PHYSICAL REVIEW D **86,** 057501 (2012)

More about excited bottomonium radiative decays

Randy Lewis
Department of Physics and Astronomy, York University, Toronto, Ontario, Canada M3J 1P3

R. M. Woloshyn
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3
(Received 23 July 2012; published 17 September 2012)

Radiative decays of bottomonium are revisited, focusing on contributions from higher-order relativistic effects. The leading relativistic correction to the magnetic spin-flip operator at the photon vertex is found to be particularly important. The combination of $\mathcal{O}(v^6)$ effects in the nonrelativistic QCD action and in the transition operator moves previous lattice results for excited Υ decays into agreement with experiment.

DOI: 10.1103/PhysRevD.86.057501 PACS numbers: 12.38.Gc, 13.20.Gd, 14.40.Pq

In a recent paper [1] we studied bottomonium radiative decays using lattice nonrelativistic QCD (NRQCD) [2]. It was shown that robust signals for transitions among the ground state and the first two excited states could be obtained using multiexponential fitting techniques [3]. The qualitative features of the decay amplitudes were in agreement with phenomenological expectations but quantitatively the values obtained were considerably larger than those determined by experiment.

In Ref. [1] the b quarks were described using an $\mathcal{O}(v^4)$ lattice NRQCD action and only the simplest magnetic spin-flip transition operator was used. In this work higher-order relativistic effects in the action and additional terms in the transition operator are investigated. It is expected that the hindered transitions, which involve states with different principal quantum numbers, are very sensitive to relativistic corrections. In comparison to Ref. [1], the additional effects considered in this work substantially reduce the amplitudes for excited Υ decays and bring them into line with experimental values.

The general setup for this study is the same as in Ref. [1]. The gauge fields come from a $2+1$-flavor dynamical simulation done on a $32^3 \times 64$ lattice by the PACS-CS collaboration [4]. An ensemble of 192 configurations is used. The light quark parameters are such that the pion mass is near physical at 156 MeV. Landau link tadpole improvement is implemented with a value of 0.8463 for the link. The b-quark bare mass is 1.945 (in lattice units) and a stability parameter n of 4, in line with Ref. [5], is used.

The three-point functions for vector to pseudoscalar transitions are constructed using a sequential source method. Starting with a vector (pseudoscalar) operator at the source t_s, the quark propagator is evolved to a time T at which a pseudoscalar (vector) operator is applied. This quantity is then evolved backward in time. At intermediate times $t_s < t' < T$ a transition operator is inserted and evolution is continued to complete the quark antiquark loop at the source.

With a vector operator at the source and pseudoscalar at the sink the three-point function is expected to have the form

$$G_{oo'}^{(VP)}(t';T) = \sum_{n,n'} c_o^{(V)}(n) A_{nn'}^{(VP)} c_{o'}^{(P)}(n') e^{-E_n^{(V)}(t'-t_s)} e^{-E_{n'}^{(P)}(T-t')}, \tag{1}$$

where the subscripts o, o' indicate the type of operator (local or smeared) that is used. As in Ref. [1], three types of operators are used: a local operator, a wavefunction smeared operator [6] and an operator with the smearing applied twice [5]. See Ref. [1] for a description of the smearing function and parameters.

The overlap coefficients c and simulation energies E are the same ones that appear in the two-point function. The quantity $A_{nn'}^{(VP)}$ is the matrix element of the transition operator between the vector state n and the pseudoscalar state n'. The three-point function with pseudoscalar source and vector sink has the same form with V and P labels reversed. The matrix elements $A_{nn'}^{(PV)}$ are related to those appearing in (1) by $A_{nn'}^{(PV)} = A_{n'n}^{(VP)}$. The matrix elements can be determined by fitting the t' dependence of the three-point function for a fixed T using overlap coefficients and energies obtained from a fit to two-point correlators.

The calculations in Ref. [1] were done using an NRQCD action including terms to $\mathcal{O}(v^4)$. In this work $\mathcal{O}(v^6)$ terms are also considered. The complete action can be found, for example, in Ref. [7]. Here we display only the spin-dependent terms linear in chromoelectric and chromomagnetic fields which are relevant for subsequent discussion (see Ref. [7] for detailed explanation of notation),

$$\delta H^{(4)} = -\frac{c_4 g}{2M_0}\boldsymbol{\sigma}\cdot\tilde{\mathbf{B}} - \frac{c_3 g}{8M_0^2}\boldsymbol{\sigma}\cdot(\tilde{\boldsymbol{\Delta}}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\boldsymbol{\Delta}}), \tag{2}$$

$$\delta H^{(6)} = -\frac{c_7 g}{8M_0^3}\{\tilde{\Delta}^{(2)}, \boldsymbol{\sigma}\cdot\tilde{\mathbf{B}}\} - \frac{c_8 3g}{64M_0^4}\{\tilde{\Delta}^{(2)}, \boldsymbol{\sigma}\cdot(\tilde{\boldsymbol{\Delta}}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\boldsymbol{\Delta}})\}, \tag{3}$$

where $\delta H^{(4)}$ gives the $\mathcal{O}(v^4)$ chromomagnetic coupling and spin-orbit terms and $\delta H^{(6)}$ their leading relativistic

