Generalized Friedberg-Lee model for *CP* violation in neutrino physics

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We propose a phenomenological model of Dirac neutrino mass operator based on the Friedberg-Lee neutrino mass model to include *CP* violation. By considering the most general set of complex coefficients, and imposing the condition that the mass eigenvalues are real, we find a neutrino mass matrix which is non-Hermitian, symmetric, and magic. In particular, we find that the requirement of obtaining real mass eigenvalues by transferring the residual phases to the mass eigenstates self-consistently dictates the following relationship between the imaginary part of the mass matrix elements B and the parameters of the Friedberg-Lee model: $B = \pm\sqrt{\frac{3}{4}(a-b_r)^2\sin^2 2\theta_{13}\cos^2\theta_{12}}$. We obtain inverted neutrino mass hierarchy $m_3 = 0$. Making a correspondence between our model and the experimental data produces stringent conditions on the parameters as follows: $35.06° \lesssim \theta_{12} \lesssim 36.27°$, $\theta_{23} = 45°$, $7.27° \lesssim \theta_{13} \lesssim 11.09°$, and $82.03° \lesssim \delta \lesssim 85.37°$. We get mildly broken μ-τ symmetry, which reduces the resultant neutrino mixing pattern from tri-bimaximal to trimaximal. The *CP* violation as measured by the Jarlskog parameter is restricted by $0.027 \lesssim J \lesssim 0.044$.

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I. INTRODUCTION

In 1957 B. Pontecorvo [1] suggested that, similar to the K-meson system, neutrino weak eigenstates are not mass eigenstates but are superpositions of its mass eigenstates. Therefore, as neutrinos propagate they would undergo oscillations. The full theory of neutrino oscillation was worked out in several papers [2].

The results of the neutrino oscillation experiments (solar [3], atmospheric [4], reactor [5], and accelerator [6] neutrino experiments) have shown that mixing among three generations in the lepton sector exists, analogous to the quark mixing, and at least two neutrinos are massive. After the discovery of neutrino oscillations, there have been many works for determining the values of the neutrino mass-squared differences and the mixing angles that relate the flavor eigenstates to the mass eigenstates. The lepton mixing matrix in the standard parametrization is given by [7,8]

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\sigma} \end{pmatrix}, \quad (1)$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ (for $ij = 12$, 13, and 23). The phase δ is called the Dirac phase, analogous to the Cabibbo-Kobayashi-Maskawa phase, and the phases ρ and σ are called the Majorana phases that are relevant for Majorana neutrinos. However, we should mention that recently the advantages of the original symmetrical form of the parametrizations of the lepton mixing matrix is discussed in Ref. [8]. The results of the Daya Bay and RENO collaborations have shown that $\theta_{13} = 0$ is now rejected at a significance level higher than 8σ. Analysis of current experimental data [9] yields $31.31° \lesssim \theta_{12} \lesssim 37.46°$, $38.64° < \theta_{23} < 53.13°$, and $7.27° < \theta_{13} < 11.09°$ at the 3σ confidence level. Also a combined analysis of the data coming from T2K, MINOS, Double Chooz, and Daya Bay experiments shows that the best-fit value of θ_{13} is $\sin^2\theta_{13} = 0.026(0.027)^{+0.003}_{-0.004}$ for normal (or inverted) mass hierarchy.

One important aspect of the neutrino mixing phenomena is that it could, in principle, provide new keys to understanding the flavor problem, particularly since it contains large mixing angles in contrast to the quark sector. Also, the disparity between the neutrino and the charged lepton masses is more pronounced than the analogous one in the quark sector. Therefore, the mass and mixing problem in the lepton sector is a fundamental problem. Also some interesting questions to be solved by future experiments are: What are the masses of neutrinos? How close to 45° is the 2–3 mixing angle? What are the values of three *CP*-violating phases of the Pontecorvo-Maki-Nakagawa-Sakata matrix (i.e., the Dirac phase δ and the Majorana phases ρ and σ)? The simplest way to introduce massive neutrinos is to add right-handed chiral neutrino fields to the standard model and to introduce the neutrino masses in the same way as the quarks and charged leptons. In this paper, we mainly focus on the Dirac neutrinos. However, we should mention that the determination of the nature of neutrinos is still a controversial subject which could eventually be decided by experimental observation, such as nonzero magnetic dipole moment of neutrinos ruling out Majorana neutrinos or neutrinoless double beta decay

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ruling out Dirac neutrinos. We believe that at this point in time the study of both types of neutrinos is justified (see, for example, Ref. [10]). A Dirac mass term for the neutrinos and charged leptons is written as

$$\mathcal{L}_{\mathbf{m}} = -\bar{\ell}_{Li}\mathcal{M}_{\mathbf{e}}{}^{ij}\ell_{Lj} - \bar{\nu}_{Li}\mathcal{M}_{\mathbf{D}}{}^{ij}\nu_{Rj} + \text{H.c.} \qquad (2)$$

A successful phenomenological neutrino mass model with flavor symmetry that is suitable for the Dirac neutrinos was proposed by Friedberg and Lee (FL) [11]. In this model the mass eigenstates of three charged leptons are identified with their flavor eigenstates. Therefore, neutrino mixing matrix can be simply described by a 3×3 unitary matrix U which transforms the neutrino mass eigenstates to the flavor eigenstates, $(\nu_e, \nu_\mu, \nu_\tau)$. As we shall show in the pure FL model, one of the neutrino masses is exactly zero, partially justifying the smallness of neutrino masses. Moreover when μ-ν symmetry is assumed, the matrix U reduces to the experimentally favored U_{TBM}.

