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Exact self-accelerating cosmologies in ghost-free bigravity and massive gravity

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Within the recently proposed ghost-free bigravity theory, we present the most general cosmological solution for which the physical metric is homogeneous and isotropic, while the second metric is inhomogeneous. The solution includes a matter source and exists for generic values of the theory parameters. The physical metric describes a universe with an effective cosmological term mimicked by the graviton mass, which causes the late time acceleration. When perturbed, this universe should rest approximately homogeneous and isotropic in space regions small compared to the graviton Compton length. In the limit where the massless graviton decouples, the solution fulfills the equations of the ghost-free massive gravity.

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Considering theories with massive gravitons [1] is motivated by the observation of the current acceleration of our Universe [2], since the graviton mass can induce an effective cosmological term. Although such theories can exhibit unphysical features, as for example the Boulware-Deser ghost [3], the recent discovery of the special massive gravity [4] and its bigravity generalization [5] which are ghost-free [6] suggests that such theories can indeed be good candidates for interpreting the observational data. This motivates studying cosmological solutions with massive gravitons.

The first self-accelerating cosmologies in the ghost-free massive gravity were obtained without matter and describe the pure de Sitter universe [7]. The matter source was then included but only for special values of the theory parameters [8]. For these solutions the physical metrics is of the Friedmann-Robertson-Walker (FRW) type, but the fiducial metric is inhomogeneous. This means that background perturbations can give rise to inhomogeneity, although this effect should be suppressed by the smallness of the graviton mass [9]. One more similar solution was found in Ref. [9], where it was argued that solutions of this type should be generic for massive gravity. The theory also admits solutions for which both metrics are FRW, but these show a nonlinear instability and seem to be unphysical [10].

The generalizations of results of Refs. [7,8] within the ghost-free bigravity were obtained in Refs. [11,12]. For these solutions, both metrics are dynamical but are not simultaneously diagonal, and the second metric does not share the translational symmetries of the first one. In the bigravity, too, only special solutions of this type are known—either without matter or for constrained values of the theory parameters.

In what follows, we present the most general cosmological solution for which the physical metric is FRW but the second metric is inhomogeneous. We construct this solution within the ghost-free bigravity of Ref. [5], but it describes the massive gravity case as well, since its second metric becomes flat in the limit where the massless graviton decouples. The solution includes the matter source, it exists for all values of the theory parameters, and its physical metric describes a FRW universe that can be spatially flat, open or closed, and which shows the late-time acceleration due to the effective cosmological term mimicked by the graviton mass.

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I. THE GHOST-FREE BIGRAVITY

The generic bigravity theory [13] is defined on a spacetime manifold equipped with two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$, whose kinetic terms are chosen to be of the standard Einstein-Hilbert form. To describe the ghost-free bigravity [5], it is convenient to use the tetrad formulation [14], in which the inverse of $g_{\mu\nu}$ and the $f_{\mu\nu}$ are parameterized as

$$g^{\mu\nu} = \eta^{AB} e^{\mu}_{A} e^{\nu}_{B}, \qquad f_{\mu\nu} = \eta_{AB} \omega^{A}_{\mu} \omega^{B}_{\nu}, \tag{1}$$

with $\eta_{AB} = \mathrm{diag}[1, -1, -1, -1]$. The action is

$$S = -\frac{1}{16\pi G}\int R\sqrt{-g}\,d^4x - \frac{1}{16\pi \mathcal{G}}\int \mathcal{R}\sqrt{-f}\,d^4x + S_{\rm int} + S_{\rm m}, \tag{2}$$

where R and $\mathcal{R}$ are the Ricci scalars for $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively, and G and $\mathcal{G}$ are the corresponding gravitational couplings. $S_{\rm m}$ describes the ordinary matter, which is supposed to directly interact only with $g_{\mu\nu}$. The interaction between the two metrics is parameterized as

$$S_{\rm int} = \frac{\sigma}{8\pi G}\int \mathcal{L}_{\rm int}\sqrt{-g}\,d^4x, \tag{3}$$

where

$$\mathcal{L}_{\rm int} = \frac{1}{2}((K^{\mu}_{\mu})^2 - K^{\nu}_{\mu}K^{\mu}_{\nu}) + \frac{c_3}{3!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\sigma}K^{\mu}_{\alpha}K^{\nu}_{\beta}K^{\rho}_{\gamma} + \frac{c_4}{4!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}K^{\mu}_{\alpha}K^{\nu}_{\beta}K^{\rho}_{\gamma}K^{\sigma}_{\delta}, \tag{4}$$

