

Towards a model of population of astrophysical sources of ultrahigh-energy cosmic raysO. E. Kalashev,¹ K. V. Ptitsyna,^{2,1} and S. V. Troitsky¹¹*Institute for Nuclear Research, Russian Academy of Sciences, 60th October Anniversary Prospect 7a, Moscow 117312, Russia*²*Physics Department, M.V. Lomonosov Moscow State University, Moscow 119899, Russia*

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We construct and discuss a toy model of the population of numerous nonidentical extragalactic sources of ultrahigh-energy cosmic rays. In the model, cosmic-ray particles are accelerated in magnetospheres of supermassive black holes in galactic nuclei, the key parameter of acceleration being the black-hole mass. We use astrophysical data on the redshift-dependent black-hole mass function to describe the population of these cosmic-ray accelerators, from weak to powerful, and confront the model with cosmic-ray data.

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I. INTRODUCTION

The origin of ultrahigh-energy (UHE) cosmic rays (CRs), that is, of cosmic particles with energies $E \geq 10^{19}$ eV, is presently unknown. However, there are numerous hints, both in data and in theory, which might help to constrain possible models of UHECR sources. In particular, the observation [1–3] of the spectrum steepening consistent with the Greisen-Zatsepin-Kuzmin [4,5] (GZK) cutoff, together with the global isotropy of the arrival directions (see e.g., Ref. [6]) and the fact that the UHE particles are not expected to be confined by the Milky Way magnetic field (see e.g., Ref. [7]), suggest that the bulk of CRs at these energies have extragalactic origin.¹ Next, the lack of clustering of arrival directions at small scales is a powerful tool [9] to constrain the number density of sources. In recent data of the Pierre Auger Observatory, no evidence for clustering at $E \geq 5 \times 10^{19}$ eV is seen [10] which translates into the number density of $n \geq 10^{-4}$ sources per cubic megaparsec, which means that sources of these extreme particles should not be exceptional and, most probably, some of them should be located relatively nearby. The latter fact gets further support, though limited by statistical significance, from the shape of the GZK feature in the spectrum, which does not seem to be very sharp (cf., Ref. [11]).

On the other hand, it is a nontrivial task to find particular astrophysical objects which could serve as UHECR accelerators. Even without a detailed modeling of the acceleration process, a number of simple estimates rule out many classes of potential sources. These simple criteria include in particular the geometrical (Hillas) criterion [12] and estimates of radiative energy losses of particles being accelerated (see e.g., Refs. [13,14]). Analysis of the modern astrophysical data demonstrates [15] that the combination of these constraints leaves just a few candidate classes of sources capable of acceleration of particles to UHE energies. Leaving aside large-scale structures where

interaction losses are expected to suppress the energy gain, the conventional diffusive (e.g., relativistic or nonrelativistic shock) acceleration may work only in ultrarelativistic jets, hot spots and lobes of exceptional active galaxies (powerful radio galaxies and blazars) which are not that abundant in the nearby Universe. For very special field configurations when synchrotron losses are suppressed and the curvature radiation dominates, possible acceleration sites also include gamma-ray bursts (GRBs) and the immediate neighborhood of supermassive black holes (SMBHs) in the galactic nuclei. While it is unclear whether these field configurations may be present in GRBs, recent ICECUBE results disfavor the GRB scenario anyway [16] (see, however, Refs. [17,18]). At this level of reasoning, the SMBH environment remains a viable option.

A natural assumption is that numerous UHECR sources are not identical—there should be less and more powerful accelerators where the maximal energies, injection spectra and fluxes of accelerated particles are different. Until now, numerous attempts to model the sources of UHECRs and to confront theoretical predictions with experiments often assumed that these parameters are fixed once and for the entire Universe (see e.g., Refs. [19,20] and numerous other works; see, however, Ref. [21] where acceleration in non-identical jets was considered). While, for numerous sources, the assumption of equal fluxes is well justifiable (in the sense that only the mean flux of a large sample of sources is important, and this mean flux does not vary significantly from one region in the Universe to another) and the injection spectrum is often fixed by the acceleration model, the maximal energies are expected to vary significantly. As it was recognized in Ref. [22], these variations affect the observable spectrum seriously. In this work, we attempt to present a toy model of numerous and different sources of UHECRs and, within certain assumptions, to confront it with the experimental data.

To this end, we choose a simple toy model of particle acceleration in the immediate vicinity of SMBHs put forward in Refs. [23,24]. The reason to choose this particular model is twofold. First, unlike many other models, it allows [15] for UHECR acceleration in numerous nearby

¹See, however, Ref. [8] and references therein where possible exceptions are discussed.

sources. Second, as we will see below, within some realistic assumptions, the acceleration capabilities of a source are determined by a single parameter, the SMBH mass. At the same time, the demography of SMBHs is well studied by astrophysicists, and we take this advantage to describe the population of sources easily.