The Dirac neutrino mass operator of the FL model can be written as

$$\mathcal{M}_{\mathbf{FL}} = a(\bar{\nu}_\tau - \bar{\nu}_\mu)(\nu_\tau - \nu_\mu) + b(\bar{\nu}_\mu - \bar{\nu}_e)(\nu_\mu - \nu_e) + c(\bar{\nu}_e - \bar{\nu}_\tau)(\nu_e - \nu_\tau) + m_0(\bar{\nu}_e\nu_e + \bar{\nu}_\mu\nu_\mu + \bar{\nu}_\tau\nu_\tau). \qquad (3)$$

All the parameters in this model (a, b, c, and m_0) are assumed to be real. For $m_0 = 0$, this Lagrangian has the following symmetry: $\nu_e \to \nu_e + z$, $\nu_\mu \to \nu_\mu + z$, and $\nu_\tau \to \nu_\tau + z$, where z is an element of the Grassmann algebra. For constant z, this symmetry is called FL symmetry [11] in which case the kinetic term is also invariant. However the other terms of the electroweak Lagrangian do not have such symmetry. The m_0-term breaks this symmetry explicitly. However, we may add that the FL symmetry leads to a magic matrix and this property is not spoiled by the m_0-term. The magic property has many manifestations which we shall discus in details. Also it has been reasoned that the FL symmetry is the residual symmetry of the neutrino mass matrix after the $SO(3) \times U(1)$ flavor symmetry breaking [12]. The mass matrix can be displayed by

$$M_{FL} = \begin{pmatrix} b + c + m_0 & -b & -c \\ -b & a + b + m_0 & -a \\ -c & -a & a + c + m_0 \end{pmatrix}, \qquad (4)$$

where $a \propto (Y_{\mu\tau} + Y_{\tau\mu})$, $b \propto (Y_{e\tau} + Y_{\tau e})$, and $c \propto (Y_{\tau e} + Y_{e\tau})$ and $Y_{\alpha\beta}$ denote the Yukawa coupling constant. The proportionality constant is the expectation value of the Higgs field. It is apparent that M_{FL} possesses exact μ-τ symmetry only when $b = c$. Setting $b = c$ and using the Hermiticity of M_{FL}, a straight forward diagonalization procedure yields $U_{\text{TBM}}^T M_{\text{FL}} U_{\text{TBM}} = \text{Diag}\{m_1, m_2, m_3\}$, where

$$m_1 = 3b + m_0, \qquad m_2 = m_0, \qquad m_3 = 2a + b + m_0, \qquad (5)$$

and the experimentally favored tri-bimaximal (TBM) neutrino mixing matrix can be reproduced and is given by

$$U_{\text{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \qquad (6)$$

Obviously the requirement that all of the mass eigenvalues are positive puts conditions on the parameters of this model. In particular $m_{\mathbf{0}}$ must be positive. It is interesting to note that U_{TBM} was first proposed on theoretical grounds by Harrison, Perkins, and Scott in 2002 [13]. For a general, exact TBM neutrino mixing, regardless of the model, the mixing angles are $\theta_{12} \approx 35.3°$, $\theta_{23} = 45°$, $\theta_{13} = 0$ and the CP-violating phases can be chosen to be zero. In general, in order to have CP-violation, the necessary condition is $\delta \neq 0$ and $\theta_{13} \neq 0$. In this model these conditions necessarily mandate that μ-τ symmetry should be broken. Another interesting question is whether $\theta_{23} = 45°$ remains correct after the μ-τ symmetry breaking.

There are four independent CP-even quadratic invariants, which can conveniently be chosen as $U_{11}^* U_{11}$, $U_{13}^* U_{13}$, $U_{21}^* U_{21}$, and $U_{23}^* U_{23}$ and three independent CP-odd quadratic invariants [14],

$$J = \Im(U_{11}U_{12}^* U_{21}^* U_{22}), \qquad I_1 = \Im[(U_{11}^* U_{12})^2], \qquad I_2 = \Im[(U_{11}^* U_{13})^2]. \qquad (7)$$

The Jarlskog rephasing invariant J [15] is relevant for CP violation in lepton number conserving processes like neutrino oscillations. I_1 and I_2 are relevant for CP violation in lepton number violating processes like neutrinoless double-beta decay. Oscillation experiments cannot distinguish between the Dirac and Majorana neutrinos. The detection of neutrinoless double-beta decay would provide direct evidence of lepton number nonconservation and the Majorana nature of neutrinos. Many theoretical and phenomenological works have discussed massive neutrino models that break μ-τ symmetry as a prelude to CP violation [16].

