Light Higgs scenario based on the TeV-scale supersymmetric strong dynamics

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(Received 5 June 2012; published 21 September 2012)

We consider a model based on the supersymmetric QCD theory with $N_c = 2$ and $N_f = 3$. The theory is strongly coupled at the infrared scale Λ_H. Its low-energy effective theory below Λ_H is described by the supersymmetric standard model with the Higgs sector that contains four isospin doublets, two neutral isospin singlets and two charged isospin singlets. If Λ_H is at the multi-TeV to 10 TeV, coupling constants for the F terms of these composite fields are relatively large at the electroweak scale. Nevertheless, the standard model-like Higgs boson is predicted to be as light as 125 GeV because these F terms contribute to the mass of the standard model-like Higgs boson not at the tree level but at the one-loop level. A large nondecoupling effect due to these F terms appears in the one-loop correction to the triple Higgs boson coupling, which amounts to a few tens percent. Such a nondecoupling property in the Higgs potential realizes the strong first-order phase transition, which is required for a successful scenario of electroweak baryogenesis.

DOI: 10.1103/PhysRevD.86.055023 PACS numbers: 12.60.Fr, 12.60.Jv

I. INTRODUCTION

Recently, the ATLAS and CMS experiments at the LHC [1] have reported an excess in the gamma-gamma mode at about 125 GeV, which may be a signal of the Higgs boson. In the standard model (SM), a light Higgs boson is the evidence of the weakly coupled Higgs sector. In models for physics beyond the SM, however, the light Higgs boson does not always correspond to a weakly coupled theory. The scenario based on little Higgs models [2] is an example of a strongly coupled theory with a light Higgs boson, where the Higgs boson arises as a pseudo Nambu-Goldstone boson originating from the breaking of some strongly interacting global symmetry at the TeV scale, and the Higgs boson mass is kept to be light.

Supersymmetry (SUSY) is one of the most attractive candidates for the physics beyond the SM. SUSY can solve the gauge hierarchy problem, as the quadratic divergence in the radiative correction to the Higgs boson mass is cancelled owing to the nonrenormalization theorem. In addition, elementary scalar fields are automatically introduced in the SUSY theory. The Higgs sector of the minimal SUSY extension of the SM (MSSM) necessarily contains two Higgs doublets. In the MSSM, the coupling constants in the Higgs potential are determined by the electroweak gauge couplings, and the mass of the SM-like Higgs boson is less than the Z boson mass at the tree level. With significant radiative corrections due to the large top Yukawa coupling [3], the Higgs mass can be pushed up to around 125 GeV in the case of very large stop masses or very large left-right stop mixing.

Even within the framework based on SUSY, models with strongly coupled light Higgs boson can be constructed. A possible way is to introduce additional R-parity-even chiral superfields which strongly couple to the Higgs sector but the F terms of which do not contribute to the Higgs boson four-point coupling. In this case, the SM-like Higgs boson is kept to be light. The strong couplings have rich phenomenological implications. First, radiative corrections involving the strongly coupled new fields can raise the SM-like Higgs boson mass to 125 GeV with rather natural choice of the stop masses and mixing. Second, the strongly coupled fields significantly contribute to the triple SM-like Higgs boson coupling through loop effects, so that it deviates by a few tens percent from the SM prediction [4,5]. A similar nondecoupling effect tends to enhance the first-order electroweak phase transition [6,7].

In the MSSM, it is not easy to make the first-order electroweak phase transition (EWPT) strong enough to satisfy the sphaleron decoupling condition [8], which is required by the successful electroweak baryogenesis [9,10]. The nondecoupling quantum effect of the additional scalar bosons through strong F-term coupling with the Higgs boson enhances the first-order EWPT, and the difficulty in the MSSM can be significantly relaxed. The enhancement requires a light SM-like Higgs boson because its mass works as a suppression factor, and the 125 GeV Higgs boson is consistent with the scenario. For example, the first-order EWPT can be strong enough in a SUSY model whose Higgs sector contains four doublets and two charged singlets [7]. In this model, the coupling among the Higgs boson and the extra bosons in the scalar potential can be taken to be strong, while the quartic self-coupling constants of the Higgs boson are determined only by the D term; i.e., by the electroweak gauge couplings, and the Higgs mass remains light.

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When we explore a more fundamental picture of models with strong couplings and a light Higgs boson, a quite different landscape from the grand unified theory over the grand desert presents itself to us. In models with strong couplings, coupling constants tend to blow up quickly through the renormalization group running. With sufficiently strong coupling consistent with successful electroweak baryogenesis, the Landau pole appears at the energy scale of multi-TeV to 10 TeV, which is much lower than the grand unified theory scale, $O(10^{16})$ GeV. There should be a cutoff scale below the energy scale where one comes across the Landau pole. The physics above the cutoff scale might be controlled by some strong dynamics.

The minimal SUSY fat Higgs model [11] is an example of the theory above the cutoff scale of a SUSY model with strongly coupled Higgs sector. This model is based on the strong $SU(2)_H$ SUSY gauge theory with three pairs of doublets, (T_1, T_2), (T_3, T_4) and (T_5, T_6). Below the cutoff scale, Higgs isospin doublets, H_u and H_d, and a neutral singlet, N, appear as composite fields of T_i's, and the other composite fields are decoupled due to their heavy masses. The low-energy effective theory is described by the superpotential $W = \lambda N(H_d H_u - v_0^2)$, where λ is a coupling constant and v_0 is a dimensionful parameter. The effective theory is identical to the nearly MSSM [12]. The quartic coupling of the Higgs boson gets a $|\lambda|^2$ contribution through the F term. Since the Higgs mass is dominated by the $|\lambda|^2 v^2$ term with $v \simeq 174$ GeV, the strong coupling tends to enhance the Higgs mass. For $\lambda \simeq 2$ and $\tan\beta \simeq 2$, the SM-like Higgs boson mass is as large as 200 GeV, which weakens the first-order EWPT too much. Extensions of the minimal SUSY fat Higgs model to $N_c = 3$, $N_f = 4$ and $N_c = 4$, $N_f = 5$ are discussed in Ref. [13]. Compositeness in SUSY models is discussed in Ref. [14].

In this paper, we propose a new UV-complete model whose low-energy effective theory accommodates strong couplings and a light Higgs boson. The model is based on $SU(2)_H$ SUSY gauge theory with three pairs of $SU(2)_H$ doublets. This model leads to two pairs of $Y = +1/2$ and $Y = -1/2$ isospin doublet composite superfields as well as several isospin singlet composite superfields in the low-energy description, which cause flavor-changing neutral currents. To avoid such dangerous flavor-changing neutral currents, we here impose an additional Z_2 parity on the model, which is unbroken spontaneously. This Z_2 parity can supply a new candidate for dark matter, in addition to the R parity. In our model, unlike the minimal SUSY fat Higgs model, the Z_2-even singlet field N can be heavy enough to decouple from the low-energy effective theory, but many composite fields remain light. The Higgs sector contains two Z_2-even doublets which are identical to the MSSM-like Higgs doublets and various extra Z_2-odd superfields such as a pair of doublets, two charged singlets and two neutral singlets. The SM-like Higgs boson mass of 125 GeV can be realized in a natural way because the F terms of the Z_2-odd superfields contribute to the mass not at the tree level but at the one-loop level. On the other hand, nondecoupling contributions of these fields in radiative corrections affect the triple Higgs boson coupling significantly [5], and can make the first-order EWPT strong enough through the large F-term coupling constants [7].

In Sec. II, we present the basic framework of $SU(2)_H$ SUSY QCD theory with the Z_2 parity, whose low-energy description gives a composite SUSY Higgs model. In Sec. III, we investigate general features of the composite SUSY Higgs sector. Generally, F terms involving a large coupling λ contribute to the SM-like Higgs boson mass at the tree level, giving rise to a SM-like Higgs boson much heavier than 125 GeV, as in the minimal SUSY fat Higgs model. In Sec. IV, we consider an extended model where we obtain the SM-like Higgs boson as light as 125 GeV in a natural way. It turns out that its low-energy effective theory describes the phenomenological model in Ref. [7]. Section V is devoted to the conclusion.

