Cutoff estimate in Lifshitz five-dimensional field theories

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We analyze if and to what extent the high energy behavior of five-dimensional (5D) gauge theories can be improved by adding certain higher dimensional operators of "Lifshitz" type, without breaking the ordinary four-dimensional Lorentz symmetries. We show that the UV behavior of the transverse gauge field polarizations can be improved by the Lifshitz operators, while the longitudinal polarizations get strongly coupled at energies lower than the ones in ordinary 5D theories, spoiling the usefulness of the construction in non-Abelian gauge theories. We conclude that the improved behavior as effective theories of the ordinary 5D models is not only related to locality and 5D gauge symmetries, but is a special property of the standard theories defined by the lowest dimensional operators.

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I. INTRODUCTION

Field theories in more than four space-time dimensions have received a lot of attention in recent years. They have given us a new perspective on various aspects of high energy physics and cosmology. For instance, a fundamental TeV-sized quantum gravity scale might arise from extra dimensions [1] or a TeV scale can naturally emerge from a redshift effect in a warped extra dimension [2]. Combined with the AdS/CFT idea [3], five-dimensional (5D) theories also give us a new handle to approach strongly coupled quantum field theories (QFT) [4].

Field theories in extra dimensions are nonrenormalizable and, as such, they should be seen as effective theories valid up to a maximum energy Λ, above which they break down. Estimates based on (not too much) naïve dimensional analysis (NDA) and unitarity bounds both give, for a simple 5D gauge theory on a flat segment of length $L = \pi R$,

$$\Lambda \sim \frac{16\pi}{g^2 R}, \tag{1.1}$$

where g is the four-dimensional (4D) gauge coupling. Based purely on four-dimensional considerations, a 5D gauge theory can be seen as an infinite number of gauge symmetries, nonlinearly realized by the pseudo Nambu-Goldstone bosons (pNGB's) coming from the gauge field components $A_y^{(n)}$ along the extra dimension y, with $m_n = n/R$. According to this picture, one would naively expect

$$\Lambda_{\rm Naive} \sim 4\pi f, \tag{1.2}$$

where $f = m_1/g$ is the decay constant of the lightest pNGB. We see that $\Lambda = 4/g \Lambda_{\rm Naive}$, and for a sufficiently weak coupling g, the 5D theory remains weakly coupled up to energies parametrically higher than those expected from a generic 4D effective theory [5]. This is also seen in the 4D deconstructed versions of 5D theories [6,7], where the delay in the unitarity breakdown in scattering amplitudes with respect to the naive estimate arises from nontrivial cancellations among different contributions [8].

Aim of this work is to show to what extent the improved high energy behavior of 5D theories holds and whether it is even possible to improve the situation by modifying the theory by adding higher dimensional operators. We technically address these questions by analyzing a specific class of nonstandard 5D theories with anisotropic scaling symmetry (also called, with some abuse of language, Lifshitz field theories, see [9] for a review and references). The reason to consider these theories is twofold. First, a symmetry principle allows to restrict the class of higher dimensional operators to consider in studying generalizations of ordinary 5D theories. Second, Lifshitz field theories are known to possibly have an improved UV behavior with respect to ordinary theories, exploiting the improved UV behavior of the particle propagators. In fact, simple UV completions of 5D theories based on Lifshitz field theories have been shown to be possible [10]. The price to be paid is however high. In these theories the 4D Lorentz invariance is broken at high energies and is generally recovered in the IR only at the price of extreme fine-tunings [11,12]. For these reasons, we consider here 4D Lorentz-invariant theories, where the anisotropy involves the extra dimension only.[1] We focus on pure non-Abelian gauge theories compactified on a plain $S^1/\mathbf{Z}_2$ orbifold, the addition of matter field being straightforward.

We separately study the UV behavior of the transverse and longitudinal gauge field polarizations. In the former case we estimate the cutoff by a one-loop computation of the gauge coupling corrections induced by the Kaluza-Klein (KK) modes to the zero mode gauge fields. After showing in some detail the form of this correction in the ordinary 5D case, leading to the cutoff estimate (3.8), we

[1]Five-dimensional Lifshitz theories where the anisotropy of the scaling symmetry shows up only in the extra dimension has been considered in [13,14] for a $\lambda\phi^4$ theory and a model of Gauge-Higgs unification, respectively.

show how the Lifshitz operators lead to a parametrically higher cutoff, Eq. (3.20). The UV behavior of longitudinal gauge bosons, on the other hand, is analyzed by looking at their elastic scattering amplitudes, $\mathcal{A}(W_L^{(n)}W_L^{(n)} \to W_L^{(n)}W_L^{(n)})$, where n is the KK mode of the longitudinal gauge field in the scattering process. Building on previous results [15], we see that, contrary to the ordinary 5D theories, the $\mathcal{O}(E^2)$ term in $\mathcal{A}(W_L^{(n)}W_L^{(n)} \to W_L^{(n)}W_L^{(n)})$ (E being the center of mass energy), no longer cancels. This leads to the breakdown of unitarity at energies lower that those obtained in ordinary theories, with an associated cutoff given by Eq. (4.6), spoiling the usefulness of the Lifshitz construction for non-Abelian gauge theories. In other words, we get that Λ can be parametrically higher than the estimate (1.1) in Abelian gauge theories, while in non-Abelian theories the addition of the Lifshitz operators at a sufficiently low scale would result in a decrease of Λ to $\Lambda_{\rm Naive}$.

We conclude that the improved behavior as effective theories of 5D theories is not only related to locality and 5D gauge symmetries, but is a special property of the standard theories defined by the lowest dimensional operator F_{MN}^2. We also deconstruct the simplest version of our Lifshitz 5D theory. We show that, as expected, the higher dimensional Lifshitz operators are reproduced in the 4D deconstructed model by next-nearest-neighbor terms in field space. The precocious breakdown of unitarity induced by the Lifshitz terms is particularly clear from this perspective using the equivalence theorem [16].[2]

The structure of the paper is as follows. In Sec. II we introduce the class of theories we consider. In Sec. III we estimate the cutoff Λ of these theories by a one-loop vacuum polarization computation. In Sec. IV we estimate again the cutoff Λ, but this time by considering the breakdown of unitarity in scattering amplitudes of longitudinal gauge bosons. In Sec. V we deconstruct a simple 5D Lifshitz theory and show the form of the terms corresponding to the higher dimensional Lifshitz operators. In Sec. VI we conclude. We report in the Appendix some details of the one-loop vacuum polarization amplitude.

