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Natural 125 GeV Higgs boson in the MSSM from focus point supersymmetry with *A*-terms

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We show that a 125 GeV Higgs boson and percent-level fine-tuning are simultaneously attainable in the minimal supersymmetric standard model, with no additional fields and supersymmetry breaking generated at the grand unification theory scale. The Higgs mass is raised by large radiative contributions from top squarks with significant left-right mixing, and naturalness is preserved by the focus point mechanism with large A-terms, which suppresses large log-enhanced sensitivities to variations in the fundamental parameters. The focus point mechanism is independent of almost all supersymmetry-breaking parameters, but is predictive in the top sector, requiring the grand unification theory-scale relation $m_{H_u}^2 \colon m_{U_3}^2 \colon m_{Q_3}^2 \colon A_t^2 = 1 \colon 1 + x - 3y \colon 1 - x \colon 9y$, where x and y are constants. We derive this condition analytically and then investigate three representative models through detailed numerical analysis. The models generically predict heavy superpartners, but dark matter searches in the case of nonunified gaugino masses are promising, as are searches for top squarks and gluinos with top- and bottom-rich cascade decays at the LHC. This framework may be viewed as a simple update to mSUGRA/CMSSM to accommodate both naturalness and current Higgs boson constraints, and provides an ideal framework for presenting new results from LHC searches.

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I. INTRODUCTION

For three decades, weak-scale supersymmetry (SUSY) has been strongly motivated by three main promises: a natural solution to the gauge hierarchy problem, an excellent dark matter candidate, and force unification. Recent results from the LHC with center-of-mass energy $\sqrt{s} = 7$ TeV and integrated luminosity $\sim 5\ \mathrm{fb}^{-1}$ have begun to challenge this paradigm. The challenge arises from two sources: first, null results from superpartner searches require the gluino and some squarks to have masses around 1 TeV or above in conventional SUSY scenarios [1,2], and second, the ATLAS and CMS experiments have excluded much of the Higgs boson mass range and, at the same time, have reported excesses consistent with a standard model (SM)-like Higgs boson at masses of 126 GeV [3] and 124 GeV [4], respectively.

The null results from superpartner searches are the most direct constraints, but they are not especially problematic for weak-scale SUSY. Thermal relic neutralinos and gauge coupling unification provide only weak upper bounds on the masses of superpartners, and are completely consistent with heavy scalars far above the TeV scale [5,6]. With regard to naturalness, TeV-scale superpartners generically require percent-level fine-tuning of the weak scale. Although naturalness is a notoriously subjective and brittle concept, we consider such fine-tuning acceptable. Less subjective is the fact that, for many superpartners, there are good reasons to expect them to be far above current LHC bounds. For example, for the first two generations of squarks, 10 TeV masses are natural [7–10], flavor constraints generically require masses far above the TeV scale, and even in models that automatically conserve flavor, electric dipole moments typically require superpartner masses well above 1 TeV [11–14]. In light of these long-standing facts, the fact that superpartners have not yet been discovered at the LHC should not be a great surprise.

The Higgs boson results, although still tentative, are more troubling. Although a 125 GeV Higgs boson is, broadly speaking, in the range expected for SUSY, it typically requires large radiative corrections from top squarks, and the required stop properties generically induce large fine-tuning. For example, in the case of the minimal supersymmetric standard model (MSSM), with SUSY breaking mediated at the grand unification theory (GUT) scale and negligible left-right stop mixing, the required stop masses are around 10 TeV, typically corresponding to a fine-tuning of roughly 1 part in 10^4. With maximal mixing, the stops may be lighter, and the fine-tuning may be reduced to roughly 1 part in 1000 [15]. Such results are extremely sensitive to the actual value of the Higgs boson mass, as well as to still significant uncertainties in the theoretical calculation of the Higgs boson mass in SUSY, and their interpretation is again subject to individual taste. The tension has, however, motivated numerous reexaminations of naturalness (see, for example, Refs. [16–19]), as well as many reconsiderations of extensions of the MSSM, with all their attendant difficulties.

It is important to note, however, that the apparent conflict between the 125 Higgs boson and naturalness assumes that the soft SUSY parameters are uncorrelated. SUSY theories with uncorrelated soft parameters are, however, excluded—a wealth of experimental data implies that if weak-scale SUSY exists, there must be structure behind the soft parameters. The possibility that correlations between soft parameters reconcile naturalness with heavy

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superpartners is formalized in the framework of focus point (FP) SUSY [20,21]. In FP SUSY, correlations reduce the sensitivity of the weak scale to variations in the fundamental parameters, even if they are large and above the TeV scale. This insensitivity may be understood in a number of equivalent ways. Graphically, in FP SUSY, the insensitivity may be understood as a property of renormalization group (RG) trajectories, which focus to a fixed value at the weak scale independent of their value in the ultraviolet. Alternatively, FP SUSY may be understood as suppressing the large log-enhanced sensitivity to GUT-scale parameters, leaving only the "irreducible" quadratic sensitivity, and thereby reducing fine-tuning by factors of roughly $\ln(m_{\rm GUT}/m_{\tilde{t}}) \sim 30$. From any view, identifying naturalness with insensitivity, all natural theories with high-scale mediation and multi-TeV top squarks are FP models, and current results from Higgs boson searches at the LHC provide a strong motivation for FP SUSY.

Early analyses of the FP mechanism [20–22] considered the MSSM with multi-TeV scalar masses, but small A-terms. This framework was reviewed recently in light of the LHC Higgs results [23]. Depending on Higgs mass uncertainties, regions of parameter space that are fine-tuned to as little as 1 part in 500 are consistent with the Higgs search results, as we will review below. Although some scalars are very heavy, these models have some superpartners well within the reach of the LHC, notably the gluino, and excellent weakly-interacting massive particle dark matter candidates. The collider and cosmological implications have been explored in many studies (see, for example, Refs. [5,23–31]), and have implications not only for FP SUSY, but also for all other models with heavy scalars. FP SUSY has also been investigated in many other related contexts, including hyperbolic branch SUSY [32,33], gauge-mediated supersymmetry-breaking models [34], mirage mediation [35], models with large gaugino masses [36–38], and the MSSM with right-handed neutrinos [39].

In this work, we consider FP models in which both scalar masses and A-terms are multi-TeV. The large A-terms allow for significant stop mixing, but of course, require a reanalysis of the FP mechanism, since the A-terms can no longer be neglected in the RG evolution. We begin by reviewing the standard definition of fine-tuning in Sec. II, and comment on its implications and some alternative definitions used in the literature. We then present an analytic derivation of focus points with large A-terms in Sec. III, deriving the necessary relationships between the soft SUSY-breaking parameters. With these analytic results as a guide, we perform fully detailed numerical analyses in Sec. IV. We consider three representative models in detail, and show that a 125 GeV Higgs mass may be obtained with only percent-level fine-tuning in the MSSM, without additional field content and without invoking a low mediation scale. We present implications for collider and dark matter searches in Sec. V and summarize our results in Sec. VI.

