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$|V_{cd}|$ from D meson leptonic decays

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We present an update of the D meson decay constant f_D using the highly improved staggered quark action for valence charm and light quarks on MILC $N_f = 2 + 1$ lattices. The new determination incorporates HPQCD's improved scale $r_1^{N_f=2+1} = 0.3133(23)$ fm, accurately retuned bare charm quark masses and data from an ensemble that is more chiral than in our previous calculations. We find $f_D = 208.3(3.4)$ MeV. Combining the new f_D with $D \to \mu\nu_\mu$ branching fraction data from CLEO-c, we extract the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cd}| = 0.223(10)_{\rm exp}(4)_{\rm lat}$. This value is in excellent agreement with $|V_{cd}|$ from D semileptonic decays and from neutrino scattering experiments and has comparable total errors. We determine the ratio between semileptonic form factor and decay constant and find $[f_+^{D\to\pi}(0)/f_D]_{\rm lat} = 3.20(15)\ \text{GeV}^{-1}$ to be compared with the experimental value of $[f_+^{D\to\pi}(0)/f_D]_{\rm exp} = 3.19(18)\ \text{GeV}^{-1}$. Finally, we mention recent preliminary but already more accurate $D \to \mu\nu_\mu$ branching fraction measurements from BES III and discuss their impact on precision $|V_{cd}|$ determinations in the future.

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I. INTRODUCTION

Determinations of individual elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix allows for many cross checks and consistency tests of the Standard Model. In most cases there are several processes that can be used to extract the same CKM matrix element each involving very different experimental and theory inputs. For the CKM matrix element $|V_{cd}|$, Particle Data Group (PDG2010) [1] quotes values coming from $D \to \pi$, $l\nu$ semileptonic decays and from neutrino/antineutrino scattering. The HPQCD Collaboration recently published a new calculation of $|V_{cd}|$ that reduced errors in the semileptonic decay determination by more than a factor of two [2], making it competitive with the neutrino scattering result. In the current article we present a third, independent determination based this time on D meson leptonic decays. We find a value for $|V_{cd}|$ in complete agreement with the other two determinations and with comparable total errors.

The branching fraction for the leptonic decay of a charged D or D_s meson via a virtual W boson is given to lowest order by,

$$\mathcal{B}(D_q \to l\nu) = \frac{G_F^2}{8\pi} f_{D_q}^2 m_l^2 M_{D_q}\left(1 - \frac{m_l^2}{M_{D_q}^2}\right)^2 |V_{cq}|^2, \quad (1)$$

where m_l is the charged lepton mass and $q = d$ or s. Electromagnetic corrections to this formula are known and routinely taken into account by experimentalists in their analyses [3,4]. Equation (1) tells us that determination of $|V_{cd}|$ from D leptonic decays requires theory to provide the D meson decay constant f_D which is a pure QCD nonperturbative quantity. The first $N_f = 2 + 1$ lattice QCD calculations [5] of f_D and f_{D_s} were carried out by the Fermilab Lattice and MILC Collaborations [8] and this predated experimental studies of these decays. Subsequent experimental measurements were consistent with the lattice predictions within errors that were more substantial then than they are today for both theory and experiment. The initial lattice calculations employed an effective theory approach (the heavy clover action [9]) for the charm quark on the lattice. In 2007 the HPQCD Collaboration introduced the highly improved staggered quark (HISQ) action which represents not only an extremely accurate lattice quark action for light quark physics, but also serves as an accurate relativistic action for heavier quarks [10]. The HISQ action has since been used very successfully in simulations involving the charm quark such as for charmonium, and for D and D_s meson decay constants and semileptonic form factors [2,11–13]. In Ref. [11] HPQCD published the first f_π, f_K, f_D and f_{D_s} results from HISQ valence quarks, including HISQ charm quarks, on the MILC AsqTad $N_f = 2 + 1$ lattices [14], all with sub 2% errors. At around the same time experimental measurements of D and D_s meson leptonic decay branching fractions were improving significantly [15–17]. And by the middle of 2008 we were facing an interesting situation where there was good agreement between experiment and theory for f_D but a close to 4σ discrepancy in f_{D_s}. Further improvements and scrutiny became crucial.

