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Brans-Dicke theory and the Pioneer anomaly

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Scalar-tensor theory offers the possibility of a modification of Newtonian gravity due to the presence of a 4d scalar dilaton field. The prototypical version of such a theory, massless Brans-Dicke theory, is considered here in the Einstein frame representation. The acceleration of a test mass is obtained from the exact 4d Xanthopoulos-Zannias solutions with spherical symmetry. The deviation of this acceleration from the pure Newtonian gravitational acceleration is examined to see if it can account for the anomalous Pioneer acceleration, while satisfying solar system constraints. Theoretical considerations, along with limits inferred from Pioneer 10 data, suggest that Brans-Dicke gravity could account for no more than a small fraction of the Pioneer anomaly, so that a complete explanation of the anomaly must lie elsewhere.

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I. INTRODUCTION

It is possible that some extension or generalization of general relativity (GR), such as scalar-tensor theory, may give rise to a modified form of gravity with observable consequences, while satisfying existing solar system constraints. A prototypical theory of this type is the massless Brans-Dicke (BD) theory[1]. An extended form of gravitation theory, in a weak field, nonrelativistic limit, can exhibit observable deviations from Newtonian gravity, which could provide guidance toward a more complete understanding of gravity. One such possible deviation from Newtonian gravity may reside in the Pioneer anomaly [2–4].

The Pioneer anomaly is a small anomalous, i.e., unmodeled, acceleration of the Pioneer 10 and 11 spacecraft that has been measured for large heliocentric distances of ~20–70 AU. This anomalous acceleration is inferred from an anomalous Doppler shift [2–4]. It is possible that this small anomalous Pioneer acceleration $\vec{a}_P$ may indicate a deviation from the Newtonian acceleration $\vec{a}_N$, and it had previously been taken to have been an essentially constant acceleration with a magnitude of $a_P = 8.74 \pm 1.33 \times 10^{-10}\ \mathrm{m/s^2}$. Recently, however, the analysis of larger data sets has supported the conclusion that the anomalous Pioneer acceleration $\vec{a}_P$ actually decreases with time with a temporal decay rate (jerk term) of magnitude $\dot{a}_P \approx 1.7 \times 10^{-11}\ \mathrm{m/s^2/yr}$ [5]. (This point of view has recently been challenged, however, see Refs. [6,7], for example.) This anomalous acceleration is seen to act on both the Pioneer 10 and 11 spacecraft, directed approximately sunward. In contrast, there appear to be no such anomalous accelerations exhibited by planetary motions.

In Refs. [8,9] the acceleration of a test mass in a spherically symmetric, static, weak field due to a source was examined in the Einstein frame representation of massless BD theory, making use of the exact Xanthopoulos-Zannias (X-Z) solutions [10] for a massless scalar field minimally coupled to the (Einstein frame) gravitational metric field. The acceleration of a test mass in the BD theory differs from the Newtonian acceleration, and depends upon the parameters of the BD theory and therefore the parameters of the X-Z solutions. We examine the possibility of whether this "anomalous" acceleration can account for the observed Pioneer anomaly, while satisfying existing solar system constraints on the BD theory, namely the constraint on the BD parameter ω_{BD}, where $\omega_{BD} > 40,000$ at the 2σ confidence level, and $\omega_{BD} > 21,000$ at the 3σ confidence level according to Ref. [11], and $\omega_{BD} > 9,000$ at the 95% confidence level according to Ref. [12].

A brief presentation of the Einstein frame representation of BD theory, along with the acceleration $\vec{a}$ of a test mass in the Newtonian limit, is presented in Sec. II. The results obtained here for $\vec{a}$ agree with those of Ref. [8], and are then used in Sec. III to establish a constraint equation for the X-Z parameters. It is concluded that this constraint, along with the constraint on ω_{BD}, cannot be satisfied simultaneously when the deviation $\Delta\vec{a} = \vec{a} - \vec{a}_N$ is identified with $\vec{a}_P$, and that the anomalous BD acceleration $\Delta\vec{a}$ can account for, at most, only a few percent of the anomalous Pioneer acceleration $\vec{a}_P$. On the other hand, in Sec. IV inferences made from Pioneer 10 data in conjunction with Cassini constraints on the Post-Newtonian parameter γ_{PPN} suggest even stronger constraints on a possible contribution to a_P due to Brans-Dicke gravity. A brief summary is presented in Sec. V.

