta E \equiv E - E_{\rm Mink}$ between the energies of a system in a nontrivial background and Minkowski space-time geometry, similar to the well-known Casimir effect when the observed energy is a difference between the energy computed for a system with conducting boundaries (positioned at finite distance $\mathbb{L}$) and infinite Minkowski space.[1] This paradigm is based on the conjecture that gravity, as described by the Einstein equations, is a low-energy effective interaction which, as such, should not be sensitive to the microscopic degrees of freedom in the system but to some effective scale. Thus, the energy density that enters the semiclassical Einstein equations should not be the "bare" energy as computed in quantum field theory but rather a "renormalized" energy density. We propose the renormalization scheme given above which sets the vacuum energy to zero in Minkowski space, wherein the Einstein equations are automatically satisfied as the Ricci tensor identically vanishes.

The above prescription is in fact the standard subtraction procedure that is normally used for the description of horizon thermodynamics [5,6] as well as in a course of computations of a different Green's function in a curved background by subtracting infinities originated from the flat space [7]. In the present context, such a definition $\Delta E \equiv (E - E_{\rm Mink})$ for the vacuum energy was first advocated in 1967 by Zeldovich [8], who argued that $\rho_{\rm vac} \sim G m_p^6$ with m_p being the proton's mass. Later on such a definition for the relevant energy $\Delta E \equiv (E - E_{\rm Mink})$ which enters the Einstein equations has been advocated from different perspectives in a number of papers; see, for example, the relatively recent works [9–16] and references therein.

We study the scaling behavior of ΔE when the background deviates slightly from Minkowski space. The difference ΔE must obviously vanish when any deviations (parametrized by Hubble constant or inverse size of the visible universe, $H \sim \mathbb{L}^{-1}$) go to zero as this corresponds to the transition to flat Minkowski space. A naive expectation based on common sense suggests that $\Delta E \sim \exp(-\Lambda_{\rm QCD}/H) \sim \exp(-10^{41})$ as QCD has a mass gap $\sim \Lambda_{\rm QCD} \sim 100$ MeV, and, therefore, ΔE must not be sensitive to the size of our Universe $\mathbb{L} \sim H^{-1}$. Such a naive expectation formally follows from the dispersion relations, which dictate that a sensitivity to very large distances must be exponentially suppressed when the mass gap is present in the system.[2]

However, as emphasized in Refs. [3,4] in strongly coupled gauge theories along with conventional dispersive contribution, there exists a nondispersive contribution, not related to any physical propagating degrees of freedom. This nondispersive (contact) term generally emerges as a result of topologically nontrivial sectors in four-dimensional QCD. The variation of this contact term with variation of the background may lead to a powerlike

[1]Here and in what follows, we use the term "Casimir effect" to emphasize the powerlike sensitivity to large distances irrespective of their nature. A crucial distinct feature that characterizes the system we are interested in is the presence of dimensional parameter $\mathbb{L} \sim H^{-1}$ (where H is a Hubble constant) in the system, which distinguishes it from infinitely large Minkowski space-time.

[2]The Casimir effect, due to the massless $E\&M$ field, obviously shows such power dependence $\Delta E = -\frac{\pi^2}{720\mathbb{L}^4}$. Similar computations for a massive scalar particle with mass m lead to an exponentially suppressed result $\Delta E \sim \exp(-m\mathbb{L})$ as expected, see e.g., Ref. [17].

scaling $\Delta E \sim H + \mathcal{O}(H)^2$ rather than to an exponential-like $\Delta E \sim \exp(-\Lambda_{\rm QCD}/H)$. If true, the difference between two metrics (Friedmann-Lemaitre-Robertson-Walker (FLRW) and Minkowski) would lead to an estimate

$$\Delta E \sim \frac{\Lambda_{\rm QCD}^3}{\mathbb{L}} \sim (10^{-3}\,{\rm eV})^4, \quad 1/\mathbb{L} \sim H \sim 10^{-33}\,{\rm eV}, \quad (1)$$

which is amazingly close to the observed DE value today. It is interesting to note that expression (1) reduces to Zeldovich's formula $\rho_{\rm vac} \sim Gm_p^6$ if one replaces $\Lambda_{\rm QCD} \to m_p$ and $H \to G\Lambda_{\rm QCD}^3$. The last step follows from the solution of the Friedman equation

$$H^2 = \frac{8\pi G}{3}(\rho_{\rm DE} + \rho_M), \qquad \rho_{\rm DE} \sim H\Lambda_{\rm QCD}^3 \qquad (2)$$

when the DE component dominates the matter component, $\rho_{\rm DE} \gg \rho_M$. In this case the evolution of the Universe approaches a de Sitter state with constant expansion rate $H \sim G\Lambda_{\rm QCD}^3$ as follows from (2).

Another motivation to study the Casimir-like behavior in QCD is a proposal [18,19] that the $\mathcal{P}$ odd correlations observed at Relativistic Heavy Ion Collider and LHC are, in fact, another manifestation of long-range order advocated in this work. Furthermore, an apparently universal thermal spectrum, observed in all high-energy collisions when the statistical thermalization could never be reached in the systems, might also be related to the same contact term, not related to any physical propagating degrees of freedom; see Refs. [18,19] and references therein for the details.