II. BASIC FRAMEWORK

We introduce a new $SU(2)$ gauge group, denoted by $SU(2)_H$, and six chiral superfields, denoted by $T_i (i = 1, 2, \ldots, 6)$, which are doublets of $SU(2)_H$. T_i's are also charged under SM gauge groups $SU(2)_L \times U(1)_Y$. We further assign a Z_2 parity to them. SM charge and Z_2 parity assignments to T_i's are described in Table I. Regarding $SU(2)_H$ gauge group, this model is nothing but the SUSY QCD theory with two colors and three flavors, which is investigated in Ref. [15]. $SU(2)_H$ gauge coupling becomes strong at an infrared scale, denoted by Λ_H. The most general tree-level superpotential that is invariant under $SU(2)_H \times SU(2)_L \times U(1)_Y \times Z_2$ symmetry is given by

$$\begin{aligned} W_{\rm tree} &= \frac{1}{2} m_1 \,\mathrm{tr}\left[\begin{pmatrix} T_2^2 & -T_2^1 \\ -T_1^2 & T_1^1 \end{pmatrix}\begin{pmatrix} T_1^1 & T_2^1 \\ T_1^2 & T_2^2 \end{pmatrix}\right] \\ &\quad + m_3 T_3 T_4 + m_5 T_5 T_6 \\ &= m_1 T_1 T_2 + m_3 T_3 T_4 + m_5 T_5 T_6, \end{aligned} \tag{1}$$

where we assume $m_1, m_3, m_5 < \Lambda_H$ so that the theory remains the SUSY QCD theory with $N_c = 2$ and $N_f = 3$ at the scale Λ_H. T_1^a and $T_2^a (a = 1, 2)$ respectively indicate the ath components of the $SU(2)_H$ doublets T_1 and T_2. In the right-hand side of the first line, the first matrix

TABLE I. SM charge and Z_2 parity assignments on the $SU(2)_H$ doublets, T_i.

Field	$SU(2)_L$	$U(1)_Y$	Z_2
$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix}$	2	0	+
T_3	1	$+1/2$	+
T_4	1	$-1/2$	+
T_5	1	$+1/2$	−
T_6	1	$-1/2$	−

transforms as $(2^*, 2^*)$ and the second one does as (2,2) under $SU(2)_H \times SU(2)_L$. The trace of their product is thus invariant under $SU(2)_H \times SU(2)_L$.

Below Λ_H, the theory is described in terms of composite chiral superfields, $M'_{ij} = T_i T_j (i \neq j)$, which are singlets of $SU(2)_H$. Following the arguments in Ref. [15], we see that we have the following dynamically generated superpotential below Λ_H:

$$W_{\rm dyn} = -\frac{1}{\Lambda^3}\epsilon^{ijklmn} M'_{ij} M'_{kl} M'_{mn}, \tag{2}$$

where Λ is some dynamically generated scale. Thanks to holomorphy, the net effective superpotential is simply the sum of $W_{\rm dyn}$ and $W_{\rm tree}$:

$$\begin{aligned} W_{\rm eff} &= W_{\rm dyn} + W_{\rm tree} \\ &= W_{\rm dyn} + m_1 M'_{12} + m_3 M'_{34} + m_5 M'_{56}. \end{aligned} \tag{3}$$

Since we cannot determine the Kähler potential only from holomorphy, we take advantage of naïve dimensional analysis (NDA) [16]. Before using NDA, we note that the terms in $W_{\rm tree}$ are exactly proportional to m_1/Λ_H, m_3/Λ_H, m_5/Λ_H because of holomorphy. In NDA, it is assumed that the other couplings in the effective Lagrangian are $O(1)$ in unit of Λ_H and that the effective theory also becomes strongly coupled at the scale Λ_H. Therefore the effective Lagrangian at Λ_H is expressed as

$$\begin{aligned} \mathcal{L}_{\rm eff} \simeq \frac{1}{(4\pi)^2}\bigg[\int d^4\theta \Lambda_H^2 \hat{K}\bigg(\frac{M'}{\Lambda_H^2}, \frac{D^\alpha}{\Lambda_H}, \frac{M'^\dagger}{\Lambda_H^2}, \frac{\bar{D}_{\dot\alpha}}{\Lambda_H}\bigg) \\ + \int d^2\theta \Lambda_H^3 \bigg\{\hat{W}\bigg(\frac{M'}{\Lambda_H^2}, \frac{D^\alpha}{\Lambda_H}\bigg) + \frac{m_1}{\Lambda_H}\frac{M'_{12}}{\Lambda_H^2} \\ + \frac{m_3}{\Lambda_H}\frac{M'_{34}}{\Lambda_H^2} + \frac{m_5}{\Lambda_H}\frac{M'_{56}}{\Lambda_H^2}\bigg\} + \text{H.c.}\bigg], \end{aligned} \tag{4}$$

where the SM gauge interactions are omitted. We rewrite the theory in terms of canonically normalized composite fields, M_{ij}, which are given by

$$M_{ij} \simeq \frac{1}{4\pi\Lambda_H} M'_{ij} \quad \text{at the scale } \Lambda_H, \tag{5}$$

and obtain the following canonical effective superpotential below Λ_H expressed in terms of M_{ij}'s:

$$W_{\rm eff} \simeq -\lambda \epsilon^{ijklmn} M_{ij} M_{kl} M_{mn} + \xi_\Omega M_{12} + \xi_\Phi M_{34} + \xi M_{56}, \tag{6}$$

where λ, ξ_Ω, ξ_Φ, ξ satisfy at the scale Λ_H

$$\lambda(\Lambda_H) \simeq 4\pi, \tag{7}$$

$$\begin{aligned} \xi_\Omega(\Lambda_H) \simeq \frac{m_1 \Lambda_H}{4\pi}, \qquad \xi_\Phi(\Lambda_H) \simeq \frac{m_3 \Lambda_H}{4\pi}, \\ \xi(\Lambda_H) \simeq \frac{m_5 \Lambda_H}{4\pi}. \end{aligned} \tag{8}$$

Below the scale Λ_H, the physical couplings that correspond to λ, ξ_Ω, ξ_Φ, ξ are regulated by the following renormalization group equations:

$$\mu \frac{d}{d\mu}\left(\frac{1}{\lambda^2}\right) \simeq -\frac{9}{16\pi^2}, \tag{9}$$

$$\begin{aligned} \mu \frac{d}{d\mu}\xi \simeq \frac{3}{32\pi^2}\lambda^2 \xi, \qquad \mu \frac{d}{d\mu}\xi_\Phi \simeq \frac{3}{32\pi^2}\lambda^2 \xi_\Phi, \\ \mu \frac{d}{d\mu}\xi_\Omega \simeq \frac{3}{32\pi^2}\lambda^2 \xi_\Omega. \end{aligned} \tag{10}$$

Figure 1 shows the renormalization group running of the physical coupling λ from the scale Λ_H to lower scales. For example, if $\Lambda_H \simeq 10$ TeV, λ at the scale M_Z is ~ 2. The runnings of ξ_Ω, ξ_Φ, ξ are not so drastic and can be neglected.

