

Mass of the Higgs boson in the trinification subgroup of E_6

Berthold Stech*

Institut für Theoretische Physik, Philosophenweg 16, D-69120 Heidelberg, Germany

(Received 19 June 2012; published 5 September 2012)

The extension of the standard model to $SU(3)_L \times SU(3)_R \times SU(3)_C$ is considered. Spontaneous symmetry breaking requires two Higgs field multiplets with a strong hierarchical structure of vacuum expectation values. These vacuum expectation values, some of them known from experiment, are used to construct invariant potentials in the form of a sum of individual potentials relevant at the weak scale. As in a previous suggestion one may normalize the most important individual potentials such that their mass eigenvalues agree with their very large vacuum expectation values. In this case (for a wide class of parameters) the scalar field corresponding to the standard model Higgs turns out to have the precise mass value $m_{\text{Higgs}} = \frac{v}{\sqrt{2}} = 123$ GeV at the weak scale. The physical mass (pole mass) is larger and found to be 125 ± 1.4 GeV.

DOI: [10.1103/PhysRevD.86.055003](https://doi.org/10.1103/PhysRevD.86.055003)

PACS numbers: 12.10.Dm, 12.60.-i, 14.80.Ec

I. INTRODUCTION

The imbedding of the standard model into a larger group implies an extended Higgs structure. The corresponding potential should give rise to spontaneous symmetry breaking leading to a large spectrum of scalar boson masses. The aim of this article is to consider the structure one encounters by extending the standard model to E_6 [1] or rather to its maximal subgroup $SU(3)_L \times SU(3)_R \times SU(3)_C$. This group has to be combined with the discrete group Z_3 exchanging left, right and color symmetries [2–7]. It has been named trinification group by the author of Ref. [3]. The subgroup with Z_2 (exchanging left and right) can appear as an intermediate symmetry at and above the scale where the gauge couplings g_1 and g_2 unite [8]. In this model, one has to deal with masses and vacuum expectation values (vevs) of scalar fields which show an extreme hierarchical structure extending over many orders of magnitudes. To break the group down to the standard model, two scalar matrix fields are necessary H and \tilde{H} . These two 3×3 matrices of fields transform according to $(3^*, 3)$ with respect to $SU(3)_L$ and $SU(3)_R$ [8,9]. Thus, we have to deal with 36 real scalar fields. With respect to the $SU(2)_L$ subgroup, they form six complex Higgs doublets and six complex singlets.

The vev of the first matrix (H), which couples directly to the fermions, can be chosen diagonal. Its structure is known from experiment: The element $\langle H \rangle_1^1$ describes the vev $v = 174$ GeV of the conventional Higgs field. The $\langle H \rangle_2^2$ element is very small and of the order of the mass of the b quark. For simplicity, we will set it to zero here. The $\langle H \rangle_3^3$ element, on the other hand, is huge and its value M is expected to be close to the scale of electroweak unification, i.e., the meeting point of the gauge couplings g_1 and g_2 which occurs at $\approx 2 \times 10^{13}$ GeV.

The second matrix of scalar fields \tilde{H} is not directly coupled to fermions [8,9]. It needs to have a sizable vev for the element $\langle \tilde{H} \rangle_2^2$ which we denote by \tilde{M} [4,8]. This vev determines the masses of the right-handed vector bosons. The experimental limit on right-handed currents provides for a lower limit for \tilde{M} of a few TeV. Already the non-vanishing of M and \tilde{M} is sufficient to break the trinification group down to the Glashow-Weinberg-Salam group. Finally, the finite value for v leaves only the electromagnetic $U(1)_e$ symmetry.

In a previous article, the fields of the matrix H have been discussed leaving the matrix \tilde{H} aside [10]. The required vevs of these fields could be obtained with the help of logarithmic contributions for the $SU(3)_L \times SU(3)_R$ invariant potentials. The input vevs mentioned above provide for scalar masses and the 15 Goldstone bosons. The article contains in addition the suggestion for a speculative normalization of the potentials. This hypothesis led to a prediction for the mass of the Higgs meson of the standard model, namely to $m_{\text{Higgs}} = v/\sqrt{2} = 123$ GeV [10]. After the more recent experimental indications for a Higgs-like structure in the 125 GeV region [11], this prediction may have some significance. Therefore a more detailed discussion is indicated. It should deal with all 36 fields and should show, in an example, the full spectrum of now $36 - 15 = 21$ scalars.