There are a number of arguments supporting the power-like behavior $\Delta E \sim H + \mathcal{O}(H)^2$ in gauge theories. See Sec. IV where we present some general arguments suggesting the Casimir-like corrections in gauge theories with nontrivial topological structure. However, it is always desirable and very instructive to see how the general arguments work in some simplified settings. This is precisely the goal for the present study: we want to consider a simplified ("deformed") version of QCD which, on one hand, would be a weakly coupled gauge theory wherein computations can be performed in a theoretically controllable manner. On other hand, this deformation would preserve all the relevant elements of strongly coupled QCD, such as confinement, degeneracy of topological sectors, nontrivial θ dependence, presence of nondispersive contribution to topological susceptibility, and other crucial aspects, necessary for this phenomenon to emerge. Such a "deformed" theory has recently been developed [20]. All computations in this work (excluding those in Sec. III A) are performed within this framework.

II. TOPOLOGICAL SUSCEPTIBILITY IN THE DEFORMED QCD

In the deformed theory an extra term is put into the Lagrangian in order to prevent the center symmetry breaking that characterizes the QCD phase transition between "confined" hadronic matter and "deconfined" quark-gluon plasma. Thus, we have a theory that remains confined at high temperature in a weak coupling regime and for which, it is claimed [20], there does not exist an order parameter to differentiate the low-temperature confined regime from the high-temperature confined regime. The nontrivial topological sectors of the theory are described in this model in terms of the weakly coupled monopoles. The monopoles in this framework are not real particles; they are pseudo-particles that live in Euclidean space and describe the physical tunneling processes between different topological sectors $|n\rangle$ and $|n+1\rangle$. In particular, the monopole fugacity ζ should be understood as a number of tunneling events per unit time per unit volume

$$\left(\frac{\text{number of tunnelling events}}{VL}\right) = \frac{N_c\zeta}{L}, \qquad (3)$$

where extra factor N_c in (3) accounts for N_c different types of monopoles present in the system and L is the size of the circle along $\tau = it$ and plays the role of the inverse temperature. The monopole gas experiences Debye screening so that the field, due to any static charge, falls off exponentially with characteristic length m_σ^{-1}. The number density $\mathcal{N}$ of monopoles is given by the monopole fugacity, $\sim\zeta$, so that the average number of monopoles in a "Debye volume" is given by

$$\mathcal{N} \equiv m_\sigma^{-3}\zeta = \left(\frac{g}{2\pi}\right)^3 \frac{1}{\sqrt{L^3\zeta}} \gg 1. \qquad (4)$$

The last inequality holds since the monopole fugacity is exponentially suppressed, $\zeta \sim e^{-1/g^2}$, and in fact we can view (4) as a constraint on the validity of the approximation.

The topological susceptibility in this model can be explicitly computed and is given by [21]

$$\chi_{\rm YM} = \int d^4x \langle q(\mathbf{x})q(\mathbf{0})\rangle = \frac{\zeta}{N_cL}\int d^3x[\delta(\mathbf{x})]. \qquad (5)$$

The light quarks can be easily inserted into the system. The corresponding generalization of Eq. (5) reads [21]

$$\chi_{\rm QCD} = \int d^4x\langle q(\mathbf{x})q(\mathbf{0})\rangle = \frac{\zeta}{N_cL}\int d^3x\left[\delta(\mathbf{x}) - m_{\eta'}^2\frac{e^{-m_{\eta'}r}}{4\pi r}\right]. \qquad (6)$$

The first term in this equation has a nondispersive nature, similar to Eq. (5) and has the positive sign. This contact term (which is not related to any physical propagating degrees of freedom) has been computed in this model using monopoles describing the transitions between the degenerate topological sectors.[3] The positive sign of this term is the crucial element of the resolution of the $U(1)_A$ problem. The second term in Eq. (6) is a standard dispersive contribution, can be restored through the absorptive part using conventional dispersion relations, and has a negative sign in accordance with general principles. This conventional physical contribution is saturated in this model by the lightest η' field. It enters $\chi_{\rm QCD}$ precisely in such a way that the Ward identity expressed as $\chi_{\rm QCD}(m_q = 0) = 0$ is automatically satisfied as a result of cancellation between the two terms. If the contact nondispersive term with "wrong sign" was not present in the system, the Ward identity could not be satisfied as physical states always contribute with negative sign in Eq. (6).

One should note that the number of tunneling events per unit time per unit volume (3) in pure gauge theory in this model (with no quarks) precisely concurs with the absolute value of the energy density of the system. Furthermore, the topological susceptibility in pure gauge theory calculated as the second derivative of $E_{\rm YM}(\theta)$ with respect to θ precisely coincides with the nondispersive contact term with "wrong sign" explicitly and directly computed in (6). Indeed,

$$E_{\rm YM}(\theta) = -\frac{N_c\zeta}{L}\cos\left(\frac{\theta}{N_c}\right),$$
$$\chi_{\rm YM}(\theta = 0) = \frac{\partial^2 E_{\rm YM}(\theta)}{\partial\theta^2}\bigg|_{\theta=0} = \frac{\zeta}{N_c L}, \qquad (7)$$

where we keep only the lowest branch $l = 0$ in expression for $\cos(\frac{\theta+2\pi l}{N_c})$ to simplify formula (7). See detailed discussions with a complete set of references on this matter in Ref. [21]. In other words, the contact term in pure gauge theory $\chi_{\rm YM} = \frac{\zeta}{N_c L}$ can be interpreted in terms of the number tunneling events between different topological sectors in the system. Therefore, there is no surprise that it has the "wrong sign" as the relevant physics cannot be described in terms of propagating physical degrees of freedom but rather is described in terms of the tunneling events between different (but physically equivalent) topological sectors in the system.

