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Perturbative generation of θ_{13} from tribimaximal neutrino mixing

Biswajoy Brahmachari[1,*] and Amitava Raychaudhuri[2,†]

[1]*Department of Physics, Vidyasagar Evening College, 39 Sankar Ghosh Lane, Kolkata 700006, India*
[2]*Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata 700009, India*

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Solar and atmospheric neutrino oscillations are consistent with a tribimaximal form of the mixing matrix U of the lepton sector. Exact tribimaximal mixing leads to $\theta_{13} = 0$. Recent results from the Daya Bay and RENO experiments have established a nonzero value of θ_{13}. Keeping the leading behavior of U as tribimaximal we perform a model-independent perturbative calculation to incorporate a nonvanishing θ_{13}. We identify the nature of the perturbation matrix and consider the possibility of the solar neutrino splitting also resulting from it. We calculate up to first order in perturbation theory and evaluate the deviations proportional to $\sin\theta_{13}$ while including CP nonconservation. Finally, we briefly discuss a gauge model where such an addition to the neutrino mass matrix arises through one-loop effects.

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Experimental data of solar, atmospheric, accelerator, and reactor neutrinos [1] translate to information about neutrino masses and mixing which can be summarized as [2,3]

$$\begin{aligned} &\Delta m^2_{21} = (7.59 \pm 0.20) \times 10^{-5}\,\text{eV}^2, \quad \theta_{12} = (34.4 \pm 1.0)^\circ, \\ &|\Delta m^2_{31}| = (2.46 \pm 0.12) \times 10^{-3}\,\text{eV}^2, \quad \theta_{23} = (42.8^{+4.7}_{-2.7})^\circ, \\ &\theta_{13} = (5.6^{+3.0}_{-2.7})^\circ, \quad \delta\,\text{unknown}. \end{aligned} \tag{1}$$

These values of the mixing angles are consistent with a mixing matrix of tribimaximal form [4],

$$U^0 = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}, \tag{2}$$

which predicts the third mixing angle θ_{13} to be exactly vanishing.

Of late, the situation has taken a different turn. Results from the Double Chooz [5] Collaboration and more recently the Daya Bay [6] experiment indicate that θ_{13} is, in fact, inconsistent with zero[1] by more than $5\sigma - \sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat.}) \pm 0.005(\text{syst.})$ [6]. Therefore, in view of these significant findings, it has to be concluded that the simple-minded tribimaximal picture fails to adequately capture the observed neutrino mixing. The smallness of θ_{13} compared to the other two mixing angles encourages us to examine here whether the former could arise from a small perturbation on the basic tribimaximal structure and could lead to a realistic neutrino mixing matrix.

We work in a flavor basis in which the charged lepton mass matrix is diagonal.[2] If the left-handed neutrino Majorana masses are m_1, m_2, m_3 then from Eq. (2) the mass matrix M^0, satisfying tribimaximal mixing, when expressed in the flavor basis has the general form,

$$\begin{aligned} M^0 &= U^0 \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} U^{0T} \\ &= \begin{pmatrix} \frac{2m_1+m_2}{3} & \frac{m_2-m_1}{3} & \frac{m_1-m_2}{3} \\ \frac{m_2-m_1}{3} & \frac{m_1+2m_2+3m_3}{6} & -\frac{m_1+2m_2-3m_3}{6} \\ \frac{m_1-m_2}{3} & -\frac{m_1+2m_2-3m_3}{6} & \frac{m_1+2m_2+3m_3}{6} \end{pmatrix} \\ &= \begin{pmatrix} m_0 - \frac{\Delta_{31}}{3} & \frac{(\Delta_{31}-\Delta_{32})}{3} & -\frac{(\Delta_{31}-\Delta_{32})}{3} \\ \frac{(\Delta_{31}-\Delta_{32})}{3} & m_0 + \frac{\Delta_{31}}{6} & \frac{(\Delta_{31}+2\Delta_{32})}{6} \\ -\frac{(\Delta_{31}-\Delta_{32})}{3} & \frac{(\Delta_{31}+2\Delta_{32})}{6} & m_0 + \frac{\Delta_{31}}{6} \end{pmatrix}, \end{aligned} \tag{3}$$

where we have set

$$m_0 = (m_1 + m_2 + m_3)/3, \qquad \Delta_{32} \equiv (m_3 - m_2), \quad \text{and} \quad \Delta_{31} \equiv (m_3 - m_1). \tag{4}$$