Invariants which combine H and \tilde{H} fields together now play an essential role. Without those invariants, several charged and neutral scalars would remain massless. In general, the inclusion of such invariants will change the previously obtained mass values from the single Higgs field H . However, as we will see, it turns out that for the special field, which corresponds to the standard model Higgs, no noticeable changes occur for a large class of parameters. Thus, the Higgs mass close to 123 GeV can still be the consequence of the following assumption [10]: the fields in H are determined by two separate potentials according to the two relevant invariants. Each potential is

*B.Stech@ThPhys.Uni-Heidelberg.DE

normalized such that the mass obtained from it coincides with the input vev to leading order in M . As a motivation, we note that only with this assumption the difference between vev and mass eigenvalue remains finite in the large M limit.

At present it seems too difficult, at least for the author, to obtain a potential valid for the high and the low scale regions which provides for spontaneous symmetry breaking of the trinification group, fulfilling thereby all theoretical requirements for this multi-Higgs problem. However, for the region of the weak scale, effective potentials can be constructed which give some insight into the structure of this Higgs system. Coming down from high scales which involve very large masses, the effective potential at low scales is certainly expected to include logarithmic terms of the same order as the H^4 and \tilde{H}^4 terms. In fact, only the inclusion of logarithmic terms allow the potentials to have their minimum at the wanted places. Shifting the appropriate fields according to their vevs and neglecting terms with inverse powers of M , one gets potentials of the tree type. No powers of fields higher than 4 appear anymore and the potential minimum remains unchanged. Moreover, the important terms containing M and \tilde{M} occur with shifted and unshifted fields of lower powers. This way the task is simplified and one gets, at low scales, a model only slightly different from the standard model. As we will see, the potential constructed leads to the required spontaneous symmetry breaking. All Higgs fields obtain masses in agreement with experimental bounds.

II. SCALAR FIELDS WITH LOW AND VERY HIGH SCALE VACUUM EXPECTATION VALUES IN THE SAME REPRESENTATION

An interesting example of a scalar Higgs field in a grand unified model is the field H_{27} , the irreducible “27” representation of E_6 . In a nonsupersymmetric version of E_6 [8,9] the maximal subgroup of E_6 , the symmetry $SU(3)_L \times SU(3)_R \times SU(3)_C$ (the trinification group) plays an important role together with the symmetry Z_3 which allows the exchange of “left” with “right” and “color.” Apart from the breaking $Z_3 \rightarrow Z_2$, this symmetry holds from the point of electroweak unification up to the complete gauge group unification. In this region which starts at $\approx 2 \times 10^{13}$ GeV, the gauge couplings $g_1^n = \sqrt{5/3}g_1$ and g_2 are identical.

Let us consider for the moment the color singlet part of a single H_{27} multiplet field only, neglecting the influence of the other fields. It is a 3×3 matrix field H_k^i where the index i transforms as an upper index with respect to $SU(3)_L$ while the lower index k transforms according to $SU(3)_R$. The indices $i = 1, 2$ are the $SU(2)_L$ indices of the standard model. H_k^i contains 18 real fields.

The vev of H can be chosen to be a diagonal matrix with real and positive elements by absorbing transformation matrices and phases by the fermion fields. In the following

we use $H = h + if$ where h and f are matrices of real fields:

$$\langle H \rangle = \langle h \rangle = \begin{pmatrix} v & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & M \end{pmatrix}. \quad (2.1)$$

Phenomenology tells us the values of v and b : v can be identified with the vev of the Higgs field of the standard model, $v = 174$ GeV. This element couples to the top quark. b couples to the bottom quark and is much smaller, roughly equal to the mass of the bottom quark at the weak scale. We will set $b = 0$ for simplicity. The value M on the other hand must be huge. It is the mass of the heavy down quark D , which is likely to be of the order of the scale where the $SU(3)_L \times SU(3)_R \times Z_2$ symmetry sets in Ref. [8]: $\langle H \rangle_3^3 = M \approx m_D \approx 2 \times 10^{13}$ GeV

For the fields in H , only two $SU(3)_L \times SU(3)_R$ invariants with nonvanishing vev exist:

$$J_1 = (Tr[H^\dagger \cdot H])^2, \quad J_2 = Tr[H^\dagger \cdot H \cdot H^\dagger \cdot H]. \quad (2.2)$$

Their vevs are $\langle J_1 \rangle = (M^2 + v^2)^2$ and $\langle J_2 \rangle = M^4 + v^4$, respectively.