II. GENERAL SETUP

Lifshitz theories are typically taken to be invariant under anisotropic scale transformations under which the time coordinate scales differently from the spatial coordinates. In this way higher derivative terms in the spatial derivatives and quadratic in the fields can be introduced without violation of unitarity. The improved UV behavior of the propagator turns otherwise nonrenormalizable theories in renormalizable ones.

Along the lines of [13], we consider here Lifshitz models where time and the ordinary spatial directions scale in the same way, so that this symmetry can be made compatible with the 4D Lorentz symmetry, while the extra dimension scales differently. We focus on pure 5D non-Abelian $SU(m)$ gauge theories (the addition of matter being straightforward) compactified on an $S^1/\mathbf{Z}_2$ orbifold of length $L = \pi R$ parametrized by the coordinate y, where the terms odd under the parity symmetry $y \to -y$ are forbidden and no localized boundary terms are inserted. In this case, the higher derivative Lifshitz terms introduced below do not lead to uncancelled boundary terms in the action variation and the eigenfunctions of a KK field mode n is the usual $\cos ny/R$ or $\sin ny/R$, depending on the parity symmetry of the field.[3] For simplicity, we take in the following Neumann (Dirichlet) boundary conditions for the gauge fields A_μ (A_y). We assume an anisotropic scale invariance of the form

$$x_\mu = \lambda x'_\mu, \quad y = \lambda^{(1/Z)} y', \quad \phi(x^\mu, y) = \lambda^{(Z-d)/2} \phi'(x^{\mu\prime}, y'), \tag{2.1}$$

where $\mu = 0, \ldots, 3$ parametrizes the ordinary $3+1$ space-time directions, ϕ denotes a generic field and Z is a positive integer. According to Eq. (2.1), we can assign to the coordinates and to the fields a "weighted" scaling dimension:

$$[x^\mu]_w = -1, \quad [y]_w = -\frac{1}{Z}, \quad [\phi]_w = \frac{d-Z}{2}. \tag{2.2}$$

Power-counting renormalizability arguments apply, provided one substitutes the standard scaling dimensions of the operators by their "weighted scaling dimensions" [18], i.e., by the dimensions implied by the assignment (2.2). The weighted dimensions of the gluons A_μ are fixed by looking at their ordinary 4D kinetic term components. One gets

$$[A_\mu]_w = 1 + \frac{1}{2Z}. \tag{2.3}$$

Gauge invariance fixes the weighted dimensions of the 5D gauge coupling g_5 and of the gluon components A_y:

$$[g_5]_w = [\partial_\mu]_w - [A_\mu]_w = [\partial_y]_w - [A_y]_w \longrightarrow [g_5]_w = -\frac{1}{2Z},$$
$$[A_y]_w = \frac{3}{2Z}. \tag{2.4}$$

For any finite Z the theory remains nonrenormalizable, but with a coupling that is less and less irrelevant as Z

[2] The importance of locality in field space in deconstructed theories has recently been analyzed in [17], where it has been shown that nonlocal terms always lead to a smaller cutoff Λ. Contrary to the Lifshitz terms considered here, the nonlocal terms in [17] remain nonlocal in the 5D limit.

[3] We have not systematically studied the effects of the Lifshitz terms for more general interval compactifications. We expect that new consistency constrains should be imposed in this case and more drastic modifications to the spectrum of the theory might arise.

increases. Notice that the scaling dimensions of A_μ and A_y are different, with the latter being smaller than one for $Z > 1$. This difference will play a crucial role in what follows.

The most general Lagrangian involving weighted marginal and relevant operators only (i.e., operators $\mathcal{O}$ with $[\mathcal{O}]_w \leq 4 + 1/Z$) is

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}F_{\mu\nu}^2 + \sum_{i=0}^{Z-1}\frac{a_i}{\Lambda_L^{2i}}\mathrm{Tr}D_y^i F_{\mu y} D_y^i F_{\mu y}, \tag{2.5}$$

where the $SU(m)$ generators T^a in the fundamental representation are normalized as $\mathrm{Tr}T^aT^b = \delta^{ab}/2$ and Λ_L is the energy scale above which the theory effectively behaves as a Lifshitz theory.[4] By properly rescaling the internal dimension and the scale Λ_L, without loss of generality, we can set $a_0 = a_{Z-1} = 1$. We do not consider here the problem of understanding where the anisotropic symmetry (2.1) comes from but simply assume its presence in the effective theory.

The quadratic mixing terms between A_μ and A_y coming from the second term in Eq. (2.5) can be canceled by choosing a generalized R_ξ gauge-fixing term of the form

$$\mathcal{L}_{g.f.} = \frac{1}{\xi}\mathrm{Tr}\left(\partial_\mu A_\mu + \xi\sum_{i=0}^{Z-1}\frac{a_i}{\Lambda_L^{2i}}(-1)^i\partial_y^{2i+1}A_y\right)^2. \tag{2.6}$$

The ghost Lagrangian associated to the gauge-fixing (2.6) can easily be derived, though it is not explicitly needed in our analysis. The spectrum of states of the Lagrangian (2.5) is the standard infinite tower of KK modes labelled by an integer n, with the usual wave functions of the form

$$A_\mu(x,y) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x)\sqrt{\frac{2}{2^{\delta_{n,0}}\pi R}}\cos(ny/R),$$
$$A_y(x,y) = \sum_{n=1}^{\infty} A_y^{(n)}(x)\sqrt{\frac{2}{\pi R}}\sin(ny/R). \tag{2.7}$$

At the quadratic level, the only effect of the higher derivative Lifshitz terms is to modify the masses of the KK modes:

$$M_n^2 = \frac{n^2}{R^2}\sum_{i=0}^{Z-1} a_i\frac{n^{2i}}{(\Lambda_L R)^{2i}}. \tag{2.8}$$

The schematic behavior of the theory is the following. For energies $E < 1/R$, it is effectively an ordinary Lorentz-invariant 4D gauge theory. For $1/R < E < \Lambda_L$, the theory behaves as an ordinary 5D gauge theory and for $E > \Lambda_L$ it behaves as a Lifshitz theory where operators are effectively classified by their weighted dimensions. If we take $\Lambda_L \sim 1/R$, the theory is never in the "ordinary" 5D regime. We will study in the next two sections the impact of the higher derivative Lifshitz operators on the cutoff of the theory, estimated by using gauge coupling corrections and by unitarity bounds on scattering amplitudes.

[4]It should be clear that, being the theory nonrenormalizable, the irrelevant operators we have not written in Eq. (2.5) cannot be kept to zero at any scale. When quantum corrections are included, they will be generated.