c_3, c_4 are parameters, and $K^\mu_\nu = \delta^\mu_\nu - \gamma^\mu_\nu$ with

$$\gamma^\mu{}_\nu = e^\mu_A \omega^A_\nu. \tag{5}$$

These expressions define the theory with two gravitons, one of which is massless and the other one is massive, with the mass $m^2 = \sigma(1 + \mathcal{G}/G)$. One can introduce the angle η such that the parameters σ, $\mathcal{G}$ are expressed as $\sigma = m^2\cos^2\eta$ and $\mathcal{G} = G\tan^2\eta$.

Varying the action with respect to e^μ_A and ω^A_ν gives the field equations [11],

$$G^\rho_\lambda = m^2\cos^2\eta T^\rho_\lambda + 8\pi G T^{(\mathrm{m})\rho}{}_\lambda, \tag{6}$$

$$\mathcal{G}^\rho_\lambda = m^2\sin^2\eta \mathcal{T}^\rho_\lambda. \tag{7}$$

Here G^ρ_λ and $\mathcal{G}^\rho_\lambda$ are the Einstein tensors for $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively, while

$$T^\rho_\lambda = \tau^\rho_\lambda - \delta^\rho_\lambda \mathcal{L}_{\mathrm{int}}, \qquad \mathcal{T}^\rho_\lambda = -\frac{\sqrt{-g}}{\sqrt{-f}}\tau^\rho_\lambda, \tag{8}$$

with

$$\tau^\rho_\lambda = (\gamma^\sigma_\sigma - 3)\gamma^\rho_\lambda - \gamma^\rho_\sigma\gamma^\sigma_\lambda - \frac{c_3}{2}\epsilon_{\lambda\mu\nu\sigma}\epsilon^{\alpha\beta\gamma\sigma}\gamma^\rho_\alpha K^\mu_\beta K^\nu_\gamma - \frac{c_4}{6}\epsilon_{\lambda\mu\nu\sigma}\epsilon^{\alpha\beta\gamma\delta}\gamma^\rho_\alpha K^\mu_\beta K^\nu_\gamma K^\sigma_\delta. \tag{9}$$

These equations should be supplemented by the conservation condition for the matter energy-momentum tensor, $\overset{(g)}{\nabla}_\rho T^{(\mathrm{m})\rho}{}_\lambda = 0$, where $\overset{(g)}{\nabla}_\rho$ is the covariant derivative with respect to $g_{\mu\nu}$.

The field equations require that $T_{\mu\nu} = g_{\mu\rho}T^\rho_\nu$ is symmetric, hence, so is $\gamma_{\mu\nu} = g_{\mu\rho}\gamma^\rho{}_\nu$, which implies that

$$e^\mu_A \omega_{B\mu} = e^\mu_B \omega_{A\mu}, \tag{10}$$

with $\omega_{A\mu} = \eta_{AB}\omega^B_\mu$. The latter property guarantees that $\gamma^\mu{}_\sigma\gamma^\sigma{}_\nu = g^{\mu\sigma}f_{\sigma\nu}$ and so $\gamma^\mu{}_\nu = \sqrt{g^{\mu\sigma}f_{\sigma\nu}}$, in agreement with the original formulation of the theory [5].

If $\mathcal{G} \to 0$ then the massless graviton decouples and, if only $f_{\mu\nu}$ becomes flat in this limit, the theory reduces to the massive gravity of Ref. [4].

II. SPHERICAL SYMMETRY

Introducing the spherical coordinates $x^\mu = (t, r, \vartheta, \varphi)$, the most general expression for the two tetrads subject to the condition (10) is [11]

$$e_0 = \frac{1}{Q}\frac{\partial}{\partial t}, \qquad e_1 = \frac{1}{N}\frac{\partial}{\partial r}, \qquad e_2 = \frac{1}{R}\frac{\partial}{\partial\vartheta},$$
$$e_3 = \frac{1}{R\sin\vartheta}\frac{\partial}{\partial\varphi}, \qquad \omega^0 = aQdt + cNdr,$$
$$\omega^1 = -cQdt + bNdr, \qquad \omega^2 = uRd\vartheta,$$
$$\omega^3 = uR\sin\vartheta d\varphi, \tag{11}$$