TABLE I. Transition operators used in calculating three-point functions. The momentum $\mathbf{k}$ is chosen to have a component only in the one direction and $\Delta_{1k} \equiv (\Delta_1 e^{i\mathbf{k}\cdot\mathbf{x}} + e^{i\mathbf{k}\cdot\mathbf{x}}\Delta_1)$.

	Magnetic	Electric
$\mathcal{O}(v^4)$	$\sum_{\mathbf{x}} \sigma_3 e^{i\mathbf{k}\cdot\mathbf{x}}$	$\frac{1}{4M_0}\sum_{\mathbf{x}} i\sigma_3\Delta_{1k}$
$\mathcal{O}(v^6)$	$\frac{1}{4M_0^2}\sum_{\mathbf{x}}\{\Delta^{(2)}, \sigma_3 e^{i\mathbf{k}\cdot\mathbf{x}}\}$	$\frac{3}{32M_0^2}\sum_{\mathbf{x}}\{\Delta^{(2)}, i\sigma_3\Delta_{1k}\}$

corrections. The fields are tadpole-improved and in our calculations the action coefficients take the tree-level values. The transition operator should be constructed in a way which is consistent with the action. This can be achieved by replacing in (2) and (3) the SU(3) color electric and magnetic fields by external electromagnetic fields and the strong coupling constant by the charge. See for example Ref. [8].

For an M1 decay, the photon momentum, polarization vector and quark spin operator should be mutually orthogonal. The explicit choice made for the operators used in our lattice simulation is given in Table I. The normalization is such that in the infinite mass limit the transition matrix element goes to 1, matching the nonrelativistic wavefunction overlap (see Eq. (1) in Ref. [1]).

Results for the three-point transition matrix elements are given in Table II for different source-sink time separations and different values of recoil momentum. The calculations are done with the $\mathcal{O}(v^4)$ action and the leading magnetic operator σ_3. The values for $T - t_s$ equal to 19 and 27 are from Ref. [1]. For the excited to ground state transitions, there is very good agreement between results using different time separations. However, using a smaller time separation $T - t_s = 15$ allows for a better determination of the $2 \rightarrow 2$ transition and we use only this for the present study.

Changing the action from $\mathcal{O}(v^4)$ to $\mathcal{O}(v^6)$ results in a decrease in the strength of the spin-dependent interactions. This is evidenced by a decrease in the mass difference between Υ and η_b states [7]. For our calculation, done at a single lattice spacing of about 0.09 fm the Υ-η_b mass difference is 56(1) MeV and 24(3) MeV for 1S and 2S states respectively using the $\mathcal{O}(v^4)$ action. These values are reduced to 43(2) and 14(2) MeV when $\mathcal{O}(v^6)$ effects are included. It is expected that inclusion of radiative corrections to the coefficients of the NRQCD action would raise these values somewhat [9]. For reference, the experimental values are 69.3 ± 2.8 MeV (from the Particle Data Group [10]) and $59.3 \pm 1.9^{+2.4}_{-1.4}$ MeV (from Adachi *et al.* [11]) for 1S. For 2S there are claims of $48.7 \pm 2.3 \pm 2.1$ MeV [12] and $24.3^{+4}_{-4.5}$ MeV [13] for the spin splitting.

Decreasing the strength of the spin-dependent interactions leads to greater overlap of wavefunctions with the same principal quantum number and decreases the overlap of states with different principal quantum numbers. This expectation is reflected in the lattice simulations. Figure 1 compares transition amplitudes between states with the same principal quantum number calculated with the $\mathcal{O}(v^4)$ and $\mathcal{O}(v^6)$ actions using only the leading magnetic $\mathcal{O}(v^4)$ transition operator. The decreased spin-dependent interaction at $\mathcal{O}(v^6)$ shows up as an increase of the transition amplitude. The triangles in Fig. 1 show the effect of adding the relativistic correction to the magnetic transition operator in the $\mathcal{O}(v^6)$ calculation. The effect is very small for these transitions.