Ab initio, the mass eigenvalues m_1, m_2, m_3 can be complex in which case they can be rendered real and positive by a diagonal phase transformation, $D = \text{diag}(e^{i\lambda_1}, e^{i\lambda_2}, 1)$, where the λ_i are Majorana phases, which do not affect neutrino oscillations.

We approximate $\Delta_{32} \simeq \Delta_{31} \equiv \Delta$, which is not unreasonable since $|\Delta_{32}| \gg \Delta_{21} \equiv (m_2 - m_1)$. Δ sets the scale for

*biswa.brahmac@gmail.com
†palitprof@gmail.com

[1]Very recently the RENO Collaboration has measured $\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat.}) \pm 0.019(\text{syst.})$ [7].

[2]This fixes the singlet right-handed charged leptons and the left-handed lepton doublets in flavor space. In this basis, mixing in the lepton sector is determined entirely by the neutrino mass matrix.

atmospheric neutrino oscillations.[3] We start with this limit and write the unperturbed mass matrix in the flavor basis as

$$M^0 \simeq \begin{pmatrix} m_0 - \frac{\Delta}{3} & 0 & 0 \\ 0 & m_0 + \frac{\Delta}{6} & \frac{\Delta}{2} \\ 0 & \frac{\Delta}{2} & m_0 + \frac{\Delta}{6} \end{pmatrix}. \quad (5)$$

At this level, $m_1^{(0)} = m_2^{(0)} = m_0 - \frac{\Delta}{3}$ and $m_3^{(0)} = m_0 + \frac{2\Delta}{3}$ and the solar mass splitting is absent. Our goal is to also generate this splitting through the same perturbation Hamiltonian that is responsible for $\theta_{13} \neq 0$. We take $m_1^{(0)}$, $m_2^{(0)}$, and $m_3^{(0)}$ to be real and positive.

The purpose of this paper is not to explain how M^0 emerges from a fundamental model even though there is no doubt that we consider it as the dominant part of the neutrino mass matrix. There are many models from which one can obtain the tribimaximal form of the mixing matrix [8]. Our discussion below will be independent of the specific mechanism by which M^0 arises.

In terms of the three mixing angles and the complex phase δ the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix is conventionally parametrized as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (6)$$

As noted, the tribimaximal mixing matrix U^0 in Eq. (2) fixes the element $U^0_{e3} = 0$. The role of a nonvanishing U_{e3}, or equivalently θ_{13}, is manifold. It is essential for *CP* nonconservation in neutrino oscillations and may be invoked to explain leptogenesis.[4] Also, $\theta_{13} \neq 0$ will be similar to the quark sector where mixing between all three generations and *CP* violation is a well-verified result, though the mixing angles in the two sectors are vastly different. For *CP* violation, of course, both θ_{13} and the complex phase δ should be nonvanishing. Besides, a reasonably large θ_{13} opens the door for an easier measurement of the neutrino mass ordering, i.e., the sign of Δm^2_{31}.

A large number of attempts have been made to generate $\theta_{13} \neq 0$ in diverse ways starting from an initial tribimaximal form. Some of these are the following. A perturbative analysis in which one of the columns or rows of U^0 is left unchanged has been examined in Ref. [9]. An alternative which involves a sequential "integrating out" of heavy neutrino states has been proposed in Ref. [10]. Another approach has been to parametrize the deviation from the tribimaximal form in a particular way [11]. Deviations from tribimaximal mixing due to charged lepton effects and renormalization group running have been other directions of study [12]. Alternative explorations have been based on the $A(4)$ symmetry in Refs. [13,14], and on other discrete symmetries in Refs. [15,16].