II. FINE-TUNING IN THE MSSM

For $\tan\beta \gtrsim 5$, the tree-level condition for electroweak symmetry breaking in the MSSM is

$$m_Z^2 \approx -2\mu^2 - 2m_{H_u}^2(m_W), \tag{1}$$

where $m_{H_u}^2(m_W)$ is the up-type Higgs mass parameter at the weak scale $m_W \sim 100\ \text{GeV}^{-1}$ TeV, and μ is the Higgsino mass parameter. Natural SUSY theories generally fall into two classes: conventional theories in which the fundamental parameters determining $m_{H_u}^2(m_W)$ have values at or below the TeV scale throughout their RG evolution, and FP theories, in which natural values of $m_{H_u}^2(m_W)$ are dynamically generated and insensitive to the values of the GUT-scale parameters, even if these GUT-scale parameters are significantly above the TeV scale. In this study, we restrict ourselves to the MSSM, that is, the supersymmetric model with minimal field content, with soft SUSY-breaking scalar and gaugino masses generated at the GUT scale $m_{\rm GUT} \simeq 2.4 \times 10^{16}$ GeV.

To evaluate naturalness, we define the sensitivity coefficients

$$c_a \equiv \left| \frac{\partial \ln m_Z^2}{\partial \ln a^2} \right|, \tag{2}$$

where a^2 is one of the input GUT-scale parameters, including m_0^2, $M_{1/2}^2$, μ_0^2, and m_3^2, the $H_u^0 H_d^0$ mass parameter.[1] The overall fine-tuning of a model is defined as

$$c \equiv \max\{c_a\}. \tag{3}$$

In the models we will consider, either c_{m_0} or $c_{M_{1/2}}$ determines c in the interesting regions of parameter space.

Note that it is quite possible to have large values of the GUT-scale parameters, but to arrange for $m_{H_u}^2(m_W) \sim m_Z^2$ by suitably fine-tuning values for these GUT parameters. In such scenarios, Eq. (1) may appear natural, and $c_{\mu_0} \propto \mu^2/m_Z^2$ will be low, but this should not obscure the fact that the model has been fine-tuned to get low $m_{H_u}^2$ and the weak scale is nevertheless unnaturally sensitive to variations in the GUT-scale parameters. Here we require not just

[1] We choose the GUT-scale parameter to be m_0^2, not m_0, because we consider m_0^2 to be more fundamental (it may be negative, for example Ref. [40]), and because we consider it more reasonable to compare squared masses against one another, given Eq. (1). For this reason, we also choose m_Z^2 instead of m_Z in the numerator of Eq. (2), and for uniformity, choose all a^2 to be mass dimension 2. As a result, our definition differs from the original definition of $c_a = \partial \ln m_Z^2/\partial \ln a$ [41,42] by a factor of 2. Such factors are clearly unimportant in judging whether a scenario is natural or not, given the subjective nature of the definition, but are important to keep in mind when comparing numerical results.

low c_{μ_0}, but low c, and the FP models discussed below will be natural according to this stricter definition.

In the sections below, we will consider models with heavy scalars. Below the scalar masses, the Higgs mass receives quadratic contributions of the form $(6/8\pi^2)y_t^2 m_{\tilde{t}}^2$, where y_t is the top Yukawa coupling. This contribution is *not* the usual source of the fine-tuning problem—it is one-loop suppressed, and if this were all there were, even values as large as $m_{\tilde{t}} \sim 3$ TeV would only be percent-level fine-tuned. The dominant source of fine-tuning, and the apparent conflict between naturalness and the 125 GeV Higgs boson, is the large log-enhanced contributions. These result from RG evolution from the GUT scale and are of the order of $\sim(6/8\pi^2)y_t^2 m_{\tilde{t}}^2 \ln(m_{\rm GUT}/m_{\tilde{t}})$. The large logarithm roughly cancels the loop-suppressed prefactor, leading to the conventional wisdom that multi-TeV top squarks imply sub-percent-level fine-tuning. To reduce the dominant source of fine-tuning, then, one may consider the RG equations (RGEs) and look for correlations that reduce the sensitivity of the weak scale to variations in the GUT-scale parameters. This is the possibility formalized in the FP framework, to which we turn in the next section. Of course, the "irreducible" quadratic contribution will remain, and will be accounted for when fine-tuning is evaluated through the full numerical analysis of Sec. IV, in which two-loop RGEs and one-loop threshold corrections are used and superpartners are integrated out at the appropriate mass scale.

III. FOCUS POINTS FOR LARGE SCALAR MASSES AND A-TERMS

We begin in this section with a simple analytic discussion to extract the desired FP behavior. In Sec. IV, we verify the validity of the results derived here through a full numerical analysis.

A. Renormalization group equations

The one-loop RGEs for SUSY parameters have the schematic form

$$\frac{dg}{d\ln Q} \sim -g^3, \tag{4}$$

$$\frac{dy}{d\ln Q} \sim -g^2 y + y^3, \tag{5}$$

$$\frac{dM}{d\ln Q} \sim -g^2 M, \tag{6}$$

$$\frac{dA}{d\ln Q} \sim g^2 M + y^2 A, \tag{7}$$

$$\frac{dm^2}{d\ln Q} \sim -g^2 M^2 + y^2 A^2 + y^2 m^2, \tag{8}$$

where positive numerical coefficients have been omitted, and g, y, M, A, and m are generic symbols for gauge couplings, Yukawa couplings, gaugino masses, trilinear scalar couplings, and scalar masses, respectively.

Because the scalar masses and A parameters do not enter the gaugino mass RGEs, it is self-consistent to assume m^2, $A^2 \gg M^2$ through the RG evolution. With this assumption, and further neglecting all Yukawa couplings other than y_t,[2] the RGEs reduce to

$$\frac{d}{d\ln Q}\begin{bmatrix} m_{H_u}^2 \\ m_{U_3}^2 \\ m_{Q_3}^2 \\ A_t^2 \end{bmatrix} = \frac{y_t^2}{8\pi}\begin{bmatrix} 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 12 \end{bmatrix}\begin{bmatrix} m_{H_u}^2 \\ m_{U_3}^2 \\ m_{Q_3}^2 \\ A_t^2 \end{bmatrix}. \tag{9}$$

Here, we have neglected not only the gaugino masses, but also terms proportional to $g_1^2 S$, where

$$S = m_{H_u}^2 - m_{H_d}^2 + \mathrm{tr}[\boldsymbol{m_Q^2} - \boldsymbol{m_L^2} - 2\boldsymbol{m_U^2} + \boldsymbol{m_D^2} + \boldsymbol{m_E^2}]. \tag{10}$$

These contributions will be discussed further in Sec. IV.