The rest of the paper is organized as follows: In Sec. II, we give a brief review of the acceleration model of Refs. [23,24] and make a bridge between the parameters which determine the maximal energy of accelerated particles and the SMBH mass. In Sec. III, we discuss the astrophysical data on the SMBH population, merge them with the acceleration model, calculate the spectrum of UHECRs with the account of propagation from source to the observer and compare it with the experimental cosmic-ray data. We obtain a good agreement with the observed spectrum by fitting the spectrum with only two continuous parameters, the overall normalization and a single free parameter of the model. Section IV demonstrates that the population model of Sec. III satisfies simple observational constraints: it does not produce too much secondary gamma radiation, it results in an acceptable number density of sources and, with the best-fit normalization, it does not require the enormous luminosity of a single source. We give our conclusions and discuss our results in Sec. V.

II. A TOY MODEL OF PARTICLE ACCELERATION IN THE BLACK-HOLE MAGNETOSPHERE

A toy model of particle acceleration in the black-hole magnetosphere was proposed by Neronov *et al.* [23,24]. Let us briefly discuss the model and its parameters.

Assume that a stationary rotating black hole without electric charge is embedded into the external magnetic field, homogeneous at the horizon distance scale. In general, the magnetic field is inclined at some angle χ with respect to the black-hole rotation axis. There is a well-known exact solution of Maxwell's equations in the Kerr metric for each inclination angle χ of an asymptotically homogeneous magnetic field [25,26]. For instance, if $\chi = 0$, then a rotation-induced electric field is parallel to the magnetic one on the symmetry axis, and its direction depends on the directions of both the magnetic field and the black hole's rotation velocity. Thus, in the region near the rotation axis, particles moving along magnetic lines are accelerated by the electric field.

In this case, radial components of the electric and magnetic fields in units $\hbar = c = G = 1$ in a locally nonrotating frame in Boyer-Lindquist coordinates on the symmetry axis are

$$B_{\hat{r}} = B_0 \left(1 - \frac{4a^2 M r}{(r^2 + a^2)^2} \right), \quad E_{\hat{r}} = -\frac{2a M B_0 (r^2 - a^2)}{(r^2 + a^2)^2}, \quad (1)$$

where M is the black hole's mass, $a \leq M$ is its angular momentum per unit mass, r is the radial coordinate and B_0 is the external homogeneous magnetic field.

Neglecting for the moment the energy losses, the maximal energy gain of the accelerating particle with a charge Ze is determined by the available potential difference in the region along the rotation axis,

$$\mathcal{E}_{\max}(a) = \int_{r_{\text{hor}}}^{r_{\text{max}}} Ze E(r, a) dr, \quad (2)$$

where $r_{\text{hor}} = M + \sqrt{M^2 - a^2}$ is the radius of the black-hole horizon where particle acceleration starts, while r_{max} limits the size of the region along the rotation axis where acceleration is possible (in Ref. [24], it is called ‘‘the vacuum gap’’ due to the absence of numerous charged particles in this region except of the single test particle being accelerated, which does not change the electromagnetic field).

Of course, one can rewrite Eq. (2) for the potential difference and \mathcal{E}_{\max} in terms of the distance-averaged electric field \bar{E} ,

$$\mathcal{E}_{\max}(a) = Ze \bar{E}(a) H,$$

$$\bar{E}(a) = \frac{1}{H} \int_{r_{\text{hor}}}^{r_{\text{max}}} Ze E(r, a) dr,$$

where $H = r_{\text{hor}} - r_{\text{max}}$.

By making use of Eq. (1), one obtains

$$\begin{aligned} \mathcal{E}_{\max}(a) &= 2ZeMB_0 \frac{ra}{r^2 + a^2} \Big|_{r_{\text{hor}}}^{r_{\text{max}}} \\ &= 2aZeMB_0 \left[\frac{(r_{\text{hor}} + H)}{(r_{\text{hor}} + H)^2 + a^2} - \frac{r_{\text{hor}}}{r_{\text{hor}}^2 + a^2} \right]. \end{aligned} \quad (3)$$

It is easy to see that the expression in square brackets equals $-H(H + 2Mr_{\text{hor}} - 2a^2)$ and therefore is always negative for any value of H (remember that the angular momentum per unit mass a varies between zero and M). Thus, for parallel magnetic field and angular momentum, $aM > 0$, and the radial component of the electric field $E_{\hat{r}}$ on the black-hole rotation axis is negative. So if there are negative charges near the rotation axis, they will be accelerated away from the black hole. According to Eq. (3), the difference between their energies at r_{max} and r_{hor} is positive, so they gain energy while moving along the rotation axis away from the black hole. In the opposite case, when the magnetic field and angular momentum are antiparallel, the radial component of the electric field on the black-hole rotation axis is positive. So positive-charge particles, situated near the rotation axis, are accelerated away from the black hole; according to Eq. (3), the energy gain in this case is positive for the positive-charge particles.

