

**Bilarge neutrino mixing and the Cabibbo angle**S. M. Boucenna,<sup>1,\*</sup> S. Morisi,<sup>1,†</sup> M. Tórtola,<sup>1,‡</sup> and J. W. F. Valle<sup>1,§</sup><sup>1</sup>*AHEP Group, Instituto de Física Corpuscular-C.S.I.C./Universitat de València Edificio de Institutos de Paterna, Apartado 22085, E-46071 Valencia, Spain*

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Recent measurements of the neutrino mixing angles cast doubt on the validity of the so-far popular tribimaximal mixing *Ansatz*. We propose a parametrization for the neutrino mixing matrix where the reactor angle seeds the large solar and atmospheric mixing angles, equal to each other in first approximation. We suggest such a bilarge mixing pattern as a model-building standard, realized when the leading order value of  $\theta_{13}$  equals the Cabibbo angle  $\lambda_C$ .

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**I. INTRODUCTION**

Understanding the structure of neutrino mixing from first principles is part of the flavor problem of the Standard Model, one of the deepest in particle physics. A partially useful strategy is to formulate attractive mixing patterns that may help to seek out possible underlying flavor symmetries. The tribimaximal mixing (TBM) *Ansatz* [1] for describing the neutrino mixing matrix [2] assumes a maximal atmospheric angle and zero reactor angle, as was suggested by the experimental data. Such striking features point toward an underlying symmetry and, indeed, many models based on (mostly discrete) non-Abelian flavor symmetries were successful in reproducing this *Ansatz* [3,4]. Small deviations from the TBM are expected on theoretical grounds.

However, the recent results published by the Double-Chooz [5], Daya Bay [6], RENO [7], T2K [8] and MINOS [9] Collaborations indicate that the reactor angle is relatively large so that corrections to the TBM pattern should be, in fact, quite large, casting doubt on its validity as a good first approximation reproducing the neutrino mixing pattern. To be more precise, on theoretical grounds a small deviation of order of the square of the Cabibbo angle was expected for the reactor angle, while recent observations indicate a much larger value of about the order of the Cabibbo angle. To evade this problem, different *Ansätze* have been considered like the bimaximal mixing [10] or the Golden ratio, see Ref. [11] for a review. However, most of these models assume a  $\mu$ - $\tau$ -invariant structure in order to predict a maximal atmospheric mixing angle. On the other hand, at the Neutrino 2012 Conference the MINOS Collaboration also gave hints for a nonmaximal atmospheric mixing angle.

Here, we propose a different approach where we take the reactor-mixing angle as the fundamental parameter. As will be shown below, the resulting parametrization does not reproduce the TBM pattern as a limiting case, though a maximal atmospheric angle can be obtained. The main

idea is that, since the reactor angle is the only small mixing parameter for the leptons, we can use it to seed both the solar and atmospheric mixing angles, as follows,

$$\sin\theta_{13} = \lambda; \quad \sin\theta_{12} = s\lambda; \quad \sin\theta_{23} = a\lambda, \quad (1)$$

where the small parameter  $\lambda$  is the reactor angle, while  $s \simeq a$  are free parameters of order few. Solar and atmospheric mixings are expressed in terms of a linear dependence on the reactor angle. In the limit where  $\lambda \rightarrow 0$ , neutrinos are unmixed.

Using the general symmetric parametrization of the neutrino mixing matrix [2], one can trivially obtain a simple approximate description by expanding only in the small parameter  $\lambda$ . For example, the Jarlskog-like invariant describing  $CP$  violation in neutrino oscillations is then given by:

$$J_{CP} \approx as\lambda^3 \sqrt{1 - a^2\lambda^2} \sqrt{1 - s^2\lambda^2} \sin(\phi_{13} - \phi_{12} - \phi_{23}), \quad (2)$$

given explicitly in terms of the rephase-invariant Dirac combination. Likewise, the effective mass parameter describing the amplitude for neutrinoless double-beta decay is given in terms of the two Majorana  $CP$  phases.