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II. ACCELERATION OF A TEST MASS

The Jordan frame action for massless BD theory is given by Ref. [1] ($G = 1$)

$$S = \frac{1}{16\pi}\int d^4x\sqrt{\tilde{g}}\left\{\tilde{\phi}\,\tilde{R} + \frac{\omega_{BD}}{\tilde{\phi}}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}\right\} + S_m(\tilde{g}_{\mu\nu}). \tag{2.1}$$

A metric with signature $(+,-,-,-)$ is used and $\kappa = \sqrt{8\pi G} = 1/M_0$, where M_0 is the reduced Planck mass. A conformal transformation to the Einstein frame is given by Ref. [13]

$$\begin{aligned} &g_{\mu\nu} = \tilde{\phi}\tilde{g}_{\mu\nu}, \qquad g^{\mu\nu} = \tilde{\phi}^{-1}\tilde{g}^{\mu\nu}, \qquad \sqrt{g} = \tilde{\phi}^2\sqrt{\tilde{g}}, \\ &\phi = \sqrt{2a_0}\ln\tilde{\phi}, \qquad a_0 = \omega_{BD} + \frac{3}{2}, \end{aligned} \tag{2.2}$$

and the action in the Einstein frame then takes the form

$$S = \frac{1}{16\pi}\int d^4x\sqrt{g}\left\{R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right\} + S_m(\tilde{\phi}^{-1}g_{\mu\nu}). \tag{2.3}$$

(Here, ϕ is the scalar field used with the action and conventions used by Cai and Myung [13] differing by a factor of $1/2k$, $k = \kappa^2 = 8\pi G$ from the scalar field used by Xanthopoulos and Zannias. I.e., $\phi_{\text{X-Z}} = \frac{1}{\sqrt{2k}}\phi_{\text{C-M}} = \frac{1}{\sqrt{2}\kappa}\phi_{\text{C-M}}$. See, for example, Refs. [8–10,13].)

A classical test particle of mass m in the Einstein frame (EF) representation, described by the action

$$S_m = -\int m(g_{\mu\nu}u^\mu u^\nu)^{1/2}ds \quad \left(u^\mu = \frac{dx^\mu}{ds}\right), \tag{2.4}$$

is related to the corresponding (constant) mass m_0 in the Jordan frame (JF) by Ref. [8,14],

$$\begin{aligned} m &= A(\phi)m_0 = \tilde{\phi}^{-1/2}m_0, \\ A(\phi) &= \tilde{\phi}^{-1/2}(\phi) = e^{-\phi/(2\sqrt{2a_0})}, \end{aligned} \tag{2.5}$$

where $A(\phi)$ connects the EF and JF metrics by $\tilde{g}_{\mu\nu} = \tilde{\phi}^{-1}g_{\mu\nu} \equiv A^2(\phi)g_{\mu\nu}$. Therefore the EF mass m will generally have a spacetime dependence, since $\phi = \phi(x^\mu)$.

The "geodesic" equation for the test mass m in the EF representation can be written in the form[8]

$$\frac{du^\nu}{ds} + \Gamma^\nu_{\alpha\beta}u^\alpha u^\beta - \frac{1}{m}\partial_\mu m[g^{\mu\nu} - u^\mu u^\nu] = 0, \tag{2.6}$$

and from (2.5) there is a term involving $\partial_\mu(\ln m) = \partial_\mu(\ln A)$ which causes the particle to deviate from true geodesic motion in the EF when $A \neq \text{const}$. In the Newtonian limit (weak field, static limit, with nonrelativistic particle motion) (2.6) reduces to

$$\frac{d^2\vec{x}}{dt^2} = -\frac{1}{2}\nabla h_{00} - \nabla(\ln A), \tag{2.7}$$

where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ contains a small correction $h_{\mu\nu}$ to flat spacetime. We now consider $\phi = \phi(r)$ to be a static, weak field with a dependence upon the radial distance from some source mass M located at $\vec{x} = 0$, which generates a static, spherically symmetric metrical gravity field $g_{\mu\nu}(r)$, so that $h_{00} = h_{00}(r)$.