In this paper we generalize the FL model by introducing complex parameters which can ultimately be linked to complex Yukawa coupling constants. We concentrate on the massive FL Dirac model, imposing the obvious constraint that mass eigenvalues be real. Using this model we obtain CP violation, mild μ-τ symmetry breaking [17], and inverted mass hierarchy for neutrinos. Moreover, the measures of these two symmetry breakings turn out to be proportional to each other. This paper is organized as follows. In Sec. II, we introduce our model and show how the constraint of reality of masses along with the minimal breaking of μ-τ symmetry, and the overall self-consistency of the model produces relationships between the free parameters of the model. We find that in our model $0 < \theta_{13} < 24°$, $35.24° < \theta_{12} < 39.20°$, $\theta_{23} = 45°$, and $71.56° < \delta < \frac{\pi}{2}$. Notice that we have ruled out the case $\delta = \frac{\pi}{2}$ as we shall explain. In Sec. III, we show that our model is, in general, consistent with the experimental data and show that implementing all of the constraints coming from the experimental data severely restricts the parameters of our model, and, in fact, almost pinpoints the relevant parameters. Section IV is devoted to a summary.

II. MODEL

In this section, we generalize the FL model by adding complex Yukawa coupling constants in order to obtain *CP* violation. This is accomplished by obtaining nonzero values for $\sin\theta_{13}$ and δ. We first let all of the coefficients in the $M_{\rm FL}$ matrix Eq. (4) except m_0 be complex. However, we demand the eigenvalues of the mass matrix to be real. We find that only one particular choice allows for minimal breaking of μ-τ symmetry, i.e., ($a \in \Re; b,\ c \in \mathbb{C}$, and $b = c^\star$). This requirement leads to a non-Hermitian mass matrix. For simplicity of notation we define the parameters as follows: $\Re(b) = \Re(c) = b_r$ and $\Im(b) = -\Im(c) = B$. The parameters indicating the measure of *CP* violation and μ-τ symmetry breaking turn out to be proportional to B and therefore we expect it to be small.

The neutrino mass matrix M'_ν is given by

$$M'_\nu = \begin{pmatrix} 2b_r + m_0 & -b_r & -b_r \\ -b_r & a + b_r + m_0 & -a \\ -b_r & -a & a + b_r + m_0 \end{pmatrix} + iB\begin{pmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}. \quad (8)$$

Notice that M'_ν and $M_{\rm FL}$ are both magic and symmetric matrices since they both commute with the magic S matrix defined by

$$S = \begin{pmatrix} F & T & T \\ T & F & T \\ T & T & F \end{pmatrix}. \quad (9)$$

Therefore one of the eigenstates is $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$, and we choose it to be ν_2 in order to be consistent with Eq. (6). In the special case $F = T = \frac{1}{3}$, $S = D$ where D is called the democracy operator. It is obvious that the real part of M'_ν has *CP* and μ-τ symmetry, separately. The imaginary part of M'_ν breaks both of these symmetries, while preserving the product of these two operations. Therefore $|M'_\nu|^2 = M'_\nu M'^\dagger_\nu$ is invariant under simultaneous *CP* and μ-τ reflection operations, which is called the mutativity operation [18]. This reduction of the symmetry, causes the symmetry of neutrino mixing matrix to reduce from TBM to trimaximal (TM).

A naive diagonalization of M'_ν yields

$$\check{m}_1 = (a + 2b_r + m_0) + \sqrt{(a - b_r)^2 - 3B^2}, \quad \check{m}_2 = m_0,$$
$$\check{m}_3 = (a + 2b_r + m_0) - \sqrt{(a - b_r)^2 - 3B^2}. \quad (10)$$

However, the usual diagonalization procedure is correct only for Hermitian matrices, where a similarity transformation by a unitary operator, i.e., $M'_{\rm diag} = U^\dagger M'_\nu U$ diagonalizes the matrix. Therefore, the results indicated in Eq. (10) are correct only in the limit $B \longrightarrow 0$, and the results in this limit suffice for our analysis to follow. Comparing our results in this limit with those shown in Eq. (5), we conclude $a < b$.

Since M'_ν is a non-Hermitian matrix, we need two distinct unitary matrices U and V to diagonalize it. These matrices can be easily obtained by diagonalizing $M'_\nu M'^\dagger_\nu$ and $M'^\dagger_\nu M'_\nu$, separately. U and V are the conventional transformation matrices for the left-handed and right-handed neutrinos, respectively. We do not display the explicit form of U and V and only mention that $V = U^*$. The resulting correct diagonal matrix is obtained by $M'_{\rm diag} = U^\dagger M'_\nu V$ and its elements are as follows:

$$m'_1 = \frac{iB(a - b_r) + 3B^2 + (a + 2b_r + m_0)^2 - (a - b_r + iB)\sqrt{3B^2 + (a + 2b_r + m_0)^2}}{a + 2(b_r + iB) + m_0}, \qquad m'_2 = m_0,$$
$$m'_3 = \frac{iB(a - b_r) + 3B^2 + (a + 2b_r + m_0)^2 + (a - b_r + iB)\sqrt{3B^2 + (a + 2b_r + m_0)^2}}{a + 2(b_r + iB) + m_0}. \quad (11)$$