It is easily seen that a tree potential with the two invariants (2.2) cannot produce the wanted vevs. To have a potential with the required properties at the weak scale, the logarithmic dependence on J_1 and J_2 must be included. In contrast to the usually very small Coleman-Weinberg term [12], one expects here the log term to be of the same order as the tree part because of the extremely long way down from the very heavy states. Indeed, large log terms follow necessarily: Our requirements are satisfied by a linear combination of the two separate potentials:

$$V_1 = \frac{1}{8} J_1 \left(\log \frac{J_1}{\langle J_1 \rangle} - 1 \right) \quad \text{and} \quad V_2 = \frac{1}{8} J_2 \left(\log \frac{J_2}{\langle J_2 \rangle} - 1 \right). \quad (2.3)$$

Higher powers of the log terms can also be used. For instance,

$$V_1 = \frac{1}{8} \frac{J_1}{1 + \log \frac{J_1}{\langle J_1 \rangle}} \quad \text{and} \quad V_2 = \frac{1}{8} \frac{J_2}{1 + \log \frac{J_2}{\langle J_2 \rangle}}. \quad (2.4)$$

Equations (2.3) and (2.4) are equivalent in our treatment. First and second derivatives at the minimum are the same and thus lead to the same scalar particle spectrum. The scalar potential taken as a linear combination of V_1 and V_2 (with positive coefficients) has the wanted properties. This potential is fully invariant and provides for the spontaneous symmetry breaking to $U(1) \times U(1)_e$. The derivatives of this potential with respect to the 18 fields vanish at the minimum and the 18×18 matrix for the second derivatives at the minimum leads to two massive states, one massless state and 15 Goldstone bosons. In Eqs. (2.3) and (2.4), the

factors $1/8$ serve as a normalization of the potentials. They are chosen such that—to order M —the mass values obtained from the second derivatives coincide with the input vevs according to our postulate. This postulate insures that the difference between mass and vev is not proportional to the huge value M . It remains finite in the limit $M \rightarrow \infty$. The normalization will be of significance as shown below.

The potentials obtained so far cannot be used to calculate radiative corrections. One has to extract a tree potential which can then be improved by renormalization group methods in the conventional way. To do this, an expansion in M is necessary. It can be done after the shift of the fields according to their vevs ($h_1^1 \rightarrow v + h_1^1$, $h_3^3 \rightarrow M + h_3^3$) is performed. By neglecting inverse powers of M , the above potentials then become polynomials with field configurations up to the fourth power only. Moreover, the interesting terms proportional to M^2 , $v \cdot M$ and v^2 occur with fields to second order only. The symmetry breaking properties remain unchanged. The relevant scale for this potential is the weak scale.

$$\begin{aligned} V_1 &= (h_3^3)^2 M^2 + 2h_1^1 h_3^3 v M + (h_1^1)^2 v^2 + O(H^3) + O(H^4), \\ V_2 &= (h_3^3)^2 M^2 + O(H^3) + O(H^4). \end{aligned} \quad (2.5)$$

Constant terms are left out and the field combinations of third and fourth order are not shown explicitly. They are not relevant at the minimum but are needed for renormalization. V_1 and V_2 of Eq. (2.5) can now be used as the tree potential at the weak scale. It replaces the standard model potential in the presence of a huge hierarchy. The Higgs field h_1^1 and the $SU(2)_L$ singlet field h_3^3 are both real fields.

As in Ref. [10], we make the assumption to use the two potentials with equal weight because of their identity in the limit $v \rightarrow 0$. The normalization is decisive and kept.

$$V_{\text{tree}} = V_1 + V_2. \quad (2.6)$$

This potential has a minimum for zero values of all (shifted) fields. The two massive states have the masses (shown to order v^2)

$$m_{\text{Higgs}}^2 = \frac{1}{2} v^2 \quad \text{and} \quad M_H^2 = 2M^2 + \frac{1}{2} v^2 \quad (2.7)$$

Thus, the prediction for the Higgs mass at the weak scale is as in Ref. [10]

$$m_{\text{Higgs}} = \frac{1}{\sqrt{2}} v \simeq 123 \text{ GeV}. \quad (2.8)$$

However, the influence of the fields of the second Higgs multiplet \tilde{H} can change this result. This is the subject of the next section together with the attempt to obtain the full mass spectrum of all scalars.