III. CASIMIR-TYPE BEHAVIOR IN DEFORMED QCD

Up to this point the theory was formulated on $\mathbb{R}^3 \times S^1$ with small compactification size L for compact time coordinate S^1 and infinitely large space $\mathbb{R}^3$ describing three other dimensions. As explained in Sec. I, we are actually interested in behavior of the system when a space with large dimensions $\mathbb{R}^3$ receives some small modifications, for example the theory is defined in a ball $\mathbb{R}^3 \to \mathbb{B}^3$ with $\mathbb{L}$ being a very large size of the compact dimension of the sphere $\mathbb{S}^2$ which is a boundary of the ball $\mathbb{B}^3$. Such a modification can be thought of as the simplest way to model and test the sensitivity of our theory to arbitrary large distances, such as the size of our visible Universe determined by the Hubble constant $H/\Lambda_{\rm QCD} \sim 10^{-41}$. We want to know how the topological susceptibility of the system which describes the θ-dependent portion of the vacuum energy $E_{\rm vac}(\theta = 0)$ changes with slight variation of the size of the system. We assume that $\mathbb{L} \sim H^{-1} \sim$ 10 Gpc is much larger than any other scales of the problem. Essentially, we want to see if our deformed QCD model with a mass gap m_σ predicts an exponential scaling typical for a free massive particle,

$$\Delta E(\mathbb{L}) \equiv [E(\mathbb{B}^3) - E(\mathbb{R}^3)] \sim \exp(-m_\sigma\mathbb{L}), \qquad (8)$$

or if it demonstrates a Casimir-type behavior,

$$\Delta E(\mathbb{L}) \equiv [E(\mathbb{B}^3) - E(\mathbb{R}^3)] \sim \frac{1}{\mathbb{L}} + \mathcal{O}\left(\frac{1}{\mathbb{L}}\right)^2. \qquad (9)$$

If we did not have a nondispersive contribution in our system, we would immediately predict the behavior (8) as the only available option for a gapped theory in close analogy with conventional Casimir computations for a massive particle $\Delta E(\mathbb{L}) \sim \exp(-m\mathbb{L})$; see, e.g., review paper [17]. However, our system is much richer, more complicated, and more interesting, as it exhibits a nondispersive term resulting from degeneracy of topological sectors in gauge theory as discussed in the text. This contact term, being unrelated to any physical degrees of freedom, may provide different scaling properties since conventional dispersion relations do not dictate its behavior at very large distances. As we shall argue below, the deformed QCD indeed exhibits the Casimir-type behavior (9) in drastic contrast with the conventional viewpoint represented by Eq. (8). As we reviewed in Sec. I, we interpret a tiny deviation of the θ-dependent vacuum energy $E_{\rm vac}$ in an expanding universe (in comparison with Minkowski space-time) as a main source of the observed DE. The Casimir-type behavior (9) plays a key role in possibility of such an identification.

We start our discussions in Sec. III A with conventional four-dimensional instanton computations [22] in which infrared regularization for some gauge modes is required and achieved by putting the system into a sphere with finite radius $\mathbb{L}$. It allows us to compute powerlike corrections to the standard instanton density [22]. However, the corresponding corrections being computed for a fixed instanton size ρ cannot be interpreted as a physically observable quantity because the integral $\int d\rho$ over large-size instantons

[3] In the context of this paper, the "degeneracy" implies the existence of winding states $|n\rangle$ constructed as follows: $\mathcal{T}|n\rangle = |n+1\rangle$. In this formula the operator $\mathcal{T}$ is the large gauge transformation operator which commutes with the Hamiltonian $[\mathcal{T}, H] = 0$. The physical vacuum state is *unique* and constructed as a superposition of $|n\rangle$ states. In quantum field theory approach the presence of n different sectors in the system is reflected by summation over $n \in \mathbb{Z}$ in definition of the path integral in QCD. It should not be confused with the conventional term "degeneracy" when two or more physically *distinct* states are present in the system.

diverges for this system when semiclassical approximation for large ρ breaks down. Nevertheless, this example explicitly shows when and why a Casimir-type correction (to conventional formula computed in infinite $\mathbb{R}^4$ space) emerges.

Next, we compute a similar correction for the "deformed QCD" model in Sec. III B, wherein a Casimir-type correction also appears, resulting from the same physics related to topological sectors of the theory. In contrast with the previous case, the correction computed in this system is a physically "observable" quantity as it represents the vacuum energy of the system. Indeed, the tunneling transitions in this case are described by weakly coupled monopoles, such that semiclassical computations of the vacuum energy (3) and (5) expressed in terms of the density ζ of pseudoparticles are fully justified. The size of pseudoparticles (fractionally charged monopoles) which describe the tunneling events in this model is fixed by construction; see Refs. [20,21] for the details.