Figure 2 shows the comparison of the calculations for the case of excited Υ decaying to the ground state η_b. Here going from $\mathcal{O}(v^4)$ (circles) to $\mathcal{O}(v^6)$ (squares) leads to a decrease of the amplitude in line with the notion that the overlap of wavefunctions is decreased. What is surprising is that including the relativistic correction to the magnetic spin-flip (triangles) leads to such a large additional decrease of the amplitude. For the decay of an excited η_b to the ground state of Υ the situation is more complicated. The effect of spin-dependent interactions on the wavefunctions leads to a relative negative sign of the transition amplitude compared to excited Υ decay. Figure 3 shows that changing the action from $\mathcal{O}(v^4)$ (circles) to $\mathcal{O}(v^6)$

TABLE II. Three-point matrix elements from simultaneous fits to 12 correlation functions with different source-sink separation.

$T - t_s$	$A_{11}^{(VP)}$	$A_{21}^{(PV)}$	$A_{31}^{(PV)}$	$A_{21}^{(VP)}$	$A_{31}^{(VP)}$	$A_{22}^{(VP)}$
			Momentum 0			
15	0.916(2)	−0.068(3)	−0.050(3)	0.071(5)	0.070(7)	0.863(67)
19	0.915(2)	−0.068(2)	−0.050(4)	0.071(4)	0.065(3)	1.11(23)
27	0.916(2)	−0.068(3)	−0.051(6)	0.071(4)	0.062(4)	1.9(1.8)
			Momentum 1			
15	0.907(2)	−0.060(6)	−0.047(5)	0.082(5)	0.072(5)	0.799(68)
19	0.907(1)	−0.062(6)	−0.047(7)	0.079(5)	0.067(5)	0.95(21)
27	0.907(2)	−0.061(5)	−0.048(8)	0.079(5)	0.066(6)	1.6(1.5)
			Momentum 2			
15	0.877(1)	−0.031(4)	−0.040(5)	0.103(6)	0.083(6)	0.791(60)
19	0.877(1)	−0.031(4)	−0.041(6)	0.102(5)	0.078(6)	1.02(20)
27	0.878(2)	−0.029(5)	−0.039(7)	0.100(5)	0.068(6)	1.0(1.6)

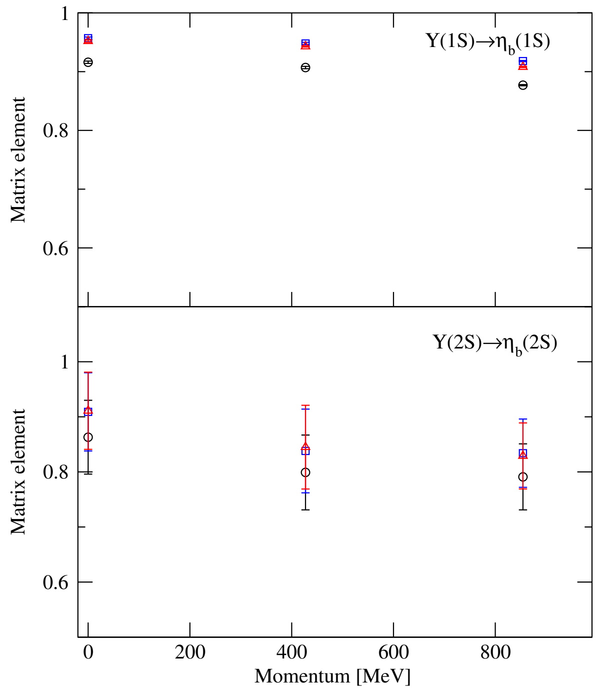

FIG. 1 (color online). The matrix elements for decay of Υ to η_b with the same principal quantum number, as a function of momentum. Shown are $\mathcal{O}(v^4)$ (circles) and $\mathcal{O}(v^6)$ (squares) action results with the leading $\mathcal{O}(v^4)$ magnetic operator. Triangles include the relativistic correction to the magnetic operator in the $\mathcal{O}(v^6)$ calculation.