After the diagonalization we find that only m'_1 and m'_3 are complex. We can extract the phases and transfer them to the mass eigenstates in the Dirac case [19]. The most general form of the diagonal mass matrix can be written as

$$M'_{\rm diag} = e^{i\alpha} e^{i\beta\lambda_3} e^{i\gamma\lambda_8} M'^{\rm real}_{\rm diag}. \quad (12)$$

In our model α automatically turns out to be zero. We would dispense with the overall phase even if it was not zero. Using the fact that m'_2 is real, we obtain $\beta = \gamma$. This implies that the $\arg(m'_1) = -\arg(m'_3) = 2\beta$. Using these conditions in Eq. (11) we obtain

$$B = \pm\sqrt{\frac{-(2a + b_r + m_0)(3b_r + m_0)}{3}}. \quad (13)$$

Note that disregarding the overall phase amounts to the following: $\mathrm{Det}(M'_{\rm diag})$ is real and $\mathrm{Det}(U) = 1$ [19].

From the requirement of the reality of B we obtain $-\frac{m_0}{3} \leq b_r \leq -(2a + m_0)$. Notice that the lower bound of b_r is simply a check on the condition $m_1 > 0$ in Eq. (5). The requirement that in the limit $B \to 0$, U and V should both approach U_{TBM} given in Eq. (6), yields $2b_r + a + m_0 > 0$. Combining the condition for reality of B with $a < b$, we obtain $3a + m_0 < 0$. From this and the overall symmetry of the FL model we conclude that $b < 0$. This conclusion is consistent with the results of experiments on solar neutrino oscillation, which indicate that $m_2 > m_1$. Notice that the occurrence of CP violation is possible only in a restricted region in the a-b_r plane where $B \neq 0$. Figure 1 illustrates the region of the parameter space where CP violation occurs.

Substituting the expression for B given by Eq. (13) into the expression we have obtained for U by diagonalizing $M'_\nu M'^{\dagger}_\nu$, we obtain

$$\begin{aligned}
U_{11} &= \sqrt{\frac{1}{(a-b_r)(3a+m_0)}}\left(\frac{-(3b_r+m_0)}{\sqrt{6}} - i\sqrt{\frac{-(2a+b_r+m_0)(3b_r+m_0)}{2}}\right),\\
U_{12} &= U_{22} = U_{32} = \frac{1}{\sqrt{3}},\\
U_{13} &= \sqrt{\frac{-1}{(a-b_r)(-a+4b_r+m_0)}}\left(\frac{(2a+b_r+m_0)}{\sqrt{6}} + i\sqrt{\frac{-(2a+b_r+m_0)(3b_r+m_0)}{2}}\right),\\
U_{21} &= \sqrt{\frac{1}{(a-b_r)(3a+m_0)}}\left(\frac{3(a+b_r)+2m_0}{\sqrt{6}} + i\sqrt{\frac{-(2a+b_r+m_0)(3b_r+m_0)}{2}}\right),\\
U_{23} &= \sqrt{\frac{-1}{(a-b_r)(-a+4b_r+m_0)}}\left(\frac{-(a+5b_r+2m_0)}{\sqrt{6}} - i\sqrt{\frac{-(2a+b_r+m_0)(3b_r+m_0)}{2}}\right),\\
U_{31} &= \sqrt{\frac{1}{(a-b_r)(3a+m_0)}}\left(\frac{-(3a+m_0)}{\sqrt{6}}\right),\\
U_{33} &= \sqrt{\frac{-1}{(a-b_r)(-a+4b_r+m_0)}}\left(\frac{-a+4b_r+m_0}{\sqrt{6}}\right).
\end{aligned} \tag{14}$$

Notice that our generalized M'_ν given in Eq. (8) has retained its magic property, since we have insisted on having real mass eigenvalues. Therefore the mixing matrices U and V are necessarily TM, which requires, for example, $|\nu_2\rangle = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$ and their first and third columns add up to zero [20]. However the TBM structure is broken down to TM since $U_{13} \neq 0$, and therefore the exact μ-τ symmetry is broken. As we shall show this symmetry is only softly broken. We can conclude from the TM nature of the U matrix that $3(a + b_r) + m_0 > 0$.

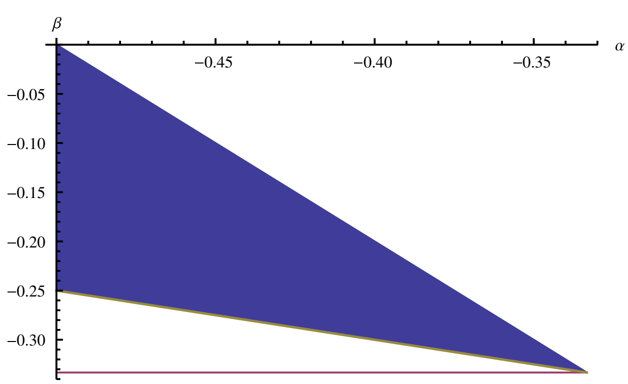

FIG. 1 (color online). CP violation is possible only in the right-angled triangle in our parameter space. The axes are defined by $\alpha \equiv \frac{a}{m_0}$ and $\beta \equiv \frac{b_r}{m_0}$. The dark triangle displays the allowed region within our model. (The line above the base of the triangle is given by $2b_r + a + m_0 = 0$.)