The largest systematic error for f_{D_s} in Ref. [11] came from the uncertainty in the scale r_1. HPQCD was using an r_1 extracted from Υ splittings namely $r_1 = 0.321(5)$ fm with 1.56% errors [18]. In 2010 HPQCD published a much more accurate r_1 determination, $r_1 = 0.3133(23)$, based on several physical quantities and an improved continuum extrapolation (from 5 lattice spacings) [19]. A change in the scale affects quantities such as f_{D_s} in two ways: 1. the bare strange and charm quark masses must be retuned on each ensemble and 2. the conversion from dimensionless decay constant (e.g., in units of r_1) to the decay constant in physical units is modified. In Ref. [12] HPQCD updated its value for f_{D_s} together with f_π and f_K using the new r_1. Although f_π and f_K hardly shifted at all upon going from old to new r_1, the updated f_{D_s} came out about 2.3σ (3%) higher than before. As a consequence, taking into account also that experimental results were changing and moving closer to theory numbers, the discrepancy in f_{D_s} between theory and experiment has now shrunk to a 1.6σ effect. Reference [12] did not present a new calculation of f_D. Instead we took the previous ratio f_{D_s}/f_D from Ref. [11] and combined this with the new f_{D_s} to estimate a new f_D.

In this article we complete the process of switching to the new r_1 scale for meson decay constants and present a direct calculation of f_D consistently using the new scale. Since the time of Ref. [11] experimental errors in the $D \rightarrow \mu, \nu_\mu$ branching fraction have improved from ~7.8% down to ~4.3% in the case of CLEO-c [15] and new even more accurate measurements are appearing now from BES III [20]. Together with the new f_D of this article with its ~1.66% error, one can now extract a $|V_{cd}|$ from D meson leptonic decays that is as accurate as those from semileptonic decays or neutrino scattering and that promises to become even more precise in the near future.

II. THE LATTICE SETUP

Table I lists the three coarse ($a \approx 0.12$ fm) and three fine ($a \approx 0.09$ fm) MILC ensembles used in this study together with some lattice details. And in Table II we show the values for valence quark masses. For f_D we have focused more on ensuring better control over chiral extrapolations by adding a more chiral fine ensemble (Set F0) rather than going to finer lattices as we did in Ref. [12] for f_{D_s}. The bare charm quark mass is tuned using the physical η_c mass adjusted for the absence of electromagnetic, charm sea and annihilation contributions in our simulations which leads to a target value of $M_{\eta_c}^{\rm target} = 2.985(3)$ GeV [21] rather than the experimental value of $M_{\eta_c}^{\rm exp} = 2.980(1)$ GeV. Most of the charm quark mass tuning had been done already in Ref. [13] for our $D \rightarrow K, l\nu$ studies. For the present calculations we needed to add tunings only on ensemble F0. Figure 1 shows the tuned η_c masses for all 6 ensembles. The bulk of the errors shown comes from the ~0.1% uncertainty in r_1/a, whereas the tiny black error bars represent the statistical errors on each data point. A similar plot for tuning of the strange quark mass via the η_s (fictitious) meson mass is given in Fig. 3 of Ref. [13]. And in Refs. [11–13] we have demonstrated that once quark masses have been fixed by η_c and η_s then masses for the D and D_s mesons can be derived with zero adjustable parameters in good agreement with experiment. We do not repeat those calculations here. However, since we have new data for the mass difference $\Delta M_D \equiv M_{D_s} - M_D$, we summarize them in an Appendix and compare with $\Delta M_B \equiv M_{B_s} - M_B$ in the B system taken from Ref. [22].

Having fixed the quark masses we evaluated D and D_s correlators on each of the 6 ensembles. We use random

TABLE I. Simulation details on three "coarse" and three "fine" MILC ensembles.

Set	r_1/a	m_l/m_s (sea)	$N_{\rm conf}$	$N_{\rm tsrc}$	$L^3 \times N_t$
C1	2.647	0.005/0.050	1200	2	$24^3 \times 64$
C2	2.618	0.010/0.050	1200	2	$20^3 \times 64$
C3	2.644	0.020/0.050	600	2	$20^3 \times 64$
F0	3.695	0.0031/0.031	600	4	$40^3 \times 96$
F1	3.699	0.0062/0.031	1200	4	$28^3 \times 96$
F2	3.712	0.0124/0.031	600	4	$28^3 \times 96$

TABLE II. Valence quark masses.