In Ref. [24], a simple expression $\mathcal{E}_{\max} \sim ZeB_0H$ rather than Eq. (3) was used; however, more precisely, Eq. (3) implies $\mathcal{E}_{\max} < ZeB_0H$. The dependence of $\xi = \mathcal{E}_{\max}/ZeB_0H$ from the angular momentum a for different H is shown in Fig. 1. For black holes with angular

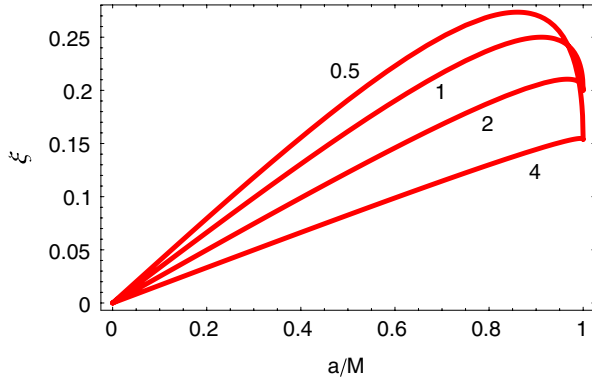


FIG. 1 (color online). Dependence of the normalized maximal energy ξ from the angular momentum a of the black hole with mass M for different values of the vacuum-gap size H (curves labeled by values of H/M ; see the text for definitions of parameters).

momentum $a > 0.1M$ and $H \sim (1-2)R_S = (2-4)M$, we have $\mathcal{E}_{\max} \sim 0.1ZeB_0H$; however, for slowly rotating black holes, $a < 0.1M$, \mathcal{E}_{\max} varies from zero to $\sim 0.01ZeB_0H$. Hereafter, R_S denotes the Schwarzschild radius.

However, the precise value of \mathcal{E}_{\max} is often irrelevant, since particles cannot achieve this maximal energy because of inevitable energy losses associated with the accelerated motion of the particle. The particle energy is determined by the balance between the energy losses and the energy gain per unit time,

$$\frac{d\mathcal{E}_+}{dt} = \frac{d\mathcal{E}_-}{dt}.$$

It was shown in Ref. [24] that protons can be accelerated to energies of about 10^{20} eV only if the magnetic field is almost aligned with the rotation axis. In this case, only the curvature radiation is relevant for an accelerated particle, and not the synchrotron one. We should also note here that we do not consider energy losses related to interactions of accelerated particles in the source. The maximal energy of an accelerated particle is

$$\mathcal{E}_{\text{curv}} = \left(\frac{3}{2}\right)^{1/4} \frac{A}{Z^{1/4}} \frac{m}{e^{1/4}} E^{1/4} R^{1/2},$$

$$\mathcal{E}_{\text{curv}} \approx 1.23 \times 10^{22} \text{ eV} \frac{A}{Z^{1/4}} \left(\frac{B_0}{1\text{G}}\right)^{1/4} \left(\frac{R}{1\text{kpc}}\right)^{1/2} \kappa^{1/4}, \quad (4)$$

where B_0 is the external magnetic field, $R \sim R_S/\chi$ is the curvature radius of magnetic field lines, Ze is the particle charge, Am is the particle mass (A is the atomic number and m is the nucleon mass) and κ is a coefficient between the electric field and the external magnetic field, $|E_{\hat{r}}| = \kappa B_0$ [see Eq. (1)]. Note that κ is a function of the angular momentum a and the coordinate r . We point out that numerically, $\mathcal{E}_{\text{curv}} < \mathcal{E}_{\max}$, cf., Ref. [15], and therefore it is $\mathcal{E}_{\text{curv}}$ which determines the actual maximal energy. For simplicity, we assume that all of the particles start with

equal initial conditions and so all of them are accelerated to the same energy $\mathcal{E}_{\text{curv}}$.

The original model of Ref. [24] treated the magnetic field B_0 as a free parameter. However, one may note that the field is constrained and, in particular, cannot be too high (see Ref. [15] for a detailed discussion). The maximal value of the magnetic field is determined [27,28] by the so-called Eddington limit, $B_{\text{Ed}} = 10^4 \left(\frac{M}{10^9 M_{\odot}}\right)^{-1/2}$ G.

Though this estimate may be obtained in several different ways, its most transparent meaning is that the magnetic field energy density is equal to that of the accreting plasma, corresponding to the Eddington luminosity. To obtain the maximal possible particle energy, we should assume that the external magnetic field is equal to the Eddington limit, $B_0 = B_{\text{Ed}}$. However, below, we will find the spectrum of CRs in the framework of this model. For this purpose we are interested in the actual maximal particle energy as a function of black-hole mass M rather than in the upper limit. So we should recognize that the realistic magnetic field can differ from the Eddington limit. In a general case, we can parametrize the external magnetic field as follows:

$$B_0 = kB_{\text{Ed}} \left(\frac{M}{10^9 M_{\odot}}\right)^{\alpha},$$

where α and k are parameters. We note in passing that several realistic models predict this kind of dependence, e.g., the Shakura-Syunyaev model [29,30] ($k \approx 0.31$, $\alpha = 0$) or the model of Ref. [31] ($k \approx 0.0093$, $\alpha \approx -0.31$; see Fig. 2 of Ref. [15] for comparison with scarce observational data). We will consider these options below. We have

$$\mathcal{E}_{\text{curv}} \approx 2.9 \times 10^{20} \text{ eV} \frac{A}{Z^{1/4}} \left(\frac{M}{10^9 M_{\odot}}\right)^{\frac{3}{8} + \frac{\alpha}{4}} \left(\frac{\chi}{1^\circ}\right)^{-\frac{1}{2}} (k\kappa)^{\frac{1}{4}}. \quad (5)$$

