

Doubly heavy baryon production at a high luminosity e^+e^- collider

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Within the framework of nonrelativistic QCD, we present a detailed discussion on doubly heavy baryon production through the e^+e^- annihilation channel, $e^+e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{QQ'} + \bar{Q} + \bar{Q}'$, at a high luminosity e^+e^- collider. Here $Q^{(\prime)}$ stands for the heavy b or c quark. In addition to the channel through the usually considered diquark state $(QQ')[^3S_1]_3$, contributions from the channels through other equally important diquark states such as $(QQ')[^1S_0]_6$ are also discussed. Uncertainties for the total cross sections are predicted by taking $m_c = 1.80 \pm 0.30$ GeV and $m_b = 5.10 \pm 0.40$ GeV. At a super- Z factory running around the Z^0 mass and with a high luminosity up to $\mathcal{L} \propto 10^{34} \sim 10^{36}$ cm $^{-2}$ s $^{-1}$, we estimate that about $1.1 \times 10^{5-7}$ Ξ_{cc} events, $2.6 \times 10^{5-7}$ Ξ_{bc} events, and $1.2 \times 10^{4-6}$ Ξ_{bb} events can be generated in one operation year. Such a Z factory, thus, will provide a good platform for studying doubly heavy baryons in a manner comparable to the CERN Large Hadron Collider.

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I. INTRODUCTION

Theoretically, the production of doubly heavy baryons, $\Xi_{QQ'}$, where the symbol $Q^{(\prime)}$ stands for the heavy b or c quark accordingly, has been analyzed in Refs. [1–15]. In particular, the computer program GENXICC, which is written in a PYTHIA-compatible format [16] for simulating the hadronic production of Ξ_{cc} , Ξ_{bc} , and Ξ_{bb} events, has been completed and upgraded in Refs. [13,14]. Throughout the paper, $\Xi_{QQ'}$, corresponding to Ξ_{cc} , Ξ_{bc} , or Ξ_{bb} , is a short notation for the baryon $\Xi_{QQ'q}$, with the light quark q equal to u , d , or s , respectively.¹ Experimentally, among the doubly heavy baryons, only Ξ_{cc} has been observed, by the SELEX collaboration [17–19]. Neither the BABAR collaboration nor the Belle collaboration has found evidence for Ξ_{cc} in related experiments [20,21], to say nothing of the Ξ_{bc} and Ξ_{bb} baryons. We hope the CERN Large Hadron Collider (LHC), due to its high collision energy and high luminosity, will change the present situation, and especially improve our understanding of those baryons' hadronic production properties.

For comparing pp , ep , and $\gamma\gamma$ collisions, a e^+e^- collider is helpful and has some advantages for performing precise measurements for certain processes. To seek $\Xi_{QQ'}$ events at the LHC is feasible, but its hadronic background is much noisier in comparison to a e^+e^- collider. In hadronic production, the baryons are produced through scattering, annihilation, or fusion of two initial partons inside the incident hadrons. In addition to the dominant gluon-gluon fusion mechanism, one also needs to take the extrinsic or intrinsic heavy quark mechanisms into consideration, especially at the small- p_t regions [9,10]. Hence,

the hadronic production becomes much more complicated due to the introduction of nonperturbative parton distribution functions and intrinsic components of the hadron (even though they are universal). By contrast, at the e^+e^- collider, one only needs to consider the e^+e^- annihilation channel, $e^+e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{QQ'} + \bar{Q} + \bar{Q}'$, which allows one to study the $\Xi_{QQ'}$ baryons' own properties.

If the luminosity of a e^+e^- collider is $\mathcal{L} \propto 10^{34-36}$ cm $^{-2}$ s $^{-1}$ and its colliding energy is around the Z^0 peak (this is called a super- Z factory Ref. [22]), it will raise the production rate by several orders of magnitude in comparison to the previous LEP, Belle, and BABAR experiments. This increment has already been observed in doubly heavy meson production due to the Z^0 -boson resonance effect [23–31]. It is thus natural to estimate that such a super- Z factory will also open new opportunities for studying the $\Xi_{QQ'}$ baryon properties, such as their spectroscopy and their inclusive and exclusive decays. In the present paper, we study the semi-inclusive production of doubly heavy $\Xi_{QQ'}$ baryons at a super- Z factory.

With the nonrelativistic QCD (NRQCD) framework [32], the production of $\Xi_{QQ'}$ baryons can be factorized into two steps: The first step is to produce two free heavy quark pairs $Q\bar{Q}$ and $Q'\bar{Q}'$, which is perturbative QCD (pQCD) calculable. This is due to the fact that the intermediate γ , gluon, or Z^0 should be hard enough to generate a heavy quark pair. The second step is to make the two heavy quarks Q and Q' into a bounding diquark (QQ') in the $[^3S_1]$ (or $[^1S_0]$) spin state and the $\bar{\mathbf{3}}$ (or $\bar{\mathbf{6}}$) color state accordingly; it will then be hadronized into a $\Xi_{QQ'}$ baryon, whose probability is described by the NRQCD matrix element. More explicitly, the intermediate diquarks in Ξ_{cc} and Ξ_{bb} have two spin and color configurations, $[^3S_1]_3$ and $[^1S_0]_6$; while for the intermediate diquark (bc) in Ξ_{bc} , there are four spin and color configurations: $\Xi_{bc}[^3S_1]_3$, $\Xi_{bc}[^3S_1]_6$, $\Xi_{bc}[^1S_0]_3$, and $\Xi_{bc}[^1S_0]_6$.

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¹In the present paper, we will ignore the isospin-breaking effect; for instance, Ξ_{cc} denotes Ξ_{ccd}^+ or Ξ_{ccu}^+ or Ω_{ccs}^+ accordingly.

It has been observed that, for the hadronic production channels, the contributions from other spin and color configurations of the diquark, such as the $(QQ')[^1S_0]_6$ configuration, can also provide sizable contributions in addition to the dominant $(QQ')[^3S_1]_3$ [8,9,12]. We will show that this is also the case for the present considered e^+e^- annihilation channel, $e^+e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{QQ'} + \bar{Q} + \bar{Q}'$. Thus, one needs to take all these states into consideration for a sound estimation.

The remaining parts of the paper are organized as follows: In Sec. II, we present the detailed formulation for dealing with the process $e^+ + e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{QQ'} + \bar{Q} + \bar{Q}'$. In Sec. III, we give the numerical results and uncertainty discussion. Section IV is reserved for a summary.