The acceleration $\vec{a} = d^2\vec{x}/dt^2$ has two contributions, one from the metric field $g_{\mu\nu}$, i.e., an acceleration $\vec{a}_g = -\nabla(\frac{1}{2}h_{00})$, and another from the EF scalar dilaton field, $\vec{a}_\phi = -\nabla(\ln A)$. The total acceleration of a test mass is therefore $\vec{a} = \vec{a}_g + \vec{a}_\phi$, where

$$\begin{aligned} \vec{a}_g &= -\nabla\left(\frac{1}{2}h_{00}\right) = -\hat{r}\partial_r\left(\frac{1}{2}h_{00}\right), \\ \vec{a}_\phi &= -\nabla(\ln A) = -\hat{r}\partial_r(\ln A). \end{aligned} \tag{2.8}$$

An "anomalous" acceleration $\Delta\vec{a}$ of a nonrelativistic test mass is interpreted as the deviation from the usual Newtonian acceleration $\vec{a}_N$, i.e., $\Delta\vec{a} = \vec{a} - \vec{a}_N = (\vec{a}_g + \vec{a}_\phi) - \vec{a}_N$. This anomalous acceleration can receive contributions from both the metric part $\vec{a}_g$ (when the metric is not the Schwarzschild metric) and the dilaton part $\vec{a}_\phi$.

III. APPLICATION TO BRANS-DICKE THEORY

Let us now consider the motion of a test mass in the background of a static, spherically symmetric spacetime of massless BD theory. The vacuum solutions of the BD theory, due to a central mass M, in the Einstein frame representation, are provided by the Xanthopoulos-Zannias (X-Z) solutions [10], which were also studied by Cai and Myung [13]. These solutions describe the region exterior to some neutral, nonrotating astrophysical object of BD theory, and we want to look at the asymptotic limit, far away from the location of the central mass, i.e., regions where $r \gg r_0$, where r_0 is a constant. (For the case of a point mass M, the point $r = r_0$ is a naked singularity, except in the case of the Schwarzschild limit, where the solution coincides with the Schwarzschild solution [10,13]. However, the solution inside an astrophysical object will not be a vacuum solution, so that there is no physical singularity.) For an astrophysical object like a star, we have $r/r_0 \gg 1$ for all regions outside the surface where the X-Z solutions apply.

The static neutral solutions, with isotropic coordinates, are discussed in Ref. [13], and are presented here for the special 4d case:

$$ds^2 = e^f dt^2 - e^{-h}(dr^2 + r^2 d\Omega^2), \tag{3.1}$$

$$e^f = g_{00} = \xi^{2\gamma}; \qquad \xi = \left(\frac{r - r_0}{r + r_0}\right), \tag{3.2}$$

$$e^{-h} = |g_{rr}| = \left(1 - \frac{r_0^2}{r^2}\right)^2\xi^{-2\gamma} = e^{-f}\left(1 - \frac{r_0^2}{r^2}\right)^2, \tag{3.3}$$

$$\phi = \pm\tilde{\gamma}\ln\xi = \sqrt{2a_0}\ln\tilde{\phi}; \quad \tilde{\gamma} = [4(1-\gamma^2)]^{1/2}, \tag{3.4}$$

$$A(\phi) = \tilde{\phi}^{-1/2} = e^{-\phi/(2\sqrt{2a_0})} = \xi^{-\Gamma/2}, \tag{3.5}$$

where r_0 and γ are integration constants ($r_0 > 0$), the constant a_0 is related to the Brans-Dicke parameter ω_{BD} by (2.2), $a_0 = \omega_{BD} + \frac{3}{2}$, with $A(\phi) = \tilde{\phi}^{-1/2} = \xi^{-\Gamma/2}$, and we have defined

$$\xi = \left(\frac{r - r_0}{r + r_0}\right) \le 1, \qquad \tilde{\gamma} = [4(1-\gamma^2)]^{1/2},$$
$$\Gamma = \pm|\Gamma| = \pm\frac{\tilde{\gamma}}{\sqrt{2a_0}} = \pm\left[\frac{2}{a_0}(1-\gamma^2)\right]^{1/2}. \tag{3.6}$$

These are the Einstein frame fields and solutions, with $0 \le \gamma \le 1$ for the description of physical (nonegative ADM mass) solutions.