During acceleration, the particle emits curvature photons. In what follows, we will need to obtain an upper bound on this emission. The peak energy of the photons is determined by the particle energy \mathcal{E} ,

$$\mathcal{E}_{\gamma} = \frac{3}{2} \frac{\mathcal{E}^3}{m^3 R},$$

and for the upper limit, we take $\mathcal{E} = \mathcal{E}_{\text{curv}}$,

$$\mathcal{E}_{\gamma} \sim 50 \text{ TeV} \frac{A^3}{Z^{3/4}} \left(\frac{M}{10^9 M_{\odot}}\right)^{\frac{1}{8} + \frac{3\alpha}{4}} \left(\frac{\chi}{1^\circ}\right)^{-1/2} (k\kappa)^{\frac{3}{4}}.$$

The ratio of luminosities in photons, L_{γ} , and in CRs, L_{CR} , may be estimated by comparing the total available potential difference in the acceleration region along the rotation axis to its fraction spent on the particle acceleration:

$$\eta = \frac{L_\gamma}{L_{\text{CR}}} = \frac{\mathcal{E}_{\text{max}}}{\mathcal{E}_{\text{curv}}}, \quad (6)$$

$$\eta = 3.12 \left(\frac{M}{10^9 M_\odot} \right)^{\frac{1}{8} + \frac{3a}{4}} \left(\frac{\chi}{1^\circ} \right)^{1/2} \frac{Z^{5/4}}{A} \xi \kappa^{-1/4} k^{3/4},$$

where $(\xi \kappa^{-1/4}) \sim (0.1-2)$ for $0 < a < M$ and $R_S < H < 6R_S$. In numerical calculations presented below, we use $\kappa^{1/4} = 0.7$ and $\xi = 0.25$, cf., Figs. 1 and 3.

To summarize, the model we use assumes a monochromatic spectrum of accelerated particles with $\mathcal{E} = \mathcal{E}_{\text{curv}}$, Eq. (4), in each particular source. The value of $\mathcal{E}_{\text{curv}}$ depends, within the assumed magnetic field model, on the SMBH mass M only (in what follows, we do not consider the acceleration of particles other than protons). The overall flux from the source remains a free parameter.

III. POPULATION OF THE SOURCES AND THE OBSERVED SPECTRUM

Having discussed the model of particle acceleration in a particular source, we switch now to the population of sources. As we have seen, the properties of a single source are determined, within the magnetic field model we choose to study, by the SMBH mass (the dependence from the spin is weak). To reconstruct the UHECR spectrum, one has to consider the population of SMBHs distributed in mass and luminosity. For simplicity we will assume that the mean SMBH luminosity in CRs is related to its mass,

$$L_{\text{CR}} \propto M^\beta, \quad (7)$$

where β is an additional model parameter. Note that not every black hole can work as a source, because the source should possess some special properties (for example, small inclination angle, the vacuum gap larger than R_S , absence of numerous charged particles in the vicinity of the black hole which might imply a thin or even absent accretion disk). The fraction of sources where the mechanism works is also encoded in the mean luminosity, Eq. (7). The observed spectrum can be obtained by convolving the SMBH mass function with the (monochromatic) single-source spectrum and Eq. (7) and taking into account the propagation effects.

A. The SMBH mass function

Since dynamical measurements of SMBH masses are available for a very limited number of cases only; it is a difficult task to find the mass function precisely. Fortunately, a number of indirect methods to estimate the SMBH mass are available (see e.g., Ref. [32] for a review). Despite having large uncertainties in individual measurements, these methods are suitable for obtaining average characteristics of the SMBH population, see e.g., Ref. [33] for a review.

An important feature of the SMBH mass function is its evolution: SMBHs grow fast, cf., e.g., Ref. [34]. For our

calculation, we use one of the most recently published redshift-dependent mass functions [35]. Of the two functions presented, we choose to use the one based on the stellar mass functions, because it has smaller statistical uncertainties. The systematic uncertainties of the mass function may be judged from Ref. [33] and are well within the overall precision of our toy model.

B. The observed spectrum

Before reaching the Earth, the accelerated protons may interact with the cosmic microwave background. The main two processes modifying the shape of the propagated cosmic-ray spectrum are photopion production and e^+e^- -pair production. The former leads to a strong suppression of the proton flux above a few tens of EeV, known as the GZK effect [4,5], while the latter mostly dominates the attenuation below 10 EeV, leading to the so-called ‘‘dip’’ feature in the spectrum [36,37]. We use the numerical code developed in Ref. [38]. The code also traces secondary particles produced in the interactions. It makes use of the kinetic equation approach and calculates the propagation of nucleons, stable leptons and photons using the standard dominant processes (see e.g., Ref. [39]).