II. CALCULATION TECHNOLOGY

Typical Feynman diagrams for $e^+(p_2)e^-(p_1) \rightarrow (QQ')[n](q_1) + \bar{Q}(q_2) + \bar{Q}'(q_3)$ interactions are presented in Fig. 1. According to NRQCD factorization formulae [33], the differential cross section of this process can be written in the following factorization form:

$$d\sigma(e^+e^- \rightarrow \Xi_{QQ'} + \bar{Q} + \bar{Q}') = \sum_n d\hat{\sigma}(e^+e^- \rightarrow (QQ')[n] + \bar{Q} + \bar{Q}') \langle \mathcal{O}^H(n) \rangle, \quad (1)$$

where the matrix element $\langle \mathcal{O}^H(n) \rangle$ is proportional to the inclusive transition probability of the perturbative state

$$\begin{aligned} \mathcal{M}((QQ')[n]) &= \bar{u}_{s_1} \left(\frac{m_Q}{M_{QQ'}} q_1 \right) \Gamma_\rho s_f(k_{\rho-1}, m_Q) \cdots s_f(k_1, m_Q) \Gamma_1 v_{s_4}(q_2) \mathcal{B}(S, s_1, s_2; q_1, M_{QQ'}) \\ &\times \bar{u}_{s_2} \left(\frac{m_{Q'}}{M_{QQ'}} q_1 \right) \Gamma'_1 s_f(k'_1, m_{Q'}) \cdots s_f(k'_{\kappa-1}, m_{Q'}) \Gamma'_\kappa v_{s_3}(q_3) \times \mathcal{C} \times \mathcal{G} \times \mathcal{D} \times \mathcal{L}_{rr'}, \end{aligned} \quad (2)$$

where s_i is the spin state of the outgoing (anti)quark, S is the spin state of the diquark, and $M_{QQ'}$ is the diquark mass. The parameters $\Gamma_1, \cdots, \Gamma_\rho, \Gamma'_1, \cdots, \Gamma'_\kappa$ are sequential interaction vertexes (the γ -matrix elements only) along the corresponding spinor lines, and $s_f(k_i^{(j)}, m_{Q^{(j)}})$ is the fermion propagator between the interaction vertexes. As a special case, when there is only one interaction vertex ($\rho = 1$ or $\kappa = 1$), there is no fermion propagator. $\mathcal{B}(S, s_1, s_2; q_1, M_{QQ'})$ is the wave function of the $(QQ')[n]$ diquark. Here, \mathcal{C} is the color factor of the process, \mathcal{G} is the gluon propagator, \mathcal{D} is the photon or Z^0

$(QQ')[n]$ pair into the heavy baryon $\Xi_{QQ'}$, and $[n]$ represents the spin and color quantum numbers for the intermediate diquark state. The short-distance cross section $d\hat{\sigma}(e^+e^- \rightarrow (QQ')[n] + \bar{Q} + \bar{Q}')$ takes the following form:

$$d\hat{\sigma}(e^+e^- \rightarrow (QQ')[n] + \bar{Q} + \bar{Q}') = \frac{\bar{\sum} |\mathcal{M}|^2 d\Phi_3}{4\sqrt{(p_1 \cdot p_2)^2 - m_e^4}},$$

where \mathcal{M} is the hard scattering amplitude, and $\bar{\sum}$ means we need to average over the spin states of the electron and positron and sum over the color and spin of all final particles. The three-particle phase space is

$$d\Phi_3 = (2\pi)^4 \delta^4 \left(p_1 + p_2 - \sum_f q_f \right) \prod_{f=1}^3 \frac{d^3 q_f}{(2\pi)^3 2q_f^0}.$$

The phase space can be generated and integrated with the help of the FormCalc program [34], or a combination of RAMBOS [35] and VEGAS [36], which can be found in the generators GENXICC [13,14] and BCVEGPY [37].

A. $\mathcal{M}(e^+e^- \rightarrow (QQ')[n] + Q + \bar{Q}')$

The hard scattering amplitude \mathcal{M} for diquark production can be related with the familiar meson production through a proper correspondence.

The hard scattering amplitude \mathcal{M} for the production channel $e^+(p_2)e^-(p_1) \rightarrow (QQ')[n](q_1) + \bar{Q}(q_2) + \bar{Q}'(q_3)$ can be written as

propagator, and $\mathcal{L}_{rr'}$ is the leptonic part, which can be expressed as

$$\mathcal{L}_{rr'} = \bar{v}_r(p_2) \Gamma u_{r'}(p_1),$$

in which $r^{(j)}$ stands for the spin state of the electron (positron), $\Gamma = \gamma^\mu$ for the γ propagator, and $\Gamma = \gamma^\mu (\frac{1}{4} - \sin^2 \theta_w - \frac{1}{4} \gamma^5)$ for the Z^0 propagator.

As a comparison, the hard scattering amplitude for meson $(Q'\bar{Q})[n]$ production through the channel $e^+(p_2)e^-(p_1) \rightarrow (Q'\bar{Q})[n](q_1) + Q(q_2) + \bar{Q}'(q_3)$ can be written as

$$\begin{aligned} \mathcal{M}((Q'\bar{Q})[n]) &= \bar{u}_{s_4}(q_2) \Gamma_1 s_f(-k_1, m_Q) \cdots s_f(-k_{\rho-1}, m_Q) \Gamma_\rho v_{s_1} \left(\frac{m_Q}{M_{Q'\bar{Q}}} q_1 \right) \mathcal{B}(S, s_1, s_2; q_1, M_{Q'\bar{Q}}) \\ &\times \bar{u}_{s_2} \left(\frac{m_{Q'}}{M_{Q'\bar{Q}}} q_1 \right) \Gamma'_1 s_f(k'_1, m_{Q'}) \cdots s_f(k'_{\kappa-1}, m_{Q'}) \Gamma'_\kappa v_{s_3}(q_3) \times \mathcal{C}' \times \mathcal{G} \times \mathcal{D} \times \mathcal{L}_{rr'}, \end{aligned} \quad (3)$$

where $M_{Q'\bar{Q}}$ is the meson mass, $\mathcal{B}(S, s_1, s_2; q_1, M_{Q'\bar{Q}})$ is the wave function of the $(Q'\bar{Q})[n]$ meson, and \mathcal{C}' is the color factor for the present case. Other parameters have the same meaning as above.