Comparing Eq. (14) with Eq. (1), we immediately obtain all of the mixing angles $(\theta_{13}, \theta_{12}, \theta_{23})$ and the CP-violating phase in terms of a, b_r, and m_0 as follows:

$$\begin{aligned}
\sin^2\theta_{13} &= \frac{2a+b_r+m_0}{3(a-b_r)},\\
\sin^2\theta_{12} &= \frac{1}{3\cos^2\theta_{13}} = \frac{a-b_r}{a-4b_r-m_0},\\
\sin^2\theta_{23} &= \frac{1}{2},\\
\tan\delta &= \sqrt{\frac{-3(3b_r+m_0)}{2a+b_r+m_0}}.
\end{aligned} \tag{15}$$

In other words, the phase difference between b and c results in a kind of μ-τ symmetry breaking with a manifestation $\theta_{13} \neq 0$, while maintaining $\theta_{23} = \frac{\pi}{4}$ ($|U_{23}| = |U_{33}|$). From Eq. (1), (13), and (15) we obtain

$$B = \pm\sqrt{\frac{3}{4}(a-b_r)^2\sin^2 2\theta_{13}\cos^2\theta_{12}}. \tag{16}$$

Using Eq. (14) and the transformation $U^\dagger M'_\nu V$, or by substituting Eq. (13) in to Eq. (11), it is easy to obtain the three neutrino masses

$$M'_{\rm diag} = e^{i\beta\lambda_3}e^{i\beta\lambda_8}\begin{pmatrix} -2(a-b_r) & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (17)$$

where

$$\beta = \arctan\left(-\frac{3B}{3(a+b_r)+2m_0}\right). \quad (18)$$

Notice that we get inverse hierarchy for neutrino masses. In the Dirac case one can choose, without loss of generality, the phases of the mass eigenstates ($\acute{\nu}^i_{L,R}$) so that $M'_{\rm diag}$ reduces to $M'^{\rm real}_{\rm diag}$ as in Eq. (12).

Since M'_ν is a symmetric matrix, it could also be used as a Majorana mass matrix. If we work with Majorana neutrinos, every element in the mixing matrix will be same as the Dirac case. The phases can be rewritten as

$$e^{i\beta\lambda_3}e^{i\beta\lambda_8} = \begin{pmatrix} e^{i2\beta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i2\beta} \end{pmatrix}. \quad (19)$$

In the Majorana case only the phase factor $e^{-2i\beta}$ in Eq. (19) can be rotated away into the charged lepton sector and the other phase factor remains. These phases can be transferred to $U_{\rm PMNS}$, and comparing Eqs. (1) and (19) one can conclude that the $\rho = -\sigma = -\beta$. These phases also contribute to the *CP* violation. Therefore, in Majorana case we obtain three nonzero *CP*-violating phases δ and $\rho = -\sigma$, with inverted hierarchy, and the masses are

$$(m'_1)_M = -2(a-b_r), \quad (m'_2)_M = m_0, \quad (m'_3)_M = 0. \quad (20)$$

Therefore $\det M'_\nu = 0$ and M'_ν has no inverse. This shows that the use of our model for Majorana neutrinos cannot be