Figure 2 presents the predicted cosmic-ray fluxes in the best-fit model for the Auger spectrum [40] for different dependencies of the magnetic field B_0 on SMBH mass mentioned in the previous section. The solid curves correspond to B_0 given by the Eddington limit, the dashed one describes the Shakura-Syunyaev model [29,30] and the dotted one corresponds to the model of Ref. [31]. The overall flux normalization is a free parameter. Besides this, we tried two values of angle $\chi = 1^\circ$ and 5° and varied the luminosity dependence parameter β [Eq. (7)] in the range $-1 < \beta < 2$. The best-fit parameter values are indicated on the plot. One can see that the first two models produce satisfactory spectral fits above 10 EeV.

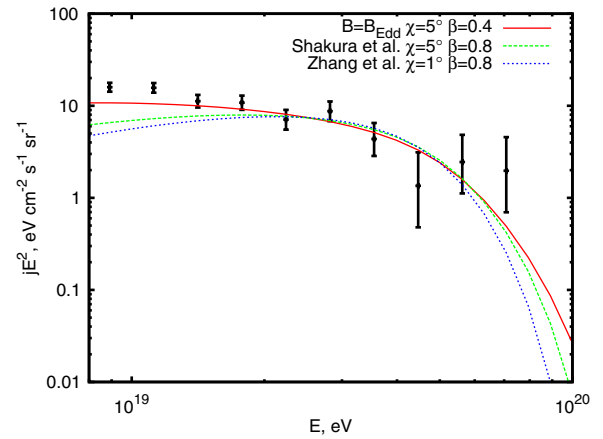


FIG. 2 (color online). The cosmic-ray flux predicted by the model for three different assumptions about the SMBH magnetic field, see labels on the plot and explanations in the text, versus the Auger experimental data [40].

IV. CONSTRAINTS

In this section, we discuss additional consistency checks of the model. They include estimates of the accompanying gamma radiation, which should not be in conflict with the measured diffuse gamma-ray background, and estimates of the concentration of sources and of the luminosity of a single source. We will see that the model passes these tests. For order-of-magnitude estimates in this section, we assume $B_0 \sim B_{\text{Ed}}$.

A. Concentration of the sources

Let us check that the local concentration of sources of CRs with energies $\mathcal{E} \gtrsim 6 \times 10^{19}$ eV is not in conflict with the lower limit [10] based on the statistics of clustering. To this end, we integrate the SMBH mass function over the range of masses corresponding to these energies.

The dependence of the particle energy from the black-hole mass is given by Eq. (5). Every black-hole mass corresponds to a range of particle energies due to the variations in the value of κ . The black-hole mass function includes all the black holes with fixed masses, and so all the black holes with every value of κ . Thus, the lower limit of the required mass interval is determined by $\mathcal{E} = 6 \times 10^{19}$ eV and the maximum value of κ . It was shown in Ref. [24] that particles can be accelerated to energies of about 10^{20} eV only if the size of the vacuum gap is not smaller than the Schwarzschild radius. In this case, the highest value of $\kappa^{1/4}$ is ~ 0.7 (see Fig. 3). Black-hole masses corresponding to particle energies $\mathcal{E} \gtrsim 6 \times 10^{19}$ eV are then $M \gtrsim 10^7 M_{\odot}$. Integrating the mass function in this range of masses, we obtain

$$n = \int_{M_{\min}}^{M_{\max}} \frac{dn}{d \log M} d \log M.$$

The integral is saturated at its lower limit; that is, M_{\max} can be taken arbitrarily high to obtain the following estimate:

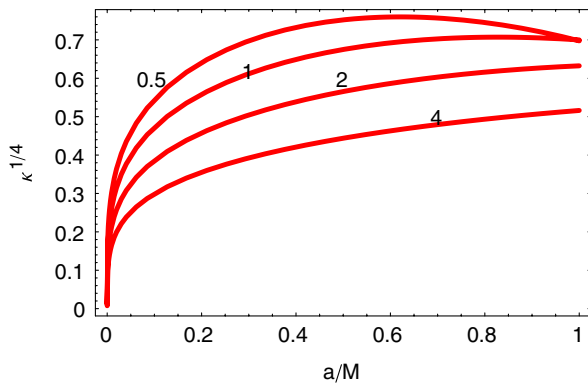


FIG. 3 (color online). Dependence of the coefficient $\kappa^{1/4}$ from the angular momentum a of the black hole with mass M for different values of the vacuum-gap size H (curves labeled by values of H/M ; see the text for definitions of parameters).

$$n \sim 10^{-3} \frac{1}{\text{Mpc}^3}. \quad (8)$$

As we see, the total concentration of sources is larger than the clustering lower bound of $10^{-4} \frac{1}{\text{Mpc}^3}$. As we have discussed above, only a fraction of the calculated concentration n corresponds to the true concentration of the sources. Our estimate tells us that this fraction should not be less than a few percent, which is reasonable.