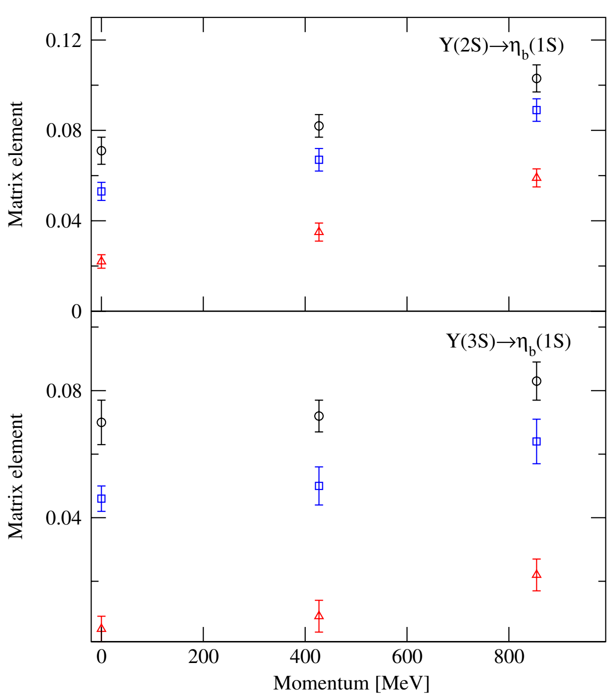

FIG. 2 (color online). The matrix elements for decay of an excited Υ to the η_b ground state, as a function of momentum. Shown are $\mathcal{O}(v^4)$ (circles) and $\mathcal{O}(v^6)$ (squares) action results with the leading $\mathcal{O}(v^4)$ magnetic operator. Triangles include the relativistic correction to the magnetic operator in the $\mathcal{O}(v^6)$ calculation.

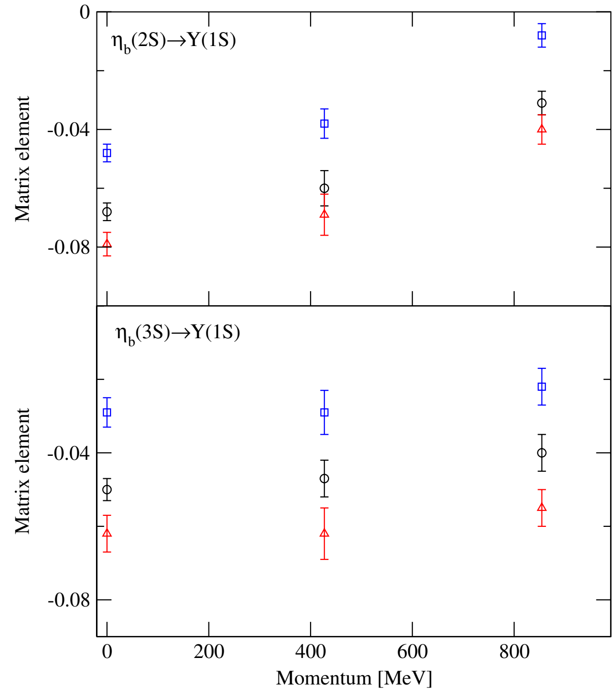

FIG. 3 (color online). The matrix elements for decay of an excited η_b to the Υ ground state, as a function of momentum. Shown are $\mathcal{O}(v^4)$ (circles) and $\mathcal{O}(v^6)$ (squares) action results with the leading $\mathcal{O}(v^4)$ magnetic operator. Triangles include the relativistic correction to the magnetic operator in the $\mathcal{O}(v^6)$ calculation.

(squares) results in a decrease in the magnitude of the amplitude but including the relativistic correction to the operator (triangles) has the opposite effect.

Up to now we have discussed only the transition due to magnetic spin-flip operators. There are additional contributions that have to be considered, that is, due to the electric terms. These are the electromagnetic counterparts of the spin-orbit terms in (2) and (3) (see also Eq. 2 in Ref. [8]). These terms are suppressed by a factor of (photon energy$/M_0$) relative to the magnetic terms at the same order in v^2. They make no contribution to the transition matrix element at zero photon momentum. For vector to pseudoscalar transitions between states with the same principal quantum number, where the mass difference is small, their effect is not important. For excited state to ground state transitions they make a non-negligible contribution.

Table III gives our final results for excited state decay amplitudes interpolated (for 2S decays) or extrapolated (for 3S decays) to the physical momentum. The values in the first two rows were obtained using an NRQCD action with terms up to the indicated order but only the leading magnetic spin-flip transition operator. For the third and fourth rows all terms in the transition operator up to the order of those included in the NRQCD action were evaluated. The difference between the first and third rows is due to the $\mathcal{O}(v^4)$ electric term. The difference between the

TABLE III. Three-point matrix elements for excited states decays at the physical momentum.