Equation (9) may be solved in terms of the eigenvalues and eigenvectors of the 4×4 matrix of numerical coefficients. The solution is

$$\begin{bmatrix} m_{H_u}^2(Q) \\ m_{U_3}^2(Q) \\ m_{Q_3}^2(Q) \\ A_t^2(Q) \end{bmatrix} = \kappa_{12}\begin{bmatrix} 3 \\ 2 \\ 1 \\ 6 \end{bmatrix} e^{12I(Q)} + \kappa_6 \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} e^{6I(Q)} + \kappa_0 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} + \kappa_0' \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \tag{11}$$

where

$$I(Q) = \int_{\ln Q_0}^{\ln Q} \frac{y_t^2(Q')}{8\pi^2} d\ln Q' \tag{12}$$

is a renormalization factor related to the running of the top Yukawa coupling from the mass generation scale Q_0 to the scale Q. It takes the value $e^{6I(m_W)} \simeq \frac{1}{3}$ for renormalization between the GUT and weak scales [21].

To consider the possibility that a large value of $m_{H_u}^2$ at the GUT scale evolves to a much smaller value at the weak scale, we set

[2] This assumption is valid for low and moderate values of $\tan\beta$. Extending this discussion to high $\tan\beta$ is possible, but requires unified A-terms for third generation sfermions ($\tilde{t}$, $\tilde{b}$, $\tilde{\tau}$) for the RGEs to be linear. Doing so further requires the inclusion of a right-handed neutrino supermultiplet for consistent RGEs.

$$m_0^2 = m_{H_u}^2(m_{\rm GUT}) = 3\kappa_{12} + 3\kappa + \kappa_0 \tag{13}$$

$$0 = m_{H_u}^2(m_W) = 3\kappa_{12}e^{12I(m_W)} + 3\kappa e^{6I(m_W)} + \kappa_0. \tag{14}$$

With these conditions, and using the above approximation for $e^{6I(m_W)}$, the parameters evolve from the GUT scale to the weak scale through

$$\begin{bmatrix} m_{H_u}^2(m_{\rm GUT}) \\ m_{U_3}^2(m_{\rm GUT}) \\ m_{Q_3}^2(m_{\rm GUT}) \\ A_t^2(m_{\rm GUT}) \end{bmatrix} = m_0^2 \begin{bmatrix} 1 \\ 1+x-3y \\ 1-x \\ 9y \end{bmatrix} \to \begin{bmatrix} m_{H_u}^2(m_W) \\ m_{U_3}^2(m_W) \\ m_{Q_3}^2(m_W) \\ A_t^2(m_W) \end{bmatrix} \simeq m_0^2 \begin{bmatrix} 0 \\ \frac{1}{3}+x-3y \\ \frac{2}{3}-x \\ y \end{bmatrix}. \tag{15}$$

There is, of course, freedom in choosing the parametrization. We choose x to parametrize the splitting between $m_{Q_3}^2(m_{\rm GUT})$ and $m_{U_3}^2(m_{\rm GUT})$, and y to be directly related to $A_t(m_{\rm GUT})$.

B. Model parameter space

The parameter space therefore consists of an overall scale m_0, and the two numbers x and y. The physically viable region of (x, y) parameter space is determined by two constraints. First, we require $A_t^2(m_W) \geq 0$, and so $y \gtrsim 0$. Second, we require that both stops are not tachyonic at the weak scale. In the limit of large scalar masses, $m_{U_3}^2(m_W)$, $m_{Q_3}^2(m_W) \gg m_t A_t(m_W)$ and one-loop corrections are subdominant, so to a good approximation, the physical stop masses are $m_{U_3}(m_W)$ and $m_{Q_3}(m_W)$. Using this approximation, we find that the viable region is

$$0 \lesssim y \lesssim \frac{1}{3} \tag{16}$$

$$-\frac{1}{3} + 3y \lesssim x \lesssim \frac{2}{3}. \tag{17}$$

This parameter space is shown in Fig. 1.

For $y = 0$, this set of solutions reduces to the FP SUSY models with $m^2 \gg A, M$ discussed previously [5,6,20,21]. For $x = y = 0$, the set reduces further to mSUGRA/CMSSM with $A_t(m_{\rm GUT}) = 0$, which also exhibits FP behavior for large scalar mass m_0^2. The mSUGRA/CMSSM case is highlighted in Fig. 1, along with two other representative models that will be examined in detail below. The blue dashed and green dot-dashed lines correspond to the special cases where $m_{Q_3}^2 = m_{U_3}^2$ at the GUT scale and $m_{Q_3}^2 \simeq m_{U_3}^2$ at the weak scale, respectively.

C. Higgs boson mass

It is well known that radiative corrections to the Higgs boson mass are enhanced both by large stop masses and significant left-right stop mixing at the weak scale. In the limit $m_{Q_3}(m_W) = m_{U_3}(m_W)$, the one-loop and dominant

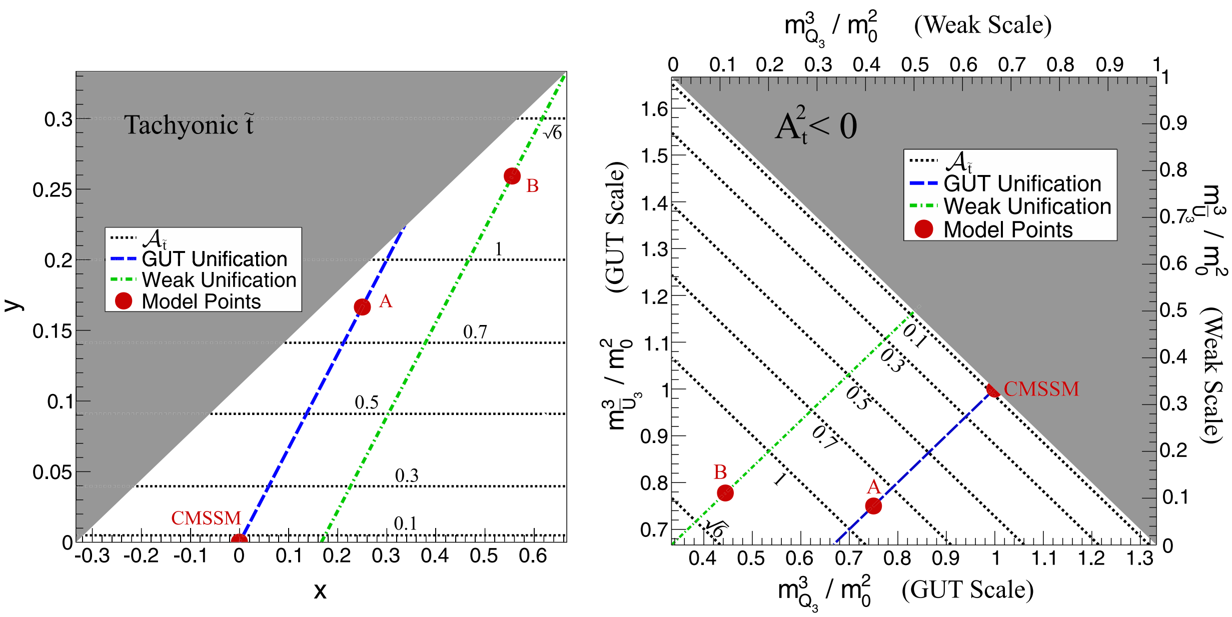


FIG. 1 (color online). The model parameter space in the (x, y) plane (left) and in the $(m_{Q_3}^2(m_{\rm GUT}), m_{U_3}^2(m_{\rm GUT}))$ [or alternatively, $(m_{Q_3}^2(m_W), m_{U_3}^2(m_W))$] plane (right). The blue dashed and green dot-dashed lines correspond to the special cases where $m_{Q_3}^2 = m_{U_3}^2$ at the GUT scale and $m_{Q_3}^2 \simeq m_{U_3}^2$ at the weak scale, respectively, and red dots correspond to the three models $(x, y) = (0, 0)$ (mSUGRA/CMSSM), $(\frac{1}{4}, \frac{1}{6})$, and $(\frac{5}{9}, \frac{7}{27})$ examined in detail. Black dotted lines of constant stop mixing parameter $\mathcal{A}_{\tilde{t}}$ are also shown.