B. Luminosity of a single source

A simple estimate of the luminosity of a single source may be obtained as follows: Consider the observed flux of CRs with energies $\mathcal{E} \gtrsim 6 \times 10^{19}$ eV. The value $j(\mathcal{E})$, which is often reported, is

$$j(\mathcal{E}) = \frac{1}{4\pi} \frac{1}{\mathcal{E}} \frac{dF}{d\mathcal{E}} = \frac{1}{4\pi} \frac{dN}{d\mathcal{E}}, \quad (9)$$

where $N(\mathcal{E})$ is the number of particles with energy $\geq \mathcal{E}$ per unit area per unit time. Then, for the flux F we have

$$\mathcal{E} \cdot j(\mathcal{E}) = \frac{1}{4\pi} \frac{dF}{d\mathcal{E}},$$

and, from the recent data [2,3], we obtain the estimate $F \sim 2.5 \times 10^5 \frac{\text{eV}}{\text{m}^2 \cdot \text{s}}$.

On the other hand, this flux is produced by the sources situated in the GZK sphere, where the GZK horizon radius is $R_{\text{GZK}} \sim 130$ Mpc for energies $\mathcal{E} \gtrsim 6 \times 10^{19}$ eV, see e.g., Ref. [41]. We assume that all these sources have approximately equal cosmic-ray luminosities $L_0 [\frac{\text{eV}}{\text{s}}]$, independent from the source black-hole mass (in reasonable agreement with the best-fit values of β in Sec. III). Because in this case the GZK radius corresponds to very small redshifts $z < 0.1$, we can neglect the changing of the source concentration with the redshift. For the order-of-magnitude estimate, we neglect also the difference between the energy with which the particle was emitted and the final particle's energy with which we detect it on the Earth. Let us also note that, because we are interested in all sources of the accelerated particles with energies $\mathcal{E} \gtrsim 6 \times 10^{19}$ eV, in the expression for the total flux we have to substitute the total concentration of sources, Eq. (8), that was obtained by integrating the mass function.

The flux from every single source situated at distance d from us is $F_0 = L_0 / (4\pi d^2)$. Thus, for the total flux we have

$$F = \int_0^{R_{\text{GZK}}} F_0 n dV = L_0 \cdot n \cdot R_{\text{GZK}}.$$

Using $n \sim 10^{-3} \frac{1}{\text{Mpc}^3}$, we obtain $L_0 \sim 10^{39} \frac{\text{erg}}{\text{s}}$. The corresponding luminosity in photons is $L_\gamma = \eta \cdot L_0$, where η is given by Eq. (6). We have

$$L_\gamma \sim 10^{39} \left(\frac{M}{10^9 M_\odot} \right)^{1/8} \frac{\text{erg}}{\text{s}}. \quad (10)$$

This value is much smaller than the typical bolometric luminosity of an active galaxy nuclei (AGN), $L_{\text{AGN}} \sim (10^{41} - 10^{43}) \frac{\text{erg}}{\text{s}}$, and by far does not exceed the Eddington limit, $L_{\text{Ed}} = 10^{47} \left(\frac{M}{10^9 M_\odot} \right) \frac{\text{erg}}{\text{s}}$. Taking the concentration of the sources of order of the lower limit, $n \sim 10^{-4} \frac{1}{\text{Mpc}^3}$, does not result in a conflict either.

In all the cases, the luminosity is not that far from the luminosity of an AGN, and a natural question arises: can we see our sources in TeV as point sources? To answer this question, one may estimate the flux of TeV photons associated with the particle acceleration in one of the nearest sources, $D \sim 10$ Mpc. It is given by $L_\gamma / (D^2 \Omega)$, where Ω is the beaming solid angle into which the emission goes. Neglecting the $M^{1/8}$ dependence, one gets the flux of $F_\gamma \sim 0.6 \Omega^{-1} \frac{\text{eV}}{\text{cm}^2 \cdot \text{s}}$, which transforms into the counting rate of few-TeV photons of order $\sim 10^{-13} \Omega^{-1} \frac{1}{\text{cm}^2 \cdot \text{s}}$, which is beyond the reach of current TeV telescopes unless the radiation is strongly beamed (which is the case [24]) and the beam points directly to the observer (which is not that probable for particular nearby sources).² Of course, this does not mean that strong TeV sources cannot accelerate UHE particles by this mechanism: the TeV emission may have a totally different origin.

C. Diffuse gamma-ray background

While individual cosmic-ray sources have quite low gamma-ray luminosities, one may wonder about the total emission of all sources in the Universe (beyond the GZK sphere). The emitted curvature photons have energies of order a few TeV and interact with the infrared background radiation to produce electromagnetic cascades in which the energy of the leading gamma rays downgrade to the GeV band. Electrons in the cascade are deflected by cosmic magnetic fields, so distant sources contribute to the diffuse gamma-ray background. Let us check that this contribution does not exceed the measured value of the diffuse flux.

A simple estimate may be obtained as follows: Consider particles with energies $\mathcal{E} \sim 10^{20}$ eV. Cosmic rays with these energies arrive to us from the interior of the GZK sphere only, but the associated photons come from all sources at all distances. Knowing how much cosmic radiation comes from the interior of the GZK sphere, we can estimate how much is present in the Universe (keeping in mind that the number of sources depends on the distance). And supposing that the luminosity of a source in CRs is connected with its luminosity in associated photons,

²Even the beamed radiation may be isotropized by electromagnetic cascades in the source [24].