In what follows, for simplicity, we take all parameters to be real. In order to fix the values of the large parameters  $s$  and  $a$ , we consider the latest experimental results on neutrino oscillation parameters. At the Neutrino 2012 Conference in Kyoto, the MINOS Collaboration reported a nonmaximal value for the atmospheric mixing angle [9]<sup>1</sup>:

$$\sin^2 2\theta_{23} = 0.94_{-0.05}^{+0.04}, \quad (3)$$

with maximal mixing disfavored at 88% C.L. This result comes from the analysis of  $\nu_\mu$  disappearance in the MINOS accelerator beam and corresponds to two degenerate points for  $\sin^2\theta_{23}$ , namely

<sup>1</sup>In a three-neutrino analysis this quantity will be equal to  $4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) = 4\cos^2\theta_{13}\sin^2\theta_{23}(1 - \cos^2\theta_{13}\sin^2\theta_{23})$ .

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$$\sin^2\theta_{23} = 0.38 \quad \text{and} \quad \sin^2\theta_{23} = 0.62. \quad (4)$$

This comes from the fact that the disappearance channel is octant-symmetric and therefore MINOS data by themselves can not show a preference for a given octant of  $\theta_{23}$ . However, one may expect that in combination with the searches for electron-neutrino appearance at the long-baseline experiments, MINOS and T2K, and with the recent measurements of  $\theta_{13}$  at reactor experiments, the degeneracy in Eq. (3) will be broken and one octant will be preferred over the other. Global analysis so far has not been able to give a completely clear picture about the true octant of  $\theta_{23}$ . The analysis given in Ref. [12] indicates small deviations of maximality, with the octant preference correlated with the mass hierarchy. This correlation is also seen in the recent analysis of atmospheric neutrino data in Super-Kamiokande [13]. However, the analyses given in Refs. [14,15] have shown a preference for  $\theta_{23} < \pi/4$  with different levels of significance. All previous neutrino oscillation global fits agree in getting  $\theta_{23}$  in the first octant for normal mass hierarchy, so for the purpose of this article we will assume that this is the case.

## II. BILARGE MIXING

Here, we discuss the recent neutrino oscillation results in terms of our proposed *Ansatz*. As already pointed out above, the recent experimental data provide a robust measurement of a relatively large  $\theta_{13}$ .

Using the best fit values of the mixing angles in Ref. [12] or Ref. [14], we can fix the three parameters in Eq. (1):  $\lambda \sim 0.16(0.15)$ ,  $a \sim 4.13(4.21)$  and  $s \sim 3.53(3.65)$ . Then, from the data we can directly read that

$$\sin\theta_{12} = \mathcal{O}(\sin\theta_{23}). \quad (5)$$

Now, we go a step further and assume the following

$$\sin\theta_{12} = \sin\theta_{23}, \quad (6)$$

which in our parametrization means

$$s = a. \quad (7)$$

Since both solar and atmospheric angles are large, we call this case the bilarge mixing *Ansatz*.

Suppose now that we are given a model predicting bilarge mixing  $a = s$  at leading order. Next-to-leading order operators in the Lagrangian, in general, induce deviations from the reference values in Eq. (1), which may be reliably determined within a given model. Here, we present a simple model-independent estimate of such corrections, obtained as follows. Typically, it is expected that the corrections to the three mixing angles from next-to-leading order terms are of the same order, that is  $\sin\theta_{ij} \rightarrow \sin\theta_{ij} \pm \epsilon$  where we have introduced a new

TABLE I. Best fit values and  $1\sigma$  ranges for the parameters  $\lambda$ ,  $s$  and  $\epsilon$  in Eq. (8) according to the global fits to neutrino oscillations.