where Q, N, R, a, b, c, u are functions of t, r. It is straightforward to compute $\gamma^\mu{}_\nu$ in (5) and obtain the following nonzero components of τ^μ_ν in (9):

$$\tau^0_0 = ab + 2au - 3a + c^2 + c_4(u-1)^2(a - ab - c^2) + c_3(u-1)(au + 2ab - 3a + 2c^2),$$
$$\tau^\vartheta_\vartheta = u(u + a + b - 3) + c_4u(1-u)[c^2 + (a-1)(b-1)] + c_3u[(a+b-2)u + c^2 + ab - 2a - 2b + 3], \tag{12}$$

with τ^r_r obtained from τ^0_0 via $a \leftrightarrow b$, also $\tau^\varphi_\varphi = \tau^\vartheta_\vartheta$, and

$$\tau^0_r = \frac{cN}{Q}[(c_3 + c_4)u^2 + 2(1 - 2c_3 - c_4)u + 3c_3 + c_4 - 3]. \tag{13}$$

The interaction Lagrangian (4) reduces to

$$\mathcal{L}_{\mathrm{int}} = u(u + 2a + 2b - 6) + c^2 + ab - 3a - 3b + 6 + c_3(u-1)[(a+b-2)u + 2c^2 + 2ab - 3a - 3b + 4] - c_4(u-1)^2(c^2 + ab - a - b + 1). \tag{14}$$

Noting that $\sqrt{-f}/\sqrt{-g} = |e^\mu_A||\omega^A_\nu| = (ab + c^2)u^2$, it is straightforward to evaluate the two energy-momentum tensors in (8).

III. HOMOGENEITY AND ISOTROPY

Let us assume the metric $g_{\mu\nu}$ and the matter distribution to be homogeneous and isotropic. This can be achieved by setting $Q = N = \mathbf{a}(t)$ and $R = \mathbf{a}(t)f_{\mathrm{k}}(r)$ with $f_{\mathrm{k}}(r) = \{r, \sin(r), \sinh(r)\}$ for $\mathrm{k} = 0,\ 1,\ -1$, respectively. We choose the matter to be a perfect fluid with $8\pi G T^{(\mathrm{m})\rho}{}_\lambda = \mathrm{diag}[\rho(t), -P(t), -P(t), -P(t)]$.

Since the Einstein tensor for $g_{\mu\nu}$ is diagonal, so should be the energy-momentum tensor T^μ_ν on the right in (6); therefore, one should have $T^0_r = 0$, which requires that $\tau^0_r = 0$. Now, τ^0_r in (13) will vanish if either $c = 0$ or if the expression between the brackets vanishes. We shall be considering below the case where $c \neq 0$ and the metrics are not simultaneously diagonal, since the $c = 0$ case has already been studied in detail [11].

If $c \neq 0$, then τ^0_r in (13) will vanish if

$$u = \frac{1}{c_3 + c_4}\left(2c_3 + c_4 - 1 \pm \sqrt{c_3^2 - c_3 + c_4 + 1}\right). \tag{15}$$

Inserting this into the above formulas, we find that the energy-momentum tensors in (8) become diagonal, with constant 00 and rr components,

$$T^0_0 = T^r_r = (u-1)(c_3u - u - c_3 + 3) \equiv \lambda,$$
$$\mathcal{T}^0_0 = \mathcal{T}^r_r = \frac{1-u}{u^2}(c_3u - c_3 + 2) \equiv \tilde{\lambda}. \tag{16}$$

The Bianchi identities for Eq. (6) then imply the conservation condition $\overset{(g)}{\nabla}_\rho T^\rho_\lambda = 0$, whose only nontrivial component is (we denote $\dot{} \equiv \partial_t$ and $' \equiv \partial_r$)

$$\overset{(g)}{\nabla}_\mu T^\mu_0 = 2\frac{\dot{\mathbf{a}}}{\mathbf{a}}(T^0_0 - T^\vartheta_\vartheta) = 0. \qquad (17)$$

It is worth noting that a similar condition for $\mathcal{T}^\mu_\nu$ follows identically, due to the invariance of $S_{\rm int}$ under diffeomorphisms, so that there is no need to impose it separately.