Our strategy here is to use perturbation theory to identify the structure of the Majorana mass matrix $M = M^0 + M'$ where $M' \ll M^0$, so that θ_{13} and the solar mass splitting are obtained. Both M^0 and M' will be symmetric and could, in general, be complex. However, M^0 as obtained in Eq. (5) from the tribimaximal mixing form is real and symmetric, i.e., Hermitian. We will consider the cases of real and complex M' separately.

For our later discussions, the eigenstates of M^0, the unperturbed mass eigenstates, in the *mass* basis are found useful. These are simply

$$\psi_1^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \psi_2^{(0)} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \psi_3^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad (7)$$

of which the first two are degenerate. So, the basis vectors $\psi_1^{(0)}$ and $\psi_2^{(0)}$ are not unique and are chosen with the knowledge that they reproduce the correct solar mixing. The physical basis is fixed by the perturbation. When we discuss lifting of the degeneracy, we consider M' to be such that $\psi_1^{(0)}$ and $\psi_2^{(0)}$ are its nondegenerate eigenstates: $\langle \psi_i^{(0)} | M' | \psi_j^{(0)} \rangle = m_i^{(1)} \delta_{ij} (i, j = 1, 2)$, with $m_1^{(1)} \neq m_2^{(1)}$. We also take $(M')_{33} = 0$ in this mass basis, so what remain are $(M')_{13}$ and $(M')_{23}$ to which we will first turn.

It is helpful to bear in mind that eigenstates in Eq. (7) when expressed in the *flavor* basis are simply the columns of U^0 [Eq. (2)], namely,

$$\psi_1^{(0)} = \begin{pmatrix} \sqrt{\frac{2}{3}} \\ -\sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{6}} \end{pmatrix}, \qquad \psi_2^{(0)} = \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} \\ -\sqrt{\frac{1}{3}} \end{pmatrix},$$

$$\psi_3^{(0)} = \begin{pmatrix} 0 \\ \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \end{pmatrix} \text{in flavor basis.} \quad (8)$$

[3] Δ is positive (negative) for the normal (inverted) ordering of neutrino masses.

[4] The Majorana phases alluded to earlier could produce *CP* violation in $\Delta L = 2$ processes.

The goal we have set for ourselves is to obtain as the perturbed mass eigenstates, when written in the flavor basis, the columns of the matrix in Eq. (6) with $\theta_{13} \neq 0$. To this end, initially, let us take M', which is symmetric, to be real and therefore Hermitian. Needless to say, this may generate a nonzero θ_{13} but will have no CP violation and hence yield[5] $\delta = 0$. For the perturbation expansion we retain terms up to linear in s_{13}. To first order we have

$$\psi_3 = \psi_3^{(0)} + \sum_{j\neq 3} O_{3j}\psi_j^{(0)}. \quad (9)$$

Here,

$$O_{3j} = \frac{\langle \psi_j^{(0)}|M'|\psi_3^{(0)}\rangle}{m_3^{(0)} - m_j^{(0)}} = -O_{j3}, \qquad (j \neq 3). \quad (10)$$

The coefficients O_{3j} are real in this case. In the mass basis, O_{ij} is proportional to M_{ij}.

The eigenstate ψ_3 should correspond to the third column of the mixing matrix U in Eq. (6) with $\delta = 0$. O_{31} and O_{32} are readily determined using Eq. (9) in the flavor basis. Written explicitly we get the matrix equation,

$$\begin{pmatrix} s_{13} \\ s_{23}c_{13} \\ c_{23}c_{13} \end{pmatrix} = \begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} \end{pmatrix} \begin{pmatrix} 1 \\ O_{31} \\ O_{32} \end{pmatrix}. \quad (11)$$

By inverting the above equation one obtains, to order linear in s_{13}, $O_{31} = \sqrt{\frac{2}{3}}s_{13}$ and $O_{32} = \sqrt{\frac{1}{3}}s_{13}$, where maximality of the atmospheric mixing angle ($s_{23} = c_{23} = 1/\sqrt{2}$) has been used. This translates to $M'_{13} = \sqrt{\frac{2}{3}}s_{13}\Delta$ and $M'_{23} = \sqrt{\frac{1}{3}}s_{13}\Delta$ in the mass basis.