We rewrite the composite superfields M_{ij} in the following way to clarify their SM charges:

$$\begin{aligned} &H_u \equiv \begin{pmatrix} M_{13} \\ M_{23} \end{pmatrix}, \qquad H_d \equiv \begin{pmatrix} M_{14} \\ M_{24} \end{pmatrix}, \\ &\Phi_u \equiv \begin{pmatrix} M_{15} \\ M_{25} \end{pmatrix}, \qquad \Phi_d \equiv \begin{pmatrix} M_{16} \\ M_{26} \end{pmatrix}, \\ &N \equiv M_{56}, \qquad N_\Phi \equiv M_{34}, \qquad N_\Omega \equiv M_{12}, \\ &\Omega^+ \equiv M_{35}, \qquad \Omega^- \equiv M_{46}, \qquad \zeta \equiv M_{36}, \qquad \eta \equiv M_{45}. \end{aligned} \tag{11}$$

Their SM charges and Z_2 parities are summarized in Table II. The effective superpotential is then written as

$$\begin{aligned} W_{\rm eff} = \lambda\{&N(H_u H_d + v_0^2) + N_\Phi(\Phi_u \Phi_d + v_\Phi^2) \\ &+ N_\Omega(\Omega^+\Omega^- + v_\Omega^2) - N N_\Phi N_\Omega - N_\Omega \zeta \eta \\ &+ \zeta H_d \Phi_u + \eta H_u \Phi_d - \Omega^+ H_d \Phi_d - \Omega^- H_u \Phi_u\}, \end{aligned} \tag{12}$$

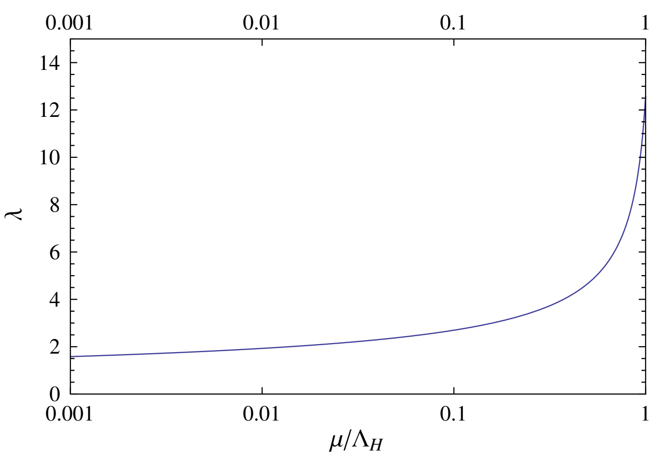

FIG. 1 (color online). The scale dependence of the physical coupling λ.

TABLE II. Properties of the composite fields under the SM gauge groups and the Z_2 parity.

Field	$SU(2)_L$	$U(1)_Y$	Z_2
H_u	2	$+1/2$	$+$
H_d	2	$-1/2$	$+$
Φ_u	2	$+1/2$	$-$
Φ_d	2	$-1/2$	$-$
Ω^+	1	$+1$	$-$
Ω^-	1	-1	$-$
N, N_Φ, N_Ω	1	0	$+$
ζ, η	1	0	$-$

where v_0^2, v_Φ^2, v_Ω^2 are defined as

$$v_0^2 \equiv \xi/\lambda, \qquad v_\Phi^2 \equiv \xi_\Phi/\lambda, \qquad v_\Omega^2 \equiv \xi_\Omega/\lambda. \tag{13}$$

We note that all the three-point couplings are of the same magnitude in this model.

III. STRUCTURE OF THE EFFECTIVE THEORY

First of all, we look for vacua of the Higgs potential in the effective theory. We study in the SUSY limit and then with soft SUSY-breaking terms. In the SUSY limit, the absolute minima of the superpotential (12) are determined from the tadpole conditions: $(\partial/\partial\phi)W_{\rm eff} = 0$ for each field ϕ at the tree level. Since we are only interested in charge-conserving vacua, we set $H_u^+ = H_d^- = \Phi_u^+ = \Phi_d^- = \Omega^+ = \Omega^- = 0$ and study whether the tadpole conditions can be satisfied. Charge-conserving vacua are determined from the following tadpole conditions:

$$0 = \frac{1}{\lambda}\frac{\partial W_{\rm eff}}{\partial N_\Omega} = -NN_\Phi - \zeta\eta + v_\Omega^2, \tag{14}$$

$$0 = \frac{1}{\lambda}\frac{\partial W_{\rm eff}}{\partial N} = -H_u^0 H_d^0 - N_\Omega N_\Phi + v_0^2, \tag{15}$$

$$0 = \frac{1}{\lambda}\frac{\partial W_{\rm eff}}{\partial N_\Phi} = -\Phi_u^0\Phi_d^0 - N_\Omega N + v_\Phi^2, \tag{16}$$

$$0 = \frac{1}{\lambda}\frac{\partial W_{\rm eff}}{\partial \zeta} = H_d^0\Phi_u^0 - N_\Omega\eta, \tag{17}$$

$$0 = \frac{1}{\lambda}\frac{\partial W_{\rm eff}}{\partial \eta} = -H_u^0\Phi_d^0 - N_\Omega\zeta, \tag{18}$$

$$0 = \frac{1}{\lambda}\frac{\partial W_{\rm eff}}{\partial H_u^0} = -NH_d^0 - \eta\Phi_d^0, \tag{19}$$

$$0 = \frac{1}{\lambda}\frac{\partial W_{\rm eff}}{\partial \Phi_u^0} = -N_\Phi\Phi_d^0 + \zeta H_d^0, \tag{20}$$

$$0 = \frac{1}{\lambda}\frac{\partial W_{\rm eff}}{\partial H_d^0} = -NH_u^0 + \zeta\Phi_u^0, \tag{21}$$

$$0 = \frac{1}{\lambda}\frac{\partial W_{\rm eff}}{\partial \Phi_d^0} = -N_\Phi\Phi_u^0 - \eta H_u^0, \tag{22}$$

$$\begin{aligned} 0 &= D \\ &= -\frac{1}{2}g_1(H_u^{0\dagger}H_u^0 - H_d^{0\dagger}H_d^0) - \frac{1}{2}g_1(\Phi_u^{0\dagger}\Phi_u^0 - \Phi_d^{0\dagger}\Phi_d^0), \end{aligned} \tag{23}$$

$$\begin{aligned} 0 &= D^{a=3} \\ &= -\frac{1}{2}g_2(-H_u^{0\dagger}H_u^0 + H_d^{0\dagger}H_d^0) \\ &\quad - \frac{1}{2}g_2(-\Phi_u^{0\dagger}\Phi_u^0 + \Phi_d^{0\dagger}\Phi_d^0), \end{aligned} \tag{24}$$

Since no symmetry forbids the term $m_1T_1T_2$ in the fundamental Lagrangian, we assume $v_\Omega^2 \neq 0$. Then the only solution to Eqs. (14) and (19)–(22) is $H_d^0 = \Phi_d^0 = H_u^0 = \Phi_u^0 = 0$, i.e., the electroweak symmetry is unbroken in the absolute SUSY vacua. At this point, our model distinctively differs from the minimal SUSY fat Higgs model [11], where nonanomalous $U(1)_R$ charges are assigned to forbid the term $m_1T_1T_2$ so that the electroweak symmetry breaking does occur in the SUSY limit. The D terms are all zero in the absolute SUSY vacua because we have $H_d^0 = \Phi_d^0 = H_u^0 = \Phi_u^0 = 0$. The conditions in Eqs. (14)–(16) determine the vacuum expectation values (VEVs) of N, N_Φ and N_Ω as follows:

$$\langle N\rangle\langle N_\Phi\rangle = v_\Omega^2, \qquad \langle N_\Omega\rangle\langle N_\Phi\rangle = v_0^2, \qquad \langle N_\Omega\rangle\langle N\rangle = v_\Phi^2. \tag{25}$$

We assume $v_0^2 \neq 0$ and $v_\Phi^2 \neq 0$ as no symmetry forbids these terms. We then have $\langle N_\Omega\rangle \neq 0$, which leads to $\eta = \zeta = 0$ through Eqs. (17) and (18). Note that the Z_2 parity is unbroken in the absolute SUSY vacua.

Since the conditions in Eq. (25) have only one solution and the other neutral components are derived to be zero, we conclude that there is only one charge-conserving absolute SUSY vacuum provided v_0^2, v_Φ^2 and v_Ω^2 are all nonzero. This vacuum respects the electroweak symmetry and the Z_2 parity. The nonzero VEVs of N, N_Φ and N_Ω give rise to effective μ terms.