Set	am_l	am_s	am_c
C1	0.0070	0.0489	0.6207
C2	0.0123	0.0492	0.6300
C3	0.0246	0.0491	0.6235
F0	0.00339	0.0339	0.4130
F1	0.00674	0.0337	0.4130
F2	0.0135	0.0336	0.4120

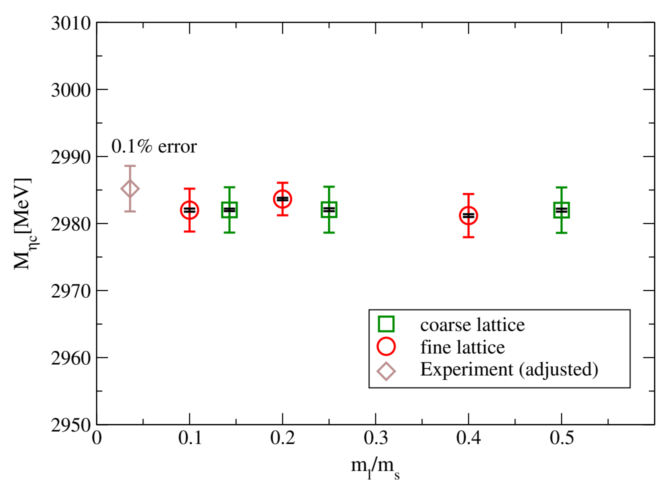

FIG. 1 (color online). Checking the tuning of the charm quark mass to the η_c meson mass. Errors on the simulation results include statistical (black error bars) plus errors arising from the uncertainty in r_1/a for each ensemble. The "experimental" η_c mass has been adjusted to take into account the lack of annihilation and electromagnetic effects in our lattice calculation.

wall sources with a different set for each color component in order to improve statistical errors. In the next section we describe how we extract meson decay constants from these correlators.

III. CORRELATORS AND FITTING STRATEGIES

The decay constant f_D of a pseudoscalar meson made out of a charm quark and a light antiquark of mass m_q is defined in terms of the matrix element of the heavy-light axial vector current $A_\mu = \bar{\Psi}_q \gamma_\mu \gamma_5 \Psi_c$ between the hadronic vacuum and the D meson state,

$$\langle 0|A_\mu|D\rangle = p_\mu f_D. \tag{2}$$

Since we employ the relativistic HISQ action for all quarks we are able to take advantage of PCAC, as is routinely done for f_π and f_K, and express the decay constant in terms of the pseudoscalar density $PS = \bar{\Psi}_q \gamma_5 \Psi_c$,

$$f_D = \frac{m_c + m_q}{M_D^2} \langle 0|PS|D\rangle. \tag{3}$$

The relevant hadronic matrix element in Eq. (3) can be extracted from the D meson two-point correlator,

$$C_D^{\rm 2pnt}(t) = \frac{1}{L^3} \sum_{\vec{x}} \sum_{\vec{y}} \langle 0|\Phi_D(\vec{y}, t)\Phi_D^\dagger(\vec{x}, 0)|0\rangle, \tag{4}$$

where $\Phi_D \equiv PS \times a^3$ is the same as the pseudoscalar density in lattice units, and is used here also as an interpolating operator for the D meson. We fit $C_D^{\rm 2pnt}(t)$ to the form,

$$C_D^{\rm 2pnt}(t) = \sum_{k=0}^{N_D-1} |b_k^D|^2 (e^{-E_k^D t} + e^{-E_k^D (N_t - t)}) + \sum_{k=0}^{N'_D-1} |d_k^D|^2 (-1)^t (e^{-E_k'^D t} + e^{-E_k'^D (N_t - t)}). \tag{5}$$

The ground state amplitude b_0^D is related to the matrix element of interest as,

$$|b_0^D|^2 \equiv \frac{|\langle 0|\Phi_D|D\rangle|^2}{2M_D a^3} = \frac{|\langle 0|PS|D\rangle|^2 a^3}{2M_D}, \tag{6}$$

and hence,

$$af_D = \frac{m_c + m_q}{M_D} \sqrt{\frac{2}{aM_D}} |b_0^D|. \tag{7}$$

Our initial goal is to extract the amplitude $b_0^D \equiv |b_0^D|$ as accurately as possible. In Refs. [2,13] we found that fit results for two-point energies and amplitudes are improved significantly if one carries out simultaneous fits to two-point and three-point correlators. Three-point correlators are calculated, for instance, when one studies $D \to \pi, l\nu$ semileptonic decays. For pions at zero momentum one has,

$$C_{D\to\pi}^{\rm 3pnt}(t, T) = \frac{1}{L^3} \sum_{\vec{x}} \sum_{\vec{y}} \sum_{\vec{z}} \langle \Phi_D(\vec{y}, T)\tilde{S}(\vec{z}, t)\Phi_\pi^\dagger(\vec{x}, 0)\rangle, \tag{8}$$

where $\tilde{S}$ is the heavy-light scalar density $\bar{\Psi}_c \Psi_q$ in lattice units. $C_{D\to\pi}^{\rm 3pnt}$ must be fit to the form,