two-loop corrections to the Higgs mass from the stop sector are [43]

$$\Delta m_h^2 = \frac{3}{4\pi^2}\frac{m_t^2}{v^2}\left\{\ln\left(\frac{M_S^2}{m_t^2}\right) + \tilde{X}_t + \frac{3}{32\pi^2}\frac{m_t^2}{v^2} \times \left[2\tilde{X}_t \ln\left(\frac{M_S^2}{m_t^2}\right) + \ln^2\left(\frac{M_S^2}{m_t^2}\right)\right]\right\}, \tag{18}$$

where $v \simeq 246$ GeV, M_S^2 is given in terms of the physical stop masses $m_{\tilde{t}_{1,2}}$ by $M_S^2 = (m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)/2$, and

$$\tilde{X}_t = \frac{[A_t(m_W) - \mu\cot\beta]^2}{M_S^2}\left[1 - \frac{[A_t(m_W) - \mu\cot\beta]^2}{12M_S^2}\right]. \tag{19}$$

The radiative contribution generated by stop mixing is contained in $\tilde{X}_t$, which increases with $A_t(m_W)$ up to a maximal value at $[A_t(m_W) - \mu\cot\beta]^2/M_S^2 = 6$ [44]. Although this enhancement of the Higgs mass depends on the stop mass, it can easily exceed ~10 GeV [45]. We note also that three-loop contributions to the Higgs mass have been calculated [46,47]. They are generically positive and may be more than 3 GeV in the focus point region with large scalar masses [48]. These three-loop corrections have not been included in our analysis, but they imply that our results are conservative, and FP SUSY models with even less fine-tuning than those presented here may also fit the Higgs mass data.

In the focus point scenario $m_{Q_3}(m_W) = m_{U_3}(m_W)$ does not generically hold, but it can reasonably be assumed that the radiative contribution to the Higgs mass from mixing will be increased with $|A_t(m_W)|$ until $|A_t(m_W)| \sim m_{Q_3,U_3}(m_W)$. The enhancement may be parametrized in terms of the mixing parameter

$$\mathcal{A}_{\tilde{t}} = \frac{|A_t(m_W)|}{\sqrt{\frac{1}{2}[m_{Q_3}^2(m_W) + m_{U_3}^2(m_W)]}} \simeq \frac{\sqrt{y}}{\sqrt{\frac{1}{2} - \frac{3}{2}y}}. \tag{20}$$

Here we have neglected the contribution from $\mu\cot\beta$, typically small for natural theories with moderate to large $\tan\beta$, and used $m_{Q_3,U_3}(m_W)$ instead of $m_{\tilde{t}_1,\tilde{t}_2}$ to allow for comparison between models with varying m_0.[3]

Figure 1 contains contours of $\mathcal{A}_{\tilde{t}}$ over the range of allowed value of x and y. Our expression for the radiative corrections to the Higgs mass are approximately correct along the green (dot-dashed) line corresponding to $m_{Q_3}(m_W) = m_{U_3}(m_W)$, and $\mathcal{A}_{\tilde{t}} = \sqrt{6}$ is only possible when this condition is approximately satisfied. For $m_{Q_3}(m_{\rm GUT}) = m_{U_3}(m_{\rm GUT})$, the maximum value of $\mathcal{A}_{\tilde{t}}$ is roughly 1. As we will see from the numerical analysis of the next section, even such large, but nonmaximal, values of $\mathcal{A}_{\tilde{t}}$ lead to significant enhancements of the Higgs mass.[4]

[3]Other definitions of $\mathcal{A}_{\tilde{t}}$ can be used, such as $|A_t(m_W)|/[\frac{1}{2} \times (m_{Q_3} + m_{U_3})]$ or $|A_t(m_W)|/\sqrt{m_{Q_3}m_{U_3}}$. All are roughly equivalent for $m_{Q_3}(m_W) \approx m_{U_3}(m_W)$.

IV. NUMERICAL RESULTS

A. Three representative cases

We now determine to what extent the analytic results of the previous sections are realized when all numerical details are included. We do this by choosing three representative models to analyze.

For reference and comparison to previous studies, we include mSUGRA/CMSSM with $A_t(m_{\rm GUT}) = 0$ as one of these cases. In addition, we would like to consider models with significant stop mixing. The special case of $\mathcal{A}_{\tilde{t}} = \sqrt{6}$ is achievable for $y \approx 0.3$. At this point, both stop masses are roughly 20% of their GUT-scale values, which results in a moderate reduction in the mixing-independent radiative contribution to the Higgs mass. Moreover, this point is near the tachyonic stop boundary for both $m_{Q_3}^2$ and $m_{U_3}^2$, where the neglected effects from proper diagonalization of the stop mass matrix, one-loop corrections, and the RG contributions of the gauginos become important. Inclusion of these effects can easily produce tachyonic stop masses in a particular model, or they may raise one stop mass and thus reduce $\mathcal{A}_{\tilde{t}}$ significantly. A detailed analysis of the maximal mixing scenario was recently made in Ref. [49].

To avoid these issues, the additional two models we consider are far from the tachyonic stop boundaries with significant, but nonmaximal, $\mathcal{A}_{\tilde{t}}$. Heavy stop masses can still be natural in FP SUSY, and so even with nonmaximal mixing, the radiative contribution to the Higgs boson mass may be substantial. The models are

$$\text{mSUGRA/CMSSM} \quad \text{with} \quad A_0 = 0\text{: } (x, y) = (0, 0), \tag{21}$$

$$\text{Model A: } (x, y) = \left(\frac{1}{4}, \frac{1}{6}\right), \tag{22}$$

$$\text{Model B: } (x, y) = \left(\frac{5}{9}, \frac{7}{27}\right), \tag{23}$$

as indicated in Fig. 1. Model A has $\mathcal{A}_{\tilde{t}} = 0.82$ and equal stop masses at the GUT scale, $m_{Q_3}^2(m_{\rm GUT}) = m_{U_3}^2(m_{\rm GUT}) = \frac{3}{4}m_0^2$. Model B has $\mathcal{A}_{\tilde{t}} = 1.53$ and equal stop masses at the weak scale, $m_{Q_3}^2(m_W) \simeq m_{U_3}^2(m_W) \simeq \frac{1}{9}m_0^2$. Both models will produce $m_{\tilde{t}_1,\tilde{t}_2} \gtrsim 1$ TeV for $m_0 \geq 3$ TeV. We emphasize, however, that models near the tachyonic stop boundaries will, of course, predict lighter