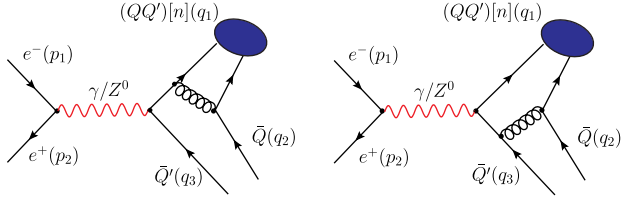


FIG. 1 (color online). Typical Feynman diagrams for $e^+e^- \rightarrow (QQ')[n] + \bar{Q} + Q'$ interactions, where Q and Q' stand for the heavy c or b quarks, and $[n]$ represents the spin and color quantum number of the intermediate diquark state. Two new Feynman diagrams for the case of $Q \neq Q'$ can be obtained by exchanging the quark lines of Q and Q' .

A comparison of Eqs. (2) and (3) tells us that if we can transform the fermion line of the diquark production case to an antifermion line of the meson production case, then

$$\begin{aligned} a &= a^T = v_{s_4}^T(q_2) \Gamma_1^T s_f^T(k_1, m_Q) \cdots s_f^T(k_{\rho-1}, m_Q) \Gamma_\rho^T \bar{u}_{s_1}^T\left(\frac{m_Q}{M_{QQ'}} q_1\right) \\ &= v_{s_4}^T(q_2) C C^- \Gamma_1^T C C^- s_f^T(k_1, m_Q) C C^- \cdots C C^- s_f^T(k_{\rho-1}, m_Q) C C^- \Gamma_\rho^T C C^- \bar{u}_{s_1}^T\left(\frac{m_Q}{M_{QQ'}} q_1\right) \\ &= (-1)^{(\rho+1)} \bar{u}_{s_4}(q_2) \Gamma_1 s_f(-k_1, m_Q) \cdots s_f(-k_{\rho-1}, m_Q) \Gamma_\rho v_{s_1}\left(\frac{m_Q}{M_{QQ'}} q_1\right). \end{aligned}$$

Thus, Eq. (2) can be transformed as

$$\begin{aligned} \mathcal{M}((QQ')[n]) &= (-1)^{(\rho+1)} \bar{u}_{s_4}(q_2) \Gamma_1 s_f(-k_1, m_Q) \cdots s_f(-k_{\rho-1}, m_Q) \Gamma_\rho v_{s_1}\left(\frac{m_Q}{M_{QQ'}} q_1\right) \mathcal{B}(S, s_1, s_2; q_1, M_{QQ'}) \\ &\quad \times \bar{u}_{s_2}\left(\frac{m_{Q'}}{M_{QQ'}} q_1\right) \Gamma_1' s_f(k_1', m_{Q'}) \cdots s_f(k_{\rho-1}', m_{Q'}) \Gamma_\rho' v_{s_3}(q_3) \times \mathcal{C} \times \mathcal{G} \times \mathcal{D} \times \mathcal{L}_{rr'}. \end{aligned} \quad (4)$$

Comparing Eq. (3) with Eq. (4), one can, thus, follow the same procedures as for meson production to finish the calculation.

By taking the Lorentz indices and the color indices explicitly, the hard scattering amplitude \mathcal{M} in Eq. (4) can be rewritten as

$$i\mathcal{M}((QQ')[n]) = \kappa \sum_{S=1}^4 \mathcal{A}_S^\mu \times \mathcal{D}_{\mu\nu} \times \mathcal{L}_{rr'}^\nu. \quad (5)$$

For the production through the γ propagator, the overall parameter $\kappa = e_{Q^{(0)}} e^2 g_s^2 C_{ij}$, where $e_{Q^{(0)}}$ is the electric charge of the quark $Q^{(0)}$ in units of e , and C_{ij} is the color factor with i and j the color indices of the outgoing antiquarks; the propagator $\mathcal{D}_{\mu\nu} = \frac{-i}{k^2} g_{\mu\nu}$; and the leptonic vector

$$\mathcal{L}_{rr'}^\nu = \bar{v}_r(p_2) \gamma^\nu u_{r'}(p_1).$$

For the production through the Z^0 propagator, $\kappa = \frac{g^2 g_s^2}{\cos^2 \theta_w} C_{ij}$;

$$\mathcal{D}_{\mu\nu} = \frac{i}{k^2 - m_Z^2 + im_Z \Gamma_z} \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} \right),$$

we can derive the diquark amplitude by following the same method as we would for meson production. More definitely, we need to deal with the following fermion line:

$$a = \bar{u}_{s_1} \left(\frac{m_Q}{M_{QQ'}} q_1 \right) \Gamma_\rho s_f(k_{\rho-1}, m_Q) \cdots s_f(k_1, m_Q) \Gamma_1 v_{s_4}(q_2).$$

Setting T to be the operation of matrix transportation and $C = -i\gamma^2 \gamma^0$ to be the charge conjugation matrix, we obtain the following equations:

$$\begin{aligned} v_{s_4}^T(q_2) C &= -\bar{u}_{s_4}(q_2), & C^- \Gamma_i^T C &= -\Gamma_i, \\ C C^- &= 1, & C^- s_f^T(k_i, m_Q) C &= s_f(-k_i, m_Q), \end{aligned}$$

$$C^- \bar{u}_{s_1}^T \left(\frac{m_Q}{M_{QQ'}} q_1 \right) = v_{s_1} \left(\frac{m_Q}{M_{QQ'}} q_1 \right).$$

Then, the fermion line a changes to

with Γ_z the total decay width of the Z^0 boson; and

$$\mathcal{L}_{rr'}^\nu = \bar{v}_r(p_2) \gamma^\nu \left(\frac{1}{4} - \sin^2 \theta_w - \frac{1}{4} \gamma^5 \right) u_{r'}(p_1).$$

The vectors \mathcal{A}_S^μ ($S = 1, \dots, 4$) can be read from the Feynman diagrams in Fig. 1. More explicitly, \mathcal{A}_S^μ can be written as

$$\begin{aligned} \mathcal{A}_1^\mu &= \bar{u}_s(q_2) \gamma_\rho \frac{\Pi_{(QQ')[n]}^{0(1)}(q_1)}{(q_{11} + q_2)^2} \gamma_\rho \frac{\not{q}_1 + \not{q}_2 + m_{Q'}}{(q_1 + q_2)^2 - m_{Q'}^2} \\ &\quad \times \Gamma_{zQ'}^\mu v_{s'}(q_3), \end{aligned} \quad (6)$$