To extend this discussion to the case of $\delta \neq 0$, we have to bear in mind that now M' is complex symmetric and *not* Hermitian and the same holds for the total Majorana mass matrix $M = M^0 + M'$. The columns of the mixing matrix U [Eq. (6)] are eigenvectors of $M^\dagger M = M^{0\dagger}M^0 + M^{0\dagger}M' + M'^\dagger M^0$, where we have dropped a term which is $\mathcal{O}(M')^2$. To proceed, we recall that M^0 is Hermitian and therefore the eigenstates of the unperturbed $M^{0\dagger}M^0$ are the same $\psi_i^{(0)}$ considered earlier [Eq. (8)] but now corresponding to eigenvalues $(m_1^{(0)})^2$, $(m_2^{(0)})^2$, and $(m_3^{(0)})^2$. In place of Eq. (10) we have

$$O_{3j} = \frac{\langle \psi_j^{(0)}|(M^{0\dagger}M' + M'^\dagger M^0)|\psi_3^{(0)}\rangle}{(m_3^{(0)})^2 - (m_j^{(0)})^2} = -O_{j3}^*, \qquad (j \neq 3), \quad (12)$$

[5]Note that a negative s_{13} with $\delta = 0$ is equivalent to a positive s_{13} and $\delta = \pi$.

which is to be used in Eq. (9) now. Requiring that ψ_3 be reproduced to first order and using the appropriate variant of Eq. (11), we get in this case $O_{31} = \sqrt{\frac{2}{3}}s_{13}e^{-i\delta}$ and $O_{32} = \sqrt{\frac{1}{3}}s_{13}e^{-i\delta}$.

To relate the above to the elements of the perturbation M' one notes:

$$\langle \psi_j^{(0)}|(M^{0\dagger}M' + M'^\dagger M^0)|\psi_i^{(0)}\rangle = m_j^{(0)}\langle \psi_j^{(0)}|M'|\psi_i^{(0)}\rangle + m_i^{(0)}\langle \psi_j^{(0)}|M'^\dagger|\psi_i^{(0)}\rangle, \quad (13)$$

and thus in the mass basis

$$O_{3j}[(m_3^{(0)})^2 - (m_j^{(0)})^2] = m_j^{(0)}(M')_{j3} + m_3^{(0)}(M')_{j3}^*, \quad (j \neq 3), \quad (14)$$

where the symmetric nature of M' has been used. Writing $M'_{13} = \sqrt{\frac{2}{3}}s'_{13}\Delta e^{i\phi}$ and $M'_{23} = \sqrt{\frac{1}{3}}s'_{13}\Delta e^{i\phi}$ using Eq. (14) one finds to leading order in Δ/m_0

$$\delta = \tan^{-1}\left(\frac{\Delta}{2m_0}\tan\phi\right), \quad \text{and} \quad s_{13} = f(\phi)s'_{13}, \quad (15)$$

where

$$f(\phi) = \frac{[(m_1^{(0)})^2 + (m_3^{(0)})^2 + 2m_1^{(0)}m_3^{(0)}\cos 2\phi]^{1/2}}{(m_1^{(0)} + m_3^{(0)})}. \quad (16)$$

The approximate formulas in (15) indicate that $s_{13} \leq s'_{13}$ with the equality holding only when $\phi = 0$, and though the range of ϕ (which is $\{0, 2\pi\}$) is the same as that of δ, the latter is suppressed compared to the corresponding ϕ. The suppression is higher as the neutrino masses approach the quasidegenerate regime ($\Delta \ll m_0$).