Note: In the set of solutions presented in Ref. [10], only the solution with the + sign in (3.4), i.e., $\phi = +\tilde{\gamma}\ln\xi$, is presented. However, the second solution $\phi = -\tilde{\gamma}\ln\xi$ is seen to exist from the invariance of the action and equations of motion under the transformations $g_{\mu\nu} \to g_{\mu\nu}$, $\phi \to -\phi$. Therefore, if ϕ is a solution to the equations of motion, then so is $-\phi$ (see, for example, Refs. [13,15]). Therefore, ϕ can be either positive or negative, and the Brans-Dicke scalar $\tilde{\phi} = A^{-2} = \xi^{\Gamma} = \xi^{\pm|\Gamma|}$ can be either a decreasing or an increasing function of r and ξ. The Einstein frame mass m of a test particle is given by (3.5),

$$m = A(\phi)m_0 = m_0\xi^{-\Gamma/2}, \tag{3.7}$$

where m_0 is the (constant) Jordan frame mass.

We now consider the asymptotic forms of these solutions for which $r_0/r \ll 1$. In this case we have the following approximations to $O(r_0/r)$:

$$\xi \approx 1 - 2\frac{r_0}{r}, \qquad g_{00} \approx 1 - 4\gamma\frac{r_0}{r}, \qquad |g_{rr}| \approx 1/g_{00},$$
$$\frac{m^2}{m_0^2} = A^2(\phi(r)) \approx \left(1 + 2\Gamma\frac{r_0}{r}\right), \qquad A \approx \left(1 + \Gamma\frac{r_0}{r}\right). \tag{3.8}$$

These were applied to the X-Z solutions in Ref. [8] to obtain the acceleration $\vec{a}$ for a test mass in pure radial motion, with the result

$$\frac{d^2r}{dt^2} \approx -\left(\gamma - \frac{1}{2}\Gamma\right)\frac{2r_0}{r^2} = -\left(\gamma - \frac{1}{2}\Gamma\right)\frac{GM}{r^2}. \tag{3.9}$$

(The Schwarzschild radial coordinate R is related to the isotropic radial coordinate r by Refs. [10,16] $R = r(1 + r_0/r)^2$, with $R \to r$ asymptotically.) The Schwarzschild case is obtained for $\gamma = 1$, $\Gamma = 0$, and the identification $r_0 = GM/2$, where M is the mass of the gravitating object [10,16].

We can apply (3.8) to the acceleration equations of (2.7) and (2.8), where $\vec{a} = \vec{a}_g + \vec{a}_\phi$, with $\Delta\vec{a} = \vec{a} - \vec{a}_N$ to compute the anomalous acceleration $\Delta\vec{a}$. From (3.8), to $O(r_0/r)$, we have $\frac{1}{2}h_{00} = -2\gamma\frac{r_0}{r}$, $A = (1 + \Gamma\frac{r_0}{r})$, $A^{-1}\partial_r A = -\Gamma\frac{r_0}{r^2}$. Therefore

$$\vec{a}_g = -\nabla\left(\frac{1}{2}h_{00}\right) = -2\gamma\frac{r_0}{r^2}\hat{r}, \qquad \vec{a}_\phi = \Gamma\frac{r_0}{r^2}\hat{r},$$
$$\vec{a} = -\hat{r}\left(\gamma - \frac{1}{2}\Gamma\right)\frac{2r_0}{r^2} = -\hat{r}\left(\gamma - \frac{1}{2}\Gamma\right)\frac{GM}{r^2}. \tag{3.10}$$

Equations (3.9) and (3.10) yield the same result, and (3.10) shows the origin of each piece of the total acceleration $\vec{a}$. The anomalous acceleration is then

$$\Delta\vec{a} = \vec{a} - \vec{a}_N = \vec{a} - \left(-\hat{r}\frac{GM}{r^2}\right) = \left((1-\gamma) - \frac{1}{2}\Gamma\right)\frac{GM}{r^2}\hat{r}. \tag{3.11}$$

Now, denote the radial component of acceleration by $\mathcal{A} = \hat{r}\cdot\vec{a}$. Then the acceleration in (3.10) indicates

$$\mathcal{A} = \left(\gamma - \frac{1}{2}\Gamma\right)\mathcal{A}_N, \tag{3.12}$$

where $\mathcal{A}_N = -GM/r^2$ is the Newtonian acceleration. The anomalous radial acceleration is then

$$\Delta\mathcal{A} = \mathcal{A} - \mathcal{A}_N = \left(\gamma - 1 - \frac{1}{2}\Gamma\right)\mathcal{A}_N;$$
$$\frac{\Delta\mathcal{A}}{\mathcal{A}_N} = -\left((1-\gamma) + \frac{1}{2}\Gamma\right) \equiv K. \tag{3.13}$$

The value of K depends upon the value of the parameter γ.