$$\begin{aligned} \mathcal{A}_2^\mu &= \bar{u}_s(q_2) \gamma_\rho \frac{\Pi_{(QQ')[n]}^{0(1)}(q_1)}{(q_{11} + q_2)^2} \Gamma_{zQ'}^\mu \frac{-\not{q}_{11} - \not{q}_2 - \not{q}_3 + m_{Q'}}{(-\not{q}_{11} - \not{q}_2 - \not{q}_3)^2 - m_{Q'}^2} \\ &\quad \times \gamma_\rho v_{s'}(q_3), \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{A}_3^\mu &= -\bar{u}_s(q_2) \Gamma_{zQ}^\mu \frac{-\not{q}_1 - \not{q}_3 + m_Q}{(-q_1 - q_3)^2 - m_Q^2} \gamma_\rho \frac{\Pi_{(QQ')[n]}^{0(1)}(q_1)}{(q_{12} + q_3)^2} \\ &\quad \times \gamma_\rho v_{s'}(q_3), \end{aligned} \quad (8)$$

and

$$\mathcal{A}_4^\mu = -\bar{u}_s(q_2)\gamma_\rho \frac{\not{q}_{12} + \not{q}_3 + \not{q}_2 + m_Q}{(\not{q}_{12} + \not{q}_3 + \not{q}_2)^2 - m_Q^2} \times \Gamma_{zQ}^{\mu(0)} \frac{\Pi_{(QQ')[n]}^{0(1)}(q_1)}{(q_{12} + q_3)^2} \gamma_\rho v_{s'}(q_3), \quad (9)$$

where for convenience, we introduce a general interaction vertex $\Gamma_{zQ}^{\mu(0)}$; the needed ones are

$$\Gamma_{zc}^\mu = \gamma^\mu \left[\alpha + \beta \left(\frac{1}{4} - \frac{2}{3} \sin^2 \theta_w - \frac{1}{4} \gamma^5 \right) \right],$$

or

$$\Gamma_{zb}^\mu = \gamma^\mu \left[\alpha + \beta \left(\frac{1}{4} - \frac{1}{3} \sin^2 \theta_w - \frac{1}{4} \gamma^5 \right) \right].$$

Here $\alpha = 1$ and $\beta = 0$ for the $\gamma - Q^{(\prime)} - Q^{(\prime)}$ vertex, and $\alpha = 0$ and $\beta = 1$ for the $Z^0 - Q^{(\prime)} - Q^{(\prime)}$ vertex. The momenta of the constituent quarks which form the bound state are

$$q_{11} = \frac{m_Q}{M_{QQ'}} q_1 + q \quad \text{and} \quad q_{12} = \frac{m_{Q'}}{M_{QQ'}} q_1 - q, \quad (10)$$

where $M_{QQ'} = m_Q + m_{Q'}$ is implicitly adopted to ensure the gauge invariance of the hard scattering amplitude, and q is the relative momentum between the two constituent quarks inside the diquark. Due to the nonrelativistic approximation, q is small and neglected in the amplitude.² For the production of (cc) and (bb) diquarks, we only need to calculate A_1^μ and A_2^μ , but we need to time the squared amplitude for (cc) and (bb) diquarks by an overall factor $(2^2/2!) = 2$, where the $1/2!$ factor is due to the symmetry of the diquark wavefunction. For the case of (bc) diquark production, however, we need to calculate A_S^μ ($S = 1, \dots, 4$).

The projector $\Pi_{(QQ')[n]}^{0(1)}(q_1)$, under the nonrelativistic approximation, takes the following form [33]:

$$\Pi_{(QQ')[n]}^{0(1)}(q_1) = \frac{1}{2\sqrt{M_{QQ'}}} (\xi_1 \gamma^5 + \xi_2 \not{\epsilon}(q_1)) (\not{q}_1 + M_{QQ'}), \quad (11)$$

where $\epsilon(q_1)$ is the polarization vector for the spin-triplet state. Here the projector Π^1 is for the case of spin triplet $[^3S_1]$, which corresponds to $\xi_1 = 0$ and $\xi_2 = 1$, and the projector Π^0 is for the case of spin singlet $[^1S_0]$, which corresponds to $\xi_1 = 1$ and $\xi_2 = 0$.

B. The color factor C_{ij}

Because of the fact that $3 \otimes 3 = \bar{3} \oplus 6$ in $SU_C(3)$ color group, the (QQ') diquark can be in either the antitriplet $\bar{3}$ or the sextuplet 6 color state. According to Fig. 1, the color factor C_{ij} of the process is

$$C_{ij} = \mathcal{N} \times \sum_{m,n} (T^a)_{im} (T^a)_{jn} \times G_{mnk}, \quad (12)$$

where $i, j, m, n = 1, 2, 3$ are the color indices of the two outgoing antiquarks \bar{Q} and \bar{Q}' and the two constituent quarks Q and Q' of the diquark, respectively; k are the color indices of the diquark (QQ') ; the indices $a = 1, \dots, 8$ are the color index for the gluon; and the normalization constant $\mathcal{N} = \sqrt{1/2}$. The function G_{mnk} is equal to the antisymmetric function ε_{mjk} or the symmetric function f_{mjk} accordingly; the antisymmetric ε_{mjk} satisfies

$$\varepsilon_{mjk} \varepsilon_{m'j'k} = \delta_{mm'} \delta_{jj'} - \delta_{mj'} \delta_{jm'},$$

and the symmetric f_{mjk} satisfies

$$f_{mjk} f_{m'j'k} = \delta_{mm'} \delta_{jj'} + \delta_{mj'} \delta_{jm'}.$$

Then, the square of C_{ij}^2 equals $\frac{4}{3}$ for the color antitriplet diquark production and $\frac{2}{3}$ for the color sextuplet diquark production.

C. Transition from $(QQ')[n]$ to $\Xi_{QQ'}$

According to NRQCD, the $\Xi_{QQ'}$ baryon can be expanded as a series of Fock states according to the relative velocity (v) of the constituent heavy quarks in the baryon rest frame:

$$|\Xi_{QQ'}\rangle = c_1 |(QQ')q\rangle + c_2 |(QQ')qg\rangle + c_3 |(QQ')qgg\rangle + \dots,$$

where c_i ($i = 1, 2, \dots$) is a function of the small velocity v , where v is the relative velocity of the heavy quarks in the rest frame of the diquark. As for Ξ_{cc} and Ξ_{bb} , the intermediate binding diquark in the Fock states can be in either the $(QQ)[^3S_1]_{\bar{3}}$ state or the $(QQ)[^1S_0]_6$ ($Q = c, b$) state; while for Ξ_{bc} , the intermediate binding diquark within the Fock states can be in the $(bc)[^3S_1]_{\bar{3}}$ or $(bc)[^1S_0]_{\bar{3}}$ or $(bc) \times [^3S_1]_6$ or $(bc)[^1S_0]_6$ states. Reference [8] has suggested that the baryon can be formed with the component $|(QQ')qg\rangle$ as well as the usual $|(QQ')q\rangle$: One of the heavy quarks emits a gluon, which does not change the spin of the heavy quark, and this gluon splits into a $q\bar{q}$. The light quarks can also emit gluons, and then the component can be formed with the light quark q plus one gluon. Because a light quark can emit gluons easily, one can take the transition probability for these diquark states to form the corresponding baryon to be of the same importance. Then, as a rough order estimation, one can take the transition probability for these diquark states to form the corresponding baryon to be the same; i.e.,

$$h_6 \simeq h_{\bar{3}}, \quad (13)$$

where $h_{\bar{3}}$ stands for the probability of transforming the color antitriplet diquark into the baryon, and h_6 stands for the probability of transforming the color sextuplet diquark into the baryon. These nonperturbative matrix elements

²The integration over q results in a wave function at zero which has been absorbed into the overall nonperturbative matrix element.

can be determined from the potential model, or from nonperturbative methods such as QCD sum rules and lattice QCD.