So far we have concentrated on obtaining $\theta_{13} \neq 0$ through a perturbation starting from the tribimaximal form. Now we consider the solar mass splitting. We choose the perturbation such that $(M')_{12} = (M')_{21} = 0$. The first order corrections to the neutrino mass are obtained from $m_i^{(1)} = \langle \psi_i^{(0)}|M'|\psi_i^{(0)}\rangle$. We demand that the following mass corrections arise at this order:

$$m_1^{(1)} = m_3^{(1)} = 0 \quad \text{and} \quad m_2^{(1)} \neq 0. \quad (17)$$

In the mass basis this implies that out of the diagonal elements only $(M')_{22} \neq 0$. Such a correction ensures that a nonzero solar mass splitting $m_2 - m_1 = m_2^{(1)}$ is induced. Solar neutrino observations establish $\Delta m_{21}^2 = (m_2)^2 - (m_1)^2$ is positive.

Putting all this together we have for the full perturbation matrix in the *mass* basis

$$M' = s'_{13}\Delta \begin{pmatrix} 0 & 0 & \sqrt{\frac{2}{3}}e^{i\phi} \\ 0 & x/3 & \sqrt{\frac{1}{3}}e^{i\phi} \\ \sqrt{\frac{2}{3}}e^{i\phi} & \sqrt{\frac{1}{3}}e^{i\phi} & 0 \end{pmatrix} \text{in mass basis,} \quad (18)$$

and

$$x \equiv m_2^{(1)}/s'_{13}\Delta. \quad (19)$$

The dimensionless parameter x is fixed by the solar splitting. In general it can be complex implying that the Majorana mass $m_2^{(1)} \equiv |m_2^{(1)}|\exp(i\chi)$. If we write $m_2 = m_2^{(0)} + m_2^{(1)} \equiv |m_2|\exp(i\lambda)$ and recall $m_1^{(0)} = m_2^{(0)}$ then one has

$$|m_2| = [(m_1^{(0)})^2 + (|m_2^{(1)}|)^2 + 2m_1^{(0)}|m_2^{(1)}|\cos\chi]^{1/2},$$
$$\lambda = \tan^{-1}\left[\frac{|m_2^{(1)}|\sin\chi}{m_1^{(0)} + |m_2^{(1)}|\cos\chi}\right]. \quad (20)$$

λ is a Majorana phase of ν_2 which arises from the perturbation.

There are thus two real parameters introduced here: $|m_2^{(1)}|$ and χ. For any phase angle χ demanding that the solar splitting is correctly obtained determines $|m_2^{(1)}|$ provided $m_1^{(0)}$ is known; i.e., the mass ordering is specified and the mass of the lightest neutrino $\tilde{m}$ is given. Thus χ, s'_{13}, and ϕ suffice to fix the full perturbation matrix M'.

Using M' in Eq. (18) and degenerate perturbation theory [17] we get for the mixing matrix with $\delta \neq 0$:

$$U_{\delta\neq 0} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & s_{13}e^{-i\delta} \\ -\sqrt{\frac{1}{6}} - \sqrt{\frac{1}{3}}s_{13}e^{i\delta} & \sqrt{\frac{1}{3}} - \sqrt{\frac{1}{6}}s_{13}e^{i\delta} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} - \sqrt{\frac{1}{3}}s_{13}e^{i\delta} & -\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{6}}s_{13}e^{i\delta} & \sqrt{\frac{1}{2}} \end{pmatrix}. \quad (21)$$

$U_{\delta\neq 0}$ is consistent with the observed mixing angles and is unitary up to order s_{13}. The nonzero CP-phase δ brings the lepton sector in line with the quarks, where CP violation has been established for long. δ is usually invoked for processes such as leptogenesis. A matrix of exactly the form of $U_{\delta\neq 0}$ has been discussed in Ref. [18] from a different motivation and its consistency with the experimentally required mixing angles noted.