$$\begin{aligned} C_{D\to\pi}^{\rm 3pnt}(t, T) &= \sum_j^{N_\pi-1} \sum_k^{N_D-1} A_{jk} e^{-E_j^\pi t} e^{-E_k^D (T-t)} \\ &+ \sum_j^{N_\pi-1} \sum_k^{N'_D-1} B_{jk} e^{-E_j^\pi t} e^{-E_k'^D (T-t)} (-1)^{(T-t)} \\ &+ \sum_j^{N'_\pi-1} \sum_k^{N_D-1} C_{jk} e^{-E_j'^\pi t} e^{-E_k^D (T-t)} (-1)^t \\ &+ \sum_j^{N'_\pi-1} \sum_k^{N'_D-1} D_{jk} e^{-E_j'^\pi t} e^{-E_k'^D (T-t)} (-1)^t (-1)^{(T-t)}. \end{aligned} \tag{9}$$

We will only consider the region $0 \leq t \leq T$ and take $T \ll N_t$ so that any contributions from mesons propagating "around the lattice" due to periodic boundary conditions in time can be ignored. The same energies E_k^D and $E_k'^D$ appear in (5) and (9). Doing simultaneous fits to $C_D^{\rm 2pnt}$ and $C_{D\to\pi}^{\rm 3pnt}$ places tighter constraints on these energies and this helps in reducing fitting errors in the two-point amplitudes b_k^D. In this way the three-point correlator is acting like a very complicated but effective smearing for the propagation of D mesons. Normally this would also be considered a very expensive smearing, however we already had simulation results for $C_{D\to\pi}^{\rm 3pnt}$ on five out of the six ensembles in Table I from the D semileptonic project published in Ref. [2] so we could take advantage of this. It was only necessary to create new three-point correlator data on ensemble F0 and this only for zero momentum pions.

In Fig. 2 we show some results for b_0^D on ensemble C1 versus the number of exponentials from simultaneous fits (we set $N_D = N'_D = N_\pi = N'_\pi$) and compare with fit results to just $C_D^{\rm 2pnt}$ alone. One sees the improvement in the fitting errors coming from the simultaneous fits. All our fits are done using Bayesian methods [23]. We use the "sequential method", where starting from $N = 2$ or 3 the output from an N—exponential fit becomes the initial values for the subsequent $(N + 1)$—exponential fit.

In addition to the D meson decay constant f_D we have also accumulated new data for f_{D_s} by studying D_s meson two-point correlators. Here we do not have D_s semileptonic decay three-point correlator data. So, our extraction of the relevant amplitude $b_0^{D_s}$ was carried out from just the two-point correlators. Since statistical errors are smaller for D_s than for D mesons, this lack of ability to carry out simultaneous fits in the case of D_s was not a serious

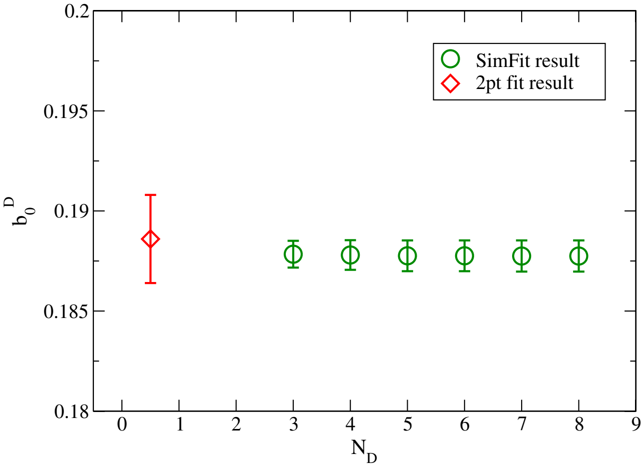

FIG. 2 (color online). Comparison of fit results for the amplitude b_0^D for ensemble C1 coming from just the two-point correlator (diamond data point to the left of figure) versus results from simultaneous fits to two-point and three-point correlators (circle data points).

problem. In Table III we list all our fit results for aM_D, af_D, aM_{D_s}, af_{D_s} and the ratio f_{D_s}/f_D.