	$\Upsilon(2S) \to \eta_b(1S)$ 611 MeV/c	$\Upsilon(3S) \to \eta_b(1S)$ 923 MeV/c	$\eta_b(2S) \to \Upsilon(1S)$ 524 MeV/c	$\eta_b(3S) \to \Upsilon(1S)$ 830 MeV/c
$\mathcal{O}(v^4)$, σ	0.088(5)	0.084(7)	−0.055(4)	−0.041(5)
$\mathcal{O}(v^6)$, σ	0.073(4)	0.067(7)	−0.033(3)	−0.023(5)
Complete $\mathcal{O}(v^4)$	0.080(5)	0.077(7)	−0.050(4)	−0.032(5)
Complete $\mathcal{O}(v^6)$	0.032(5)	0.013(7)	−0.059(4)	−0.041(5)
Experiment	0.034(7)	0.018(3)		

second and fourth rows is dominated by the correction to the magnetic spin-flip coupling. In the last row, the transition amplitude inferred from the experimentally observed $\Upsilon(2S)$ and $\Upsilon(3S)$ decays [14,15] is given. Our best estimate, complete $\mathcal{O}(v^6)$, is consistent with the experimental values.

The excited state decays are so-called hindered transitions and the amplitudes depend on the interplay of spin-dependent effects on the states, relativistic corrections and recoil (momentum) effects. The systematics of excited Υ and excited η_b decays are different and experimental information on η_b decays would provide a very stringent test of calculations. Now that there are experimental glimpses [12,13] of $\eta_b(2S)$ one may hope that a measurement of its radiative decay may be feasible in the not too distant future.

In this brief report we have updated a previous calculation [1] of bottomonium radiative decay amplitudes by extending the lattice NRQCD action to include spin-dependent terms of $\mathcal{O}(v^6)$ and by including contributions to the transition operators that are consistent with the action. These changes bring the calculated decay amplitudes into agreement with values determined from observed excited Υ decays. The NRQCD action and transition operators were constructed using tree-level coefficients (tadpole improved for the action). Radiative corrections to the coefficients were not considered here. These have been calculated for some cases and noticeable effects were found [9,16]. A further refinement of the calculation would have to apply such corrections in a complete and consistent way.

Finally, we note that the large changes in the excited state decay amplitudes found in going from $\mathcal{O}(v^4)$ to $\mathcal{O}(v^6)$ NRQCD may suggest that it would be beneficial to avoid nonrelativistic approximations altogether. The construction and use of relativistic lattice actions for heavy quarks is the subject of ongoing investigations [17,18]. The comparison of nonrelativistic and relativistic approaches to excited bottomonium radiative decays would be a worthwhile direction for future work.

We thank the PACS-CS Collaboration for making their dynamical gauge field configurations available. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1] R. Lewis and R. M. Woloshyn, Phys. Rev. D **84**, 094501 (2011).
[2] G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, and K. Hornbostel, Phys. Rev. D **46**, 4052 (1992).
[3] G. P. Lepage, B. Clark, C. T. H. Davies, K. Hornbostel, P. B. Mackenzie, C. Morningstar, and H. Trottier, Nucl. Phys. B, Proc. Suppl. **106–107**, 12 (2002).
[4] S. Aoki *et al.* (PACS-CS Collaboration), Phys. Rev. D **79**, 034503 (2009).
[5] C. T. H. Davies, E. Follana, I. D. Kendall, G. P. Lepage, and C. McNeile, Phys. Rev. D **81**, 034506 (2010).
[6] C. T. H. Davies, K. Hornbostel, A. Langnau, G. P. Lepage, A. Lidsey, J. Shigemitsu, and J. Sloan, Phys. Rev. D **50**, 6963 (1994).
[7] S. Meinel, Phys. Rev. D **82**, 114502 (2010).
[8] N. Brambilla, Y. Jia, and A. Vairo, Phys. Rev. D **73**, 054005 (2006).
[9] T. C. Hammant, A. G. Hart, G. M. von Hippel, R. R. Horgan, and C. J. Monahan, Phys. Rev. Lett. **107**, 112002 (2011).
[10] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
[11] I. Adachi *et al.* (Belle Collaboration), arXiv:1110.3934
[12] S. Dobbs, Z. Metreveli, Kamal K. Seth, A. Tomaradze, and T. Xiao, Phys. Rev. Lett. **109**, 082001 (2012).
[13] R. Mizuk *et al.* (Belle Collaboration), arXiv:1205.6351.
[14] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **101**, 071801 (2008).
[15] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **103**, 161801 (2009).
[16] A. Hart, G. M. von Hippel, and R. R. Horgan, Phys. Rev. D **75**, 014008 (2007).
[17] Y. Aoki *et al.* (RBC and UKQCD Collaborations), arXiv:1206.2554.
[18] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel, and G. P. Lepage (HPQCD Collaboration), arXiv:1207.0994 [Phys. Rev. D (to be published)].