To ascribe an extra radial inward acceleration to the Pioneer effect, we want $\mathcal{A}$ to be more negative than $\mathcal{A}_N$, and therefore, we want $\Delta\mathcal{A}/\mathcal{A}_N = K > 0$, and therefore require $((1-\gamma) + \frac{1}{2}\Gamma) < 0$. This implies that $\Gamma = -|\Gamma|$, since $(1-\gamma) \ge 0$. We therefore arrive at an algebraic equation to describe a portion of the anomalous Pioneer acceleration to the X-Z solution of BD theory,

$$(1-\gamma) - \frac{1}{2}|\Gamma| + K = 0, \tag{3.14}$$

with $\Gamma = -|\Gamma| = -[\frac{2}{a_0}(1-\gamma^2)]^{1/2}$, $\gamma \in [0, 1]$. In order to satisfy the solar system constraints on the BD theory, we require that $a_0 \approx \omega_{BD} \gtrsim 4\times 10^4$ at the 2σ confidence level [11], and therefore $|\Gamma| \lesssim \frac{1}{\sqrt{2}}\times 10^{-2}$. Now, let us approximate a space averaged value of $\Delta\mathcal{A}$ to be less than or comparable to the anomalous Pioneer acceleration $\mathcal{A}_P = -8.74\times 10^{-10}$ m/s^2, i.e., $K \sim \langle\Delta\mathcal{A}\rangle/\langle\mathcal{A}_N\rangle \lesssim \mathcal{A}_P/\langle\mathcal{A}_N\rangle$, where $\langle\mathcal{A}_N\rangle = \frac{1}{\Delta r}\int_{r_1}^{r_2}\mathcal{A}_N dr = -GM/r_1 r_2$, with $r_1 \approx 20$ AU and $r_2 = 70$ AU and $M = M_\odot$. This implies that K must take a value $K \lesssim 2\times 10^{-4}$. We therefore find that both $|\Gamma|$ and K are very small compared to unity, which by (3.14) implies that $(1-\gamma) \ll 1$. We can therefore define the small parameter $\varepsilon = (1-\gamma) \ll 1$, with $(1-\gamma^2)^{1/2} \approx \sqrt{2\varepsilon}$, and the algebraic equation (3.14) for the anomalous acceleration takes the form

$$\varepsilon - \frac{1}{\sqrt{a_0}}\sqrt{\varepsilon} + K = 0, \qquad (0 < \varepsilon \ll 1). \tag{3.15}$$

This serves to define the function $K(\varepsilon) = -\varepsilon + \sqrt{\varepsilon}/\sqrt{a_0}$. From (3.13) we see that a maximum value of the function $K(\varepsilon)$ will define a maximum anomalous radial acceleration $\mathcal{A}_{\max}/\mathcal{A}_N$. The maximum value of K, denoted $K_{\max}$, is found by determining the value of $\varepsilon = \varepsilon_0$ locating the local maximum of $K(\varepsilon)$ and then evaluating $K(\varepsilon)$ at ε_0 to obtain $K_{\max} = K(\varepsilon_0)$. Setting $dK/d\varepsilon|_{\varepsilon_0} = 0$ yields $\varepsilon_0 = 6.25 \times 10^{-6}$, and we find that $d^2K/d\varepsilon^2|_{\varepsilon_0} < 0$ showing that ε_0 does indeed locate the local maximum of $K(\varepsilon)$. Evaluating K at $\varepsilon = \varepsilon_0$ gives

$$K_{\max} = K(\varepsilon_0) = 6.25 \times 10^{-6}. \tag{3.16}$$

So we find that $(\Delta\mathcal{A}/\mathcal{A}_N)_{\max} \sim 6.25 \times 10^{-6}$. Setting $(\Delta\mathcal{A}/\mathcal{A}_N)_{\max} \sim (\langle\Delta\mathcal{A}\rangle/\langle\mathcal{A}_N\rangle)_{\max}$ and using $\mathcal{A}_P/\langle\mathcal{A}_N\rangle \sim 2 \times 10^{-4}$, yields the estimate

$$\frac{|\langle\Delta\mathcal{A}\rangle|_{\max}}{a_P} \sim \frac{6.25 \times 10^{-6}}{2 \times 10^{-4}} = 3.12 \times 10^{-2}, \tag{3.17}$$

at the 2σ confidence level, so that the anomalous acceleration provided by the X-Z solutions for BD theory can account for no more than a few percent of the anomalous Pioneer acceleration.