IV. CHIRAL AND CONTINUUM EXTRAPOLATION

The next goal is to extrapolate the entries for f_D in Table III to the continuum and chiral limit. The latter is defined as the limit $m_q/m_s \to 1/27.4$, or using $m_s/m_c = 1/11.85$ from Ref. [24], the limit $m_q/m_c \to 1/(27.4 \times 11.85)$. We carry out the simultaneous chiral/continuum extrapolation using continuum partially quenched heavy meson chiral perturbation theory (PQHMChPT) [25–27] augmented by lattice spacing dependent terms. This is the same formalism employed recently in our f_B and f_{B_s} determinations [22]. We write,

$$f_D = A(1 + \delta f + [\text{analytic}])(1 + [\text{discret}]). \qquad (10)$$

The chiral logarithm term δf is taken from the original literature on PQHMChPT [26,27] and is also summarized in the Appendix of Ref. [22]. As in that reference we take,

$$[\text{analytic}] = \beta_0(2m_u + m_s)/\tilde{m}_c + \beta_1 m_q/m_c + \beta_2(m_q/m_c)^2, \qquad (11)$$

TABLE III. Fit results.

Set	aM_D	af_D	aM_{D_s}	af_{D_s}	f_{D_s}/f_D
C1	1.1395(7)	0.1370(5)	1.1878(3)	0.1541(3)	1.1245(37)
C2	1.1591(7)	0.1421(4)	1.2014(4)	0.1566(3)	1.1018(32)
C3	1.1618(5)	0.1464(3)	1.1897(3)	0.1552(3)	1.0600(16)
F0	0.8096(3)	0.0943(2)	0.8471(1)	0.1074(1)	1.1385(22)
F1	0.8130(3)	0.0966(2)	0.8471(2)	0.1082(1)	1.1202(21)
F2	0.8189(3)	0.1001(2)	0.8434(2)	0.1076(1)	1.0750(14)

where $m_u(m_q)$ is the sea (valence) light quark mass. $\tilde{m}_c$ is the AsqTad charm quark mass tuned to the η_c meson made out of AsqTad charm quark and antiquark, and is the appropriate charm quark mass to use for sea quarks. We take $\tilde{m}_c$ from Ref. [10] where it was found that $\tilde{m}_c/m_c \approx 0.9$ for lattices employed in the current article. Using ratios of bare quark masses to parameterize the "*analytic*" terms is convenient since such ratios are scale independent. We use the valence charm quark mass as the scale to measure the dominant discretization effects and set,

$$[\text{discret}] = c_0(am_c)^2 + c_1(am_c)^4. \qquad (12)$$

We will call the chiral/continuum extrapolation ansatz given by Eq. (10) together with (11) and (12) and Eq. (A7) of Ref. [22] for δf our "basic ansatz". The result of the extrapolation to the physical point using the basic ansatz is given by the green square point in Fig. 3. We have tested the stability of this result by modifying the basic ansatz in a number of ways and redoing the extrapolation. The modifications that were tried out are the following:

(1) dropping the β_2 term in (11)
(2) adding a $(m_q/m_c)^3$ term in (11)
(3) dropping the c_1 term in (12)
(4) adding $(am_c)^n$, $n = 6, 8, 10$, to (12)
(5) replacing c_i in (12) by $c_i \times$[power series in (m_q/m_c)]
(6) using powers of (a/r_1) rather than of (am_c) in (12)
(7) using Eq. (A1) of Ref. [22] rather than (A7) for the chiral logarithm term δf
(8) allowing for a 20% error in f_π entering the chiral perturbation theory formulas

Figure 4 compares the extrapolation results with these modifications in place with the basic ansatz value at the physical point.

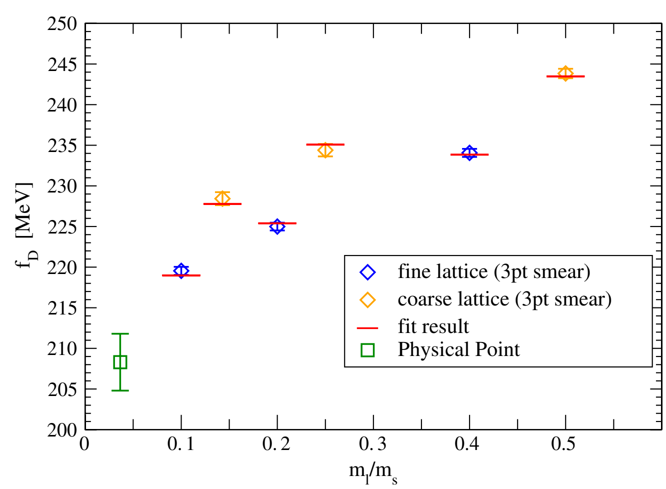

FIG. 3 (color online). Results for f_D versus m_l/m_s and at the physical point.