It is also noted in Ref. [8] that if the baryon is formed by the Fock state component $|(QQ')q\rangle$ only, the emitted gluon for the case of (QQ') in the 1S_0 spin state, which will split into a $q\bar{q}$ pair with q being combined by (QQ') to form the baryon, must change the spin of the heavy quark. Then, according to NRQCD [32], one may conclude that h_6 must be v^2 suppressed in comparison with $h_{\bar{3}}$. This provides the underlying reason why only $h_{\bar{3}}$ has been taken into consideration for doubly heavy baryon production, cf., Refs. [1–5]. Fortunately, these matrix elements are overall parameters, and their uncertainties can be conveniently discussed. We will adopt the approximation of Eq. (13) throughout the paper.³

Furthermore, the color-singlet nonperturbative matrix element $\langle \mathcal{O}^H(1S) \rangle$ in the factorization formula [Eq. (1)] can be related to the Schrödinger wave functions at the origin, $|\psi_{(Q\bar{Q}')} (0)|$ [33]:

$$h_6 \simeq h_{\bar{3}} = \langle \mathcal{O}^H(1S) \rangle \simeq |\psi_{|(Q\bar{Q}') [1S]} (0)|^2. \quad (14)$$

Since the spin-splitting effect is small, we do not distinguish the bound state parameters for the spin-singlet and the spin-triplet states; i.e., those parameters, such as the constituent quark masses, the bound state mass, and the wave function, are taken to be the same for the spin-singlet and spin-triplet states.

III. NUMERICAL RESULTS

When doing the numerical calculation, the input parameters are taken as the following values [9]:

$$\begin{aligned} |\Psi_{cc}(0)|^2 &= 0.039 \text{ GeV}^3, & |\Psi_{bc}(0)|^2 &= 0.065 \text{ GeV}^3, \\ |\Psi_{bb}(0)|^2 &= 0.152 \text{ GeV}^3, & m_c &= 1.8 \text{ GeV}, \\ m_b &= 5.1 \text{ GeV}, & M_{\Xi_{cc}} &= 3.6 \text{ GeV}, \\ M_{\Xi_{bc}} &= 6.9 \text{ GeV}, & M_{\Xi_{bb}} &= 10.2 \text{ GeV}. \end{aligned}$$

Other input parameters are taken from the Particle Data Group [38]: $\Gamma_z = 2.4952 \text{ GeV}$, $m_Z = 91.1876 \text{ GeV}$, $m_W = 80.399 \text{ GeV}$, and $\cos\theta_w = m_W/m_Z$. We take the renormalization scale to be $2m_c$ for Ξ_{cc} or Ξ_{bc} , and $2m_b$ for Ξ_{bb} , which leads to $\alpha_s(2m_c) = 0.212$ and $\alpha_s(2m_b) = 0.164$ for leading-order α_s running.

At the super- Z factory, total cross sections for the production channel $e^+ + e^- \rightarrow \Xi_{QQ'}(n) + \bar{Q} + \bar{Q}'$ (where

³Indifferent to the heavy quark fragmentation, during the fragmentation of a diquark into a baryon, the diquark may dissociate, which will decrease the baryon production cross section to a certain degree. In our present calculation, we will not take this effect into consideration. Thus, our present estimations can be treated as an upper limit for the total cross sections.

n stands for the intermediate diquark state) are presented in Table I. It is found that

- (i) When the e^+e^- collision energy is at the Z^0 peak ($E_{\text{cm}} = \sqrt{s} = m_Z$), the production channel through a γ propagator is much smaller than in the case of a Z^0 propagator; i.e., for production through the same diquark state, its cross section is always less than 10^{-3} of the Z^0 case.
- (ii) At the Z^0 peak, the contribution from the diquark state $n = [^1S_0]_6$ configuration is almost half that of the state $n = [^3S_1]_{\bar{3}}$ for Ξ_{cc} and Ξ_{bb} baryon production. For Ξ_{bc} baryon production, however, four intermediate (bc) -diquark states will make sizable contributions: the total cross-sections of $n = [^3S_1]_6$, $[^1S_0]_{\bar{3}}$, and $[^1S_0]_6$ are about 50, 73, and 37% that of $n = [^3S_1]_{\bar{3}}$, respectively. Therefore, one should take all these Fock states' contributions into consideration in order to derive a sound estimation.

To show how the total cross sections change with the variation of e^+e^- collision energy $E_{\text{cm}} = \sqrt{s}$, we present their cross sections versus the collision energy in Figs. 2–4 for Ξ_{cc} , Ξ_{bc} , and Ξ_{bb} baryon production, respectively. In the small collision energy region (e.g., $E_{\text{cm}} \lesssim 50 \text{ GeV}$), we have

$$\sigma_{e^+e^- \rightarrow \gamma \rightarrow \Xi_{QQ'}(n)} > \sigma_{e^+e^- \rightarrow Z^0 \rightarrow \Xi_{QQ'}(n)}.$$

The production cross section for the case of γ propagators dominates at small E_{cm} , there is a small peak at 10–30 GeV, and then it decreases slowly with the increment of E_{cm} . The production cross section for the case of Z^0 propagators is negligible at small E_{cm} , but it rises logarithmically: when $E_{\text{cm}} \sim m_Z$, due to the Z^0 -resonance effect, the cross section

TABLE I. Total cross section (in pb) for the production of $\Xi_{QQ'}(n)$ baryons (where n stands for the intermediate diquark state) through e^+e^- annihilation at the Z^0 peak ($\sqrt{s} = m_Z$).