Now, using the above formulas one finds

$$T^0_0 - T^\vartheta_\vartheta = \frac{c_3 u - u - c_3 + 2}{u-1}[(u-a)(u-b)+c^2]. \qquad (18)$$

In view of (17) this should vanish, so that either the first or the second factor on the right should be zero. The former case was considered in Refs. [8,11] (see also Ref. [15]). However, the condition $c_3 u - u - c_3 + 2 = 0$ constrains the possible values of the parameters c_3, c_4 so that the solutions obtained in this way are not general. We, therefore, abandon this condition in what follows and require instead that

$$(u-a)(u-b) + c^2 = 0. \qquad (19)$$

In view of this, one has $T^0_0 = T^\vartheta_\vartheta$ and $\mathcal{T}^0_0 = \mathcal{T}^\vartheta_\vartheta$, which implies that both energy-momentum tensors are proportional to the unit tensor, $T^\mu_\nu = \lambda\delta^\mu_\nu$ and $\mathcal{T}^\mu_\nu = \tilde\lambda\delta^\mu_\nu$. The field Eqs. (6) and (7) then reduce to

$$G^\rho_\lambda = \Lambda\delta^\rho_\lambda + 8\pi G T^{(\mathrm{m})\rho}{}_\lambda, \qquad (20)$$

$$\mathcal{G}^\rho_\lambda = \tilde\Lambda\delta^\rho_\lambda, \qquad (21)$$

with $\Lambda = m^2\cos^2\eta\lambda$ and $\tilde\Lambda = m^2\sin^2\eta\tilde\lambda$. As a result, the two metrics actually decouple one from another, and the graviton mass gives rise to a cosmological term separately for each metric. However, one has to remember that solutions of (20) and (21) should, in addition, fulfill the consistency condition (19).

The solution for $g_{\mu\nu}$—Eq. (20) for $g_{\mu\nu}$ comprise a closed system, with no additional conditions imposed, so that we can solve them. The metric is

$$ds^2_g = \mathbf{a}^2(t)(dt^2 - dr^2 - f^2_{\rm k}(r)d\Omega^2), \qquad (22)$$

and the Einstein equations reduce to

$$3\frac{\dot{\mathbf{a}}^2 + {\rm k}\mathbf{a}^2}{\mathbf{a}^4} = \Lambda + \rho, \qquad (23)$$

where ρ is determined by the matter conservation condition, $\dot\rho + 3(\dot{\mathbf{a}}/\mathbf{a})(\rho + P) = 0$. This describes a universe filled with ordinary matter and containing the cosmological term mimicked by the graviton mass. At early times the matter density ρ dominates, but at late times the cosmological term wins, which leads to the self-acceleration.

The solution for $f_{\mu\nu}$—Eq. (21) should determine the metric

$$ds^2_f = \mathbf{a}^2(a\,dt + c\,dr)^2 - \mathbf{a}^2(b\,dr - c\,dt)^2 - U^2 d\Omega^2. \qquad (24)$$

Here a, b, c are free functions of t, r, but $U = uR(t,r)$ is already fixed by the previous considerations. In addition, a, b, c should satisfy the constraint (19). One could, therefore, wonder if the system is not overdetermined and the freedom is enough to fulfill all conditions.

To see that the latter is indeed the case, we notice that the function U can be considered as the new radial coordinate. The temporal coordinate can also be changed in such a way that in new coordinates T, U, the metric becomes diagonal. The source term in (21) is invariant under reparameterizations. Therefore, the problem reduces to solving the Einstein equations with the cosmological constant $\tilde\Lambda$ to find a diagonal metric parameterized by the Schwarzschild coordinate U. The solution is the (anti-)de Sitter metric

$$df^2 = \Delta^2 dT^2 - \frac{dU^2}{\Delta^2} - U^2 d\Omega^2, \qquad (25)$$

where $\Delta^2 = 1 - \tilde\Lambda U^2/3$. There remains the need to establish the correspondence between the T, U and t, r coordinates and to fulfill the constraint (19).