III. ESTIMATE OF THE CUTOFF THROUGH GAUGE COUPLING CORRECTIONS

According to NDA, the coefficients of the local operators in a nonrenormalizable Lagrangian should be of the same order of the ones induced by radiative corrections at the scale Λ, where Λ is the energy above which the effective theory breaks down. One can also invert the logic and apply NDA to particularly simple operators to estimate the value of Λ itself. The obvious choice of operator in a 5D gauge theory is the kinetic term $F_{\mu\nu}^2$. The cutoff Λ can then be defined as the scale where the one-loop vacuum polarization correction to the zero mode gauge fields $A_\mu^{(0)}$ becomes of order one. A naive estimate that just takes into account the phase space of the loop integration and the number of colors would give

$$\Lambda_{\text{Naive}}^{5D} \simeq \frac{24\pi^3}{mg_5^2} = \frac{24\pi^2}{mg_4^2 R}, \tag{3.1}$$

where $24\pi^3$ is the 5D loop factor, $g_4 = g_5/\sqrt{\pi R}$ is the 4D gauge coupling and m is the quadratic Casimir operator, $C_2(G) = m$, for $SU(m)$. A more detailed computation such as the one below (see the Appendix for further details) shows that this estimate is in fact too naive and optimistic, and a more reliable one is obtained by using the 4D loop factor $16\pi^2$ in Eq. (3.1). In light of these possible discrepancies, in what follows we estimate the cutoff for the Lifshitz field theories by computing in detail the one-loop vacuum polarization for $A_\mu^{(0)}$.

Before considering the Lifshitz case, it is useful to review the ordinary 5D Lorentz-invariant computation. The 5D Lorentz-invariant model is obtained by taking $Z = 1$ in Eq. (2.5). A useful, though not necessary, way to compute the gauge+ghost contribution to the one-loop gauge coupling correction is to make use of a mass-dependent β function in 4D.[5] The whole contribution (ghosts included) of the KK resonances of mass M_n to the β function of the 4D gauge coupling is (see Appendix D of [19])

$$\beta(g_4, ER) = \frac{g_4^3}{16\pi^2}\beta_g(ER), \tag{3.2}$$

with

[5]Notice that we use in the following β functions and RG flows only as a useful technical tool to get the one-loop correction to the gauge coupling in 5D. We are not resumming logs.

$$\beta_g(ER) = m\left(\sum_{n=1}^{\infty}\int_0^1 dx\frac{x(1-x)(6x^2-9x-1)E^2}{M_n^2+E^2x(1-x)} - \frac{11}{3}\right) = m\left(\sum_{n=-\infty}^{\infty}\int_0^1 dx\frac{1}{2}\frac{x(1-x)(6x^2-9x-1)E^2}{M_n^2+E^2x(1-x)} - \frac{23}{12}\right), \tag{3.3}$$

where E is the sliding renormalization group RG (Euclidean) energy scale and $M_n^2 = n^2/R^2$ is the mass of the KK mode n. We show in the Appendix some details on how to obtain Eq. (3.3), since we are not aware of any derivation in the literature. The factor $-11/3$ in Eq. (3.3) is the zero mode contribution. When $M_n \to 0$, the integral over x is trivial and gives $-7/2$, which reproduces the contribution of a massless gauge field plus its scalar (longitudinal) component: $-7/2 = -11/3 + 1/6$. The one-loop gauge contribution can be written as

$$g_4^{-2}(E) = g_4^{-2}(E_0) - \frac{1}{8\pi^2}\int_{E_0}^{E}\frac{d\mu}{\mu}\beta_g(\mu R). \tag{3.4}$$

Performing the sum over the KK modes n, we get

$$\beta_g(ER) = m\left(\int_0^1 dx\frac{6x^2-9x-1}{2}\pi ER\sqrt{x(1-x)} \times \coth(\pi ER\sqrt{x(1-x)}) - \frac{23}{12}\right), \tag{3.5}$$

and, using Eq. (3.4), the following RG behavior for g_4^{-2} is obtained:

$$g_4^{-2}(E) = g_4^{-2}(E_0) - \frac{m}{8\pi^2}\left(\int_0^1 dx\frac{6x^2-9x-1}{2} \times\log\left(\frac{\sinh(\pi ER\sqrt{x(1-x)})}{\sinh(\pi E_0R\sqrt{x(1-x)})}\right) - \frac{23}{12}\log\frac{E}{E_0}\right). \tag{3.6}$$

For $R \to 0$, Eq. (3.6) reproduces the usual one-loop logarithmic gauge contribution. We are here interested in using Eq. (3.5) to estimate the cutoff of the theory. The latter is defined as the energy Λ where the one-loop factor is comparable to the "tree-level" term $g^{-2}(E_0)$. For $E \gg 1/R, E_0$, we get

$$g_4^{-2}(E) \simeq g_4^{-2}(E_0) + \frac{29m}{1024}ER, \tag{3.7}$$

from which one obtains

$$\Lambda^{(1)} \simeq \frac{1024}{29g_0^2m}\frac{1}{R}. \tag{3.8}$$

In Eq. (3.8), $g_0 \equiv g_4(E_0)$ and we have introduced a superscript (1) to Λ to specify that this is the value of the cutoff for the ordinary theory with $Z = 1$. In comparing the naive estimate (3.1) with the more refined (3.8) we notice that the former is too optimistic by almost one order of magnitude. If we insist in using naive estimates based on loop factors only, we see that a more reliable estimate is obtained by replacing the 5D loop factor $24\pi^3$ with the 4D loop factor $16\pi^2$ in Eq. (3.1).

Let us now consider the Lifshitz theory. Interestingly enough, the expression (3.3) for the β function still holds, provided we use the modified mass terms (2.8) for the KK gluon mode n. This is best seen in unitary gauge, $\xi \to \infty$, in which $A_y = 0$ and the Lifshitz interactions boil down to higher derivative quadratic terms for the KK gluons. For simplicity, we keep the marginal operators only, setting all couplings a_i to zero, except $a_{Z-1} = 1$. For further simplicity, let us first take $Z = 2$. Summing Eq. (3.3) over the KK modes n gives

$$\sum_{n=-\infty}^{\infty}\frac{1}{n^4+a_2^2} = \frac{1}{a_2^2}\mathrm{Re}(\pi\sqrt{a_2}e^{i\pi/4}\cot(\pi\sqrt{a_2}e^{i\pi/4})), \tag{3.9}$$

where a_2 is the value for $Z = 2$ of the variable a_Z defined for future use as

$$a_Z^2 = a_Z^2(E) \equiv x(1-x)(\Lambda_L R)^{2Z}\frac{E^2}{\Lambda_L^2}. \tag{3.10}$$