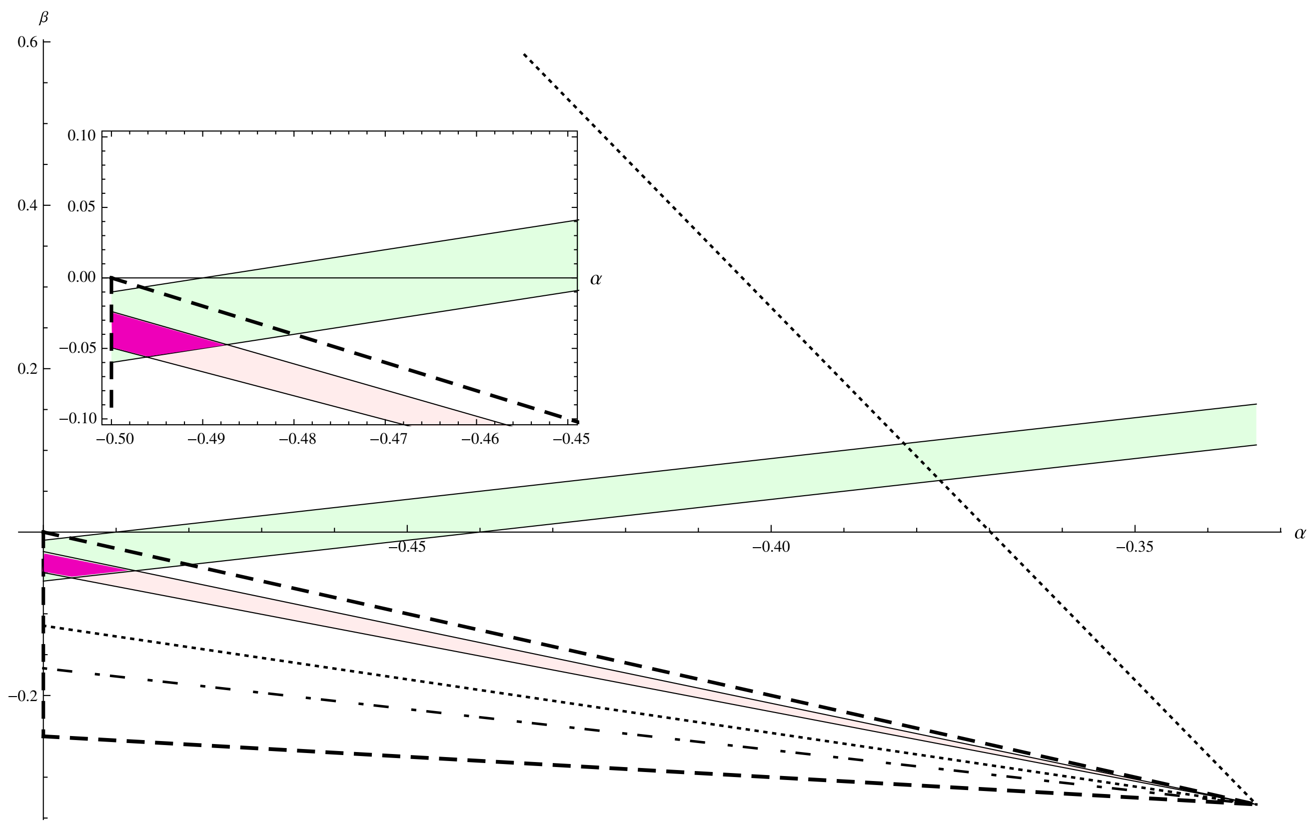

FIG. 2 (color online). In this figure we show the theoretical region of our model for *CP* violation along with all available experimental data. The axes are defined by $\alpha \equiv \frac{a}{m_0}$ and $\beta \equiv \frac{b_r}{m_0}$. The triangle delimited by the dashed lines is identical with the colored triangle shown in Fig. 1. The line immediately above the base of the triangle is the line given by $3(a+b_r)+2m_0$, and in our model only the part of the mentioned triangle above this line is allowed. This line is interesting since on this line the following parameters are all constants: $J = 0.083$, $\theta_{13} = 24.09°$, $\delta = 71.56°$, while the derivative of B in the direction perpendicular to this line is zero. Moving away from this line in the upward direction, B and θ_{13} decrease monotonically to zero, while δ increases to $\frac{\pi}{2}$ when the upper border of the triangle $(2a+b_r+m_0=0)$ is reached. The colored region with negative slope indicates the experimentally allowed region for the $\sin^2\theta_{13}$. The region delimited by dotted lines indicates the experimentally allowed region for the $\sin^2\theta_{12}$. The colored region bounded by two closely spaced parallel lines with positive slope are the result of the restriction coming from the experimental values for Δm^2_{31} and Δm^2_{21}. The overlap region of all of the experimental data is indicated by the darker region and this is completely contained within the region for our model.

consistent with the type-I seesaw mechanism proposed in 1980 [21].

A rephasing-invariant measure of CP violation in neutrino oscillation is the universal parameter J [15] given in Eq. (7), and it has a form which is independent of the choice of the Dirac or Majorana neutrinos. Using Eq. (14) the expression for J simplifies to

$$J = -\frac{B}{6(a - b_r)}. \tag{21}$$

From this expression we can conclude that the maximal CP nonconservation is not just a question of a relative phase assuming the value $\pm\frac{\pi}{2}$, the magnitudes of the coupling constants are also essential. In fact, the maximal CP nonconservation does not correspond to $\sin\delta = 1$ [15]. One can see that the soft breaking of μ-τ symmetry leads to both $\theta_{13} \neq 0$ and $J \neq 0$, but it does not affect the favorable result $\theta_{23} = 45°$, originally resulting from the TBM mixing angles.

III. COMPARISON WITH EXPERIMENTAL DATA

In this section we compare the experimental data with the results obtained from our model. We do this by mapping all of the constraints obtained from the experimental data onto our parameter space, as shown in Fig. 2. Note that there is a significant, general overlap between the experimental data and the CP-violating part of our parameter space [as originally shown in Fig. 1].