Let us proceed to the case with soft SUSY-breaking terms. For simplicity, we only introduce soft SUSY-breaking mass terms and $B\mu$ term for H_u and H_d, which we denote by $m_{H_u}^2$, $m_{H_d}^2$ and $B\mu$. We redefine the phases of T_3 and T_5 to make $B\mu$ and v_0^2 real and positive. We further rotate the phase of (T_1, T_2) so that the product of the VEVs of N_Φ and N_Ω is real. We expand the potential with respect to H_u, H_d, N, N_Ω and N_Φ, with setting $\Phi_u = \Phi_d = \Omega^+ = \Omega^- = \zeta = \eta = 0$. The tree-level potential is then expressed as

$$\begin{aligned}V = & m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + B\mu H_u H_d + \text{H.c.}\\ & + |\lambda|^2|N|^2(|H_u|^2 + |H_d|^2) + |\lambda|^2|H_u H_d\\ & - N_\Phi N_\Omega + v_0^2|^2 + |\lambda|^2|NN_\Omega - v_\Phi^2|^2\\ & + |\lambda|^2|NN_\Phi - v_\Omega^2|^2 + \frac{1}{8}g_1^2(H_u^\dagger H_u - H_d^\dagger H_d)^2\\ & + \frac{1}{8}g_2^2(H_u^\dagger \sigma^a H_u + H_d^\dagger \sigma^a H_d)^2. \end{aligned} \tag{26}$$

Using the $SU(2)_L$ gauge symmetry, we take $H_u^+ = 0$. Then the condition $(\partial/\partial H_u^+)V = 0$ leads to $H_d^- = 0$, as in the MSSM. From the conditions $(\partial/\partial H_u^{0*})V = (\partial/\partial H_d^{0*})V = (\partial/\partial H_u^0)V = (\partial/\partial H_d^0)V = 0$, we have

$$\begin{aligned}&\frac{1}{4}(g_1^2 + g_2^2)(|H_u^0|^2 - |H_d^0|^2)H_u^0 + (m_{H_u}^2 + |\lambda|^2|N|^2\\ &\quad + |\lambda|^2|H_d^0|^2)H_u^0 + (|\lambda|^2 N_\Phi N_\Omega - |\lambda|^2 v_0^2 - B\mu)H_d^{0*} = 0,\end{aligned} \tag{27}$$

$$\begin{aligned}&-\frac{1}{4}(g_1^2 + g_2^2)(|H_u^0|^2 - |H_d^0|^2)H_u^0 + (m_{H_d}^2 + |\lambda|^2|N|^2\\ &\quad + |\lambda|^2|H_u^0|^2)H_d^0 + (|\lambda|^2 N_\Phi N_\Omega - |\lambda|^2 v_0^2 - B\mu)H_u^{0*} = 0,\end{aligned} \tag{28}$$

$$H_u^0 H_d^0 = \text{real}, \tag{29}$$

by using the fact that $B\mu$, v_0^2 and the VEV of $N_\Phi N_\Omega$ are real. Since the VEV of $H_u^0 H_d^0$ is real, we can take the VEVs of H_u^0 and H_d^0 both real by using the $U(1)_Y$ gauge symmetry. We hereafter denote these VEVs by v_u and v_d, respectively. The conditions: $(\partial/\partial N)V = (\partial/\partial N_\Phi)V = (\partial/\partial N_\Omega)V = 0$ and their complex conjugates lead to

$$N(v_u^2 + v_d^2) + N_\Omega^*(NN_\Omega - v_\Phi^2) + N_\Phi^*(NN_\Phi - v_\Omega^2) = 0, \tag{30}$$

$$N_\Omega^*(N_\Phi N_\Omega + v_u v_d - v_0^2) + N^*(NN_\Phi - v_\Omega^2) = 0, \tag{31}$$

$$N_\Phi^*(N_\Phi N_\Omega + v_u v_d - v_0^2) + N^*(NN_\Omega - v_\Phi^2) = 0, \tag{32}$$

$$N^* N_\Phi^* v_\Omega^2, N^* N_\Omega^* v_\Phi^2 = \text{real}. \tag{33}$$

We derive the mass spectrum in the presence of soft SUSY-breaking terms. For simplicity, we here assume that v_Φ^2 and v_Ω^2 are also real and positive. We assume that $|v_u|, |v_d| \ll \sqrt{v_0^2}, \sqrt{v_\Phi^2}, \sqrt{v_\Omega^2}$ and we make a perturbative expansion of the masses with respect to v_u^2 and v_d^2. At the zeroth order of v_u^2 and v_d^2, the VEVs of N, N_Φ and N_Ω, denoted by $\langle N\rangle^0$, $\langle N_\Phi\rangle^0$ and $\langle N_\Omega\rangle^0$, are given by

$$\begin{gathered}\langle N\rangle^0 = \sqrt{\frac{v_\Phi^2 v_\Omega^2}{v_0^2}}, \qquad \langle N_\Phi\rangle^0 = \sqrt{\frac{v_0^2 v_\Omega^2}{v_\Phi^2}},\\ \langle N_\Omega\rangle^0 = \sqrt{\frac{v_0^2 v_\Phi^2}{v_\Omega^2}},\end{gathered} \tag{34}$$

which are the same as those in the SUSY limit. These VEVs respectively correspond to the SUSY-conserving masses of (H_u, H_d), (Φ_u, Φ_d) and (Ω^+, Ω^-). In the following discussion, we use the VEVs of N^0, N_Φ^0 and N_Ω^0 as the parameters of the model, instead of v_0^2, v_Φ^2 and v_Ω^2. The VEVs of N, N_Φ and N_Ω at the first order of v_u^2 and v_d^2, denoted by $\langle N\rangle^1$, $\langle N_\Phi\rangle^1$ and $\langle N_\Omega\rangle^1$, satisfy the following relations:

$$\begin{pmatrix} (\langle N_\Phi\rangle^0)^2 + (\langle N_\Omega\rangle^0)^2 & \langle N\rangle^0\langle N_\Phi\rangle^0 & \langle N\rangle^0\langle N_\Omega\rangle^0 \\ & (\langle N\rangle^0)^2 + (\langle N_\Omega\rangle^0)^2 & \langle N_\Phi\rangle^0\langle N_\Omega\rangle^0 \\ & & (\langle N\rangle^0)^2 + (\langle N_\Phi\rangle^0)^2 \end{pmatrix}\begin{pmatrix}\langle N\rangle^1 \\ \langle N_\Phi\rangle^1 \\ \langle N_\Omega\rangle^1\end{pmatrix} = \begin{pmatrix} -\langle N\rangle^0(v_u^2 + v_d^2) \\ 0 \\ 0\end{pmatrix}. \tag{35}$$

We next study the mass spectrum of the Z_2-even Higgs sector. At the first order of v_u^2 and v_d^2, the conditions (26) and (27) reduce to

$$\begin{aligned}&\frac{1}{4}(g_1^2 + g_2^2)(v_u^2 - v_d^2)v_u + (m_{H_u}^2 + |\lambda|^2|\langle N\rangle^0\\ &\quad + \langle N\rangle^1|^2 + |\lambda|^2 v_d^2)v_u + \{|\lambda|^2((\langle N_\Phi\rangle^1\langle N_\Omega\rangle^0\\ &\quad + \langle N_\Phi\rangle^0\langle N_\Omega\rangle^1) - B\mu\}v_d = 0,\end{aligned} \tag{36}$$

$$\begin{aligned}&-\frac{1}{4}(g_1^2 + g_2^2)(v_u^2 - v_d^2)v_d + (m_{H_d}^2 + |\lambda|^2|\langle N\rangle^0\\ &\quad + \langle N\rangle^1|^2 + |\lambda|^2 v_u^2)v_d + \{|\lambda|^2((\langle N_\Phi\rangle^1\langle N_\Omega\rangle^0\\ &\quad + \langle N_\Phi\rangle^0\langle N_\Omega\rangle^1) - B\mu\}v_u = 0.\end{aligned} \tag{37}$$