Production channel	Cross section (pb)
$e^+e^- \rightarrow \gamma \rightarrow \Xi_{cc}([^3S_1]_{\bar{3}})$	8.90×10^{-4}
$e^+e^- \rightarrow \gamma \rightarrow \Xi_{cc}([^1S_0]_6)$	4.29×10^{-4}
$e^+e^- \rightarrow Z^0 \rightarrow \Xi_{cc}([^3S_1]_{\bar{3}})$	0.727
$e^+e^- \rightarrow Z^0 \rightarrow \Xi_{cc}([^1S_0]_6)$	0.353
$e^+e^- \rightarrow \gamma \rightarrow \Xi_{bc}([^3S_1]_{\bar{3}})$	2.66×10^{-4}
$e^+e^- \rightarrow \gamma \rightarrow \Xi_{bc}([^3S_1]_6)$	1.33×10^{-4}
$e^+e^- \rightarrow \gamma \rightarrow \Xi_{bc}([^1S_0]_{\bar{3}})$	1.85×10^{-4}
$e^+e^- \rightarrow \gamma \rightarrow \Xi_{bc}([^1S_0]_6)$	9.27×10^{-5}
$e^+e^- \rightarrow Z^0 \rightarrow \Xi_{bc}([^3S_1]_{\bar{3}})$	1.00
$e^+e^- \rightarrow Z^0 \rightarrow \Xi_{bc}([^3S_1]_6)$	0.502
$e^+e^- \rightarrow Z^0 \rightarrow \Xi_{bc}([^1S_0]_{\bar{3}})$	0.730
$e^+e^- \rightarrow Z^0 \rightarrow \Xi_{bc}([^1S_0]_6)$	0.365
$e^+e^- \rightarrow \gamma \rightarrow \Xi_{bb}([^3S_1]_{\bar{3}})$	1.94×10^{-5}
$e^+e^- \rightarrow \gamma \rightarrow \Xi_{bb}([^1S_0]_6)$	8.95×10^{-6}
$e^+e^- \rightarrow Z^0 \rightarrow \Xi_{bb}([^3S_1]_{\bar{3}})$	8.03×10^{-2}
$e^+e^- \rightarrow Z^0 \rightarrow \Xi_{bb}([^1S_0]_6)$	3.86×10^{-2}

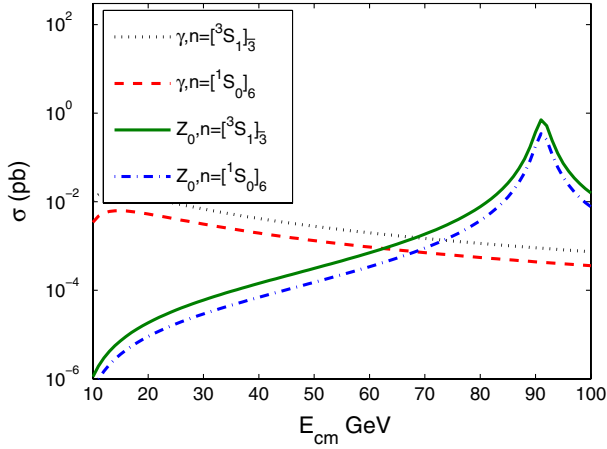


FIG. 2 (color online). Total cross section (in pb) of the production channel $e^+ + e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{cc}(n) + \bar{c} + \bar{c}$, versus the e^+e^- collision energy $E_{\text{cm}} = \sqrt{S}$, where n stands for the corresponding intermediate (cc)-diquark state.

is about 4 orders of magnitude bigger than its value at $E_{\text{cm}} = 50$ GeV and about 3 orders bigger than the total cross section (the summed-up cross sections for both the γ propagators and the Z^0 propagators) in the region of low E_{cm} , ~ 10 – 30 GeV. This shows that a e^+e^- collider running around the Z^0 peak will provide us a better chance for producing more doubly heavy baryons than the previous LEP, BABAR, and Belle platforms.

To serve as a useful reference, we calculate the combined total cross section (the summed-up cross sections for both the γ propagator and the Z^0 propagator) by taking the collision energies $E_{\text{cm}} = (1 \pm 3\%)m_Z$ and $E_{\text{cm}} = (1 \pm 5\%)m_Z$, which are shown by Table II. The table shows that when the collision energy E_{cm} is away from its center value of m_Z , the combined total cross section drops down quickly: a 5% deviation will lead to about 1 order of magnitude lower. Taking the production of $\Xi_{cc}[^3S_1]_3$ as an example, the combined total cross section for $E_{\text{cm}} = (1 \pm 3\%)m_Z$ will be lowered to $\sim \binom{18.0\%}{16.7\%}$ of its peak value; and the combined total cross section for $E_{\text{cm}} = (1 \pm 5\%)m_Z$ will be lowered to $\sim \binom{7.5\%}{6.7\%}$ of its peak value.

Considering a super- Z factory which is running around the mass of the Z^0 boson with a high luminosity $\mathcal{L} \propto 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, the number of $\Xi_{QQ'}(Q, Q' = c, b)$ events per year can be estimated:⁴

- (i) A total of 1.08×10^5 Ξ_{cc} baryon events can be generated, which includes 7.28×10^4 events coming from the intermediate $(cc)[^3S_1]_3$ diquark state and 3.53×10^4 events coming from the intermediate $(cc)[^1S_0]_6$ diquark state.

⁴Approximately, 1 year $\approx \pi \times 10^7$ s, but it is common for a collider to operate only about $1/\pi$ of the time in a year [39]; i.e., it is customary to estimate that $10^{34} \text{ cm}^{-2} \text{ s}^{-1} \approx 10^5 \text{ pb}^{-1}/\text{year}$.

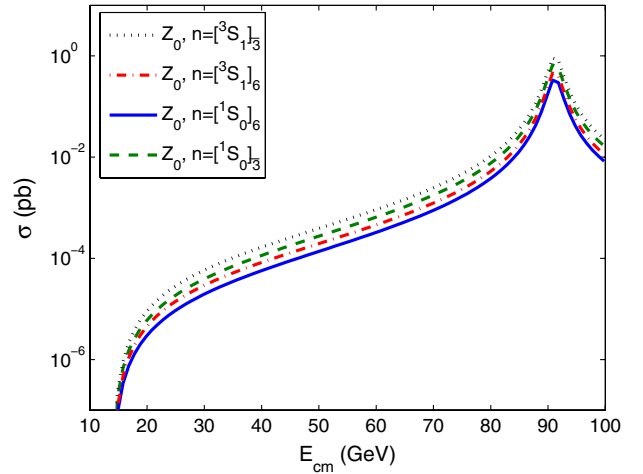
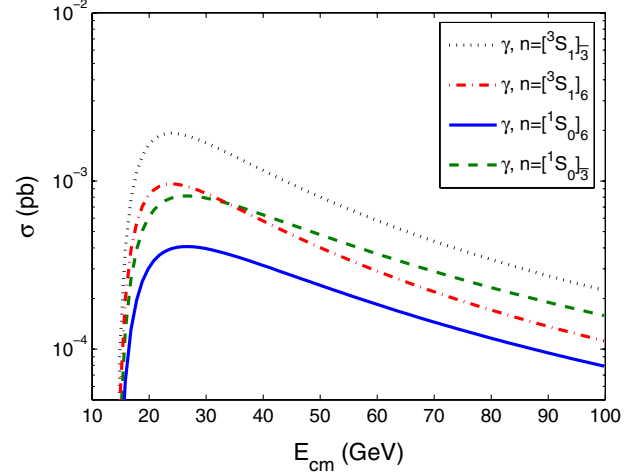