Eq. (6), we can estimate the total photon emission from all the sources.

The flux at the Earth, Eq. (9), is expressed in terms of the energy at detection, \mathcal{E} , which in general differs from the energy at injection, E_{in} . In particular, the account of the Universe's expansion (even neglecting additional energy losses) results in $\mathcal{E} = E_{\text{in}}(1+z)^{-1}$, where z is the redshift of the source. The number of emitted particles per unit time is also $(1+z)$ times higher than the number of detected particles on the Earth per unit time.

The contribution to $j(E)$ from the sources located at redshift z is

$$dj(E, z) = \frac{1}{4\pi} \frac{dn_{\text{BH}}(M(E_{\text{in}}))}{dE_{\text{in}}} \frac{dN_0}{dt} \frac{1}{S(z)} dV(z),$$

where $n_{\text{BH}}(M(E_{\text{in}}))$ is the number density of black holes with masses $\leq M(E_{\text{in}})$ [for $M(E)$, see Eq. (5)] at the redshift z , dN_0/dt is the number of detected particles per unit time from one of the sources emitted with the energy E_{in} , dV is the volume of a spherical layer at the distance z from us, and S is the area of a sphere with the radius equal to the distance from the source to the Earth (it is necessary for calculating the number of particles that pass through the unit area on the Earth). Taking into account the dependence from the redshift z , we may find the source cosmic-ray luminosity $L(E_{\text{in}})$; that is, its total energy emission in CRs per unit time,

$$L(E_{\text{in}}) = \frac{dE_{\text{tot}}}{dt} = E_{\text{in}} \frac{dN_0^{\text{in}}}{dt},$$

where dN_0^{in}/dt is the number of particles with the energy E_{in} emitted by the source per unit time and we assumed that the source emits particles with the only energy, determined by its mass M .

For a flat universe, we have $S = 4\pi a_0^2 \rho^2(z)$ and $dV = Sa_0 d\rho(z)$, where

$$\rho(z) = \int_0^z \frac{dz}{H(z)a_0}$$

is the geodesic coordinate distance from the observer to the source location. Finally,

$$j(E) = \int_0^{z_{\text{max}}} \frac{1}{4\pi} \frac{dn_{\text{BH}}(M((1+z)E))}{d \log \frac{M}{M_\odot}} \frac{L((1+z)E)}{(1+z)^2 E} \times \frac{d \log \frac{M}{M_\odot}}{dE_{\text{in}}} \Big|_{M((1+z)E)} a_0 d\rho(z), \quad (11)$$

where $dn_{\text{BH}}(M)/d \log M$ is just the mass function for a given z [35], and z_{max} is the redshift of the most distant source. Here we are interested in the values of the mass function at the points $M((1+z)E)$ as a function of z . This function could be easily constructed using the data from Ref. [35]. Using Eq. (5), we have

$$\frac{d \log_M M}{dE_{\text{in}}} \odot (M((1+z)E)) \sim \frac{1.16}{E_{\text{in}}} = \frac{1.16}{(1+z)E}.$$

For simplicity, let us suppose that all the sources have the same luminosities, $L((1+z)E) = L_0$. This assumption is in reasonable agreement with the results obtained in Sec. III. Thus, we can take the luminosity out of the integral. Let us now imagine just for a moment, that CRs with such energies could come from all distances, and calculate the ratio of the values $j(E)$ for the CRs with the energies $\sim 10^{20}$ eV from the GZK sphere ($z_{\text{max}} = z_{\text{GZK}} \sim 0.01$) and from the whole Universe (e.g., $z_{\text{max}} = 2$). The luminosities in front of these two integrals are canceled, and after integrating we obtain

$$\frac{E^2 j(E)_{\text{tot}}}{E^2 j(E)_{\text{GZK}}} = 20.$$

Taking into account the observed value of the cosmic-ray flux, we estimate

$$E^2 j(E)_{\text{tot}} \sim 10^5 \frac{\text{eV}}{\text{m}^2 \cdot \text{s} \cdot \text{sr}}.$$

For calculating the value of $E_\gamma^2 j(E)_\gamma$ for the gamma radiation from all the possible sources in the whole Universe, we should only replace $d \log M / dE$ in the integral of Eq. (11) with the corresponding expression for photons, Eq. (10), and the cosmic-ray luminosity L with the photon luminosity $L_\gamma = \eta L$, Eq. (6). The order-of-magnitude estimate then reads

$$E_\gamma^2 j(E)_\gamma^{\text{tot}} = \eta E^2 j(E)_{\text{tot}} \sim 10^5 \frac{\text{eV}}{\text{m}^2 \cdot \text{s} \cdot \text{sr}}.$$

This total gamma-ray emission associated with particle acceleration in all the sources does not exceed the observed value of the diffuse gamma-ray background [42].