They give the same conditions for the electroweak symmetry breaking as in the next-to-MSSM [17] if we define the effective $B\mu$ term as

$$B\mu_{\text{eff}} \equiv B\mu - |\lambda|^2(\langle N_\Phi\rangle^1\langle N_\Omega\rangle^0 + \langle N_\Phi\rangle^0\langle N_\Omega\rangle^1). \tag{38}$$

We comment on the range of Λ_H favored by the naturalness. NDA implies that the Lagrangian contains the following Kähler potential:

$$\mathcal{L}_{\text{eff}} \supset \int d^4\theta \frac{\Lambda_H}{4\pi}(N + N_\Phi + N_\Omega) \tag{39}$$

with a $O(1)$ factor for each coupling. We now turn on the effects of soft SUSY breaking. We introduce a singlet chiral superfield X whose F term, F_X, has a nonzero VEV. We couple X to the other superfields by contact interactions suppressed by the scale M, where M is identified with the Planck scale in the case of gravity mediation and with the messenger scale in the case of gauge mediation.[1] The soft SUSY-breaking scale is given by F_X/M. With soft SUSY-breaking effects, we have the following Kähler potential that gives extra contributions to the tadpole terms in Eq. (6):

$$\mathcal{L}_{\rm eff} \supset \int d^4\theta \frac{X^\dagger}{M}\frac{\Lambda_H}{4\pi}(N + N_\Phi + N_\Omega) + \text{H.c.} = \frac{F_X^\dagger}{M}\int d^2\theta \frac{\Lambda_H}{4\pi}(N + N_\Phi + N_\Omega) + \text{H.c.} \tag{40}$$

These extra contributions affect the VEV of N, which is given by

$$\langle N\rangle = \sqrt{\frac{1}{\lambda(M_Z)}}\sqrt{\frac{\xi'_\Phi \xi'_\Omega}{\xi'}}, \tag{41}$$

where ξ', ξ'_Φ and ξ'_Ω are the sums of the tadpoles in Eq. (6) and the extra contributions from soft SUSY-breaking effects, which are expressed as

$$\begin{aligned} \xi' &= \xi + a\frac{F_X^\dagger}{M}\frac{\Lambda_H}{4\pi}, \\ \xi'_\Phi &= \xi_\Phi + a_\Phi\frac{F_X^\dagger}{M}\frac{\Lambda_H}{4\pi}, \\ \xi'_\Omega &= \xi_\Omega + a_\Omega\frac{F_X^\dagger}{M}\frac{\Lambda_H}{4\pi}, \end{aligned} \tag{42}$$

where a, a_Φ and a_Ω are all $O(1)$. On the other hand, to break the electroweak symmetry without unnatural cancellation between the effective μ term $\lambda(M_Z)\langle N\rangle$ and the soft SUSY-breaking terms $m^2_{H_u}$ and $m^2_{H_d}$, we require that $\lambda(M_Z)\langle N\rangle$ be smaller than about 1 TeV, which is equivalent to

$$\sqrt{\frac{\xi'_\Phi\xi'_\Omega}{\xi'}} \lesssim \frac{1}{\sqrt{\lambda(M_Z)}} \times 1\ \text{TeV}. \tag{43}$$

To satisfy the above condition in the presence of soft SUSY-breaking effects, it is required that

$$\sqrt{\left|\frac{F_X}{M}\right|\frac{\Lambda_H}{4\pi}} \lesssim \frac{1}{\sqrt{\lambda(M_Z)}} \times 1\ \text{TeV}. \tag{44}$$

Assuming that the soft SUSY-breaking scale F_X/M is about 1 TeV and $\lambda(M_Z)$ is $O(1)$, we obtain the following estimate on the upper bound of Λ_H suggested by the naturalness:

$$\Lambda_H \lesssim 4\pi \times 1\ \text{TeV}. \tag{45}$$

Now that we know the favorable range of Λ_H, we discuss the mass of the SM-like Higgs boson, m_h, which depends on Λ_H via $\lambda(M_Z)$. In the decoupling limit, m_h is given at the tree level by [17]

$$m_h^2 \simeq M_Z^2\cos^2(2\beta) + \lambda(M_Z)^2 v^2 \sin^2(2\beta). \tag{46}$$

Here we introduce $\tan\beta \equiv v_u/v_d$ as usual. Since the naturalness suggests $\Lambda_H \lesssim 10$ TeV, we have $\lambda(M_Z) > 1.9$, according to Fig. 1. Therefore, to realize the Higgs boson mass of 125 GeV, we need large $\tan\beta$ so that the contribution from the second term in Eq. (46) is suppressed.

Finally we comment on how the SM Yukawa couplings of appropriate magnitudes can be derived in this model, especially for the $O(1)$ top quark Yukawa coupling. We adopt the mechanism proposed in Refs. [11,18]. First we introduce two additional $SU(2)_H$ doublets, T_7 and T_8, which have the tree-level mass term below:

$$W_7 = m_7 T_7 T_8, \tag{47}$$

where $m_7 > \Lambda_H$ is assumed. Above the scale m_7, our model is described by the SUSY QCD theory with two colors and four flavors and is in the superconformal window. We assume that this theory approaches the infrared fixed point at some scale $\Lambda_4 > m_7$ and becomes nearly conformal. At the scale m_7, the fields T_7 and T_8 decouple and the theory becomes the SUSY QCD theory with two colors and three flavors. Through renormalization group evolutions, the wavefunctions of T_i's receive large corrections of

$$Z \simeq \left(\frac{m_7}{\Lambda_4}\right)^{\gamma^*}, \tag{48}$$

where γ^* denotes the anomalous dimension at the infrared fixed point, which equals to $(3N_c - N_f)/N_f = 1/2$ for the $N_c = 2$ and $N_f = 4$ SUSY QCD theory. By introducing new $SU(2)_H$ singlets with SUSY-conserving mass $M_f \sim \Lambda_H$ and integrating them out, it is possible to have the following higher-dimensional superpotential at the scale Λ_H:

$$W_f = \frac{1}{M_f}Z^{-1}h_u^{ij}(T_1T_3, T_2T_3)Q_iU_j + \cdots, \tag{49}$$

where h_u^{ij} denotes the Yukawa coupling in the fundamental theory, whose value is at most $O(1)$ and the factor of Z^{-1} comes from the wavefunction renormalization of T_i's. Therefore the fundamental theory may not contain any Landau pole below the Planck scale, and the theory can be UV-complete. We find that, below the scale Λ_H, the term in Eq. (49) reduces to the following term:

$$W_f \simeq \frac{\Lambda_H}{4\pi M_f}Z^{-1}h_u^{ij}H_uQ_iU_j + \cdots \tag{50}$$

by using NDA. We can cancel the suppression factor of $1/4\pi$ by the enhancement factor of Z^{-1} with appropriate values of

[1]Notice that since the superfields N, N_Φ and N_Ω are composites of the fundamental superfields T_i's, which are charged under the $SU(2)_H$ gauge group, gauge mediation can induce soft SUSY-breaking terms for N, N_Φ and N_Ω even if they are gauge singlets.

Λ_4 and m_7. In particular, we can derive the $O(1)$ top quark Yukawa coupling below Λ_H.

IV. A NATURAL MODEL FOR THE LIGHT HIGGS BOSON FROM SUSY STRONG DYNAMICS

In this section, we present a simple UV-complete model where the SM-like Higgs boson strongly couples to other fields in the Higgs sector, but its mass is naturally as light as 125 GeV. This model arises as the low-energy effective theory of the SUSY QCD theory with $N_c = 2$ and $N_f = 3$ with an additional $SU(2)_H$ singlet chiral superfield. The singlet induces a large SUSY-conserving mass for N, so that N decouples from the theory below Λ_H and the term NH_uH_d disappears from the effective superpotential. The effective theory has a similar structure to the "Four Higgs doublets and two charged singlets" (4HDΩ) model [7].