It is straightforward to check that

$$\mathrm{Re}(\pi\sqrt{a_2}e^{i\pi/4}\cot(\pi\sqrt{a_2}e^{i\pi/4})) = \frac{\pi\sqrt{a_2}}{\sqrt{2}}\frac{\sinh(\sqrt{2a_2}\pi)+\sin(\sqrt{2a_2}\pi)}{\cosh(\sqrt{2a_2}\pi)-\cos(\sqrt{2a_2}\pi)} = E\frac{d}{dE}\log(\cosh(\sqrt{2a_2}\pi)-\cos(\sqrt{2a_2}\pi)). \tag{3.11}$$

Using the above relations, we get

$$g_4^{-2}(E) = g_0^{-2} - \frac{m}{8\pi^2}\left(\int_0^1 dx\frac{6x^2-9x-1}{2} \times\log\left(\frac{\cosh(\sqrt{2a_2}\pi)-\cos(\sqrt{2a_2}\pi)}{\cosh(\sqrt{2a_{2,0}}\pi)-\cos(\sqrt{2a_{2,0}}\pi)}\right) - \frac{23}{12}\log\frac{E}{E_0}\right), \tag{3.12}$$

where $a_{2,0} = a_2(E_0)$. For $E \gg \Lambda_L$, $1/R$, E_0, such that $a_2 \gg 1$, we have

$$g_4^{-2}(E) \simeq g_0^{-2} + \kappa_2(\Lambda_L R)m\sqrt{\frac{E}{\Lambda_L}}, \tag{3.13}$$

where

$$\kappa_2 = \frac{1}{16\pi^2}\frac{25\sqrt{2\pi}\Gamma(1/4)^2}{84} \simeq \frac{1}{16} \tag{3.14}$$

is a numerical factor. The transverse gauge fields $A_\mu^{(0)}$ enter in a strongly coupled regime for

$$\Lambda^{(2)} \simeq \Lambda_L \left(\frac{1}{g_0^2 m \kappa_2(\Lambda_L R)} \right)^2. \tag{3.15}$$

We can also analyze the asymptotic region $E \gg 1/R$, Λ_L, E_0 for an arbitrary, but finite, Z. We have

$$\sum_{n=-\infty}^{\infty} \frac{1}{n^{2Z} + a_Z^2} = \frac{1}{Za^2} \sum_{l=0}^{Z-1} \mathrm{Re}(\pi a_Z^{1/Z} e^{(i\pi(2l+1)/2Z)} \cot(\pi a_Z^{1/Z} e^{(i\pi(2l+1)/2Z)})). \tag{3.16}$$

For large energies (i.e., large a_Z), we also have

$$\sum_{l=0}^{Z-1} \mathrm{Re}(\pi a_Z^{1/Z} e^{(i\pi(2l+1)/2Z)} \cot(\pi a_Z^{1/Z} e^{(i\pi(2l+1)/2Z)})) = \frac{\pi a_Z^{1/Z}}{\sin\pi/(2Z)} (1 + \mathcal{O}(e^{-ca_Z^{1/Z}})), \tag{3.17}$$

where c is a positive numerical factor of $\mathcal{O}(1)$. Using the above relations, we get

$$g^{-2}(E) \simeq g_0^{-2} + m\kappa_Z(\Lambda_L R)\left(\frac{E}{\Lambda_L}\right)^{1/Z}, \tag{3.18}$$

where

$$\kappa_Z = \frac{1}{16\pi^2} \frac{(21Z+8)\pi^{3/2}\Gamma(\frac{1}{2Z})}{8\pi^2 2^{4+1/Z} Z^2 \sin(\frac{\pi}{2Z})\Gamma(\frac{1}{2}(5+\frac{1}{Z}))}. \tag{3.19}$$

For generic Z, the would-be cutoff of the theory is estimated to be

$$\Lambda^{(Z)} \simeq \Lambda_L \left(\frac{1}{g_0^2 m \kappa_Z(\Lambda_L R)} \right)^Z. \tag{3.20}$$

For $Z = 1, 2$, Eq. (3.20) reproduces the previous estimates (3.8) and (3.15). The first numerical values of $\Lambda^{(Z)}$ are

$$\Lambda^{(1)} \simeq \frac{35}{g_0^2 m} \frac{1}{R}, \qquad \Lambda^{(2)} \simeq \frac{260}{(g_0^2 m)^2} \frac{\Lambda_L}{(\Lambda_L R)^2}, \qquad \Lambda^{(3)} \simeq \frac{940}{(g_0^2 m)^3} \frac{\Lambda_L}{(\Lambda_L R)^3}, \ldots . \tag{3.21}$$

The would-be cutoff of the theory is parametrically high for a sufficiently small 't Hooft coupling $g_0^2 m$, provided that $\Lambda_L \sim 1/R$.[6]

Sometimes it is useful to consider how many KK modes $N_{\mathrm{Max}}^{(Z)}$ have a mass below the cutoff of the theory. From an effective field theory point of view, these correspond to the states that we are justified to keep in the theory.[7] Interestingly enough, $N_{\mathrm{Max}}^{(Z)}$ does not increase in the Lifshitz theory, because the spacing between the KK modes is enlarged for $Z > 1$ and compensates for the higher cutoff $\Lambda^{(Z)}$. Using Eqs. (2.8) and (3.20) we have

$$N_{\mathrm{Max}}^{(Z)} \simeq \frac{1}{g_0^2 m k_Z}, \tag{3.22}$$

and since k_Z slightly decreases for increasing values of Z, the actual number of KK states below the cutoff actually decreases with respect to the ordinary $Z = 1$ theory.

When the weighted relevant operators are considered, $a_i \neq 0$, the sums over the KK modes become rather cumbersome and complicated, but a qualitative physical description can easily be given. The gauge coupling evolution is essentially dictated by the value of $\Lambda_L R$. For $\Lambda_L R \sim 1$, all the terms appearing in Eq. (2.8) are of the same order of magnitude and the marginal coupling a_{Z-1} quickly dominates for $n > 1$. In this case, the approximation above is justified and the would-be cutoff of the theory is given by Eq. (3.20). For $\Lambda_L R \gg 1$, up to KK modes of order $n \sim \Lambda_L R$, the dominant coupling is the ordinary a_0 term, giving rise to the usual coupling behavior (3.7). The theory enters in the Lifshitz regime only for $E > \Lambda_L$. It is then obvious that the Lifshitz operators are significant only in the energy range $1/R < \Lambda_L < \Lambda^{(1)}$.