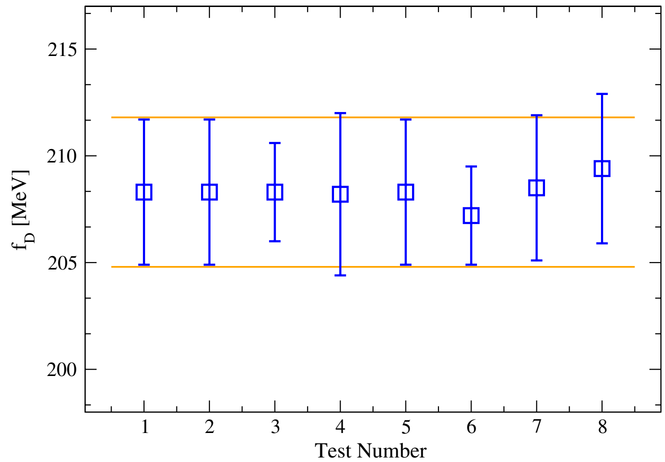

FIG. 4 (color online). Testing the stability of the chiral/continuum extrapolation. The numbers on the horizontal axis refer to the type of modification applied to the basic ansatz of (10)–(12) as listed in the text. The orange horizontal lines bracket the basic ansatz result, i.e., the physical point result in Fig. 3.

V. RESULTS

Table IV gives the error budget for f_D, f_{D_s} and f_{D_s}/f_D. For all but the last two entries we use the methods of Ref. [28] to isolate contributions from different sources that make up the total error coming out of the chiral/continuum extrapolations. For the finite volume error we take over the result from Ref. [11] where an analysis was carried out comparing finite and infinite volume chiral perturbation theory.

Taking all errors into account our final value for f_D is,

$$f_D = 208.3(1.0)_{\rm stat}(3.3)_{\rm sys}\ {\rm MeV}. \tag{13}$$

This is in good agreement with the previous result of $f_D = 207(4)$ MeV [11] based on HPQCD's old r_1, but is slightly more accurate. Equation (13) represents the most precise f_D available today.

For completeness we also give new values for f_{D_s} and f_{D_s}/f_D,

$$f_{D_s} = 246.0(0.7)_{\rm stat}(3.5)_{\rm sys}\ {\rm MeV}, \tag{14}$$

TABLE IV. Error budget.

Source	f_D (%)	f_{D_s} (%)	f_{D_s}/f_D (%)
statistics/fitting	0.5	0.3	0.3
scale r_1	0.7	0.7	...
r_1/a	0.04	0.05	...
continuum extrap.	1.2	1.2	0.9
chiral extrap. & $g_{D^*D\pi}$	0.7	0.2	0.5
mass tunings	0.1	0.2	0.2
finite volume	0.3	0.1	0.3
Total	1.7	1.5	1.1

and

$$f_{D_s}/f_D = 1.187(4)_{\rm stat}(12)_{\rm sys}. \tag{15}$$

The result for f_{D_s}, Eq. (14), is consistent with HPQCD's best updated value of $f_{D_s} = 248.0(2.5)$ MeV [12] but is not as accurate. One sees from Table IV that the dominant error comes from the continuum extrapolation. In this respect the current calculation of f_{D_s} is not competitive with Ref. [12] which employed data from five lattice spacings.

The new f_D of Eq. (13) can be combined with the $D \to \mu$, ν_μ branching fraction from CLEO-c [15] to extract a new value for $|V_{cd}|$. We find,

$$|V_{cd}|_{{\rm lepton}.d.} = 0.223(10)_{\rm exp.}(4)_{\rm lat}. \tag{16}$$

The first error, which is the experimental error, dominates the total error of 4.8%. Equation (16) agrees very well with HPQCD's recent determination of $|V_{cd}|$ from $D \to \pi$, $l\nu$ semileptonic decays [2], namely $|V_{cd}|_{{\rm semilep}.d.} = 0.225(6)_{\rm exp}(10)_{\rm lat}$, where now the lattice error dominates over the one from experiment. Both leptonic and semileptonic determinations agree with $|V_{cd}| = 0.230(11)$ [1] coming from neutrino scattering, and all three have comparable total errors.

As mentioned in the Introduction, BES III has recently announced preliminary results for the $D \to \mu\nu_\mu$ branching fraction [20]. Using their numbers we find,

$$|V_{cd}|^{\rm BES\ III}_{{\rm lepton}.d.} = 0.220(7)_{\rm exp}(4)_{\rm lat} \qquad [\text{preliminary}], \tag{17}$$

which agrees well with (16) and has smaller experimental errors.