The basis independent measure of CP violation, the leptonic Jarlskog [19] invariant, arising from $U_{\delta\neq 0}$ [Eq. (21)] is

$$J = \mathrm{Im}[U_{e1}U_{\mu 2}U^*_{e2}U^*_{\mu 1}] = -\frac{1}{3\sqrt{2}}s_{13}\sin\delta$$
$$= -\frac{1}{3\sqrt{2}}s'_{13}f(\phi)\frac{(\frac{\Delta}{2m_0})\sin\phi}{\cos^2\phi + (\frac{\Delta}{2m_0})^2\sin^2\phi}, \quad (22)$$

signifying that both s'_{13} and ϕ have to be nonvanishing in order for CP violation to be present in the lepton sector. Moreover, in the quasidegenerate regime the observation of CP violation is less likely.

The above discussion is valid when the solar mass splitting and the mixing angle θ_{13} are unrelated. In the following we do not examine mass matrices of the associated general form [Eq. (18)]. Nonetheless, we make one passing remark. It would not be unreasonable to expect that the different nonzero terms of the perturbation matrix (18) are roughly of similar order. We may then expect $x \sim \mathcal{O}(1)$. Recalling Eq. (19) one has the order of magnitude estimate[6] $s_{13} \sim \mathcal{O}[(\Delta m^2_{21}/\Delta m^2_{31})(m_3^{(0)} + m_1^{(0)})/(m_2^{(0)} + m_1^{(0)})]$. The measured values of Δm^2_{21} and $|\Delta m^2_{31}|$ are known. We illustrate two extreme limits: normal ordering with $m_3^{(0)} \gg m_1^{(0)}, m_2^{(0)}$ implies $s_{13} \sim \mathcal{O}[10^{-2}(m_3^{(0)}/2m_1^{(0)})]$ while for the inverted ordering with $m_3^{(0)} \ll m_1^{(0)}, m_2^{(0)}$ one has $s_{13} \sim \mathcal{O}[10^{-2}]$. This is the general expectation if both θ_{13} and the solar mass splitting arise from the same perturbation of the tribimaximal mass matrix.

We now identify a special limit when the perturbation mass matrix is of a texture which can be realized from a simple model and where s_{13} gets related to Δm^2_{21} resulting in restrictive predictions. To relate to mass models it is more convenient to first rewrite M' in the flavor basis. We find from Eq. (18)

$$M' = s'_{13}\Delta\left[\begin{pmatrix} 0 & \sqrt{\frac{1}{2}}e^{i\phi} & \sqrt{\frac{1}{2}}e^{i\phi} \\ \sqrt{\frac{1}{2}}e^{i\phi} & 0 & 0 \\ \sqrt{\frac{1}{2}}e^{i\phi} & 0 & 0 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} x & x & -x \\ x & x & -x \\ -x & -x & x \end{pmatrix}\right] \text{in flavor basis.} \quad (23)$$

[6]This result is only indicative. The full flexibility of variation of ϕ and χ is not taken into account.

Here the first matrix on the right-hand side is responsible for θ_{13} and the second for Δm^2_{21}.

We see from Eq. (23) that, aside from the diagonal part which is proportional to the identity matrix[7] and can be subsumed in M^0, the perturbation is of the form:

$$M' = \begin{pmatrix} 0 & A & B \\ A & 0 & C \\ B & C & 0 \end{pmatrix} \text{in flavor basis,} \qquad (24)$$

where A, B, and C are complex in general. Such a texture of M' can follow from a Zee-type model [20] as we discuss later. In such models $(M')_{\alpha\beta}$ is proportional to $(m^2_\alpha - m^2_\beta)$, where m_α is the mass of the charged lepton α. As $m_\tau \gg m_\mu, m_e$, unless other couplings are of vastly different order from each other, one must have $B \sim C \gg A$. Such a form of the mass matrix can be reproduced by the choice $\frac{3}{\sqrt{2}} e^{i\phi} + x = \epsilon$, where ϵ is small, when the perturbation matrix Eq. (23) reduces to

$$M' = \frac{s'_{13}\Delta}{3\sqrt{2}} \begin{pmatrix} 0 & \epsilon & 6e^{i\phi} + \epsilon \\ \epsilon & 0 & 3e^{i\phi} - \epsilon \\ 6e^{i\phi} + \epsilon & 3e^{i\phi} - \epsilon & 0 \end{pmatrix}. \qquad (25)$$