Similar results also apply in the presence of fermions. The anisotropic scaling (2.1) would demand the presence of higher (covariant) derivative interactions along the internal dimension, that in unitary gauge boil down to a modified KK mass formula for the fermion KK modes, similar to Eq. (2.8). The explicit contribution of a fermion to β is reported in Eq. (A14). The analysis is essentially identical to the one we did for the gauge case. In particular, the gauge coupling correction still scales as $(\Lambda_L R) \times (E/\Lambda_L)^{1/Z}$, as in Eq. (3.18). The results shown here apply then for Abelian theories as well, where at one-loop level the matter contribution is the only one.

IV. CUTOFF FROM UNITARITY BOUNDS IN SCATTERING AMPLITUDES

In the last section we have shown that the cutoff of Lifshitz field theories, as obtained by a detailed computation of the vacuum polarization correction of the transverse polarizations of the zero mode field $A_\mu^{(0)}$, can be parametrically higher than the one in ordinary theories. We show here that the estimate (3.20) does not apply in non-Abelian gauge theories, since the scattering amplitudes of longitudinal components of the gauge fields break unitarity well before the energy (3.20) is reached and even before the ordinary 4D value (3.8). This result could also be obtained by analyzing the gauge coupling corrections to the longitudinal components of $A_\mu^{(n)}$, but this computation is rather cumbersome, while we will see how it is straightforward,

[6]Notice that Eq. (3.20) does not hold for parametrically large Z—in which case it would predict that $\Lambda^{(Z)} \to 0$ for $Z \to \infty$ ($k_Z \propto Z$ for large Z)—because the limit of taking large energies does not commute with the large Z limit.

[7]Naive truncations of this kind should be considered with care, because they can lead to a breakdown of the 5D nonlinearly realized gauge symmetries.

building on previous works, to get the bounds coming from scattering amplitudes.

Let us briefly review the behavior of the scattering amplitudes of longitudinal KK gauge bosons in 5D theories, focusing for simplicity to elastic processes [5,15]. This amplitude could grow as fast as E^4, where E is the center of mass energy of the incoming fields. In [15] it has been shown that the $\mathcal{O}(E^4)$ and $\mathcal{O}(E^2)$ terms in the amplitude of longitudinal 5D gauge boson scattering vanish, whenever the following relations hold:

$$g_{nnnn}^2 = \sum_k g_{nnk}^2, \qquad 4g_{nnnn}^2 M_n^2 = 3\sum_k g_{nnk}^2 M_k^2, \tag{4.1}$$

where k and n are KK levels, g_{nnnn} is the quartic gauge coupling of KK gauge fields at level n, g_{nnk} is the trilinear coupling of two KK n and one KK k gauge fields, and M_n^2 is the mass of the KK n gauge field. In ordinary 5D theories, Eq. (4.1) are satisfied and the amplitude does not grow with the energy, as already claimed in [5,8]. Unitary violation is detected from the $\mathcal{O}(E^0)$ terms in the amplitude and arises from the multiplicity of states in a coupled channel analysis [5]. For a $SU(m)$ theory compactified on a segment, the maximum number of KK states $N_{\rm Max}$ that can enter in a scattering process without leading to a violation of unitarity is given by [5]

$$N_{\rm Max} \simeq \frac{8\pi}{m}\frac{1}{g_0^2}, \tag{4.2}$$

leading to a cutoff estimate

$$\Lambda \sim M_{N_{\rm Max}} \simeq \frac{8\pi}{g_0^2 m R}, \tag{4.3}$$

roughly in agreement with Eq. (3.8).

As we already mentioned, in the unitary gauge $A_y = 0$, no new interactions arise from the higher derivative Lifshitz terms, and the couplings g_{nnnn} and g_{nnk} are the same as in the ordinary 5D theories. The first constraint in Eq. (4.1) is then automatically satisfied. The only effect of the Lifshitz interactions is to modify the gauge boson KK masses as given in Eq. (2.8). It is straightforward to check that, due to the modification in the mass formula, the second relation in Eq. (4.1) is no longer satisfied in the Lifshitz case. For illustration, let us consider $Z = 2$. For plain $S^1/\mathbf{Z}_2$ compactifications, the sum over k in Eq. (4.1) reduces to two terms, $k = 0$ and $k = 2n$, which are the only two states that can be exchanged in the scattering process, due to the conservation of the 5D momentum, mod $\mathbf{Z}_2$. A simple computation gives

$$4g_{nnnn}M_n^2 - 3g_{nn0}M_0^2 - 3g_{nn2n}M_{2n}^2 = -\frac{18}{\pi}g_0^2\Lambda_L^2\frac{n^4}{(\Lambda_L R)^4}. \tag{4.4}$$

Neglecting the $\mathcal{O}(E^0)$ terms, the $W_L^{(n)}W_L^{(n)} \to W_L^{(n)}W_L^{(n)}$ scattering goes like

$$\mathcal{A}\,(W_L^{(n)}W_L^{(n)} \to W_L^{(n)}W_L^{(n)})_{E^2} \sim \frac{g_0^2}{(1+\frac{n^2}{(\Lambda_L R)^2})^2}\frac{E^2}{\Lambda_L^2}. \tag{4.5}$$

While the transverse components of the gauge fields remain weakly coupled for energies above the ordinary bound (4.3), the longitudinal components show a breakdown of unitarity at energies below Eq. (4.3). We get, from Eq. (4.3),[8]

$$\Lambda \sim \frac{4\pi\Lambda_L}{g_0}. \tag{4.6}$$

When $\Lambda_L \simeq 1/R$, Eq. (4.6) is the energy one would expect from 4D considerations for a pNGB with mass $M_1 \simeq 1/R$ and "pion" decay constant $f = M_1/g_0$, which would give $\Lambda \simeq 4\pi f$, equal to the naive estimate (1.2). As expected, the ordinary 5D result (4.3) is recovered for $\Lambda_L \to \infty$, in which case one has to look at the $\mathcal{O}(E^0)$ terms.

A similar result is obtained, by the equivalence theorem, by studying the scattering of the pNGB's in a different gauge, such as Landau or Feynman gauge. In these gauges, the higher derivative Lifshitz terms give rise to derivative quartic interactions among the pNGB's that reproduce the behavior (4.5). We will briefly come back to this point in the next section, when the 4D deconstructed version of the theory is considered.