Another way to check the consistency of the Standard Model and/or to test the lattice approach to heavy flavor physics is to consider the ratio between semileptonic form factor and decay constant $f_+^{D\to\pi}(0)/f_D$. We find, by combining Eq. (13) with $f_+^{D\to\pi}(0) = 0.666(29)$ from Ref. [2],

$$[f_+^{D\to\pi}(0)/f_D]_{\rm lat} = 3.20(15)\ {\rm GeV}^{-1}. \tag{18}$$

This can be compared with the experimental ratio in which $|V_{cd}|$ cancels of [15,29]

$$[f_+^{D\to\pi}(0)/f_D]_{\rm exp} = 3.19(18)\ {\rm GeV}^{-1}. \tag{19}$$

Equations (13) and (16) and the good agreement between (18) and (19) are the main results of this article.

VI. SUMMARY

In this article we presented a new determination of the CKM matrix element $|V_{cd}|$, Eq. (16), made possible by an updated calculation of the decay constant f_D, Eq. (13), and improved determinations of the $D \to \mu$, ν_μ leptonic decay branching fraction by CLEO-c [15] and BES III [20]. In

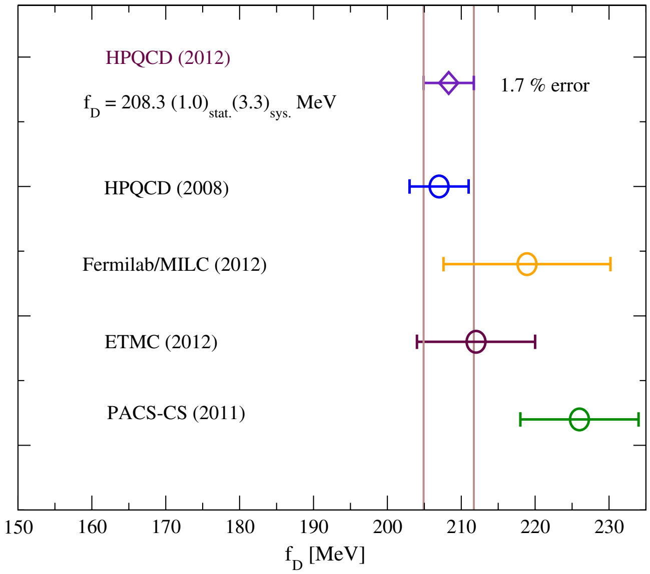

FIG. 5 (color online). Result for f_D from this work and comparisons with previous work [11,27,30,31].

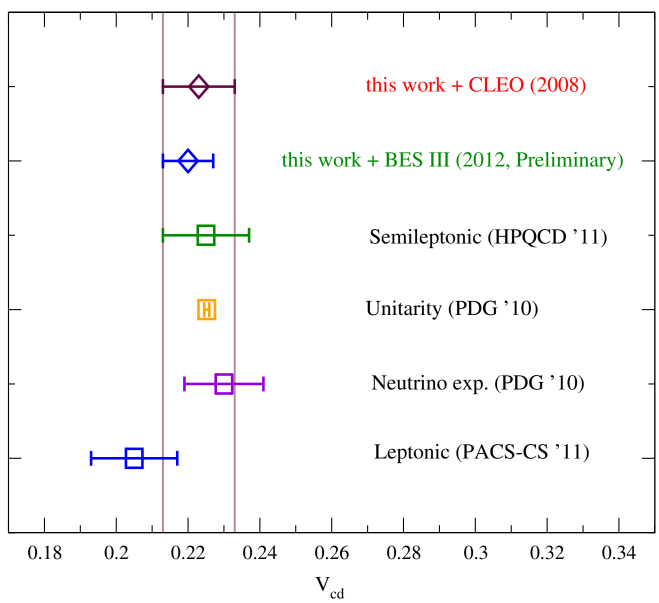

FIG. 6 (color online). Summary of $|V_{cd}|$ determinations from leptonic and semileptonic D decays, from neutrino scattering and from unitarity.

Fig. 5 we compare the new f_D with HPQCD's previous value [11] and with results from other lattice collaborations [27,30,31]. And in Fig. 6 we plot different results for $|V_{cd}|$ including the leptonic decay determination of this article, together with semileptonic decay and neutrino scattering determinations.

In the future it will be important to continue working on reducing the theory errors in Eq. (18) and the experimental errors in Eq. (19). The former is dominated by errors in the lattice determination of $f_+^{D\to\pi}(0)$ and work is underway to

TABLE V. Mass splittings in the D and B systems. The ΔM_B numbers are taken from Ref. [22].