The special case $\epsilon = 0$ is quite predictive. From Eq. (19) this requires

$$-\frac{3}{\sqrt{2}} s'_{13} \Delta e^{i\phi} = m_2^{(1)} = |m_2^{(1)}| e^{i\chi}. \qquad (26)$$

Thus $s'_{13} = \sqrt{2}|m_2^{(1)}|/3|\Delta|$ and $\phi = \pi + \chi$ ($\phi = \chi$) for the normal (inverted) ordering.

Due to these relationships, χ and $m_1^{(0)}$ besides determining $|m_2^{(1)}|$ now also fix s_{13} and δ through Eq. (15). As noted, $m_1^{(0)}$ is known when the mass ordering and the lightest neutrino mass $\tilde{m}$ are fixed. So, for any mass ordering the two remaining parameters are χ and the lightest neutrino mass.

We show now that χ and $\tilde{m}$ can be chosen such that one has consistency with both the solar mass splitting and the measured θ_{13}. In our discussion below we use the central values of the atmospheric and solar mass splittings from Eq. (1) and seek an acceptable θ_{13}. We do not attempt an exhaustive listing of the entire consistent ranges of the parameters in this work but rather present some typical solutions for both mass orderings.

We find that with $\epsilon = 0$, in the normal mass ordering case ($m_1^{(0)} = \tilde{m}$) taking $m_1^{(0)} = 10^{-2}$ eV and $115° \leq \chi \leq 137°$, $\sin^2 2\theta_{13}$ varies from 0.057 to 0.130, which includes the 1σ experimentally allowed range, while the Jarlskog CP-violation parameter J remains more or less constant around -0.026. The replacement $\chi \leftrightarrow (2\pi - \chi)$ with $m_1^{(0)}$ fixed keeps $\sin^2 2\theta_{13}$ unchanged and replaces J by $-J$.

For the inverted mass ordering[8] ($m_1^{(0)} = \tilde{m} - \Delta$), on the other hand, taking $0 \leq m_3^{(0)} \leq 10^{-3}$ eV and $\chi \sim 94°$ one obtains $\sin^2 2\theta_{13} \sim 0.054$, which is allowed at 2σ, with $J \sim -0.028$. For $m_3^{(0)} = 3 \times 10^{-2}$ eV and $\chi \sim 101°$ one has $\sin^2 2\theta_{13} \sim 0.09$ within 1σ and $J \sim -0.029$. For any $m_3^{(0)}$, replacing χ by $(2\pi - \chi)$ results in the same $\sin^2 2\theta_{13}$ but J changes sign.

We now briefly note how M' of the texture in Eq. (24) can follow from a Zee-type model.[9] It bears repetition that here the Zee model provides a subleading contribution M' to a leading tribimaximal mass matrix M^0 of a different origin.[10] The Zee model has a simple $SU(2)_L \times U(1)_Y$ invariant structure. For this, a second scalar $SU(2)_L$ doublet and a charged singlet scalar χ^+ are introduced. The latter couples to a pair of lepton doublets, where the coupling $f_{\alpha\beta}$ is antisymmetric in the generation index. Likewise due to $SU(2)$ antisymmetry the charged scalar also couples to a pair of Higgs doublets h_u and h_d antisymmetrically. In this model a contribution to the neutrino mass M' arises radiatively from one-loop diagrams such as Fig. 1 and can be expressed as

$$M'_{\alpha\beta} = \frac{1}{M_s^2} \mu (m^2_\alpha - m^2_\beta) f_{\alpha\beta} \frac{v_u}{v_d} I. \qquad (27)$$