A. Casimir-type corrections for four-dimensional instantons

Our goal here is to study a powerlike correction to the instanton density described in the classic paper [22]. As such, we adopt 't Hooft's notation and, in particular, use the same background-dependent gauge $C_4 = \mathcal{D}_\mu A_\mu^{\mathrm{a\ qu}}$, which drastically simplifies all computations. Essentially, the problem is reduced to analysis of the normalization factors for a finite number of zero modes (eight for $SU(2)$ gauge group) in this gauge wherein the system is defined in a sphere with large but finite radius $\mathbb{L}$. Essentially, we follow the construction described in section XI of Ref. [22]. The corresponding normalization factor explicitly enters the expression for the instanton density as it accompanies the integration over collective variables. The contribution from nonzero modes does not exhibit such corrections; see the few comments on this issue at the end of this section. We now concentrate on the zero modes and powerlike corrections which accompany the normalization factors if the system is defined on a large but finite space $\mathbb{B}^4_{\mathbb{L}}$ (four- dimensional interior of a ball of radius $\mathbb{L}$) rather than an infinite space $\mathbb{R}^4$.

We start with four translational zero modes which have the form

$$A_\mu^{\mathrm{a\ qu}}(\nu) \sim \eta_{a\mu\nu}(1+r^2)^{-2}, \qquad \nu = 1, \ldots, 4, \qquad (10)$$

where we literally use 't Hooft's notations for $\eta_{a\mu\nu}$ symbols and dimensionless coordinate $r^2 = x_\mu^2$ measured in units of $\rho = 1$. Computing the corresponding correction factor due to the translation zero modes κ_{tr}, we have

$$\kappa_{\mathrm{tr}} \equiv \frac{\int_0^{\mathbb{L}} d^4x[A_\mu^{\mathrm{a\ qu}}(\nu)]^2}{\int_0^{\infty} d^4x[A_\mu^{\mathrm{a\ qu}}(\nu)]^2} \simeq \left[1 - \frac{3}{\mathbb{L}^4} + \mathcal{O}\left(\frac{1}{\mathbb{L}^6}\right)\right]. \qquad (11)$$

The corresponding correction factor to the instanton density has powerlike correction as anticipated. As a result of additional rotational symmetry, one should expect, in general, $\mathbb{L}^{-2}$ corrections, while translation zero modes lead to a much smaller correction $\sim\mathbb{L}^{-4}$ as Eq. (11) shows. It will be neglected in what follows. Dilaton and global gauge rotations lead to $\sim\mathbb{L}^{-2}$ as we discuss below.

For the dilaton zero mode

$$A_\mu^{\mathrm{a\ qu}} \sim \eta_{a\mu\nu}x^\nu(1+r^2)^{-2}, \qquad (12)$$

a similar formula reads

$$\kappa_{\mathrm{dil}} \equiv \frac{\int_0^{\mathbb{L}} d^4x[A_\mu^{\mathrm{a\ qu}}(\nu)]^2}{\int_0^{\infty} d^4x[A_\mu^{\mathrm{a\ qu}}(\nu)]^2} \simeq \left[1 - \frac{3}{\mathbb{L}^2} + \mathcal{O}\left(\frac{1}{\mathbb{L}^4}\right)\right], \qquad (13)$$

such that the correction to the instanton density is proportional to $\sqrt{\kappa_{\mathrm{dil}}} \simeq (1 - \frac{3}{2\mathbb{L}^2})$.

Computing the corresponding contribution due to three zero modes related to global gauge rotations requires much more refined analysis as explained in Ref. [22]. This is due to the specific features of the background-dependent gauge $C_4 = \mathcal{D}_\mu A_\mu^{\mathrm{a\ qu}}$ when the corresponding three modes are pure gauge artifact. As shown in Ref. [22], the corresponding contribution is finite but very sensitive to the infrared regularization determined by the size R of large sphere. The corresponding contribution to the instanton density is $\sim(\lambda_4 V)^{3/2}$ where V is the four volume, while $\lambda_4 \sim V^{-1}$ is defined as follows

$$\lambda_4 = \frac{\int_V d^4x[\psi_\mu^a(b)]^2}{\int_V d^4x[\psi^a(b)]^2}, \qquad b = 1, 2, 3,$$

$$\psi^a(b) = \eta_{a\mu\nu}\bar{\eta}_{b\mu\lambda}\frac{x^\nu x^\lambda}{(1+x^2)}, \qquad (14)$$

$$\psi_\mu^a(b) = \mathcal{D}_\mu\psi^a(b) = \eta_{a\lambda\mu}\bar{\eta}_{b\lambda\nu}\frac{x^\nu}{(1+x^2)^2}.$$

The corresponding powerlike corrections can be computed in a similar manner to the other zero modes, except that we must retain the regularization since the denominator above diverges as $\sim V$. So we have the two correction factors

$$\kappa_{\mathrm{num}} \equiv \frac{\int_0^{\mathbb{L}} d^4x[\psi_\mu^a(b)]^2}{\int_0^{\infty} d^4x[\psi_\mu^a(b)]^2} \simeq \left[1 - \frac{3}{\mathbb{L}^2} + \mathcal{O}\left(\frac{1}{\mathbb{L}^4}\right)\right],$$

and

$$\kappa_{\mathrm{den}} \equiv \frac{V(R)}{V(\mathbb{L})}\frac{\int_0^{\mathbb{L}} d^4x[\psi^a(b)]^2}{\int_0^{R} d^4x[\psi^a(b)]^2} \simeq \left[1 - \frac{4}{\mathbb{L}^2} + \mathcal{O}\left(\frac{1}{\mathbb{L}^4}\right)\right].$$