We introduce a $SU(2)_H$ singlet chiral superfield, S, which is neutral under the SM gauge groups and is Z_2-even. The superpotential involving S generally takes the following form[2]:

$$\Delta W = (y_1T_1T_2 + y_3T_3T_4 + y_5T_5T_6)S + \frac{M_S}{2}S^2 + \frac{\kappa}{3}S^3. \tag{51}$$

The coupling constants y_1, y_3 and y_5 are assumed to be at most $O(1)$ at the scale Λ_4, so that they remain finite up to the Planck scale. For simplicity, we here assume $y_5 \gg y_1$, y_3.[3] At scales below Λ_4, they are enhanced by the same mechanism as the Yukawa couplings described in the previous section; renormalization group running from the scale Λ_4 to m_7 enhances them by the factor of Z^{-1}. We assume that M_S is of the order of Λ_H. Then S can be integrated out in the effective theory below Λ_H. We obtain

$$\Delta W = -\frac{y_5^2Z^{-2}}{2M_S}(T_5T_6)(T_5T_6). \tag{52}$$

After using NDA, this becomes the following term in the effective superpotential:

$$\Delta W_{\rm eff} \simeq -\frac{y_5^2Z^{-2}}{2M_S}\frac{\Lambda_H^2}{(4\pi)^2}N^2 = -\frac{M_N}{2}N^2, \tag{53}$$

where M_N is defined as

$$M_N \equiv \frac{y_5^2Z^{-2}}{M_S}\frac{\Lambda_H^2}{(4\pi)^2}. \tag{54}$$

Since the factor of Z^{-2} compensates the suppression factor of $1/(4\pi)^2$, we have the relation: $M_N \sim \Lambda_H$ when y_5 is $O(1)$.

[2]The possible tadpole term for S can be eliminated by shifting the value of S.

[3]When y_1 and y_3 are as large as y_5, we have to take into account the mixings among N, N_Φ and N_Ω in the effective theory. This complicates the model, although it does not affect the main results of our discussion.

We study how the term in Eq. (53) modifies the model with the relation $M_N \sim \Lambda_H$. First we look for charge-conserving absolute SUSY vacua, using the conditions Eqs. (14)–(24), but Eq. (15) is replaced with

$$0 = -H_u^0H_d^0 - N_\Omega N_\Phi + v_0^2 - \frac{M_N}{\lambda}N. \tag{55}$$

From Eqs. (14) and (19)–(22), we again have $H_d^0 = \Phi_d^0 = H_u^0 = \Phi_u^0 = 0$. It follows that $\zeta = \eta = 0$. We obtain the following relations:

$$NN_\Phi = v_\Omega^2, \qquad N_\Omega N_\Phi = v_0^2 - \frac{M_N}{\lambda}N, \qquad N_\Omega N = v_\Phi^2. \tag{56}$$

Let us take those values of m_1, m_3 and m_5 which satisfy the following relations:

$$m_5^2 \gtrsim 4\pi\Lambda_H m_1/\lambda(M_Z), \qquad m_5^2 \gtrsim 4\pi\Lambda_H m_3/\lambda(M_Z), \tag{57}$$

$$\lambda(M_Z)\frac{m_5\Lambda_H}{4\pi} \ll \Lambda_H^2, \tag{58}$$

or equivalently

$$\xi^2 \gtrsim \xi_\Phi M_N^2/\lambda(M_Z), \qquad \xi^2 \gtrsim \xi_\Omega M_N^2/\lambda(M_Z), \tag{59}$$

$$\lambda(M_Z)\xi \ll M_N^2, \tag{60}$$

where the scale dependences of ξ, ξ_Φ and ξ_Ω are negligible. The VEV of N is then approximately given by

$$\langle N\rangle \simeq \frac{\xi}{M_N} = \frac{\lambda v_0^2}{M_N}, \tag{61}$$

and eventually the VEVs of N_Φ and N_Ω are given by

$$\langle N_\Phi\rangle \simeq \frac{1}{\lambda}\frac{\xi_\Omega}{\xi}M_N = \frac{1}{\lambda}\frac{v_\Omega^2}{v_0^2}M_N,$$
$$\langle N_\Omega\rangle \simeq \frac{1}{\lambda}\frac{\xi_\Phi}{\xi}M_N = \frac{1}{\lambda}\frac{v_\Phi^2}{v_0^2}M_N. \tag{62}$$

Since N has the large SUSY-conserving mass of order Λ_H, we may integrate it out in the effective theory below Λ_H. We then obtain the following superpotential:

$$\begin{aligned} W_{\rm eff} = \lambda\{&N_\Phi(\Phi_u\Phi_d + v_\Phi^2) + N_\Omega(\Omega^+\Omega^- + v_\Omega^2) - N_\Omega\zeta\eta \\ &+ \zeta H_d\Phi_u + \eta H_u\Phi_d - \Omega^+H_d\Phi_d - \Omega^-H_u\Phi_u\} \\ &+ \frac{\lambda^2}{2M_N}(H_uH_d + v_0^2 - N_\Phi N_\Omega)^2. \end{aligned} \tag{63}$$

We expand the fields N_Φ and N_Ω around their VEVs and replace them respectively with $\langle N_\Phi\rangle + n_\Phi$ and $\langle N_\Omega\rangle + n_\Omega$, where n_Φ and n_Ω denote their physical components. The superpotential is then expressed as

$$W_{\rm eff} = \lambda\bigg\{\bigg(\frac{v_\Omega^2}{\lambda v_0^2}M_N + n_\Phi\bigg)(\Phi_u\Phi_d + v_\Phi^2) + \bigg(\frac{v_\Phi^2}{\lambda v_0^2}M_N + n_\Omega\bigg) \times (\Omega^+\Omega^- + v_\Omega^2) - \bigg(\frac{v_\Phi^2}{\lambda v_0^2}M_N + n_\Omega\bigg)\zeta\eta + \zeta H_d\Phi_u + \eta H_u\Phi_d - \Omega^+ H_d\Phi_d - \Omega^- H_u\Phi_u\bigg\} + \frac{\lambda^2}{2M_N}\bigg(H_uH_d + v_0^2 - \frac{v_\Phi^2 v_\Omega^2}{\lambda^2 v_0^4}M_N^2 - \frac{v_\Omega^2}{\lambda v_0^2}M_N n_\Omega - \frac{v_\Phi^2}{\lambda v_0^2}M_N n_\Phi - n_\Phi n_\Omega\bigg)^2, \tag{64}$$

which can be rewritten as

$$W_{\rm eff} = -\mu(H_uH_d - n_\Phi n_\Omega) - \mu_\Phi \Phi_u\Phi_d - \mu_\Omega(\Omega^+\Omega^- - \zeta\eta) + \lambda\{H_d\Phi_u\zeta + H_u\Phi_d\eta - H_u\Phi_u\Omega^- - H_d\Phi_d\Omega^+ + n_\Phi\Phi_u\Phi_d + n_\Omega(\Omega^+\Omega^- - \zeta\eta)\} + \cdots, \tag{65}$$

where the effective μ terms are given as

$$\mu = -\frac{\lambda^2}{M_N}\bigg(v_0^2 - \frac{v_\Phi^2 v_\Omega^2}{\lambda^2 v_0^4}M_N^2\bigg) \simeq -\frac{\lambda^2 v_0^2}{M_N},$$
$$\mu_\Phi = -\frac{v_\Omega^2}{v_0^2}M_N, \qquad \mu_\Omega = -\frac{v_\Phi^2}{v_0^2}M_N. \tag{66}$$

For μ, we make an approximation using the relations in Eqs. (59) and (60). Notice that we have $|\mu| \gtrsim |\mu_\Phi|, |\mu| \gtrsim |\mu_\Omega|$ because of the relation in Eq. (59).

So far, we have shown that

(i) N can be integrated out from the effective theory below Λ_H so that there is no three-point coupling for H_u and H_d.