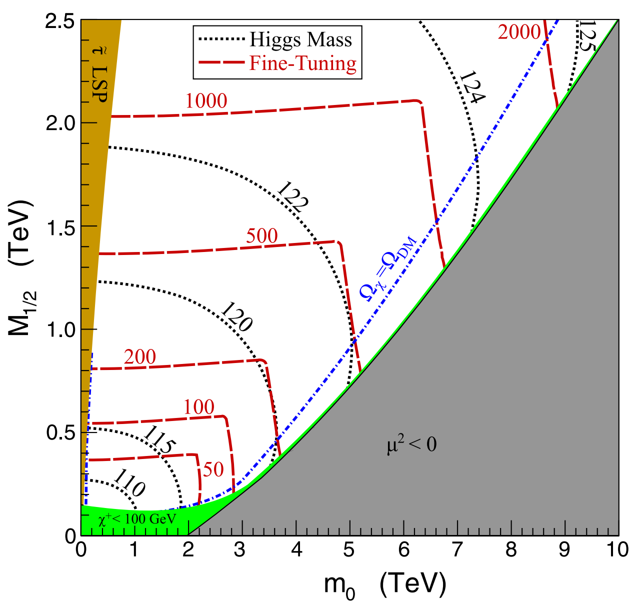
[4]For $m_{Q_3}(m_W) \neq m_{U_3}(m_W)$, the maximal correction to m_h will occur for different values of $\mathcal{A}_{\tilde{t}}$, possibly at smaller values. However, in the case of reasonably large m_0 that we will consider numerically, these effects should appear near the tachyonic stop boundary, where the approximate form of $\mathcal{A}_{\tilde{t}}$ using $m_{Q_3,U_3}(m_W)$ breaks down.

stops, and are perfectly well-defined possibilities. We do not consider them for simplicity, but some of them have near maximal mixing and may produce the 125 GeV Higgs boson with even less fine-tuning than the models we consider.

To derive numerical results, we must, of course, specify the complete SUSY model, including parameters that have little impact on electroweak symmetry breaking and naturalness. For concreteness, we fix all scalar masses (except the stops) to be degenerate with $m_{H_u}^2$ at the GUT scale, $\tan\beta = 10$, and $\mu > 0$. Although $A_t^2(m_{\rm GUT})$ is fixed, the sign of $A_t(m_{\rm GUT})$ is not. We set $A_t(m_{\rm GUT}) < 0$; this choice will be discussed further in Sec. IV B. We impose unification of the A-terms at the GUT scale, using $A_0 = A_t(m_{\rm GUT})$. Both spectra and fine-tuning are obtained using SOFTSUSY 3.1.7 [50], suitably modified to include the nonuniversal boundary conditions and correlations assumed in these FP models. The numerical analysis therefore includes two-loop RGEs, one-loop threshold corrections, minimization of the electroweak potential at M_S, and quadratic contributions to the Higgs mass at energy scales below the masses of the superpartners, where they have been integrated out. There is an uncertainty in the determination of the Higgs mass, which has been estimated to be roughly 3–5 GeV [51,52] through comparison of numerical routines.

Figure 2 contains contours of m_h and c (left panel) and $m_{\tilde g}$ and $m_{\tilde t_1}$ (right panel) in the $(m_0, M_{1/2})$ plane for the mSUGRA/CMSSM case with $A_t(m_{\rm GUT}) = 0$. In this case, $c = c_{M_{1/2}}$ for low m_0, and remains roughly constant as m_0 increases until c_{m_0} becomes the dominant contribution at large m_0, where the contours angle downward. However, even when fine-tuning due to m_0 is dominant, it is greatly suppressed relative to the naïve value of m_0^2/m_Z^2. Taking the results at face value, we find that it is possible to achieve a Higgs boson mass of 125 GeV for $m_0 \approx 10$ TeV, $M_{1/2} \approx 2$ TeV, and $c \sim 2000$, an order of magnitude less fine-tuning than would be required without the FP mechanism. We note, however, that these conclusions are extremely sensitive to uncertainties in the experimental measurements and theoretical calculations of the Higgs mass. For example, if these effects combine to imply that we have overestimated the Higgs boson mass by 5 (3) GeV [51,52], a 125 GeV Higgs mass requires $m_0 \approx 3.5(5)$ TeV, $M_{1/2} \approx 0.5(1)$ TeV, and $c \sim 200(500)$. Within current uncertainties, then, even the focus point region of mSUGRA/CMSSM with $A_t(m_{\rm GUT}) = 0$ may yield the desired Higgs boson mass with fine-tunings not far below the percent level.

The results for Models A and B are shown in Figs. 3 and 4, respectively. The basic behavior of the fine-tuning contours is similar to the mSUGRA/CMSSM case, but $c_{M_{1/2}}$ is somewhat larger and c_{m_0} slightly smaller at large m_0. Both effects are larger for Model B. The more important effects are the shifts in both the m_h contours and the region with no viable electroweak minimum. In both cases, m_h increases



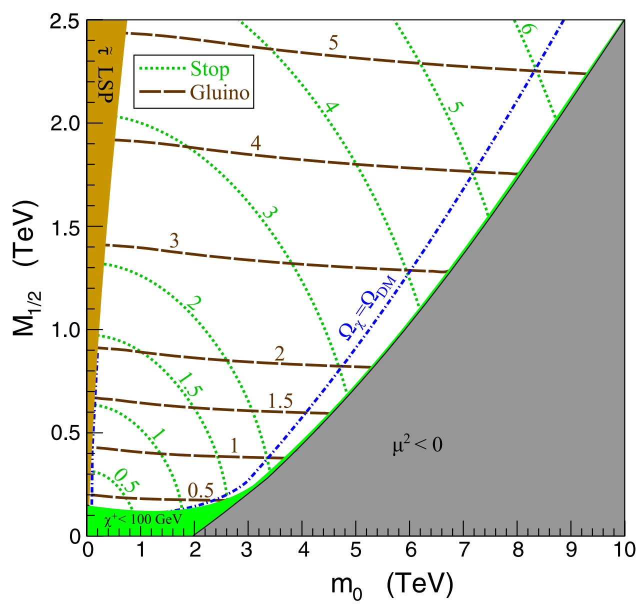


FIG. 2 (color online). mSUGRA/CMSSM with $(x, y) = (0, 0)$. Left: Contours of Higgs boson mass m_h (black dotted) and fine-tuning parameter c (red dashed) in the $(m_0, M_{1/2})$ plane. Right: Contours of gluino mass M_3 (brown dashed) and lighter stop mass $m_{\tilde t_1}$ (green dashed) in the $(m_0, M_{1/2})$ plane. In both panels, the region where neutralino dark matter has the correct thermal relic abundance is given by the blue dot-dashed line. The shaded regions are excluded because electroweak symmetry is not broken (gray, labeled $\mu^2 < 0$), charginos are too light (green, labeled $\chi^+ < 100 \sim$ GeV), or the lightest supersymmetric particle is a stau (gold, labeled $\sim \tau$ LSP). For definiteness, we assume gaugino mass unification, $\tan\beta = 10$, $\mu > 0$, and at the GUT scale, the stop masses are defined by the FP condition, and all other scalar masses are set to m_0.