IV. EXPERIMENTAL LIMITS ON A POSSIBLE SCALAR-TENSOR COMPONENT IN THE APPARENT ACCELERATION OF PIONEER 10

For purposes of solar-system tests (weak-field limit), alternative theories of gravitation are usually expressed in terms of a set of post-Newtonian PPN parameters [17–19]. The Brans-Dicke coupling constant ω_{BD} is related to a single PPN parameter $\gamma_{\rm PPN}$, which describes the effect of solar gravity on photon trajectories, causing both a bending and a group delay of an electromagnetic wave. The two parameters are related by,

$$\omega_{BD} = \frac{1 - 2\gamma_{\rm PPN}}{\gamma_{\rm PPN} - 1}. \tag{4.1}$$

A limit on $\gamma_{\rm PPN}$ inferred from radio Doppler data, between Earth and the Cassini spacecraft, implies that ω_{BD} is greater than 9000 at a 95% confidence level [12]. When viewed as a measurement of possible scalar-tensor coupling, the value of $\gamma_{\rm PPN}$ is restricted to an interval less than or equal to one, the latter value in agreement with general relativity and no scalar field. The determination of $\gamma_{\rm PPN}$ from the Cassini data by Bertotti *et al.* [11] is $\gamma_{\rm PPN} = 1 + (2.1 \pm 2.3) \times 10^{-5}$, which yields a three-sigma lower bound on ω_{BD} of 21,000.

When expressed in solar-system barycenter SSB isotropic coordinates (x, y, z, t), used by JPL for solar-system dynamics, the expression for the Shapiro time delay [20] becomes [12],

$$\Delta r_{12} = \frac{1}{2}(1 + \gamma_{\rm PPN}){\rm R_g} \ln\left(\frac{r_1 + r_2 + r_{12} + (1 + \gamma_{\rm PPN}){\rm R_g}}{r_1 + r_2 - r_{12} + (1 + \gamma_{\rm PPN}){\rm R_g}}\right), \tag{4.2}$$

where $\rm R_g$ is the solar Schwarzschild radius given by $\rm 2GM_\odot/c^2$ and equal to 2953.25 m. The radius $\rm r_1$ is the magnitude of the position vector $\mathbf{r_1}$ between the SSB and transmitting station at $\rm t_1$, $\rm r_2$ is the magnitude of the position vector $\mathbf{r_2}$ between the SSB and a spacecraft at $\rm t_2$, and $\rm r_{12}$ is the magnitude of the vector $\mathbf{r_2} - \mathbf{r_1}$. The light time $\rm t_2 - t_1$ can be found iteratively as $\rm r_{12}/c$. As this is a post-Newtonian correction, all the distances inside the log term can be taken as Euclidian distances. The small term $(1 + \gamma_{\rm PPN})R_g$ inside the log is a correction for the bending of the photon trajectory by solar gravity. It is important only for ray paths that approach closely to the solar limb, where there is a relatively large bending. For purposes of calculating the total light time between transmission from Earth to the spacecraft (uplink) and the return from spacecraft to Earth (downlink), the Shapiro delay on the downlink is given by Eq. (4.2) with 1 replaced by 2 and 2 replaced by 3.