(ii) The VEV of N still gives the effective μ term, which can take an appropriate value.

(iii) The VEVs of N_Φ and N_Ω can be of the same order as or smaller than that of N so that $\Phi_{u,d}$ and $\Omega^\pm$ do not decouple.

Consequently, the model can reduce to the "Four Higgs doublets and two charged singlets" (4HDΩ) model [7], with additional superfields n_Φ and n_Ω which have little impact on the main features of the model because their tree-level couplings to the MSSM Higgs fields, H_u and H_d, are suppressed by powers of $1/\Lambda_H$.

TABLE III. Benchmark sets of model parameters. μ_Φ, μ_Ω and the relevant B terms are set to be zero for simplicity. For the MSSM parameters, we fix $\bar{m}^2_{\tilde{t}_L} = \bar{m}^2_{\tilde{t}_R} = 1000$ GeV, the left-right mixing parameter $X_t = A_t + \mu\cot\beta = 500$ GeV, $\mu = 200$ GeV, $m_A = 500$ GeV and $\tan\beta = 3$. Masses are given in GeV.

Set	$\bar{m}_{\Phi_u^0} = \bar{m}_{\Phi_u^\pm}$	$\bar{m}_{\Phi_d^0} = \bar{m}_{\Phi_d^\pm}$	$\bar{m}_{\Omega^-}$	$\bar{m}_{\Omega^+}$	$\bar{m}_\zeta$	$\bar{m}_\eta$
A	50	350	50	350	50	350
B	50	400	50	400	50	400
C	50	450	50	450	50	450

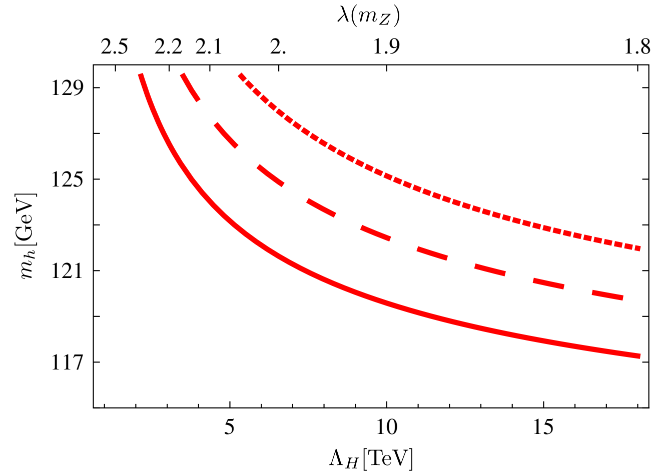

FIG. 2 (color online). The SM-like Higgs boson mass m_h. The solid, dashed, and dotted curves correspond to the benchmark sets A, B, and C given in Table IIIrespectively. The MSSM parameters are fixed as $\bar{m}^2_{\tilde{t}_L} = \bar{m}^2_{\tilde{t}_R} = 1000$ GeV, $X_t = 500$ GeV, $\mu = 200$ GeV, $m_A = 500$ GeV and $\tan\beta = 3$.

Before discussing the mass of the SM-like Higgs boson, we again comment on the range of Λ_H that is favored by the naturalness. The naturalness of the electroweak symmetry breaking requires that the effective μ term for H_u and H_d be smaller than about 1 TeV

$$\lambda(M_Z)\langle N\rangle = \mu \lesssim 1 \text{ TeV}. \tag{67}$$

Additionally, in order that the fields Φ_u, Φ_d and $\Omega^\pm$ remain in the effective theory, the terms of μ_Φ and μ_Ω should be smaller than 1 TeV

$$\lambda(M_Z)\langle N_\Phi\rangle = \mu_\Phi \lesssim 1 \text{ TeV},$$
$$\lambda(M_Z)\langle N_\Omega\rangle = \mu_\Omega \lesssim 1 \text{ TeV}. \tag{68}$$

On the other hand, soft SUSY-breaking terms in the Kähler potential

$$\mathcal{L}_{\rm eff} \supset \frac{F_X^\dagger}{M}\int d^2\theta \frac{\Lambda_H}{4\pi}(N + N_\Phi + N_\Omega) + \text{H.c.} \tag{69}$$

give extra contributions to the tadpoles in Eq. (5), as we have discussed in Sec. III. The VEVs of N, N_Φ and N_Ω are then given by

TABLE IV. The cutoff scale Λ_H and the coupling constant λ for realizing $m_h = 125$ GeV in each benchmark set.

Set	Λ_H[TeV]	$\lambda(m_Z)$
A	3.8	2.1
B	6.4	2.0
C	10.2	1.9

TABLE V. The mass spectrum of the Z_2-odd particles for $m_h = 125$ GeV in each benchmark set. Masses are given in GeV.

Set	$m_{\Phi_u^\pm}$	$m_{\Phi_d^\pm}$	$m_{\Phi_u^0}$	$m_{\Phi_d^0}$	m_{Ω_-}	m_{Ω_+}	m_ζ	m_η	$m_{\tilde{\chi}_1'^\pm}$	$m_{\tilde{\chi}_2'^\pm}$	$m_{\tilde{\chi}_1'^0}$	$m_{\tilde{\chi}_2'^0}$
A	353.6	371.7	140.2	493.5	354.1	371.2	369.2	356.2	117.6	352.7	117.6	352.7
B	331.9	417.2	134.2	516.0	332.5	416.7	414.9	334.7	110.3	331.0	110.3	331.0
C	315.6	464.0	129.7	546.0	316.2	463.7	462.1	318.5	104.9	314.6	104.9	314.6

$$\langle N\rangle \simeq \frac{\xi'}{M_N}, \qquad \langle N_\Phi\rangle \simeq \frac{1}{\lambda(M_Z)}\frac{\xi'_\Omega}{\xi'}M_N,$$
$$\langle N_\Omega\rangle \simeq \frac{1}{\lambda(M_Z)}\frac{\xi'_\Phi}{\xi'}M_N, \tag{70}$$

where ξ', ξ'_Φ and ξ'_Ω contain the extra contributions from soft SUSY-breaking effects and can be written as

$$\xi' = \xi + a\frac{F_X^\dagger}{M}\frac{\Lambda_H}{4\pi}, \qquad \xi'_\Phi = \xi_\Phi + a_\Phi\frac{F_X^\dagger}{M}\frac{\Lambda_H}{4\pi},$$
$$\xi'_\Omega = \xi_\Omega + a_\Omega\frac{F_X^\dagger}{M}\frac{\Lambda_H}{4\pi}, \tag{71}$$

where a, a_Φ and a_Ω are all $O(1)$. In order to solve the gauge hierarchy problem, the soft SUSY-breaking scale F_X/M is at most 1 TeV. Soft SUSY breaking then always respects the condition Eq. (67). To satisfy the conditions Eqs. (68) in the presence of soft SUSY-breaking effects, we need to have

$$\Lambda_H \lesssim 4\pi \times 1\ \text{TeV}. \tag{72}$$

As we know the range of Λ_H that is favored by the naturalness and that gives not so heavy Φ_u, Φ_d and $\Omega^\pm$, we calculate the mass of the SM-like Higgs boson m_h. The mass depends on the coupling constant λ through radiative corrections involving the scalar and fermion components of Z_2-odd fields $\Omega^\pm$, Φ_u^+, Φ_d^-, ζ and η. Hence m_h depends on Λ_H through the scale dependence of λ. Here we consider the case with $\mu_\Omega = \mu_\Phi = 0$ for simplicity. The charginos still obtain masses from the VEVs of the Higgs fields. In this simple case, the Z_2-odd scalars do not mix with each other. Thus the corrected mass of the SM-like Higgs boson is approximately expressed as