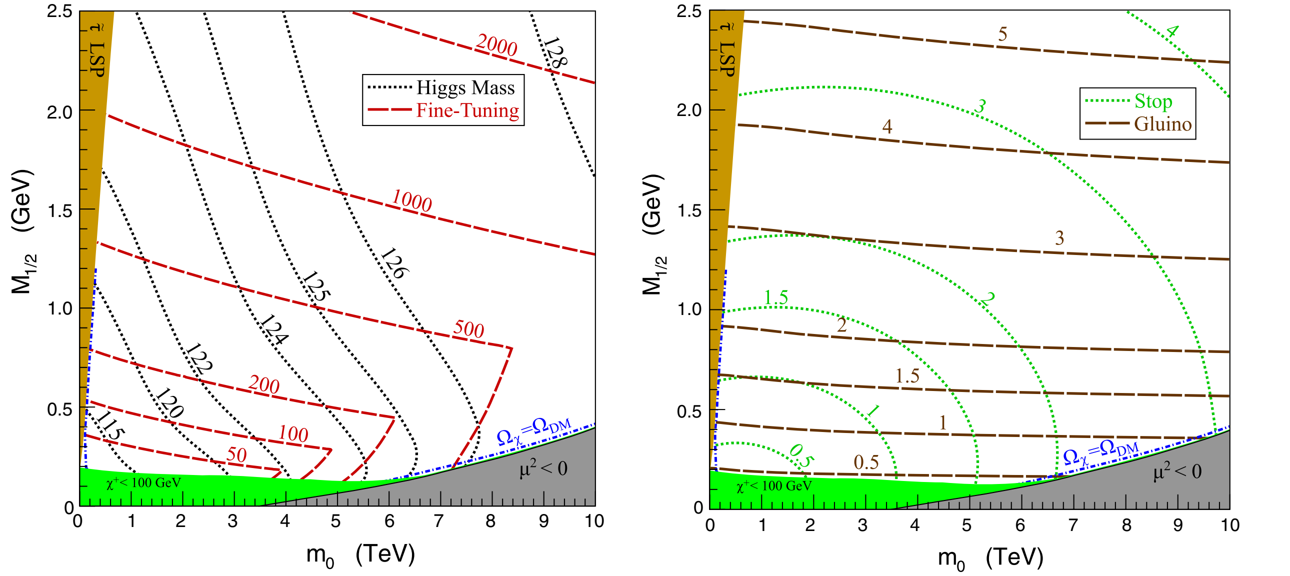


FIG. 3 (color online). As in Fig. 2, but for Model A with $(x, y) = (\frac{1}{4}, \frac{1}{6})$ and GUT-unified stops with $m^2_{Q_3}(m_{\rm GUT}) = m^2_{U_3}(m_{\rm GUT}) = \frac{3}{4}m_0^2$.

significantly due to contributions from stop mixing, with $m_h = 125$ GeV found at $m_0 \approx 3$–6 TeV for Model A and $m_0 \approx 2$–4 TeV for Model B over the given range of $M_{1/2}$. Furthermore, the position of the $\mu^2 < 0$ region has shifted to smaller $M_{1/2}$ and larger m_0, allowing relatively small gaugino masses at larger values of m_0 than would be possible in mSUGRA/CMSSM. This effect can best be understood as Eq. (9) being more approximately true when $A_t(m_{\rm GUT})$ is increased.

The desire for low fine-tuning motivates consideration of low values of $M_{1/2}$ that yield $m_h = 125$ GeV. For Model A, the $m_h = 125$ GeV contour meets the chargino bound at $M_{1/2} \approx 200$ GeV, $m_0 \approx 6.5$ TeV, and $c \approx 350$. The improvement in fine-tuning over the mSUGRA/CMSSM case is a factor of 5. Model B demonstrates an even greater improvement, with the intersection found at $M_{1/2} \approx 180$ GeV, $m_0 \approx 3.7$ TeV, and $c < 50$ (fine-tuning of 2%), a factor of 40 improvement over mSUGRA/CMSSM. In this region the change in the gluino mass is marginal between the different models, but the lightest stop mass is reduced significantly for larger y. In mSUGRA/CMSSM $m_{\tilde{t}_1} \approx 6$ TeV when $m_h = 125$ GeV, which is

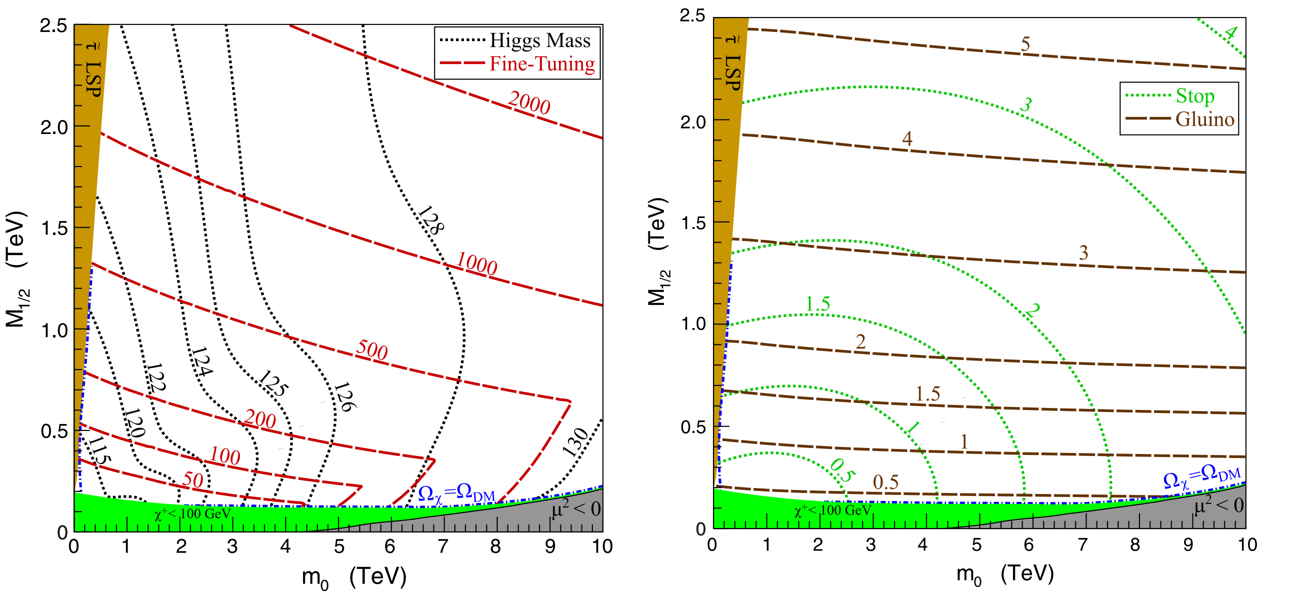


FIG. 4 (color online). As in Fig. 2, but for Model B with $(x, y) = (\frac{5}{9}, \frac{7}{27})$ and weak-scale-unified stops with $m^2_{Q_3}(m_W) = m^2_{U_3}(m_W) = \frac{1}{9}m_0^2$.

reduced to $m_{\tilde{t}_1} \approx 2$ TeV in Model A and $m_{\tilde{t}_1} \approx 900$ GeV in Model B near the chargino bound. As in the mSUGRA/CMSSM case, the fine-tunings are reduced by an order of magnitude relative to their values without the FP mechanism. As a result, percent-level fine-tunings are compatible with $m_h \approx 125$ GeV.