III. TWO $SU(3)_L \times SU(3)_R$ SCALAR FIELD MULTIPLETS

The second multiplet of scalar fields \tilde{H} is required from phenomenological considerations. In the model [8], it is not directly coupled to the fermions, only via gauge vector bosons. It can have its own paritylike symmetry: $\tilde{H} \rightarrow -\tilde{H}$. Its vevs cannot be diagonalized anymore when keeping $\langle H \rangle$ diagonal. For the breaking of the original left-right symmetry, the dominant vev has to be at the (3,2) position [4]. Defining $\tilde{H} = \tilde{h} + i\tilde{f}$, one has

$$\langle \tilde{H} \rangle = \langle \tilde{h} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \tilde{M} & 0 \end{pmatrix}. \quad (3.1)$$

The value of \tilde{M} fixes the masses m_{W_R} of new vector bosons coupled to right-hand vector currents: $m_{W_R}/m_{W_L} \simeq \tilde{M}/v$. These right-hand currents are, apart from their helicity structure, of the same form as the well-known left-hand currents. LHC experiments can discover these bosons if their masses lie in the TeV region. In the following we take $v \ll \tilde{M} \ll M$.

The wanted total potential should now provide masses for all fields. One needs new invariants depending on \tilde{H} but necessarily also invariants combining H and \tilde{H} fields. Otherwise, one would have additional symmetries and thus many massless scalar particles. Important invariants which also respect the symmetry $\tilde{H} \rightarrow -\tilde{H}$ are

$$\begin{aligned} J_3 &= \text{Tr}[\tilde{H}^\dagger \cdot \tilde{H}]^2, & J_4 &= \text{Tr}[\tilde{H}^\dagger \cdot \tilde{H} \cdot \tilde{H}^\dagger \cdot \tilde{H}], \\ J_5 &= \text{Tr}[\tilde{H}^\dagger \cdot \tilde{H} \cdot H^\dagger \cdot H], & J_6 &= \text{Tr}[\tilde{H}^\dagger \cdot H \cdot H^\dagger \cdot \tilde{H}]. \end{aligned} \quad (3.2)$$

Their vevs are $\langle J_3 \rangle = \langle J_4 \rangle = \tilde{M}^4$, $\langle J_5 \rangle = 0$, $\langle J_6 \rangle = \tilde{M}^2 M^2$.

The invariant J_5 can be directly added to the potential (2.6) since all its 36 first derivatives vanish at the proposed minimum. J_6 , on the other hand, has to be combined with J_3 or J_4 : the first derivatives of $J_4 - 2\tilde{M}^2/M^2 J_6$ are not strictly zero at the minimum but vanish for large M . Thus, this combination can be used. Therefore, a suitable and still simple tree potential for the 36 scalar fields reads

$$\begin{aligned} V_{\text{tree}} &= 2(h_3^3)^2 M^2 + 2h_1^1 h_3^3 v M + (h_1^1)^2 v^2 + r_4 \left(J_4 - 2 \frac{\tilde{M}^2}{M^2} J_6 \right) \\ &\quad + r_5 J_5 + O((H, \tilde{H})^3) + O((H, \tilde{H})^4). \end{aligned} \quad (3.3)$$

In this equation, the shifted fields (also $\tilde{h}_2^3 \rightarrow \tilde{M} + \tilde{h}_2^3$) have to be used and, after expanding in powers of M , the inverse powers of M have to be neglected. Similar to Eq. (2.6), the combination of fields not contributing to the tree level spectrum of particles are not shown explicitly. They are, however, important for the scale dependence. The coefficients r_4 and r_5 have to be positive and not too small but are otherwise arbitrary. The formula (3.3) is simple enough to obtain analytically the eigenvalues and eigenvectors of the