We have also performed a more detailed numerical simulation of the secondary gamma-ray flux. The injection spectrum of the curvature photons is similar to the synchrotron spectrum [43],

$$I(\omega) \approx 2\sqrt{3}Z^2 e^2 \gamma \frac{\omega}{\omega_c} \int_{2\omega/\omega_c}^{\infty} K_{5/3}(x) dx,$$

where

$$\omega_c = 3 \frac{\gamma^3}{R}$$

is the ‘‘critical’’ frequency (for higher frequencies, radiation is negligible), ω is the frequency of the radiated photons, $K_{5/3}$ is the Macdonald function, $\gamma = (\mathcal{E}/m)$ is the particle Lorentz factor, and R is the curvature radius of the particle trajectory (for curvature radiation, R is constant, and it is equal to the curvature radius of the magnetic lines). The total flux of the photons from a single source is related to the cosmic-ray flux by the coefficient η , Eq. (6).

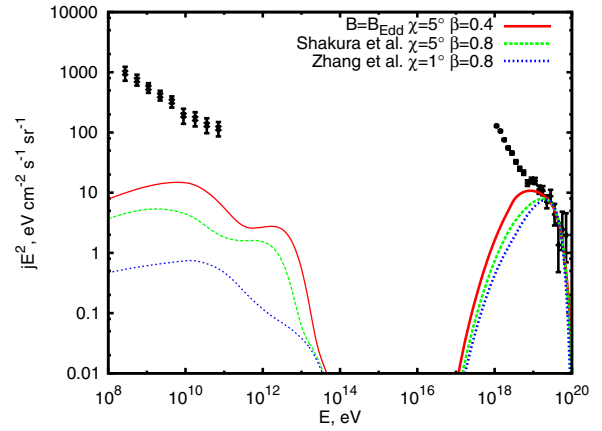


FIG. 4 (color online). Gamma-ray fluxes predicted by the same models as shown in Fig. 2.

For the best-fit spectrum of Sec. III B, we used the same code to describe the propagation of the accompanying gamma rays and to calculate the observed gamma-ray flux. The result is presented in Fig. 4. One may see that the diffuse gamma-ray upper limit is satisfied.

V. CONCLUSIONS AND DISCUSSION

We have constructed and studied a toy model of UHECR acceleration in the vicinity of numerous and various SMBHs in centers of galaxies. The model assumes that

- (1) cosmic-ray particles are accelerated by the regular electric field within a few R_S from the SMBH [24]; the field configuration is given by the solutions of Refs. [25,26] and is fully determined by the SMBH mass M , its angular momentum a and the magnetic field normalization B_0 ;
- (2) all cosmic-ray particles accelerated near a given SMBH have similar initial conditions, and therefore all are accelerated up to one and the same energy limited by the curvature radiation losses; this maximal energy is calculated in the model and depends on M only (provided B_0 is a given, model-dependent function of M ; dependence from a smooths this monochromatic spectrum insignificantly; interaction losses are not considered);
- (3) the mean flux of a source (which accounts for the fraction of the sources where this mechanism does work) depends on M in a powerlike manner; the normalization and the exponent are two free parameters of the model (the best fit to the cosmic-ray spectrum indicates that this dependence is weak);
- (4) the concentration of sources is determined by the redshift-dependent SMBH mass function taken from astrophysical literature.

Within these assumptions, and given the $B_0(M)$ relation is fixed (we considered three popular choices for it), the model has two free parameters which we find by fitting

the cosmic-ray spectrum at the Earth to the experimental data. With parameters fixed in this way, we subject the model to several further tests, which it passes successfully:

- (1) The concentration of sources is large enough to satisfy the constraints from the absence of clustering in UHECR arrival directions.
- (2) The luminosity of a particular source, determined by the flux normalization and concentration, is not too high.
- (3) Secondary gamma rays from distant sources do not overshoot the measured GeV diffuse gamma-ray background.

Given the success of the toy model, it is interesting to discuss its possible refinements. The assumptions we have made within the model are quite robust and realistic. One subtle point is related to the value of the SMBH angular momentum a , which may vary from one black hole to another. However, these variations are probably modest given the scaling relation between the mass and the angular momentum of cosmic black holes proposed in Ref. [44]. The precision of predictions may be improved with more realistic modeling of the acceleration mechanism; in particular, by taking into account the charge concentration at the SMBH and in the acceleration region, the finite thickness of the accretion disk, etc. Ultimately, this approach might give an answer to the question of which particular SMBHs are strong sources and which are not, thus determining the (presently free) parameter theoretically. However, this is a complicated task and is far beyond the scope of the present work.

One possible question concerns the low-energy part of the spectrum where, as is clearly seen from Fig. 4, the contribution of the mechanism we discuss is insufficient to explain the observed spectrum due to the depletion of the SMBH mass function at low masses. It is tempting to speculate that this depletion is compensated by a huge contribution of the SMBH in our own Galaxy which, indeed, has the appropriate mass. A quantitative analysis of this proposal requires, however, a much more precise study of physical properties and possibilities for particle acceleration close to the Galactic center.

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