Set	ΔM_D [MeV]	ΔM_B [MeV]	$\Delta M_D - \Delta M_B$ [MeV]
C1	80.4(1.1)	64.8(2.2)	15.6(2.5)
C2	69.7(1.0)	57.7(1.8)	12.0(2.1)
C3	46.5(5)	41.3(2.0)	5.2(2.1)
F0	87.3(7)	71.7(2.9)	15.6(3.0)
F1	79.4(7)	61.4(2.0)	18.0(2.1)
F2	57.4(4)	47.8(1.3)	9.6(1.4)

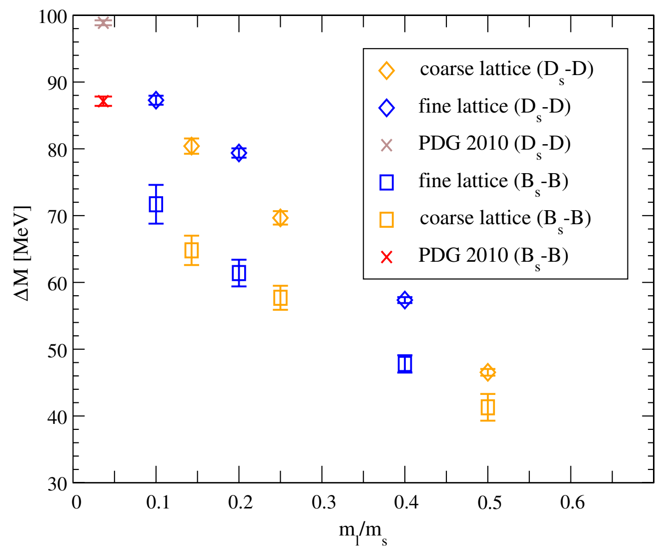

FIG. 7 (color online). Comparison of the mass differences $\Delta M_D = M_{D_s} - M_D$ and $\Delta M_B = M_{B_s} - M_B$.

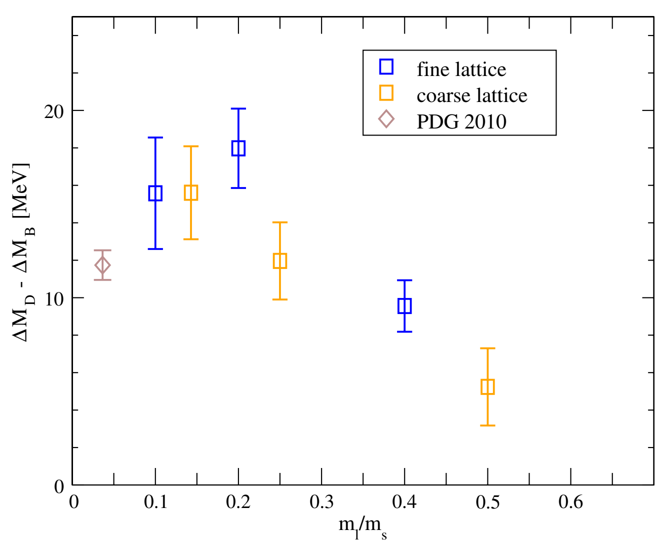

FIG. 8 (color online). $\Delta M_D - \Delta M_B$, the difference of the mass differences in the D and B systems.

significantly reduce them [32]. The experimental error in Eq. (19) comes mainly from the leptonic decay branching fraction and one can look forward to improvements there as well. In particular, the recent measurements by BES III [20] look very promising. The crucial question is whether the nice agreement seen now between Eqs. (18) and (19) will continue to hold once errors dip down to ∼2% or below.

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APPENDIX: THE D_s-D MASS DIFFERENCE

In this appendix we summarize results for the mass difference $\Delta M_D = M_{D_s} - M_D$ and compare with the analogous difference in the B system $\Delta M_B = M_{B_s} - M_B$, where the latter was calculated in Ref. [22] employing NRQCD b-quarks. This is an interesting quantity to compare since the leading heavy quark mass dependence cancels in each of the mass differences and one is testing whether the subleading contributions are accurate enough to be able to distinguish between the D and B systems. In the difference of differences $\Delta M_D - \Delta M_B$ any mistunings of the strange quark mass should also cancel out (identical strange and light quark propagators are used in the B/B_s and the D/D_s calculations). Table V lists simulation results for ΔM_D, ΔM_B and for $\Delta M_D - \Delta M_B$. The first two quantities are plotted in Fig. 7 versus m_l/m_s. For ΔM_D statistical errors are small enough so that a slight lattice spacing dependence is detected. Errors are larger for ΔM_B and no discretization effects are visible. Figure 8 shows $\Delta M_D - \Delta M_B$. There is reasonable agreement with experiment at the 1σ level for small m_l/m_s.

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