The fraction, $V(R)/V(\mathbb{L})$, is the correction to V in the instanton density factor and is included here so that we can take the regularization $R \to \infty$. The combined gauge rotation correction factor is then

$$\kappa_{\mathrm{rot}} \equiv \frac{\kappa_{\mathrm{num}}}{\kappa_{\mathrm{den}}} \simeq \left[1 + \frac{1}{\mathbb{L}^2} + \mathcal{O}\left(\frac{1}{\mathbb{L}^4}\right)\right], \qquad (15)$$

such that the correction to the instanton density is proportional to $(\kappa_{\mathrm{rot}})^{3/2} \simeq (1 + \frac{3}{2\mathbb{L}^2})$. Accidentally, for $SU(2)$

gauge group the leading $\mathbb{L}^{-2}$ corrections from the dilation (13) and global gauge rotations (15) exactly cancel each other. This accidental cancellation does not hold for the general $SU(N)$ gauge group when power of $\kappa_{\rm rot}$ enters the instanton density with a different power.

We remark here that the technique used in Ref. [22] is essentially a variational approach wherein the boundary conditions are implemented implicitly rather than explicitly. It allows us to use all the zero modes, (10), (12), and (14), as well as the standard classical instanton solution in the original form defined on $\mathbb{R}^4$ in which the conformal invariance is a symmetry of the system. So in this approach, neither the instanton itself nor its zero modes (10), (12), and (14) are solutions of the equation of motions which vanish at the boundary. This approach has been tested in many follow- up papers, and we adopt it in the present work using the same technique in the next section. We also point out that the conformal invariance is explicitly broken in the one-instanton sector by the size of the instanton ρ, such that corrections take the form $(\frac{\rho^2}{\mathbb{L}^2})^n$. It is restored by the integration $\int d\rho$. However, in this paper we are interested in the computation in the one-instanton sector only when dimensional parameter ρ is explicitly present in the system, and it is small and fixed.

The important message here is that such kind of power correction do appear in general. The source of these corrections is a long range tail of zero modes. We can not derive a definite conclusion from these computations because the integral over large size instantons $\int d\rho$ diverges and the semiclassical approximation breaks down. However, the same problem studied in the deformed QCD model considered in Sec. III B does not suffer from such deficiencies as semiclassical computations that are under complete theoretical control. Thus, a Casimir-like correction to the monopole fugacity ζ in this model is explicitly translated to the correction to the vacuum energy density and topological susceptibility (7), supporting (9) and in huge contrast with naive expectation (8). It is important that the source of the corrections in the deformed QCD model is the same as in undeformed QCD considered here, and that source is the long-range tails of the zero modes, which lead to large-distance sensitivity. The only difference is that the role of the instanton size ρ in computations above in the one-instanton sector is played by the inverse monopole's mass m_W^{-1} in Sec. III B. Because it is a true scale of the problem, however, m_W^{-1} is not integrated over as ρ is.

B. Casimir-type corrections for three-dimensional monopoles

We now turn to the deformed gauge theory described in Refs. [20,21] wherein the low-energy behavior is given by a $U(1)^N$ Coulomb gas of monopoles in Euclidean $\mathbb{R}^3$. Basically, we want to understand the dependence of the monopole fugacity, ζ, which comes out of the measure transformation to collective coordinates, on the size of the system, $\mathbb{L}$. In this case, as in the previous section, we consider the interior of a sphere of large but finite radius $\mathbb{L}$. There are four zero modes present in this system: three translations since the monopoles are in $\mathbb{R}^3$, no dilations since the monopole size is fixed by the symmetry-breaking scale in this model m_W, and one gauge rotation since the gauge group for a given monopole is $U(1)$. As in Ref. [22], we work in a regular gauge to remain sensitive to the large-distance physics. The monopole solution in the "hedgehog" regular gauge is given by

$$v_\mu^a(x) = \epsilon_{\mu\nu a}\frac{x^\nu}{|x|^2}\left[1 - \frac{m_W|x|}{\sinh(m_W|x|)}\right], \qquad (16)$$

$$\phi^a(x) = \frac{x^a}{|x|^2}[m_W|x|\coth(m_W|x|) - 1], \qquad (17)$$

where we adapted notations from Refs. [23,24], treating the monopole measure in supersymmetric Yang-Mills theory. In formula (16), v_μ^a denotes the three spacial gauge fields for the classical solution, and ϕ^a the gauge field in the compact time direction (the "Higgs" field in this model) when all fields can be combined in a single four-dimensional field v_m.