In order to model ranging and Doppler data accurately enough, additional terms are needed for the Lorentz factors $\sqrt{1 - \rm v_i^2/c^2}$ and some book keeping terms, as described by Moyer [21] with the aid of Eq. 11-7 and Eq. 13-47. For purposes of evaluating the contribution of a possible Brans-Dicke scalar coupling, we evaluate Eq. (4.2) for uplink and downlink, add the two together, and numerically differentiate twice to obtain an apparent acceleration imposed on the Doppler data. The effect is maximum near solar conjunction where the Pioneer 10 ray path approaches the Sun within 3 degrees of arc. The Pioneer 11 trajectory is highly inclined to the ecliptic, and the Shapiro time delay is significantly smaller. Also, there is no concern about trajectory perturbations from a weak scalar coupling, given that ω_{BD} is greater than 9000. This essentially solves the problem. If a scalar field is going to manifest itself by biasing the Pioneer Doppler data and producing an apparent acceleration, it must do so near solar conjunction. However, if a scalar field can be detected by Pioneer 10, it would more easily be detected by Cassini, with its dual-frequency radio data at X-Band and Ka-Band. Further, if a scalar field is just barely detected by Pioneer 10, there would be no evidence of a scalar field affecting the Pioneer 11 data. In order to illustrate the effect, we plot the acceleration in Fig. 1 over the last solar conjunction for Pioneer 10 in June 2001, even though no data were obtained at the 2001 conjunction. The signal from the spacecraft was too weak. Data are available however for the solar oppositions of March 2000 and March 2002, after which no more data are available. Data are available for earlier conjunctions [3], but excessive noise introduced by the solar corona into the S-Band carrier wave make any data inside 10 deg from the Sun

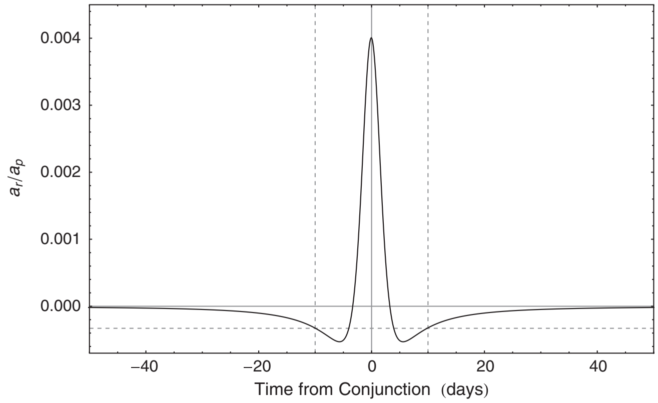

FIG. 1. Maximum three-sigma effect of a scalar coupling on Pioneer S-Band radio Doppler data, expressed as a fraction of the entire Pioneer anomaly at a magnitude of 8.74×10^{-10} m s^{-2}. The vertical dashed lines at plus and minus 10 days indicate the region around solar conjunction where noise from the solar corona make the data useless. The horizontal dashed line indicates the limit of a maximum effect on the Pioneer acceleration.

useless [3], or within a time interval of plus and minus 10 days on either side of conjunction.

Based on Fig. 1, we conclude that Cassini limits on scalar coupling limit the possible effect on an apparent Pioneer 10 acceleration to at most 0.03% of the entire Pioneer anomaly a_P.

V. SUMMARY

Massless Brans-Dicke theory has been considered in the Einstein frame representation, where exact, spherically symmetric solutions are provided by the Xanthopoulos-Zannias solutions of a scalar field ϕ minimally coupled to Einstein gravity. These X-Z solutions are characterized by the parameters $\gamma \in [0, 1]$ and $\Gamma = \pm[\frac{2}{a_0}(1 - \gamma)^2]^{1/2}$, where the constant a_0 is related to the BD parameter ω_{BD} by $a_0 = \omega_{BD} + 3/2$. We then consider the Newtonian limit for the acceleration of a test mass, finding that the net acceleration $\vec{a} = \vec{a}_g + \vec{a}_\phi$ gets contributions from both the metric field $g_{\mu\nu}$ and the dilaton field ϕ.

An anomalous radial acceleration $\Delta\mathcal{A}$, i.e., one deviating from the Newtonian gravitational field $\mathcal{A}_N$, is identified, with $\frac{\Delta\mathcal{A}}{\mathcal{A}_N} = -((1 - \gamma) + \frac{1}{2}\Gamma)$. When the X-Z solution parameters are constrained by existing solar system observations (constraints on ω_{BD}), we find that at the 2σ confidence level $\Delta\mathcal{A}$ can be, at most, only a few percent of the anomalous Pioneer acceleration $\mathcal{A}_P$.

Finally, using Pioneer 10 data in combination with the Cassini limits on the PPN parameter $\gamma_{\rm PPN}$, we argue that a maximum apparent acceleration a_r, occurring near solar conjunction, contributes no more than .03% of the entire Pioneer anomaly a_P at the 3σ confidence level.

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