$$m_h^2 \simeq m_Z^2\cos^2 2\beta + (\text{MSSM-loop}) + \frac{\lambda^4 v^2}{8\pi^2} \times \left(c_\beta^4 \ln\frac{m_{\Omega^+}^2 m_{\Phi_d^\pm}^2 m_{\Phi_u^0}^2 m_\zeta^2}{m_{\tilde{\chi}_1'^\pm}^4 m_{\tilde{\chi}_1'^0}^4} + s_\beta^4 \ln\frac{m_{\Omega^-}^2 m_{\Phi_u^\pm}^2 m_{\Phi_d^0}^2 m_\eta^2}{m_{\tilde{\chi}_2'^\pm}^4 m_{\tilde{\chi}_2'^0}^4}\right), \tag{73}$$

where $m_{\Phi_u^\pm}(m_{\Phi_d^\pm})$ and $m_{\Phi_u^0}(m_{\Phi_d^0})$ are the masses of charged scalars and neutral scalars from Z_2-odd doublet $\Phi_u(\Phi_d)$, $m_{\Omega^\pm}$ and $m_{\zeta,\eta}$ are the scalar masses of Z_2-odd charged singlets and Z_2-odd neutral singlets respectively, $m_{\tilde{\chi}_{1,2}'^\pm}$ are the Z_2-odd chargino masses, and $m_{\tilde{\chi}_{1,2}'^0}$ are the Z_2-odd neutralino masses. The mass eigenstates of the neutralinos and charginos in this simple case are written as $\tilde{\chi}_1'^0 = (\tilde{\zeta}, \bar{\tilde{\Phi}}_d^0)^T$, $\tilde{\chi}_2'^0 = (\tilde{\eta}, \bar{\tilde{\Phi}}_u^0)^T$, $\tilde{\chi}_1'^+ = (\tilde{\Omega}^+, \bar{\tilde{\Phi}}_d^+)^T$, and $\tilde{\chi}_1'^- = (\tilde{\Omega}^-, \bar{\tilde{\Phi}}_u^-)^T$. The Z_2-odd scalar masses can be typically expressed by $m_{\phi'}^2 = \bar{m}_{\phi'}^2 + (k'g'^2 + kg^2 + c\lambda^2)v^2$, where $\bar{m}_{\phi'}^2$ denotes the soft SUSY-breaking scalar mass.[4] For the soft SUSY-breaking masses, we consider three benchmark sets of parameters shown in Table III. When the contribution coming from the Higgs VEV dominates the mass, significant nondecoupling effects can arise [4,5]. Since we are interested in such nondecoupling cases, some soft SUSY-breaking parameters are taken to be as light as 50 GeV.

The SM-like Higgs boson mass for benchmark parameter sets are shown in Fig. 2. Here we fix the parameters in the stop sector as $\bar{m}_{\tilde{t}_L}^2 = \bar{m}_{\tilde{t}_R}^2 = 1000$ GeV and the left-right mixing parameter as $X_t = A_t + \mu\cot\beta = 500$ GeV. The MSSM Higgs parameters are fixed as $\mu = 200$ GeV, $m_A = 500$ GeV and $\tan\beta = 3$. With this parameter set, the SM-like Higgs boson mass in the MSSM is evaluated as $m_h \simeq 102.3$ GeV [19]. In our model, the SM-like Higgs boson mass can get significant contributions from loop diagrams involving the Z_2-odd fields due to the large coupling constant λ, in addition to the loop contributions from top quark fields. The size of the corrections depends on the soft SUSY-breaking parameters. As shown in Fig. 2, the SM-like Higgs boson mass m_h can reach to 125 GeV by the radiative corrections.

In Table IV, the values of Λ_H and λ corresponding to $m_h = 125$ GeV are displayed. Once these values are fixed, we can find out the mass spectrum for the Z_2-odd particles in each benchmark as shown in Table V. Since they are noncolored particles, linear colliders can have an advantage on the direct searches for them.

As shown in Ref. [7], the F terms from the couplings among the MSSM-like Higgs doublets, the Z_2-odd doublets and the charged singlets can significantly enhance the first-order electroweak phase transition if the coupling constant is as large as $\lambda \sim 2$. The Z_2-odd neutral singlets can also contribute to making the phase transition stronger by the precise enhancement mechanism. Then the sphaleron decoupling condition required by successful electroweak baryogenesis is satisfied more easily. On the benchmark points chosen in the analysis, the first-order electroweak phase transition is strong enough.

[4]Because $\Phi_u^\pm$ ($\Phi_d^\pm$) and $\Phi_u^0(\Phi_d^0)$ are the components of Φ_u (Φ_d), $\bar{m}_{\Phi_u^\pm}^2 = \bar{m}_{\Phi_u^0}^2$ and $\bar{m}_{\Phi_d^\pm}^2 = \bar{m}_{\Phi_d^0}^2$ are satisfied.

V. CONCLUSIONS

We have shown that the SUSY QCD theory with $N_c = 2$ and $N_f = 3$ with one fundamental singlet S can give the strongly coupled Higgs sector containing four isospin doublets, two charged singlets and two neutral singlets as the low-energy description. Since the cutoff scale Λ_H is as low as multi-TeV to 10 TeV, the coupling constant λ in the Higgs sector is as large as ~ 2. In our model, however, the SM-like Higgs boson is naturally light because the F terms do not contribute to its mass at the tree level, while radiative corrections involving strongly coupled fields in the Higgs sector are large enough to raise the SM-like Higgs boson mass to 125 GeV.

We comment on collider signatures of our model. The model contains many new Z_2-odd charged and neutral scalars and fermions with masses of several hundred GeV. The lightest one is definitely stable because of the Z_2 parity.[5] At the LHC, Z_2-odd particles are pair-produced through electroweak interactions, and decay into two lightest Z_2-odd particles, two lightest R-parity-odd particles and several SM particles. (For early studies, see Ref. [20].) The most clear signatures of the model are events with two or three leptons and large missing transverse momentum. The results from the searches for slepton, chargino and neutralino direct productions by ATLAS collaboration [21] apply to our model. The current bound [21] is mild and our benchmark spectra of Table V may not have been excluded yet.

At the tree level, the SM-like Higgs boson couples to MSSM particles in the same way as the MSSM. It also couples to Z_2-odd particles through large coupling constant λ, but its decay width and branching ratios at the tree level are not altered if the mass of the light Z_2-odd particle is larger than half the SM-like Higgs boson mass. The branching ratio into two photons can be significantly affected by loop corrections involving Z_2-odd charged scalars and fermions that strongly couple to the SM-like Higgs boson. They can enhance or suppress the branching ratio depending on the parameters. Also, the triple coupling for the SM-like Higgs boson, which has been studied in Refs. [4,5], receives large corrections from loops involving Z_2-odd doublets and singlets. It thus significantly deviates from the SM prediction, and such deviation can be observed through future collider experiments.

Finally we comment on a possible extension of the model. In the present model, the large coupling constant λ with the 125 GeV SM-like Higgs boson can make the first-order electroweak phase transition strong enough to enable electroweak baryogenesis [7]. In addition, the lightest Z_2-odd field can be another source of dark matter than the lightest R-parity-odd field as long as it is electrically neutral. Furthermore, the model can be extended to explain the tiny neutrino masses. By introducing Z_2-odd right-handed neutrino superfields whose Majorana masses are at the TeV scale, the tiny neutrino masses can be generated at loop levels [22,23]. With this extension of the model, we may be able to build the testable theory in which baryon asymmetry of the Universe, dark matter and the tiny neutrino masses can be simultaneously explained from the UV-complete SUSY strong dynamics around multi-TeV to 10 TeV without excessive fine-tuning. We leave these topics for future studies.

[5]The lightest Z_2-odd scalar and fermion can be both stable if their mass difference is smaller than the lightest R-parity-odd, Z_2-even particle.

ACKNOWLEDGMENTS

This work was supported in part by Grants-in-Aid for Scientific Research, No. 22244031 (S. K.), No. 23104006 (S. K.), No. 23104011 (T. S.) and No. 24340046 (S. K. and T. S.). The work of T. Y. was supported in part by a grant of the Japan Society for the Promotion of Science, No. 23-3599.

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