We then want to compute the correction factors for the collective coordinate measure coming from these four zero modes when the system is defined in a large but finite sphere. We closely follow the 't Hooft's treatment [22] presented in previous Sec. III A. We start by considering the translation modes defined by the spacial derivative of the classical monopole solution (16)

$$Z_m^a(\nu) = -\partial_\nu v_m^a(x-z) + \mathcal{D}_m v_\nu^a = v_{m\nu}^a, \qquad (18)$$

where the second term on the right-hand side of Eq. (18) is necessary to keep $Z_m^a(\nu)$ in the background gauge; see Refs. [23,24] for the details. This leads us to the following expression for the correction factor due to the translation zero modes

$$\kappa_{\rm tr} \equiv \frac{\int_0^{\mathbb{L}} d^4x[Z_m^a(\nu)]^2}{\int_0^\infty d^4x[Z_m^a(\nu)]^2} \simeq \left[1 - \frac{1}{m_W\mathbb{L}} + \mathcal{O}\left(\frac{1}{\mathbb{L}^2}\right)\right]. \qquad (19)$$

Next we consider the gauge rotation zero mode. As in the previous section, the contribution to the collective coordinate measure, and so the monopole fugacity, is $\sim(\lambda V)^{\frac{1}{2}}$ where V is the three-volume and λ is given by

$$\lambda = \frac{\int_V d^3x[B_\mu^a]^2}{\int_V d^3x[\phi^a]^2} \quad B_\mu^a = \frac{1}{2}\epsilon_{\mu\nu\rho}\partial_\nu v_\rho^a = \mathcal{D}_\mu\phi^a. \qquad (20)$$

Again, the denominator diverges as $\sim V$ and we look at the two correction factors

$$\kappa_{\rm num} \equiv \frac{\int_0^{\mathbb{L}} d^3x[B_\mu^a]^2}{\int_0^\infty d^3x[B_\mu^a]^2} \simeq \left[1 - \frac{1}{m_W\mathbb{L}} + \mathcal{O}\left(\frac{1}{\mathbb{L}^2}\right)\right],$$

and

$$\kappa_{\rm den} \equiv \frac{V(R)}{V(\mathbb{L})}\frac{\int_0^{\mathbb{L}} d^3x[\phi^a]^2}{\int_0^R d^3x[\phi^a]^2} \simeq \left[1 - \frac{3}{m_W\mathbb{L}} + \mathcal{O}\left(\frac{1}{\mathbb{L}^2}\right)\right].$$

The total correction factor for the gauge rotation mode is then

$$\kappa_{\rm rot} \equiv \frac{\kappa_{\rm num}}{\kappa_{\rm den}} \simeq \left[1 + \frac{2}{m_W\mathbb{L}} + \mathcal{O}\left(\frac{1}{\mathbb{L}^2}\right)\right], \tag{21}$$

and, therefore, the total correction to the monopole fugacity from (20) is $\sqrt{\kappa_{\rm rot}} \simeq (1 + \frac{1}{\mathbb{L}})$. Assembling the total correction to the fugacity,

$$\kappa_{\rm tr}^{3/2}\kappa_{\rm rot}^{1/2} \simeq \left[1 - \frac{1}{2m_W\mathbb{L}} + \mathcal{O}\left(\frac{1}{\mathbb{L}^2}\right)\right]. \tag{22}$$

Thus, the deformed QCD, when put on a manifold with a boundary, receives some corrections to the monopole fugacity compared to Minkowski space that are powerlike in the manifold size. The correction (22) to the monopole fugacity leads immediately to the same correction to the topological susceptibility and so the background energy density through the relation (7). To be more precise,

$$\zeta(\mathbb{L}) = \zeta \cdot \left[1 - \frac{1}{2m_W\mathbb{L}} + \mathcal{O}\left(\frac{1}{\mathbb{L}^2}\right)\right], \tag{23}$$

where ζ is the monopole fugacity that enters the relation (7) computed in infinite Minkowski space. We emphasize that the energy density changes in the bulk of space-time, not only in the vicinity of the boundaries, similar to the Casimir effect when the bulk energy density changes as a result of merely presence of the boundary. To reiterate, the deformed QCD, despite the presence of a mass gap, displays a surprising Casimir-like sensitivity to large-distance boundaries, such that the energy density differs from the Minkowski space value by $\Delta E \sim \frac{1}{m_W\mathbb{L}}$. Again, this is in contrast to the naive expectation based on analyzing the physical degrees of freedom, $\Delta E \sim e^{-m\mathbb{L}}$ with $m \sim m_\sigma$ being the lowest mass scale of the problem (8).

C. A few general comments

Computations of the Casimir corrections presented above were based on an analysis of the zero modes when the corresponding normalization factor explicitly enters the instanton or monopole density. Now, we want to present some arguments suggesting that a correction due to the nonzero mode contributions can be neglected, and, therefore, it cannot cancel the zero modes contribution. Indeed, the computation of the nonzero mode contribution is reduced to analysis of the phase shifts in the scattering matrix which cannot change the normalization of the wave function, itself, as the only changes that occur are the phase shifts. An absolute normalization is dropped from the final formula for the instanton or monopole density when the ratio of the eigenvalues is considered. This argument is consistent with observation that nonzero mode contribution depends on matter context of the theory as it varies when massive scalar of spinor fields in different representations are part of the consideration. At the same time, the Casimir-type corrections computed above are exclusively due to the gauge portion of the theory, not its matter context. Indeed, these Casimir corrections were derived in pure gluodynamics. So, it is difficult to imagine how a Casimir correction to nonzero mode contribution (if it is nonzero) may cancel a Casimir-type correction originated from analysis of gauge zero modes.