V. DECONSTRUCTED 4D MODEL

It is interesting to analyze the deconstructed version of our setup. Let us briefly recall the deconstruction of an ordinary 5D $SU(m)$ pure gauge theory on an interval [6,7]. The Lagrangian of a linear moose with N sites and $N-1$ link variables U_i is given by

$$\mathcal{L} = -\frac{1}{2}\sum_{i=1}^{N}{\rm Tr}F_{\mu\nu,i}^2 + f^2\sum_{i=1}^{N-1}{\rm Tr}|D_\mu U_i|^2, \tag{5.1}$$

where the U_i's transform as $U_i \to g_{i+1}U_i g_i^\dagger$ under gauge transformations and have only "nearest-neighbor" interactions with the gauge fields $A_{\mu,i+1}$ and $A_{\mu,i}$. The covariant derivative is

$$D_\mu U_i = \partial_\mu U_i - igA_{\mu,i+1}U_i + igU_iA_{\mu,i}. \tag{5.2}$$

For simplicity we have taken in Eqs. (5.1) and (5.2) a universal decay constant f and a universal coupling constant g. The gauge group $SU(m)^N$ is nonlinearly realized, because the link fields are "σ-model" fields that can be written as

$$U_i(x) = e^{(i\pi_i(x))/f}, \tag{5.3}$$

in terms of would-be Goldstone bosons $\pi_i(x) = \pi_i^a T^a$. In the unitary gauge $\langle U_i\rangle = 1$, the Lagrangian (5.1) contains an $N\times N$ mass matrix for the gauge fields of the form

[8] As before, the actual cutoff should be computed by considering inelastic channels as well, and can be smaller than the estimate (4.6).

$$M^2 = g^2 f^2 \begin{pmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ -1 & 2 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2 & -1 \\ 0 & 0 & 0 & \dots & -1 & 1 \end{pmatrix}, \quad (5.4)$$

which has eigenvalues

$$M_n^2 = 4g^2 f^2 \sin^2 \frac{\pi n}{2N}, \qquad n = 0, \dots, N-1. \quad (5.5)$$

We can define

$$L = Na, \qquad a = \frac{1}{gf}, \qquad p_5 = \frac{\pi n}{L}, \quad (5.6)$$

so that a can be interpreted as the lattice spacing of the interval and L its length. The 5D gauge coupling g_5 is given by $g_5^2 = ag^2$. For $n \ll N$, we have

$$M_n^2 = \frac{4}{a^2} \sin^2 \frac{p_5 a}{2} \simeq p_5^2 = \frac{n^2}{R^2}, \quad (5.7)$$

where $R = L/\pi$ is the radius of the 5D covering circle of $S^1/\mathbf{Z}_2$. In the unitary gauge, $\pi_1 = \pi_2 = \dots = \pi_{N-1} = 0$ and $SU(m)^N$ is spontaneously broken to the diagonal subgroup $SU(m)$. In the canonical basis, the gauge coupling g_4 of the unbroken gauge fields is $g_4^2 = g^2/N = g_5^2/L$, in agreement with what expected from a 5D theory.

Let us generalize the deconstruction above and include the higher derivative operators appearing in Eq. (2.5). For simplicity, we consider only the $Z = 2$ case. The higher derivative terms in the extra dimension suggest that in the deconstructed theory "next-nearest-neighbor" interactions should be present. The deconstructed Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{2} \sum_{i=1}^{N} \mathrm{Tr} F_{\mu\nu,i}^2 + f^2 \sum_{i=1}^{N-1} \mathrm{Tr} |D_\mu U_i|^2 + \tilde{f}^2 \sum_{i=1}^{N} \mathrm{Tr} |D_\mu U_i - D_\mu U_{i+1}|^2, \quad (5.8)$$

where $U_{N+1} = U_1$ and $U_N = 0$ in the last term, and $\tilde{f}$ is the "Lifshitz" pion decay constant. As we will shortly see, the last term in Eq. (5.8) corresponds to the $\mathrm{Tr}(D_y F_{\mu y})^2$ term in Eq. (2.5). In the unitary gauge $\langle U_i \rangle = 1$, all π_i's vanish and $SU(m)^N$ is spontaneously broken to the diagonal subgroup $SU(m)$, like in the ordinary case reviewed above. The quadratic terms for the gauge fields coming from the last term in Eq. (5.8) give rise to the following $N \times N$ mass matrix:

$$\tilde{M}^2 = g^2 \tilde{f}^2 \begin{pmatrix} 2 & -3 & 1 & 0 & \dots & 0 & 0 & 0 \\ -3 & 6 & -4 & 1 & \dots & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 6 & -4 & 1 \\ 0 & 0 & 0 & 0 & \dots & -4 & 6 & -3 \\ 0 & 0 & 0 & 0 & \dots & 1 & -3 & 2 \end{pmatrix}. \quad (5.9)$$

The total mass matrix for the gauge fields is given by

$$M_{\mathrm{Tot}}^2 = M^2 + \tilde{M}^2, \quad (5.10)$$

with M^2 as in (5.4). By explicit computation, we find that the mass matrix (5.9) has eigenvalues

$$\tilde{M}_n^2 = 16 g^2 \tilde{f}^2 \sin^4 \frac{\pi n}{2N}, \qquad n = 0, \dots, N-1. \quad (5.11)$$

Interestingly enough, the matrices (5.4) and (5.9) are simultaneously diagonalizable and hence the total mass eigenvalues are simply given by the sum of Eqs. (5.7) and (5.11):

$$M_{\mathrm{Tot},n}^2 = 4g^2 f^2 \sin^2 \frac{\pi n}{2N} + 16 g^2 \tilde{f}^2 \sin^4 \frac{\pi n}{2N}, \quad n = 0, \dots, N-1. \quad (5.12)$$

The Lifshitz scale Λ_L is defined as

$$a^2 \Lambda_L = \frac{1}{g\tilde{f}}. \quad (5.13)$$

For $n \ll N$, we have

$$M_n^2 = \frac{4}{a^2} \sin^2 \frac{p_5 a}{2} + \frac{16}{\Lambda_L^2 a^4} \sin^4 \frac{p_5 a}{2} \simeq p_5^2 + \frac{p_5^4}{\Lambda_L^2} = \frac{n^2}{R^2} \left(1 + \frac{n^2}{(\Lambda_L R)^2} \right), \quad (5.14)$$

which reproduces Eq. (2.8) for $Z = 2$.