36×36 matrix of second derivatives. The square of the masses are

$$\begin{aligned} & \frac{v^2}{2}, \quad 2M^2 + \frac{v^2}{2}, \quad (M^2 + \tilde{M}^2)r_5(2\times), \\ & M^2r_5(4\times), \quad 4\tilde{M}^2r_4, \quad (\tilde{M}^2 + v^2)r_5(2\times), \\ & 2\tilde{M}^2r_4(4\times), \quad \tilde{M}^2r_5(2\times), \quad v^2r_5(4\times), \quad 0(15\times). \end{aligned} \quad (3.4)$$

It is seen that the first and the second term are unchanged as compared to Sec. II. The 15 zero mass Goldstone particles will become the longitudinal parts of the gauge vector bosons. The other masses depend on the parameters r_4 , r_5 , M and \tilde{M} . Interestingly, there is one Higgs-like multiplet independent of M and \tilde{M} depending only on r_5 and v . It will be referred to as the ‘‘second Higgs.’’ The diagonalization of the 36×36 matrix allows us to identify all fields. After taking account of tiny mixings, the new field h_1^1 describes the field of the standard model-like Higgs and the second Higgs consists of the (also slightly changed) four fields $\tilde{h}_1^1, \tilde{f}_1^1, \tilde{h}_1^2, \tilde{f}_1^2$ from the matrix \tilde{M} . One can add to the potential the invariant $J_7 = \text{Tr}[\tilde{H}^\dagger H]\text{Tr}[H^\dagger \tilde{H}]$ and also J_3 in log form similar to V_1 . These contributions do not affect the value of the Higgs mass. Thus, for a wide class of potentials the result for the Higgs obtained here and in Ref. [10] can remain valid.

But certainly, it is also possible to devise special potentials which have quite different properties. There is then less connection with the standard model.

IV. THE LOW SCALE REGION

Let us now look at the low scale region near the mass of the Higgs. One can start from Eq. (3.3) which, if written in detail, contains a large amount of different field configurations. However, drastic simplifications occur after diagonalizing the 36×36 mass matrix and going with M and \tilde{M} to infinity. This allows us to neglect 16 heavy fields [the 16 masses are given in Eq. (3.4)]. The rest of the fields consist of the second Higgs (four states), which may or may not be very heavy compared to the Higgs, and finally the Higgs field h_1^1 itself and the 15 massless Goldstone bosons. The latter become the longitudinal components of the gauge bosons. In the low scale limit, only the coupling to the top and to the three low mass vector bosons W^+ , W^- , Z need to be considered.

The relevant part of the Lagrangian for this low scale domain in the unitary gauge turns out to be

$$\begin{aligned} \mathcal{L}_h = & -\frac{1}{4}h^4 - \frac{1}{2}vh^3 - \frac{1}{2}v^2h^2 + g_t \bar{t}th + (g_2)^2 W^+ W^- v h \\ & + \frac{1}{2}((g_1)^2 + (g_2)^2)Z^2 v h - r_4(|\tilde{H}_1^1|^2 + |\tilde{H}_2^1|^2)^2 \\ & - r_5(h^2 + 2hv + v^2)(|\tilde{H}_1^1|^2 + |\tilde{H}_2^1|^2). \end{aligned} \quad (4.1)$$

For convenience, the field h_1^1 is denoted by h . According to our derivation, the sum of the first three terms in Eq. (4.1) can be identified with the renormalized potential for the Higgs field at the weak scale. The next three terms describe the coupling of h to the top and to the vector bosons W^+ , W^- , Z . The final terms contain the fields of the second Higgs which need only be considered if their masses are relatively low. Since these fields have no vev and are not directly coupled to fermions, their neutral members are possible candidates for dark matter particles.

It is seen that our model has the form of the standard model except for the second Higgs and the larger number of Goldstone bosons. The ratio between the h^3 coupling constant and the mass term is the same as in the standard model. Only the h^4 coupling is a factor 2 larger.

Introducing the scale dependent coupling coefficient $\lambda(\mu)$ Eq. (4.1) leads to the boundary condition $\lambda(\mu_0) = 1/8$. Here, μ_0 stands for the weak scale at which the matching with h should be performed. It seems appropriate to take for this scale the sum of the square of the four boson masses caused by the Higgs particle: $\mu_0^2 = m_H^2 + 2m_W^2 + m_Z^2 \simeq (192 \text{ GeV})^2$. The Higgs mass and λ are related according to $m_{\text{Higgs}}^2 = 4\lambda v^2$.