We also comment that the correction $\mathbb{L}^{-1}$ occurs as a manifestation of a slow powerlike decay of the zero modes in the background of a topologically nontrivial gauge configuration. It should be contrasted with conventional behavior of zero modes with a mass gap present in the system from the very beginning (for example, the well-studied problem of a double-well potential). In the former case, the zero modes decay according to the power law and lead to the Casimir-type correction, while in the later case, the zero modes are well localized configurations that decay exponentially fast at large distances and cannot be sensitive to large-distance physics. The mass gap is present for all physical degrees of freedom in both models. However, in the former case the mass gap emerges as a result of the same instanton or monopole dynamics, while in the latter a mass gap was present in the system from the very beginning and it was not associated with any instanton or monopole dynamics. QCD obviously belongs to the former case, and we therefore expect this effect will persist in strongly coupled QCD.

Next, our computations of the Casimir correction to the instanton or monopole density are based on assumption of the dilute gas approximation. This is enforced in Sec. III A by a finite instanton size ρ which is kept fixed and small. On other hand, the semiclassical approximation in Sec. III B is automatically justified due to parametrically small fugacity ζ, and total neutrality in this system is automatically achieved as long as the size of the system $\mathbb{L}$ is much larger than the Debye screening length m_σ^{-1} [see (4)]. In other words, we assume $\mathbb{L} \gg m_\sigma^{-1}$ such that neutrality of the system is automatically satisfied with exponential accuracy. The finite size of the manifold does not spoil this neutrality if condition $\mathbb{L} \gg m_\sigma^{-1}$ is satisfied. Furthermore, the computation of the monopole's fugacity ζ and corresponding corrections (23) can be performed without taking into account the interaction of a monopole with other particles from the system as it would correspond to higher-order corrections in density expansion $\sim\zeta^2$. This is precisely the procedure which was followed in the original computations by Polyakov in Ref. [25] and in the deformed QCD model in Ref. [20] at weak coupling.

Also, we emphasize that in the variational approach developed in Ref. [22], neither the classical solution nor the corresponding zero modes vanish at the boundary of a finite-size manifold. The constraints related to the finite-size $\mathbb{L}$ of the manifolds are accounted for implicitly rather than explicitly in this approach. In particular, one should not explicitly cut off the classical action of the configuration as a result of finite-size $\mathbb{L}$, where instanton or monopole is defined as this contribution is implicitly taken into account by variational approach. However, even if we use an explicit cutoff for the classical solution (as some people suggest), it still cannot cancel the zero mode corrections as these terms have different behavior in N. Indeed, the correction to the classical solution would be one and the same for any N, while corrections due to zero modes depend on N as correction (21) counts the number of gauge rotations for $SU(N)$ gauge theory.

Finally, it is quite possible that we overlooked some other possible corrections (for example, some corrections due to the boundaries which may occur in close vicinity of these boundaries). We emphasize that our main result is not a computation of a specific coefficient in front of the correction to fugacity in Eq. (23). Rather, our main point is that these type of corrections do occur in a system with a gap, and it is very difficult to imagine that some boundary corrections may mysteriously cancel these computed bulk corrections (as some people suggest). Therefore, we present below some arguments and examples suggesting that Casimir-type behavior in gauge theories is, in fact, quite generic rather than a peculiar feature of the system.

IV. TOPOLOGICAL SECTORS AND THE CASIMIR CORRECTION IN QCD

In this section we want to present a few generic arguments suggesting that the emergence of a Casimir-like behavior is not an accident and not a computational error. Rather, the effect has deep theoretical roots as argued in Ref. [26]. We review these arguments starting with an analogy with the well-known Aharonov-Casher effect as formulated in Ref. [27]. The relevant part of that work can be stated as follows. If one inserts an external charge into a superconductor when the electric field is exponentially suppressed $\sim \exp(-r/\lambda)$ with λ being the penetration depth, a neutral magnetic fluxon will be still sensitive to an inserted external charge at arbitrary large distance. The effect is purely topological and nonlocal in nature. The crucial point is that this phenomenon occurs, in spite of the fact that the system is gapped, due to the presence of different topological states in the system. We do not have the luxury of solving a similar problem in strongly coupled four-dimensional QCD analytically. However, one can argue that the role of the "modular operator" of Ref. [27] (which is the key element in the demonstration of long-range order) is played by large gauge transformation operator $\mathcal{T}$ in QCD, which also commutes with the Hamiltonian $[\mathcal{T}, H] = 0$, such that our system must be transparent to topologically nontrivial pure gauge configurations, similar to transparency of the superconductor to the "modular electric field"; see Ref. [26] for the details.

We interpret the computational results in a number of systems where Casimir-like corrections have been established as a manifestation of the same physics that can be described in terms of the operator $\mathcal{T}$. We should mention that there are a few other systems, such as topological insulators, where a topological long-range order emerges in spite of the presence of a gap in the system. We refer to Ref. [26] for relevant references and details.

There are a number of simple systems in which the Casimir-type behavior $\Delta E \sim \mathbb{L}^{-1} + \mathcal{O}(\mathbb{L})^{-2}$ has been explicitly computed. In all known cases, this behavior emerges from nondispersive contributions when the dispersion relations do not dictate the scaling properties of this term.