B. Deflection of the focus point

The formulation of FP scenarios is defined by the condition $m_{H_u}^2(m_W) \simeq 0$, even when superpartner masses are significantly larger. However, a viable minimum for electroweak symmetry breaking requires $m_{H_u}^2(m_W) < 0$. This behavior could be introduced directly through an appropriate boundary condition, shifting the FP boundary condition so that $m_{H_u}^2$ will be small but negative at the weak scale. It can also arise dynamically due to subdominant terms in the RG evolution, which generically divert the RG trajectory and may generate a viable electroweak minimum.

The complete one-loop RG equations for the up-type Higgs mass parameter is

$$\frac{dm_{H_u}^2}{d\ln Q} = \frac{1}{16\pi^2}\left[2y_t^2(m_{H_u}^2 + m_{Q_3}^2 + m_{U_3}^2 + A_t^2) - 6g_2^2M_2^2 - \frac{6}{5}g_1^2M_1^2 + \frac{3}{5}g_1^2 S\right], \tag{24}$$

where S is given in Eq. (10). The FP behavior is governed by the scalar mass and A_t terms of the first line, and the gaugino masses and S terms in the second line deflect the solution. The M_1 and M_2 terms in the second line of Eq. (24) drive $m_{H_u}^2$ positive at the weak scale; the contribution of the S term can be positive or negative, but in mSUGRA/CMSSM the contribution is positive and further increases $m_{H_u}^2$. However, additional contributions are also introduced in the first line of Eq. (24) by the deflection of A_t^2 away from the FP solution. The RG evolution of A_t is given by

$$\frac{dA_t}{d\ln Q} = \frac{1}{16\pi^2}\left[12y_t^2A_t + 2y_b^2A_b + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{26}{15}g_1^2M_1\right], \tag{25}$$

where the first term is considered in the FP analysis, and the remaining terms deflect the solution. This results in a significant deflection of $A_t(m_W)$ for nonzero gluino mass, producing a corresponding deflection in $m_{H_u}^2$ that rivals the deflection from the terms in the second line of Eq. (24).

In the case of $A_t(m_{\rm GUT}) = 0$, RG evolution generates a nonzero value for $A_t(m_W)$, which in turn drives $m_{H_u}^2$ more negative at the weak scale. For scenarios with gaugino mass unification, this contribution is larger than the direct one-loop contributions from M_1 and M_2, producing an overall negative contribution to $m_{H_u}^2$ at the weak scale. This must still be balanced against the positive contribution from S and, for sufficiently large scalar masses $m_{H_u}^2$, will be positive at the weak scale. This is the origin of the phenomenologically excluded $\mu^2 < 0$ region in mSUGRA/CMSSM at high m_0, which requires larger values of m_0 with increasing $M_{1/2}$, as shown in Fig. 2.

For $A_t(m_{\rm GUT}) \gtrsim M_3$, the contribution to $m_{H_u}^2$ depends upon the relative sign of $A_t(m_{\rm GUT})$ and M_3. The deflection of $A_t^2(m_W)$ is $\Delta A_t^2(m_W) \sim -A_t(m_{\rm GUT})M_3$, with a negative deflection of $m_{H_u}^2$ when $\Delta A_t^2(m_W) > 0$. When $A_t(m_{\rm GUT})$ and M_3 have the same sign this results in no viable electroweak minimum, but when the signs are opposite, a viable minimum is reached similarly to the $A_t(m_{\rm GUT}) = 0$ case. This is the origin of the condition $A_t(m_{\rm GUT}) < 0$ in Sec. IV A. For $A_t(m_{\rm GUT}) \neq 0$, the $\mu^2 < 0$ region also shifts to higher values of m_0. This is because the RG contributions from direct deflections of $m_{H_u}^2$ are unchanged, but $\Delta A_t^2(m_W)$ is increased relative to the $A_t(m_{\rm GUT}) = 0$ case.

V. IMPLICATIONS FOR COLLIDERS AND DARK MATTER

The most immediate experimental implication of the FP SUSY models discussed here is the possibility of a natural SM-like Higgs boson in the currently allowed range around $m_h \approx 125$ GeV. If these models are realized in nature, the Higgs boson should be discovered in the very near future with properties consistent with those of a SM-like Higgs boson.

The FP SUSY models discussed here also naturally explain the nonobservation of superpartners at the LHC so far. In FP models, the squarks and sleptons of the first two generations barely RG evolve, and so have physical masses essentially set by their value at the GUT scale. As noted above, electroweak symmetry breaking is highly insensitive to these masses, and so they are not constrained by the FP mechanism. However, under the assumption that they are $\sim m_0$, in all the scenarios we consider, for $m_h \approx 125$ GeV, the squarks of the first two generations are in the multi-TeV region, well beyond the current bound of $m_{\tilde{q}} \gtrsim 1.4$ TeV [1,2]. The stop masses do RG evolve in these models, and they may be somewhat lighter, but they are also beyond current bounds, which are much weaker for third generation squarks [53,54].

In the future, probably the most promising avenues for collider discovery are stop and gluino searches. As noted in Sec. IV, the most natural FP scenario we have considered in detail (Model B) has large stop mixing and relatively light stops in the range $m_{\tilde{t}} \sim 1$ TeV. As emphasized in Sec. IV A, however, for simplicity, we have purposely avoided models near the tachyonic stop boundaries, where the stop mixing is even higher. These will produce models with lighter stops, and quite possibly even less fine-tuning. Such stops will be within reach of future LHC analyses. Of course, light stops are a general feature of many natural

SUSY theories. The new feature of FP models with light stops is that the fine-tuning is significantly reduced, even including the fine-tuning with respect to the stop mixing parameter, and the particle content is minimal, and so conventional MSSM searches are applicable. Future gluino searches are also promising. As discussed in Sec. III, for reasons related to the general structure of the RGEs, it is quite natural for the scalars to participate in the FP mechanism, but not the gauginos. For this reason, requiring less fine-tuning than 1 part in 1000 implies $m_{\tilde{g}} \lesssim 3$–4 TeV. The relatively light stops also imply that gluinos, if produced, will decay dominantly through top- and bottom-rich cascade decays through off-shell (or even on-shell) stops [25,30,55,56].