However, the most restricting experimental data comes from the values of $\sin^2\theta_{13}$ and $m'_1 = \sqrt{|\Delta m^2_{31}|}$ and $m'_2 = \sqrt{|\Delta m^2_{21} - \Delta m^2_{31}|}$. These restrictions are shown in Fig. 2 by shaded regions. Therefore, the overlap of all of the experimental data and our model is reduced to a tiny region close to the top corner of the triangle. In Fig. 3 we have plotted the values of $|\Delta m^2_{31}|$ against $\sin^2\theta_{13}$ to elucidate this important overlap region. In Table I we state all of the relevant experimental results presented at the 3σ C.L. [9], along with the restrictions that they impose on the parameters of our model. The values stated as the results of our model (combined with experimental data) in the last column of the table are obtained from the tiny region mentioned above which results in the following

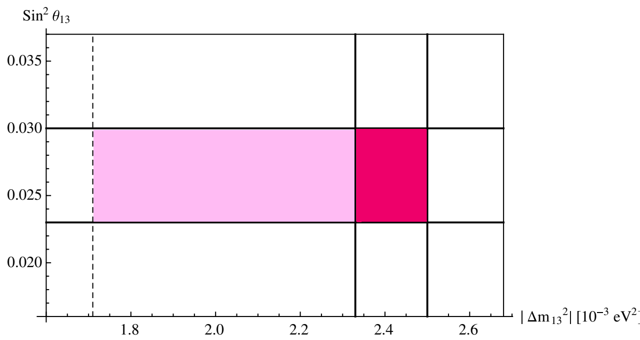

FIG. 3 (color online). In this figure the whole experimentally allowed region of the $\sin^2\theta_{13} - |\Delta m^2_{31}|$ plane is indicated by the region bounded by the largest rectangle, which includes the axes. This whole region corresponds to the two colored regions shown in Fig. 2. The best experimental fit is shown by the darker shaded region in the middle. Our model is represented by the lighter shaded region which overlaps the best experiments fit.

restricted values for the parameters of our model, and using $m'_3 = 0$:

$$m_0 \approx (4.70\text{–}5.25)10^{-2}\ \text{eV}, \qquad a \approx -(2.3\text{–}2.6)10^{-2}\text{eV},$$
$$b_r \approx -(0.1\text{–}0.3)10^{-2}\ \text{eV}, \qquad B \approx (0.34\text{–}0.62)10^{-2}\ \text{eV}. \tag{22}$$

In Fig. 2 we have a special point, $a = b_r = -\frac{m_0}{3}$ at which $B = 0$. Therefore in our model we cannot have CP violation at this point, where the neutrino mass matrix reduces to the following special form:

$$M'_\nu = (3b_r + m_0)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - b_r\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = (3b_r + m_0)\mathbf{1} - b_r\tilde{D}. \tag{23}$$

The first term is identically zero and M'_ν reduces to the democratic matrix $-b_r\tilde{D}$. M'_ν can be diagonalized by the U_{TBM} matrix defined by Eq. (6), yielding

$$M'_{\text{diag}} = U^T_{\text{TBM}}[(3b_r + m_0)\mathbf{1} - b_r\tilde{D}]U_{\text{TBM}} = \begin{pmatrix} 3b_r + m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & 3b_r + m_0 \end{pmatrix}. \tag{24}$$

TABLE I. The allowed ranges for all parameter obtained from the experimental data and our model.

Parameter	The experimental data	The best fit ($\pm 1\sigma$)	Combining our model with the experimental data
Δm^2_{21}	$(7.12\text{–}8.20)10^{-5}$ eV^2	7.62 ± 0.19	$m_1 \approx (4.14\text{–}5.00)10^{-2}$ eV $m_2 \approx (4.70\text{–}5.25)10^{-2}$ eV
Δm^2_{31}	$-(2.15\text{–}2.68)10^{-3}$ eV^2	$-(2.40^{+0.10}_{-0.07})$	$m_1 \approx (4.14\text{–}5.00)10^{-2}$ eV $m_3 = 0$
$\sin^2\theta_{12}$	0.27–0.37	$0.320^{+0.015}_{-0.017}$	0.33–0.35
$\sin^2\theta_{13}$	0.016–0.037	$0.027^{+0.003}_{-0.004}$	0.016–0.037
$\sin^2\theta_{23}$	0.39–0.64	$0.53^{+0.05}_{-0.07}$	0.5
δ	0–2π	0–2π	82.03°–85.37°
J	⋯	⋯	0.027–0.044

Note that in this case the only nonzero element is $(M'_{\rm diag})_{22} = m_0$. This matrix has attracted some attention and several authors have considered generalizations of this matrix to break the degeneracy [22]

IV. CONCLUSION

In this paper we have proposed a generalization of Friedberg-Lee neutrino mass model, in which *CP* violation is possible. In our model the coefficients are allowed to be complex, with the constraint that the mass eigenvalues be real. We find and display the region in our parameter space where *CP* violation is possible. Since the parameters of our model are related to the Yukawa coupling constants, this region determines a corresponding *CP*-violating region in the space of the Yukawa coupling constants. In this region the resulting mass matrix turns out to be non-Hermitian, symmetric, and magic. We find that the symmetry of the neutrino mixing matrix is reduced from TBM to TM with the implication that the μ-τ symmetry is mildly broken. Comparing the results of our model with experimental data, we find that the overlap region is very restricted and this narrows the allowed ranges for the parameters, as shown in Table I. In particular, we find Jarlskog parameter is restricted to $0.027 \lesssim J \lesssim 0.044$, which could be tested in future experiments such as the upcoming long baseline neutrino oscillation ones. Also $35.06° \lesssim \theta_{12} \lesssim 36.27°$, $7.27° \lesssim \theta_{13} \lesssim 11.09°$, $\theta_{23} = 45°$, and $82.03° \lesssim \delta \lesssim 85.37°$. We obtain the allowed ranges for the values of three masses $m_1 \approx (4.14\text{–}5.00)10^{-2}$ eV, $m_2 \approx (4.70\text{–}5.25)10^{-2}$ eV, and $m_3 = 0$, therefore we have inverted hierarchy.