Ref.	$\lambda$	$s$	$\epsilon$
Forero <i>et al.</i> [12]	$0.23 \pm 0.04$	$2.8^{+0.5}_{-0.4}$	$0.067^{+0.035}_{-0.025}$
Fogli <i>et al.</i> [14]	$0.19^{+0.03}_{-0.02}$	$3.0^{+0.5}_{-0.3}$	$0.038^{+0.019}_{-0.018}$

parameter  $\epsilon$  to characterize the magnitude of the correction. In this case our bilarge mixing gets corrections of the same order for the three mixing angles (given by  $\epsilon$ ), which may either increase or decrease the starting bilarge values of the mixing angles. For definiteness let us consider an example where bilarge mixing is corrected as

$$\begin{aligned} \sin\theta_{13} &= \lambda - \epsilon; & \sin\theta_{12} &= s\lambda - \epsilon; \\ \sin\theta_{23} &= a\lambda + \epsilon, \end{aligned} \quad (8)$$

where we take  $s = a$  as in Eq. (7). Since we have three free parameters, we can fix them using the best fit values reported by global analysis of neutrino oscillation data in Refs. [12,14]. The results are given in Table I.

In order to quantitatively clarify the role of the relation in Eq. (7) with respect to the reactor mixing angle, we consider here the most generic case given by Eq. (8), where the three angles are given in terms of four parameters instead of three. Three of these parameters can be fixed from the three measured mixing angles, leaving one free parameter that we choose to be  $\lambda$ . In order to quantify the deviation from our exact bilarge mixing *Ansatz* defined in Eq. (7), we plot the combination  $(a - s)/(a + s)$  as a function of the expansion parameter  $\lambda$  in Fig. 1. The colored or shaded bands are calculated from the two- and three-sigma allowed ranges for the neutrino oscillation parameters obtained in the current global fits. The solid and dashed lines indicate the best fits of Refs. [12,14], respectively. It is remarkable that the strict bilarge *Ansatz*

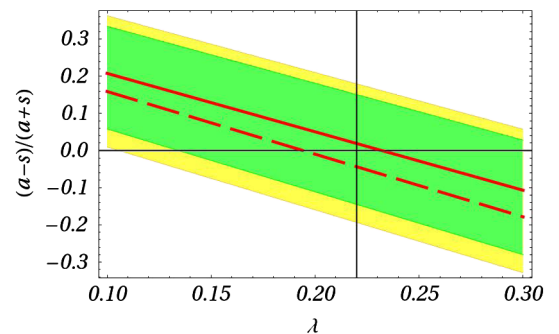


FIG. 1 (color online). Deviation from the bilarge *Ansatz* versus the expansion parameter  $\lambda$  at two and three sigma in the neutrino oscillation parameters. The solid and dashed lines indicate the best fits of Refs. [12,14], respectively. The strict bilarge *Ansatz* holds when  $\lambda \approx \lambda_C$  (vertical line).

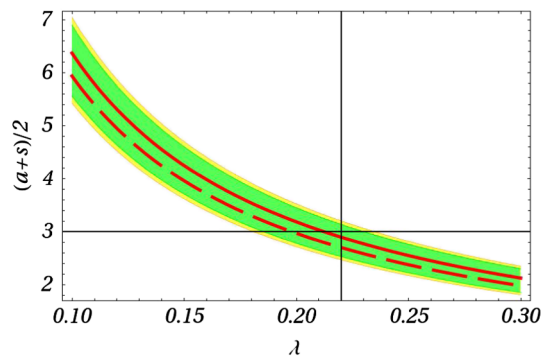


FIG. 2 (color online). Average of solar and atmospheric angles versus the expansion parameter  $\lambda$  at two and three sigma in the neutrino oscillation parameters. The solid and dashed lines indicate the best fits of Refs. [12,14], respectively.

in Eq. (7) holds when  $\lambda \approx \lambda_C$  where  $\lambda_C \approx 0.22$ . This means that  $\lambda_C$  is the leading order value of  $\sin\theta_{13}$ .