The phenomenology of dark matter for the large $A_t(m_{\rm GUT})$ models we consider is more complicated than in mSUGRA/CMSSM FP scenarios. For the mSUGRA/CMSSM case, there is an excellent thermal relic dark matter candidate, a binolike neutralino with a significant Higgsino component. It has the correct thermal relic density in the region of parameter space with $m_h \approx 125$ GeV, and will either be detected or excluded by direct detection searches in the near future [23]. For large $A_t(m_{\rm GUT})$, the preferred region with $m_h \approx 125$ GeV is at lower m_0, but, as discussed in Sec. IV B, the $\mu^2 < 0$ region is found at much larger m_0. The result is that for regions of parameter space with $m_h \approx 125$ GeV, μ is significantly above M_1, the dark matter is nearly pure bino, and its thermal relic density is typically too large. This may be fixed when $m_\chi \approx m_h/2$ and the annihilation is enhanced by the (SM-like) Higgs funnel. Unfortunately, this slice of parameter space is located at $M_{1/2} \sim 150$–200 GeV, which is inconsistent with the recent ATLAS gluino mass bound of $m_{\tilde{g}} \gtrsim 900$ GeV [1] for decoupled squarks. The bound requires $M_{1/2} \gtrsim 400$ GeV, and at such high gaugino masses, the Higgs resonance is not in effect, and relic neutralinos are overabundant.

There are, however, several possible solutions to this dark matter problem. First, one may, of course, always invoke R-parity violation or allow the neutralino to decay to another, lighter supersymmetric particle, such as a gravitino [57,58] or axino [59,60]. In the standard formulation of gravity mediation, the gravitino mass is typically of the order of the masses of SM superpartners, but light gravitinos can be introduced through a nonstandard Kähler potential [61].

More satisfying, however, is the observation that all of the analysis of the previous paragraph relies heavily on the assumption of gaugino mass unification. The gaugino masses play a subleading role in the FP analysis, and nonunified gaugino masses are perfectly possible in FP models. A thermal relic may be restored either by reducing M_1 to produce a light neutralino, which increases its annihilation cross section, or increasing M_1 to values closer to μ, thereby increasing the Higgsino component of the neutralino, and with it, the annihilation cross section. In either case, the scattering cross section can be expected on general grounds to rise with the annihilation cross section. Although more work is required, the prospects for both direct and indirect dark matter searches for neutralino dark matter should be excellent; for example, in the bino-Higgsino case, they should be similar to those for mSUGRA/CMSSM FP neutralinos [62–65]. Irrespective of shifts in M_1, considering a point with $m_h = 125$ GeV and $m_{\tilde{g}} = 900$ GeV produces a fine-tuning of $c \approx 300$ for Model A and $c \approx 110$ for Model B, and these FP scenarios provide a natural possibility for realizing $m_h \approx 125$ GeV in the MSSM consistent with weakly-interacting massive particle thermal relic dark matter.

VI. CONCLUSIONS

Many of the most cherished virtues of weak-scale SUSY, such as gauge coupling unification, radiative electroweak symmetry breaking, and the heavy top quark, are dynamically generated. In this study, we have examined FP SUSY, in which naturalness is also dynamically generated. This framework is motivated by the desire to find natural supersymmetric theories with 125 GeV Higgs masses in the MSSM, without extending the theory with additional field content or invoking low-scale SUSY-breaking mediation.

We have extended earlier works on FP SUSY to analyze the possibility that both scalar masses and A-terms are multi-TeV and hierarchically larger than all other soft parameters. We find that the FP mechanism may be realized if the soft parameters are in the relation $m^2_{H_u}: m^2_{U_3}: m^2_{Q_3}: A^2_t = 1: 1 + x - 3y: 1 - x: 9y$ at the GUT scale, where x and y are in the ranges given in Eq. (17). The FP mechanism is independent of all other scalar masses, and of the gaugino masses, provided they are smaller than these. In the $A = 0$ limit the FP boundary condition can be accommodated in the CMSSM framework, but cannot be achieved within the CMSSM for nonzero A-terms. This is in contrast to hyperbolic branch models with large A-terms [33], which achieve μ-terms with large scalar masses/A-terms at the GUT scale but do not exhibit the same reduction in fine-tuning arising from other fundamental parameters.

We examined three particular choices for (x, y) in detail. The results may be very roughly summarized, and contrasted with previous results, as follows. For general models with large stop masses, there are both quadratic contributions $(6/8\pi^2)y_t^2 m_{\tilde{t}}^2$ and large logarithm-enhanced contributions $\sim(6/8\pi^2)y_t^2 m_{\tilde{t}}^2 \ln(m_{\rm GUT}/m_{\tilde{t}})$. Without the FP mechanism, the large log terms dominate. A 125 GeV Higgs mass may be achieved with 10 TeV stop masses and no mixing, leading to a fine-tuning of roughly 1 part in 10^4, or with highly mixed, few TeV stops, with a fine-tuning of roughly 1 part in 1000 [15]. The FP mechanism suppresses the leading log terms, and so reduces fine-tuning by roughly an order of magnitude. Previously, FP models with no stop mixing achieved a 125 GeV Higgs with 10 TeV stops and fine-tuning of around 1 part in 1000 [23]. In this work, we have found new FP models with significant stop mixing, where the 125 GeV Higgs bosons are achieved with fine-tuning of 1 part in 100.

To our knowledge, the models presented here are the first with minimal field content and SUSY-breaking mediated at the GUT scale that accommodate a 125 GeV Higgs boson with only percent-level fine-tuning. General implications for SUSY searches at colliders and dark matter experiments have been summarized in Sec. V. The model framework provides a simple extension of the mSUGRA/CMSSM boundary conditions that simultaneously preserves naturalness, accommodates the new Higgs boson constraints, and predicts superpartners within reach of the LHC, and is therefore an ideal framework for presenting new results from LHC searches.

The motivation to find natural and simple theories consistent with a 125 GeV Higgs boson has led us to a predictive relation between soft parameters in the top sector. If the predictions of FP models continue to be born out, it will become increasingly interesting to explore what UV frameworks may naturally yield the required relations. Such a study is beyond the scope of this work, but we close with some speculations. The general solution presented here allows for the possibility for FP behavior for nonzero $A_t(m_{\rm GUT})$, provided there is some splitting between the masses $m_{H_u}^2$, $m_{Q_3}^2$, and $m_{U_3}^2$ at the GUT scale. Such splitting is generically possible in SUSY theories derived from superstring models with SUSY-breaking arising from the dilaton/moduli sector of the theory, and indeed there is a direct connection between the size of the A-term and the mass splitting between scalars in such frameworks [66,67]. It is interesting to note that the required boundary conditions could arise either purely from the dilaton/moduli sector of such a theory, or in combination with a direct F-term mediation scheme, providing a UV explanation for this class of models.

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