This generalization could also be used for massive Majorana neutrinos because the generalized mass matrix is still symmetric. In Majorana case all of the parameters are identical to the Dirac case except that there are two extra *CP*-violation phases $27.92° \leq \rho = -\sigma \leq 45.56°$. However, since $\det M'_\nu = 0$, the mass matrix M'_ν is not invertible. This shows that the use of our model for Majorana neutrinos cannot be consistent with the type-I seesaw mechanism.

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[1] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1957); [Sov. Phys. JETP **6**, 429 (1957)]; B. Pontecorvo, Zh. Eksp. Teor. Fiz. **34**, 247 (1958).[Sov. Phys. JETP **7**, 172 (1958)].

[2] S. M. Bilenky and B. Pontecorvo, Phys. Lett. **61B**, 248 (1976); Lett. Nuovo Cimento **17**, 569 (1976); Phys. Rep. **41**, 225 (1978); S. Eliezer and A. Swift, Nucl. Phys. **B105**, 45 (1976); H. Fritzsch and P. Minkowski, Phys. Lett. **62B**, 72 (1976); S. Nussinov, Phys. Lett. **63B**, 201 (1976); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); B. Kayser, Phys. Rev. D **24**, 110 (1981); C. Giunti, C. W. Kim, and U. W. Lee, Phys. Rev. D **44**, 3635 (1991); A. Y. Smirnov and G. T. Zatsepin, Mod. Phys. Lett. A **7**, 1272 (1992); M. Zralek, Acta Phys. Pol. B **41**, 2563 (2011).

[3] Q. R. Ahmad *et al.* (SNO Collaboration), Phys. Rev. Lett. **89**, 011301 (2002).

[4] For a review, see C. K. Jung, T. Kajita, T. Mann, and C. McGrew, Annu. Rev. Nucl. Part. Sci. **51**, 451 (2001).

[5] K. Eguchi *et al.* (KamLAND Collaboration), Phys. Rev. Lett. **90**, 021802 (2003).

[6] M. H. Ahn *et al.* (K2K Collaboration), Phys. Rev. Lett. **90**, 041801 (2003).

[7] J. Schechter and J.W.F. Valle, Phys. Rev. D **22**, 2227 (1980); H. Fritzsch and Z. Z. Xing, Phys. Lett. B **517**, 363 (2001); W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).

[8] W. Rodejohann and J.W.F. Valle, Phys. Rev. D **84**, 073011 (2011).

[9] T. Schwetz, M. Tortola, and J. W. F. Valle, New J. Phys. **13**, 109401 (2011); F.P. An *et al.* (Daya Bay Collaboration), Phys. Rev. Lett. **108**, 171803 (2012); J. K. Ahn *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012); arXiv:1205.4018v2.

[10] Y. Farzan and E. Ma, Phys. Rev. D **86**, 033007 (2012).

[11] R. Friedberg and T. D. Lee, Chinese Phys. C **30**, 591 (2007).

[12] C. S. Huang, T. Li, W. Liao, and S. H. Zhu, Phys. Rev. D **78**, 013005 (2008).

[13] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **530**, 167 (2002).

[14] E. Jenkins and A. V. Manohar, Nucl. Phys. **B792**, 187 (2008).

[15] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985).

[16] Z. Z. Xing, H. Zhang, and S. Zhou, Phys. Lett. B **641**, 189 (2006); T. Baba and M. Yasue, Phys. Rev. D **75**, 055001 (2007); Z. Z. Xing, H. Zhang, and S. Zhou, Int. J. Mod. Phys. A **23**, 3384 (2008).

[17] Z. Z. Xing, H. Zhang, and S. Zhou, Phys. Lett. B **641**, 189 (2006).

[18] P. F. Harrison and W. G. Scott., Phys. Lett. B **547**, 219 (2002).

[19] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); M. Fukugita and T. Yanagida, *Physics of Neutrinos and Applications to Astrophysics* (Springer, New York, 2003); S. Nasri, J. Schechter, and S. Moussa, Phys. Rev. D **70**, 053005 (2004).

[20] Sanjeev Kumar, Phys. Rev. D **82**, 013010 (2010).

[21] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).

[22] H. Fritzsch and Z. Z. Xing, Phys. Lett. B **372**, 265 (1996); **440**, 313 (1998); Phys. Rev. D **61**, 073016 (2000).