To get the physical mass of the Higgs (the pole mass) in our model is clearly of interest. This requires us to study the scale dependence of the mass in the low scale region. A systematic analysis of Eq. (4.1) is required here but has not been done. However, one can get an estimate by using as an approximation the standard model result for the connection between the scale-dependent $\overline{\text{MS}}$ coupling constant $\lambda(\mu)$ and the pole mass according to Ref. [13]:

$$m_{\text{Higgs}}^2|_{\text{pole}} = \frac{\lambda(\mu)}{1 + \delta(\mu)} 4v^2. \quad (4.2)$$

The scale-dependent correction term $\delta(\mu)$ can be calculated from particle couplings and masses [13]. We use in this expression for the mass of the top quark $m_t = 173 \text{ GeV}$ and for the Higgs mass 123 GeV .

An estimate of the pole mass can now be obtained by fixing $\lambda(\mu_0 = 192 \text{ GeV}) = 1/8$. Applying the renormalization group equations of the standard model and taking μ in the interval $110 \text{ GeV} < \mu < 250 \text{ GeV}$ one finds from (4.2)

$$m_{\text{Higgs}}|_{\text{pole}} = 125 \pm 1.4 \text{ GeV}.$$

ACKNOWLEDGMENTS

It is a pleasure to thank my colleagues Jan Pawłowski, Tilman Plehn, Michael Schmidt and Werner Wetzel for stimulating discussions.

- [1] F. Gursey, P. Ramond, and P. Sikivie, *Phys. Lett.* **60B**, 177 (1976); Y. Achiman and B. Stech, *Phys. Lett.* **77B**, 389 (1978); Q. Shafi, *Phys. Lett.* **79B**, 301 (1978); R. Barbieri, D. V. Nanopoulos, and A. Masiero, *Phys. Lett.* **104B**, 194 (1981). For an early review see B. Stech, *Ettore Majorana Inst. Sci.*, edited by S. Ferrara *et al.* (1980), Vol. 7, p. 23.
- [2] Y. Achiman and B. Stech, in *Advanced Summer Institute on New Phenomena in Lepton and Hadron Physics*, edited by D. E. C. Fries and J. Wess (Plenum, New York, 1979).
- [3] S. L. Glashow, in *Fifth Workshop on Grand Unification*, edited by K. Kang, H. Fried, and P. Frampton (World Scientific, Singapore, 1984), p. 88.
- [4] K. S. Babu, X. G. He, and S. Pakvasa, *Phys. Rev. D* **33**, 763 (1986).
- [5] S. Willenbrock, *Phys. Lett. B* **561**, 130 (2003).
- [6] J. Sayre, S. Wiesenfeldt, and S. Willenbrock, *Phys. Rev. D* **73**, 035013 (2006).
- [7] Ch. Cauet, H. Pas, and S. Wiesenfeldt, *Phys. Rev. D* **83**, 093008 (2011).
- [8] B. Stech and Z. Tavartkiladze, *Phys. Rev. D* **77**, 076009 (2008).
- [9] B. Stech, *Fortschr. Phys.* **58**, 692 (2010).
- [10] B. Stech, [arXiv:1012.6028v2](https://arxiv.org/abs/1012.6028v2).
- [11] G. Aad *et al.* (ATLAS Collaboration), *Phys. Lett. B* **710**, 383 (2012); G. Aad *et al.* (ATLAS Collaboration), *Phys. Rev. Lett.* **108**, 111803 (2012); G. Aad *et al.* (ATLAS Collaboration), *Phys. Lett. B* **710**, 49 (2012); G. Aad *et al.* (ATLAS Collaboration), *Phys. Rev. Lett.* **108**, 111802 (2012); S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Lett. B* **710**, 91 (2012); S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Rev. Lett.* **108**, 111804 (2012); S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Lett. B* **710**, 284 (2012); S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Lett. B* **713**, 68 (2012); CMS Collaboration, Report No. CMS-PAS-HIG-12-001; CMS Collaboration, Report No. CMS-PAS-HIG-12-007.
- [12] S. Coleman and E. Weinberg, *Phys. Rev. D* **7**, 1888 (1973).
- [13] A. Sirlin and R. Zucchini, *Nucl. Phys.* **B266**, 389 (1986).