FIG. 3 (color online). Total cross section (in pb) of the production channel $e^+ + e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{bc}(n) + \bar{c} + \bar{b}$, versus the e^+e^- collision energy $E_{\text{cm}} = \sqrt{S}$, where n stands for the corresponding intermediate (bc)-diquark state.

- (ii) A total of 2.60×10^5 Ξ_{bc} baryon events can be generated, which includes 1.00×10^5 events coming from the intermediate $(bc)[^3S_1]_3$ diquark state, 5.02×10^4 events coming from the intermediate $(bc)[^3S_1]_6$ diquark state, 7.30×10^4 events coming from the intermediate $(bc)[^1S_0]_3$ diquark state, and 3.65×10^4 events coming from the intermediate $(bc)[^1S_0]_6$ diquark state.
- (iii) A total of 1.19×10^4 Ξ_{bb} baryon events can be generated, which includes 8.03×10^3 events coming from the intermediate $(bb)[^3S_1]_3$ diquark state and 3.86×10^3 events coming from the intermediate $(bb)[^1S_0]_6$ diquark state.
- (iv) A frequency on the order of 10^4 – 10^5 events per year shows clearly that a sizable number of Ξ_{cc} , Ξ_{bc} , and even Ξ_{bb} events can be produced at the future super- Z factory. If its luminosity can be increased up to $\mathcal{L} \propto 10^{36} \text{ cm}^{-2} \text{ s}^{-1}$, the event numbers will be further increased by 2 orders of magnitude.

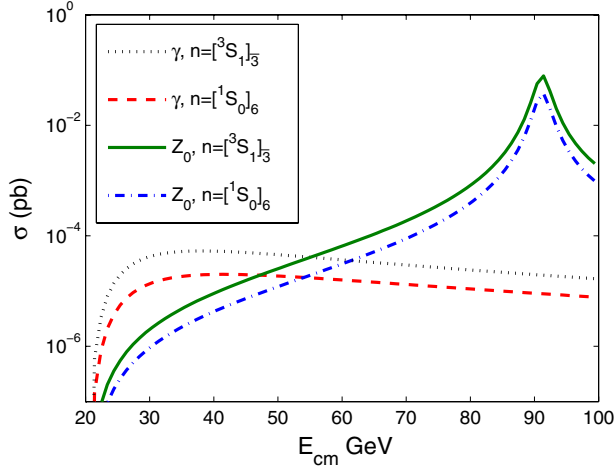


FIG. 4 (color online). Total cross section (in pb) of the production channel $e^+ + e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{bb}(n) + \bar{b} + \bar{c}$, versus the e^+e^- collision energy $E_{\text{cm}} = \sqrt{S}$, where n stands for the corresponding intermediate (bb)-diquark state.

Thus, even if we have a slight deviation of E_{cm} from m_Z , e.g., up to 5% departure, there will still be a sizable number of events.

Finally, we discuss the theoretical uncertainties from the heavy quark masses by varying $m_c = 1.80 \pm 0.30$ GeV and $m_b = 5.10 \pm 0.40$ GeV. The cross sections for $E_{\text{cm}} = m_Z$ with varying m_c and m_b are presented in Tables III and IV, which are more sensitive to m_c than to m_b . Here we only consider the cross sections for the channels through Z^0 propagators, since the cross sections for channels through γ propagators are small and negligible, as shown by Table I. By adding these two uncertainties in quadrature, we obtain

$$\sigma_{e^+ + e^- \rightarrow Z^0 \rightarrow \Xi_{cc}([^3S_1]_3) + \bar{c} \bar{c}} = 0.727_{+0.543}^{-0.275} \text{ pb}, \quad (15)$$

$$\sigma_{e^+ + e^- \rightarrow Z^0 \rightarrow \Xi_{cc}([^1S_0]_6) + \bar{c} \bar{c}} = 0.353_{+0.263}^{-0.133} \text{ pb}, \quad (16)$$

$$\sigma_{e^+ + e^- \rightarrow Z^0 \rightarrow \Xi_{bc}([^3S_1]_3) + \bar{b} \bar{c}} = 1.00_{+0.850}^{-0.402} \text{ pb}, \quad (17)$$

TABLE II. The combined total cross section (in pb) for the $\Xi_{QQ'}$ baryon through the process $e^+e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{QQ'}(n) + \bar{Q} + \bar{Q}'$ (where n stands for the intermediate diquark state) for several typical collision energies (E_{cm}).

E_{cm}	95% m_Z	97% m_Z	m_Z	103% m_Z	105% m_Z
$\sigma_{\Xi_{cc}([^3S_1]_3)}$	0.0487	0.1215	0.7283	0.1307	0.0543
$\sigma_{\Xi_{cc}([^1S_0]_6)}$	0.0236	0.0589	0.3533	0.0634	0.0263
$\sigma_{\Xi_{bc}([^3S_1]_3)}$	0.0658	0.166	1.00	0.180	0.0744
$\sigma_{\Xi_{bc}([^3S_1]_6)}$	0.0329	0.0830	0.502	0.0899	0.0372
$\sigma_{\Xi_{bc}([^1S_0]_3)}$	0.0478	0.121	0.730	0.131	0.0542
$\sigma_{\Xi_{bc}([^1S_0]_6)}$	0.0239	0.0603	0.365	0.0655	0.0271
$\sigma_{\Xi_{bb}([^3S_1]_3)}$	0.0052	0.0132	0.0803	0.0145	0.0060
$\sigma_{\Xi_{bb}([^1S_0]_6)}$	0.0025	0.0063	0.0386	0.0070	0.0029