The leading behavior of the amplitude (4.5) can be reproduced in the deconstructed model. It is actually easier, by using the equivalence theorem, to look at the $\pi\pi \to \pi\pi$ scattering. In the ordinary linear moose (5.1), the $\mathcal{O}(E^2)$ term schematically reads

$$\mathcal{A}(\pi\pi \to \pi\pi)_{E^2} \propto \frac{E^2}{f^2} = g^2 \left(\frac{EL}{N} \right)^2, \quad (5.15)$$

where we have used Eq. (5.6) in the second equality. For simplicity we have omitted in Eq. (5.15) gauge and site indices and have written only the structure of the amplitude. In the limit $N \to \infty$, the $\mathcal{O}(E^2)$ term in the amplitude vanishes, in agreement with the 5D expectation. In the linear "Lifshitz" moose (5.8), additional derivative quartic couplings arise and we get an extra term $\Delta\mathcal{A}$ contributing to the amplitude:[9]

$$\Delta\mathcal{A}(\pi\pi\to\pi\pi)_{E^2}\propto\frac{E^2\tilde{f}^2}{f^4}=g^2\left(\frac{E}{\Lambda_L}\right)^2, \tag{5.16}$$

where in the last equality we have used Eqs. (5.6) and (5.13). The factors of N now cancel and the $\mathcal{O}(E^2)$ term no longer vanishes in the 5D limit. Instead, using Eq. (5.16), we get a cutoff $\Lambda\sim 4\pi\Lambda_L/g$, in agrement with Eq. (4.6).

One might ask if the Lifshitz terms, even if not introduced at tree-level, are radiatively generated. The answer is clearly negative in the 5D limit, because of $SO(5)$ symmetry. A similar conclusion is reached in the 4D deconstructed model by using spurions [7]. When the gauge coupling g is switched off, the Lagrangian (5.1) has an $SU(m)^{2N-2}$ global symmetry under which $U_i\to L_{i+1}U_iR_i^\dagger$, where L_i and R_i are independent $SU(m)$ matrices ($i=1,\ldots,N-1$). This global symmetry is explicitly broken by the gauge fields A_i, but it can formally be restored by introducing spurion fields q_i. The gauge fields and the spurions transform as $A_i\to L_{i+1}A_iL_{i+1}^\dagger$ and $q_i\to R_iq_iL_{i+1}^\dagger$, respectively. By writing the covariant derivative (5.2) as

$$D_\mu U_i=\partial_\mu U_i-igA_{\mu,i+1}U_i+igU_iq_iA_{\mu,i}q_i^\dagger, \tag{5.17}$$

the $SU(m)^{2N-2}$ global symmetry is formally restored. The original model is eventually recovered by setting $q_i=1$. It is not difficult to see that no $SU(m)^{2N-2}$ invariant operators can be constructed by using the spurions, leading to $\mathrm{Tr}(DU_{i+1}^\dagger DU_i)$ when $q_i=1$. We conclude that the Lifshitz operators are not generated by quantum corrections.

Although we have not explicitly worked out the deconstructed version of Eq. (2.5) for general Z, we expect that the introduction of "next-next-...-nearest-neighbor" interactions should reproduce the corresponding 5D higher-derivative terms for any Z. No new results are expected to arise by considering higher values of Z.

VI. CONCLUSIONS

We have studied the cutoff estimate in 5D field theories, where certain higher derivative (Lifshitz) operators are added to the action. By a detail one-loop vacuum polarization computation, we have argued that the transverse polarizations of the gauge fields have a softer UV behavior with respect to the ones in ordinary 5D theories. On the other hand, the same higher derivative terms negatively affect the longitudinal polarizations of the gauge fields. Because of these operators, the $\mathcal{O}(E^2)$ terms in the scattering amplitude of longitudinal gauge bosons no longer vanish, in contrast to the usual 5D case, and lead to an earlier breakdown of unitarity with respect to the standard 5D situation. Of course, this problem does not occur for Abelian gauge theories, in which the Lifshitz operators do improve the UV behavior of the theory. We have then considered (for the special case $Z=2$) the deconstructed version of the 5D Lifshitz models and shown how similar conclusions are reached from this perspective. As expected, the Lifshitz terms correspond to next-nearest-neighbor interactions in field space.

Our analysis explicitly shows that the relatively good UV behavior of standard 5D theories, for which $\Lambda>\Lambda_{\mathrm{Naive}}$, as defined in Eqs. (1.1) and (1.2), do not only come from 5D locality and 5D gauge symmetries, both preserved in our Lifshitz construction, but are peculiar of the standard 5D action. The 4D deconstructed models are useful in this respect, since they show how the Lifshitz terms break the global symmetries responsible for the good UV behavior of ordinary 5D theories. From the Lifshitz field theory point of view, our results explicitly show that care has to be used in determining the UV behavior (e.g., renormalizability) of Lifshitz theories based only on the effective UV dimension of the couplings. It is crucial to also pay attention to the effective dimensions of the fields, even when they can be gauged away (like the fields A_y in our case), since they can lead to a precocious strong coupling behavior.

ACKNOWLEDGMENTS

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APPENDIX: DERIVATION OF THE 5D β FUNCTION

The β function (3.3) is conveniently computed using background field methods and a background field gauge-fixing. We write the gauge field as $A_{\mathrm{tot}}=\bar{A}+A$, where $\bar{A}$ is the classical background value and A is the quantum fluctuation. The 5D Lagrangian reads, including gauge-fixing and ghosts,

$$\mathcal{L}_{BFG}=-\frac{1}{4}F^{\mathrm{tot},2}_{MN,a}-\frac{1}{2\xi}(\bar{D}_MA^M_a)^2+(\bar{D}_M\omega_a)(\bar{D}^M\omega_a-gf_{abc}\omega_bA^c_M)-\frac{1}{4}\delta Z\bar{F}^2_{\mu\nu,a}, \tag{A1}$$

with a, b, c color indices, M, N 5D indices and

$$\bar{D}_MA_{N,a}=\partial_MA_{N,a}+gf_{abc}\bar{A}_{M,b}A_{N,c} \tag{A2}$$

[9] The Lifshitz term in Eq. (5.8) also contains additional contributions to the kinetic terms of the pions, that have thus to be canonically normalized. The net effect of this normalization is the term in the denominator appearing in Eq. (4.5). For simplicity, we neglect these corrections that do not play an important role for our purposes.

the covariant derivative with respect to the background field only. In Eq. (A1), we have explicitly included the counterterm δZ for the 4D background field strength $\bar{F}_{\mu\nu}(x)$, omitting all the others that do not play any role in the computation. We choose in the following $\xi = 1$ so that all quadratic gauge mixing terms vanish. As well known, the gauge symmetries of the classical background allow to compute the β function directly from the two-point function $\langle \bar{A}_\mu(-p)\bar{A}_\nu(p)\rangle$. The effective one-loop Lagrangian for the zero mode background $\bar{A}_\mu(x)$ reads

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} Z \bar{F}_{\mu\nu}^2 + \dots . \tag{A3}$$