The first example is an explicit computation [28] in exactly solvable two-dimensional QED, defined in a box size $\mathbb{L}$. The model has all elements crucial for the present work: a nondispersive contact term which emerges due to the topological sectors of the theory. This model is known to be a theory of a single physical massive field. Still, one can explicitly compute $\Delta E \sim \mathbb{L}^{-1}$ in contrast with naively expected exponential suppression, $\Delta E \sim e^{-\mathbb{L}}$ [28]. Another piece of support for a powerlike behavior is an explicit computation in a simple case of Rindler space-time in four-dimensional QCD [3,18,29], where Casimir-like corrections have been computed using the unphysical Veneziano ghost, which effectively describes the dynamics of the topological sectors and the contact term when the background is slightly modified. Thus, powerlike behavior is not a specific feature of two-dimensional physics, as some people (incorrectly) interpret the results of Ref. [28].

Our next example is the two-dimensional CP^{N-1} model formulated on finite interval with size $\mathbb{L}$ [30]. In this case, one can explicitly see emergence of $\Delta E \sim \mathbb{L}^{-1}$ in large N limit in close analogy to our case (23) where a theory has a gap but, nevertheless, exhibits the powerlike corrections. The correction computed in Ref. [30] also comes from a nondispersive contribution that cannot be associated with any physical propagating degrees of freedom, similar to our case (23).

Powerlike behavior $\Delta E \sim \mathbb{L}^{-1}$ is also supported by recent lattice results [31]. The approach advocated in Ref. [31] is based on the physical Coulomb gauge, in which the nontrivial topological structure of the gauge fields is represented by the so-called Gribov copies leading to a strong infrared singularity. Thus, the same Casimir-like scaling emerges in a different framework where the unphysical Veneziano ghost (used in Refs. [3,18,29]) is not even mentioned.

The very same conclusion also follows from the holographic description of the contact term presented in Ref. [26]. The key element for this conclusion follows from the fact that the contact term in holographic description is determined by the massless Ramond-Ramond gauge field defined in the bulk of five-dimensional space.

Therefore, it is quite natural to expect that the massless Ramond-Ramond field in the holographic description leads to powerlike corrections when the background is slightly modified.

To avoid any confusion with terminology, we follow Ref. [26] and call this effect the "topological Casimir effect," where no massless degrees of freedom are present in the system but, nevertheless, the system itself is sensitive to arbitrary large distances. It is very different from the conventional Casimir effect, where massless physical photons are responsible for powerlike behavior. From the holographic viewpoint discussed in Ref. [26], the "topological Casimir effect" in our physical space-time can be thought of as the conventional Casimir effect in multidimensional space when a massless propagating R-R field in the bulk is responsible for this type of behavior, although this field is not a physical asymptotic state in our four-dimensional world.

V. CONCLUSION AND FUTURE DIRECTIONS

We tested the sensitivity of a deformed QCD model with nontrivial topological features to arbitrary large distances. A naive expectation based on dispersion relations dictates that a sensitivity to very large distances must be exponentially suppressed (8) when the mass gap is present in the system. However, we argued that along with the conventional dispersive contribution, there exists a nondispersive contribution, not related to any physical propagating degrees of freedom. This nondispersive (contact) term with the "wrong sign" emerges as a result of topologically nontrivial sectors and can be explicitly computed in our model. The variation of this contact term with variation of the background leads to a powerlike "topological Casimir effect" (9) in accordance with other arguments presented in Sec. IV and in contrast with the naively expected exponential suppression (8).

The "topological Casimir effect" in QCD, if confirmed by future analytical and numerical studies, may have profound consequences for understanding of the expanding FLRW Universe we live in. We already mentioned in Sec. I that the observed DE (1) may in fact be just a manifestation of this "topological Casimir effect" without adjusting any parameters. In the adiabatic approximation, the Universe expansion can be modeled as a slow process in which the size of the system adiabatically depends on time $\mathbb{L}(t)$ which leads to extra energy as equations (9) and (23) suggest. Such a model is obviously consistent with observations if $\mathbb{L}(t)$ is sufficiently large [32]. We do not insist that this is the model of our Universe. Rather, we claim that if the effect persists in strongly coupled QCD, the energy density which cannot be identified with any physical propagating degrees of freedom is sensitive to arbitrary large distances as a result of nontrivial topological features of QCD. Different geometries (such as the FLRW Universe) obviously would lead to different coefficients. Nonetheless, the important message from these computations in our simplified model is that the energy density in the bulk is sensitive to arbitrary large distances comparable with the visible size of the universe and that this sensitivity comes not from any new physics but simply from the proper treatment of the topological structure of QCD.

We add that a comprehensive phenomenological analysis based on this idea has been recently performed in Ref. [33], where comparison with current observational data including SnIa, BAO, CMB, and BBN has been presented; see also Refs. [29,34–38] with related discussions. The conclusion was that the model (1) is consistent with all presently available data, and we refer the reader to these papers on analysis of the observational data.

Finally, what is perhaps more remarkable is the fact that the "topological Casimir effect" which is the subject of this work can be, in principle, experimentally tested in heavy ion collisions, where a similar environment can be achieved; see Refs. [18,19] for the details. In particular, the $\mathcal{P}$ odd correlations observed at Relativistic Heavy Ion Collider and LHC have been interpreted in Refs. [18,19] as a result of the long-range order represented by the "topological Casimir effect".

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