Let us introduce 1-forms

$$\theta^0 = \Delta dT, \quad \theta^1 = \frac{dU}{\Delta}, \quad \theta^2 = U d\vartheta, \quad \theta^3 = U\sin\vartheta d\varphi,$$

such that $f_{\mu\nu} = \eta_{AB}\theta^A_\mu\theta^B_\nu$. At the same time, $f_{\mu\nu}$ can be expanded with respect to ω^A_μ from (11). The two sets of 1-forms may differ from each other by a local Lorentz boost, so that

$$\omega^0 = \theta^0\sec\alpha + \theta^1\tan\alpha, \qquad \omega^1 = \theta^1\sec\alpha + \theta^0\tan\alpha, \qquad (26)$$

where α is the boost parameter. Using the explicit expressions for ω^A and θ^A and comparing the coefficients in front of dt, dr in (26) give four conditions, which determine α, a, b, c in terms of Δ, T, U. As a result, the consistency condition (19) assumes the form

$$\dot U T' - \dot T U' - u^2\mathbf{a}^2 + u\mathbf{a}\sqrt{A_+A_-}/\Delta = 0, \qquad (27)$$

with $A_\pm = \Delta^2\dot T + U' \pm (\Delta^2 T' + \dot U)$. This equation determines $T(t,r)$.

Let us first consider the $\eta \to 0$ limit, when $\tilde\Lambda = 0$ and $\Delta = 1$, in which case exact solutions of (27) can be found. For ${\rm k} = 0$, when $U = u\mathbf{a}r$, we find

$$T(t,r) = C\int^t \frac{\mathbf{a}^2}{\dot{\mathbf{a}}}dt + \left(\frac{u^2}{4C} + Cr^2\right)\mathbf{a}, \qquad (28)$$

where C is an integration constant. This solution agrees with the one obtained in Ref. [9] for $c_3 = c_4 = 0$, $u = 3/2$.

For ${\rm k} = 1$, one has $U = u\mathbf{a}\sin r$ and we find

$$T(t,r) = \int^t \sqrt{(C^2 + u^2)(\dot{\mathbf{a}}^2 + \mathbf{a}^2)}dt + C\mathbf{a}\cos(r). \qquad (29)$$

For ${\rm k} = -1$ and $U = u\mathbf{a}\sinh r$, we obtain

$$T(t,r) = \int^t \sqrt{(C^2 - u^2)(\dot{\mathbf{a}}^2 - \mathbf{a}^2)}dt + C\mathbf{a}\cosh(r). \quad (30)$$

If $\Delta \neq 1$, then exact solutions are more difficult to find; however, at least when $\tilde{\Lambda}$ is small, the solution can be constructed perturbatively as

$$T = T_0 + \sum_{n\geq 1}(-\tilde{\Lambda}/3)^n T_n. \quad (31)$$

Here, T_0 corresponds to zero-order expressions (28)–(30), while the corrections T_n can be obtained by separating the variables with the ansatz $T_n = \sum_{m=0}^{n+1} f_m(t) g^m(r)$ where $g(r) = \{r^2, \cos(r), \cosh(r)\}$ for k = 0, 1, −1, respectively. For example, for k = 0, one finds

$$T_1 = C\int^t \frac{\mathbf{a}^6}{4\dot{\mathbf{a}}^3}dt + \left(\frac{Cr^4}{4} + \frac{u^2 r^2}{8C} - \frac{u^4}{192C^3}\right)\mathbf{a}^3, \quad (32)$$

with C being the same as in (28) and similarly for $n > 1$.

This completes out constructions, since all field equations and the consistency condition are now fulfilled.

IV. DISCUSSION

We have obtained the cosmological solution in the ghost-free bigravity with matter, for generic values of the theory parameters (provided that u in (15) is real). One can choose c_3, c_4 such that $\Lambda > 0$. The metric $g_{\mu\nu}$ then describes a FRW universe, which can be spatially flat, open, or closed. It is matter-dominated at early times, but at late times it enters the accelerated phase due to the effective cosmological term mimicked by the graviton mass. The metric $f_{\mu\nu}$ is the static (anti—)de Sitter (25) parameterized by T, U.

The two metrics are not simultaneously diagonal and do not have the same Killing symmetries. In particular, the translational symmetries of $g_{\mu\nu}$ are not shared by $f_{\mu\nu}$, since the Stückelberg fields $\phi^0 = T(t,r)$, $\phi^k = U(t,r) \times \{\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\varphi\}$ are inhomogeneous functions of x^k. One can, therefore, expect the fluctuations around the background solutions to be inhomogeneous, but this effect will be sourced by terms proportional to m^2 in Eqs. (6) and (7) and so will be small in regions smaller than $1/m$, in agreement with the arguments of Ref. [9].