In Fig. 2 we show the average value of  $a$  and  $s$ , that is  $(a+s)/2$ , as a function of  $\lambda$ . As displayed in the figure, the correlation is such that  $(a+s)/2 \sim 3$  when  $\lambda \sim \lambda_C$ . It follows that one possible form of our bilarge *Ansatz*, which can be useful for model building, is

$$\sin\theta_{13} = \lambda; \quad \sin\theta_{12} = 3\lambda; \quad \sin\theta_{23} = 3\lambda. \quad (9)$$

It is remarkable to see how such a simple form is nearly consistent with current global neutrino oscillation data.

Now, a few words on model building. Given a particular mixing matrix  $U$ , the structure of the neutrino mass matrix is fixed as

$$m_\nu = U \cdot D \cdot U^T,$$

where  $D$  is a diagonal matrix. For the sake of illustration, we consider in our mixing *Ansatz* in Eq. (9) a normal and strongly hierarchical spectrum ( $m_{\nu_1} = 0$ ) for neutrino masses and fix the square mass differences at their best fit values, as given in Ref. [12]. We find that the resulting weak-basis neutrino mass matrix  $m_\nu$  has the form

$$m_\nu \sim \begin{pmatrix} 0.20 & 0.32 & 0.15 \\ & 0.75 & 0.70 \\ & & 1 \end{pmatrix} \sim \begin{pmatrix} \lambda_C & \lambda_C & \lambda_C \\ & 1 & 1 \\ & & 1 \end{pmatrix}, \quad (10)$$

where in the last step the Cabibbo angle is used as the expansion parameter and we do not specify numerical coefficients of order one.

From the form obtained in Eq. (10), one sees that the parameter  $\lambda_C$  appears only in the first row. On the other hand, in the “atmospheric sector,” there seems to be “democracy” in the choice of the neutrino mass entries.

Altogether, the above indicates the following two general features regarding the neutrino mass generation mechanism:

- (i) a Froggatt-Nielsenlike flavor symmetry [16] that could perhaps generate the required pattern given in Eq. (10).
- (ii) some gravitylike [17] “flavor-blind” mechanism operating within the “atmospheric sector.” The problem in this case is that, for reasonable values of the coefficients of the dimension-five operator, the induced neutrino masses are too small. However, there may be other well-motivated “anarchy”-type schemes [18,19].

It is beyond the scope of this paper to provide a definite model realization, but rather we stress the simplicity of the *Ansatz* in Eq. (10), which may provide a fresh model-building guideline that may perhaps replace the tribimaximal *Ansatz*.

### III. SUMMARY AND OUTLOOK

We have argued that recent experimental results on neutrino oscillations are very well described by the *Ansatz* in Eq. (1) with small deviations, as in Eq. (8). It is truly remarkable that with the expansion parameter  $\lambda$  taken as  $\lambda \approx \lambda_C$ , the Cabibbo angle characterizing the quark mixing matrix, we obtain the simplest bilarge limit,  $s = a$ , given in Eq. (7). This appears to be an important numerical “coincidence,” which may drastically change our theoretical approach for constructing neutrino mass models by moving from a geometrical interpretation of the neutrino mixing angles to one in which these are no longer associated to Clebsh-Gordan coefficients of any symmetry, in sharp contrast to the previous paradigm.<sup>2</sup> The simple correlations between *Ansatz* deviations and  $\lambda$  are illustrated in Figs. 1 and 2. The bilarge *Ansatz* holds for  $\lambda \approx \lambda_C$  (vertical line). It remains to be seen whether nature is perhaps telling us something profound regarding the ultimate theory of flavor.

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<sup>2</sup>We should stress, however, that the TBM pattern may still be tenable if the underlying theory is capable of providing sufficiently large corrections to  $\theta_{13}$  without affecting too much the solar angle, which is in principle possible.

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