Here $f_{\alpha\beta}$ is the antisymmetric coupling in $f_{\alpha\beta} L_\alpha L_\beta \chi$, where L is the left-handed lepton doublet. Also, μ is the trilinear scalar coupling in $\mu h_u h_d \chi$, M_s a typical scalar mass, and I a dimensionless factor arising from the loop integral. v_u, v_d are the vacuum expectation values of the two Higgs doublets h_u and h_d. The vertex f violates lepton

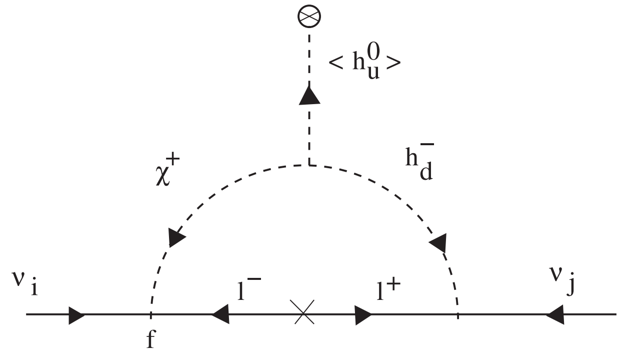

FIG. 1. One-loop contributions to the neutrino Majorana mass matrix M' in the Zee model. For $M'_{e\mu}$ the dominant contribution is proportional to m^2_μ while for $M'_{e\tau}$ and $M'_{\mu\tau}$ it is proportional to m^2_τ.

[7] Such a piece proportional to the identity does not affect the mixing and makes a constant contribution to all three neutrino mass eigenvalues.

[8] In this case, $\tilde{m} = m_3^{(0)}$ and $\Delta < 0$.

[9] An alternative way to generate a nonzero θ_{13} using the Zee model has been examined in Ref. [21].

[10] Models can be constructed which accommodate both M^0 and M'. An $A(4)$ based example can be found in Ref. [14].

number by two units. This diagram gives rise to a mass matrix M' which is off-diagonal and symmetric, as required. $(M')_{12}$ can be neglected compared to $(M')_{13}$ and $(M')_{23}$ because the latter will receive contribution from diagrams with a τ lepton.[11] Thus, the correction obtained in this fashion is naturally of the desired form with $\epsilon \sim 0$. Further, the coupling $f_{\alpha\beta}$ can be complex which can lead to an M' of the form in Eq. (24), which is complex symmetric. The interference of M' with the matrix M^0 [Eq. (12)] leads to CP violation in the neutrino sector. It is worth bearing in mind that M' is suppressed compared to the leading term in M^0 by $\mathcal{O}(s_{13}\Delta/m)$. Taking $s_{13} \sim 0.1$, $\Delta \sim 0.1$ eV, and $\mu \sim 100$ GeV, unless other factors in Eq. (27) are tuned to suppress the contribution, one requires $M_s \sim \mathcal{O}(10^6 \text{ GeV})$, which puts the additional scalars of the model beyond the reach of the current experiments.

In conclusion, we have shown that θ_{13} consistent with experiments, a CP-phase δ, and the solar mass splitting can all be the outcome of a specific perturbation to a basic neutrino mass matrix, the latter associated with tribimaximal mixing. This leads to a nonzero Jarlskog invariant and opens the door for CP violation in the lepton sector. In particular, a constrained version of this perturbation relates the neutrino Majorana phase to the solar mass splitting as well as θ_{13} and δ. Some sample solutions which meet all requirements have been presented. We have provided an example where the requisite perturbation contributions to the neutrino Majorana mass matrix can arise from a Zee-type model through radiative corrections.

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[11]This indicates that the general form of the perturbation matrix [Eq. (25)] with arbitrary ϵ will require unnatural choices for the couplings $f_{\alpha\beta}$.

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