Setting the matter density to zero, the solution for $g_{\mu\nu}$ is pure de Sitter, rewriting which in static coordinates reproduces the static bigravity solutions found in Ref. [12].

The solution exists for any value of η. When $\eta \to 0$, then $\tilde{\Lambda} \to 0$, so that $f_{\mu\nu}$ becomes flat, while $g_{\mu\nu}$ still describes the expanding universe. Therefore, we obtain in this limit the solution of the ghost-free massive gravity with flat reference metric [4], with the Stückelberg fields expressed by (28)–(30).

Our solution, therefore, describes the most general cosmology with a homogeneous and isotropic $g_{\mu\nu}$ and an inhomogeneous $f_{\mu\nu}$, and it applies equally within the bigravity and massive gravity contexts. The latter property is quite special, since in general the metric $f_{\mu\nu}$ does not necessarily become flat for $\eta \to 0$, while generic massive gravity solutions do not always extend to the bigravity [16].

All other known cosmological solutions can be obtained by setting $c = 0$ in Eqs. (11)–(14), in which case both metrics are FRW. The corresponding bigravity solutions do not always show the late-time acceleration and do not cover the massive gravity case [11,17]. The massive gravity cosmologies with two FRW metrics do not extend to the bigravity and show a nonlinear instability [10].

It seems, therefore, that our solution is the most sensible physically, since it covers all possible cases and shows the late-time acceleration expected for massive gravitons.

Note added.—When this text was being completed, there appeared the article [18] on massive gravity ($\eta = 0$) cosmologies whose analysis is partly similar to the above discussion, although it does not give the explicit solution for the Stückelberg scalars.

[1] M. Fierz and W. Pauli, Proc. R. Soc. A **173**, 211 (1939).

[2] A.G. Reiss *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999).

[3] D.G. Boulware and S. Deser, Phys. Rev. D **6**, 3368 (1972).

[4] C. de Rham, G. Gabadadze, and A.J. Tolley, Phys. Rev. Lett. **106**, 231101 (2011).

[5] S.F. Hassan and R.A. Rosen, J. High Energy Phys. 02 (2012) 126.

[6] S.F. Hassan and R.A. Rosen, Phys. Rev. Lett. **108**, 041101 (2012); A. Golovnev, Phys. Lett. B **707**, 404 (2012); J. Kluson, J. High Energy Phys. 06 (2012) 170; S.F. Hassan, A. Schmidt-May, and M. von Strauss, arXiv:1203.5283; S.F. Hassan and R.A. Rosen, J. High Energy Phys. 04 (2012) 123.

[7] K. Koyama, G. Niz, and G. Tasinato, Phys. Rev. Lett. **107**, 131101 (2011); Phys. Rev. D **84**, 064033 (2011).

[8] A.H. Chamseddine and M.S. Volkov, Phys. Lett. B **704**, 652 (2011).

[9] G. D'Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava, and A.J. Tolley, Phys. Rev. D **84**, 124046 (2011).

[10] A.E. Gumrukcuoglu, C. Lin, and S. Mukohyama, J. Cosmol. Astropart. Phys. 11 (2011) 030; arXiv:1206.1338.

[11] M.S. Volkov, J. High Energy Phys. 01 (2012) 035.

[12] M. S. Volkov, Phys. Rev. D **85**, 124043 (2012).

[13] C. J. Isham, A. Salam, and J. Strathdee, Phys. Rev. D **3**, 867 (1971).

[14] A. H. Chamseddine and V. Mukhanov, J. High Energy Phys. 08 (2011) 091; K. Hinterbichler and R. A. Rosen, J. High Energy Phys. 07 (2012) 047.

[15] T. Kobayashi, M. Siino, M. Yamaguchi, and D. Yoshida, arXiv:1205.4938.

[16] V. Baccetti, P. Martin-Moruno, and M. Visser, arXiv:1205.2158.

[17] M. von Strauss, A. Schmidt-May, J. Enander, E. Mortsell, and S. F. Hassan, J. Cosmol. Astropart. Phys. 03 (2012) 042; D. Comelli, M. Crisostomi, F. Nesti, and L. Pilo, J. High Energy Phys. 03 (2012) 067.

[18] P. Gratia, W. Hu, and M. Wyman, arXiv:1205.4241.