TABLE III. Uncertainties for the total cross section (in pb) of the production channel $e^+e^- \rightarrow Z^0 \rightarrow \Xi_{QQ'}(n) + \bar{Q} + \bar{Q}'$ (where n stands for the intermediate diquark state) with varying m_c (in GeV). Here, m_b is fixed at 5.10 GeV.

m_c (GeV)	1.50	1.65	1.80	1.95	2.10
$\sigma_{\Xi_{cc}([^3S_1]_3)}$	1.270	0.949	0.727	0.569	0.453
$\sigma_{\Xi_{cc}([^1S_0]_6)}$	0.616	0.461	0.353	0.276	0.220
$\sigma_{\Xi_{bc}([^3S_1]_3)}$	1.85	1.34	1.00	0.767	0.598
$\sigma_{\Xi_{bc}([^3S_1]_6)}$	0.926	0.672	0.502	0.383	0.299
$\sigma_{\Xi_{bc}([^1S_0]_3)}$	1.28	0.954	0.730	0.571	0.455
$\sigma_{\Xi_{bc}([^1S_0]_6)}$	0.639	0.477	0.365	0.285	0.227

TABLE IV. Uncertainties for the total cross section (in pb) of the production channel $e^+e^- \rightarrow Z^0 \rightarrow \Xi_{QQ'}(n) + \bar{Q} + \bar{Q}'$ (where n stands for the intermediate diquark state) with varying m_b (in GeV). Here, m_c is fixed at 1.80 GeV.

m_b (GeV)	4.70	4.90	5.10	5.30	5.50
$\sigma_{\Xi_{bc}([^3S_1]_3)}$	0.987	0.995	1.00	1.01	1.02
$\sigma_{\Xi_{bc}([^3S_1]_6)}$	0.494	0.498	0.502	0.506	0.509
$\sigma_{\Xi_{bc}([^1S_0]_3)}$	0.738	0.734	0.730	0.726	0.723
$\sigma_{\Xi_{bc}([^1S_0]_6)}$	0.369	0.367	0.365	0.363	0.362
$\sigma_{\Xi_{bb}([^3S_1]_3)}$	0.105	0.092	0.080	0.071	0.062
$\sigma_{\Xi_{bb}([^1S_0]_6)}$	0.051	0.044	0.039	0.034	0.030

$$\sigma_{e^+ + e^- \rightarrow Z^0 \rightarrow \Xi_{bc}([^3S_1]_6) + \bar{b} \bar{c}} = 0.502_{+0.424}^{-0.203} \text{ pb}, \quad (18)$$

$$\sigma_{e^+ + e^- \rightarrow Z^0 \rightarrow \Xi_{bc}([^1S_0]_3) + \bar{b} \bar{c}} = 0.730_{+0.55}^{-0.275} \text{ pb}, \quad (19)$$

$$\sigma_{e^+ + e^- \rightarrow Z^0 \rightarrow \Xi_{bc}([^1S_0]_6) + \bar{b} \bar{c}} = 0.365_{+0.274}^{-0.138} \text{ pb}, \quad (20)$$

$$\sigma_{e^+ + e^- \rightarrow Z^0 \rightarrow \Xi_{bb}([^3S_1]_3) + \bar{b} \bar{b}} = 0.080_{+0.025}^{-0.018} \text{ pb}, \quad (21)$$

$$\sigma_{e^+ + e^- \rightarrow Z^0 \rightarrow \Xi_{bb}([^1S_0]_6) + \bar{b} \bar{b}} = 0.039_{+0.012}^{-0.009} \text{ pb}. \quad (22)$$

IV. SUMMARY

Using the NRQCD factorization formula, we have studied the production of the doubly heavy baryon $\Xi_{QQ'}$ (Ξ_{cc} , Ξ_{bc} , Ξ_{bb}) through e^+e^- annihilation. When the e^+e^- collision energy is around the Z^0 peak, sizable cross sections for the doubly heavy baryon can be obtained. Contributions from those $(QQ')[n]$ diquark states with the same importance have been discussed. By adding all these intermediate diquark states' contributions together, we obtain the total cross section for $\Xi_{QQ'}$ at $E_{\text{cm}} = Z^0$:

$$\sigma_{e^+e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{cc} + \bar{c} \bar{c}} = 1.08_{+0.806}^{-0.409} \text{ pb}, \quad (23)$$

$$\sigma_{e^+e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{bc} + \bar{b} \bar{c}} = 2.60_{+2.10}^{-1.02} \text{ pb}, \quad (24)$$

$$\sigma_{e^+e^- \rightarrow \gamma/Z^0 \rightarrow \Xi_{bb} + \bar{b}\bar{b}} = 0.119_{+0.037}^{-0.027} \text{ pb}, \quad (25)$$

where the errors are caused by varying $m_c = 1.80 \pm 0.30$ GeV and $m_b = 5.10 \pm 0.40$ GeV.

At the super-Z factory, with a high luminosity up to $\mathcal{L} \propto 10^{34} - 10^{36} \text{ cm}^{-2} \text{ s}^{-1}$, one would expect to accumulate $1.1 \times 10^{5-7}$ Ξ_{cc} events, $2.6 \times 10^{5-7}$ Ξ_{bc} events, and $1.2 \times 10^{4-6}$ Ξ_{bb} events in one operation year. If we take the e^+e^- collision energy to run slightly off the Z^0 peak, i.e., $\sqrt{s} = 0.95m_Z$ or $1.05m_Z$, the total production cross section will be lowered by about 1 order of magnitude from its peak value. However, one may still observe sizable events. At hadronic colliders, there is much pollution from the hadronic background, and many produced baryon events will be cut off by the triggering condition. So, some alternative measurements would be helpful for a comprehensive study. In addition to the LHC, the super-Z factory would provide another good platform for studying $\Xi_{QQ'}$ baryon properties.

As a final remark, if not too many baryon events are cut off by the triggering condition at the super-Z factory, one

may have the chance to further distinguish the baryons with different light quark contents. When an intermediate diquark is formed, it will grab a light quark (with gluons if necessary) from the “environment” to form a colorless doubly heavy baryon with a relative possibility for various light quarks as $u:d:s \simeq 1:1:0.3$ [16]. For example, if the diquark $(cc)[^3S_1]_3$ is produced, then it will fragment into Ξ_{cc}^{++} with 43% probability, Ξ_{cc}^+ with 43% probability, and Ω_{cc}^+ with 14% probability. The conditions are the same for the production of Ξ_{bb}^0 , Ξ_{bb}^- , and Ω_{bb}^- or the production of Ξ_{bc}^+ , Ξ_{bc}^0 , and Ω_{bc}^0 .

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