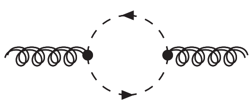
We choose a standard momentum subtraction renormalization scheme, by demanding that

$$Z(p^2 = -E^2) = 1. \tag{A4}$$

The mass-dependent β function is given by

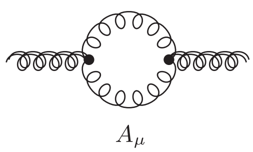
$$\beta(g_4, ER) = g_4 \frac{d \log Z}{d \log E}. \tag{A5}$$

After a lengthy computation, we get the following results for the relevant Feynman graphs, in dimensional regularization:[10]

$(a) =$ 

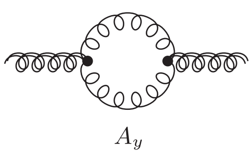
$$= -iC_2(G)\frac{g_4^2}{(4\pi)^{d/2}}\Gamma\Big(2-\frac{d}{2}\Big)\sum_{n=0}^{\infty}\int_0^1 dx\Big(\frac{\mu^2}{M_n^2 - p^2x(1-x)}\Big)^{\frac{\epsilon}{2}}$$

$$\times\Big(-\frac{4}{2-d}\Big(M_n^2 - p^2x(1-x)\Big)\eta^{\mu\nu} + (1-2x)^2p^\mu p^\nu\Big), \tag{A6}$$

$(b) =$ 

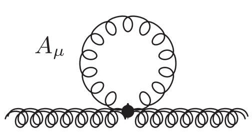
$$= -iC_2(G)\frac{g_4^2}{(4\pi)^{d/2}}\Gamma\Big(2-\frac{d}{2}\Big)\sum_{n=0}^{\infty}\int_0^1 dx\Big(\frac{\mu^2}{M_n^2 - p^2x(1-x)}\Big)^{\frac{\epsilon}{2}}$$

$$\times\Big(\Big(-4p^2 + \frac{2d}{2-d}\Big(M_n^2 - p^2x(1-x)\Big)\Big)\eta^{\mu\nu} + \Big(\frac{8-d}{2} + 2dx(1-x)\Big)p^\mu p^\nu\Big), \tag{A7}$$

$(c) =$ 

$$= -iC_2(G)\frac{g_4^2}{(4\pi)^{d/2}}\Gamma\Big(2-\frac{d}{2}\Big)\sum_{n=1}^{\infty}\int_0^1 dx\Big(\frac{\mu^2}{M_n^2 - p^2x(1-x)}\Big)^{\frac{\epsilon}{2}}$$

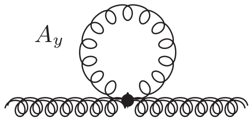
$$\times\Big(\Big(\frac{2}{2-d}\Big(M_n^2 - p^2x(1-x)\Big)\Big)\eta^{\mu\nu} + \Big(-\frac{1}{2} + 2x(1-x)\Big)p^\mu p^\nu\Big), \tag{A8}$$

$(d) =$ 

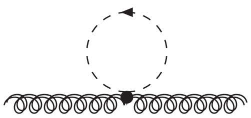
$$= -iC_2(G)\frac{g_4^2}{(4\pi)^{d/2}}\Gamma\Big(2-\frac{d}{2}\Big)\sum_{n=0}^{\infty}\int_0^1 dx\Big(\frac{\mu^2}{M_n^2 - p^2x(1-x)}\Big)^{\frac{\epsilon}{2}}$$

$$\times\Big(d\Big((1-x)^2p^2 - M_n^2\Big) - \frac{d^2}{2-d}(M_n^2 - p^2x(1-x))\Big)\eta^{\mu\nu}, \tag{A9}$$

[10] Notice that dimensional regularization is typically used in association with a mass-independent renormalization scheme, such as MS or $\overline{\rm MS}$, in which μ, coming from $g \to g\mu^{\epsilon/2}$, is the RG scale. In our (unconventional) use of dimensional regularization with a mass-dependent scheme, μ becomes irrelevant and the RG scale is identified with the subtraction scale E.



$$(e) = \quad = -iC_2(G)\frac{g_4^2}{(4\pi)^{d/2}}\Gamma\Big(2-\frac{d}{2}\Big)\sum_{n=1}^{\infty}\int_0^1 dx\Big(\frac{\mu^2}{M_n^2-p^2x(1-x)}\Big)^{\frac{\epsilon}{2}} \times\bigg((1-x)^2p^2 - M_n^2 - \frac{d}{2-d}(M_n^2-p^2x(1-x))\bigg)\eta^{\mu\nu}\,, \tag{A10}$$



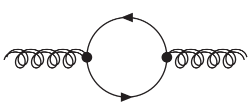
$$(f) = \quad = -iC_2(G)\frac{g_4^2}{(4\pi)^{d/2}}\Gamma\Big(2-\frac{d}{2}\Big)\sum_{n=0}^{\infty}\int_0^1 dx\Big(\frac{\mu^2}{M_n^2-p^2x(1-x)}\Big)^{\frac{\epsilon}{2}} \times\bigg(-2\Big((1-x)^2p^2 - M_n^2\Big) + \frac{2d}{2-d}(M_n^2-p^2x(1-x))\bigg)\eta^{\mu\nu}\,. \tag{A11}$$

In the above graphs $M_n = n/R$ is the mass of the KK mode running into the loop, the single wiggly lines represent the gluon fluctuations A_μ and A_y, dashed lines represent the ghost fields and the double wiggly lines represent the background field $\bar{A}_\mu$. Notice the presence of a quartic interaction among ghost and gauge fields in this gauge, leading to the graph (f). Summing all the contributions, we get

$$(a)+(b)+(c)+(d)+(e)+(f) = iC_2(G)\frac{g_4^2}{16\pi^2}(\eta^{\mu\nu}p^2 - p^\mu p^\nu)\int_0^1 dx\bigg((-2-6x+4x^2)\times\log\Big(\frac{-p^2x(1-x)}{\mu^2}\Big) + \sum_{n=1}^{\infty}(6x^2-9x-1)\log\Big(\frac{M_n^2-p^2x(1-x)}{\mu^2}\Big) + C\bigg) + \mathcal{O}(\epsilon), \tag{A12}$$

where C is an irrelevant divergent constant. The finite wave function correction Z is determined by the renormalization condition (A4), that fixes the counter-term δZ. Using Eq. (A5), we can finally get Eqs. (3.2) and (3.3).

For completeness, we also report the contribution from a massless 5D fermion in a representation r of $SU(m)$:



$$= -iT(r)\frac{g_4^2}{(4\pi)^{d/2}}\Gamma\Big(2-\frac{d}{2}\Big)\sum_{n=0}^{\infty}\int_0^1 dx\Big(\frac{\mu^2}{M_n^2-p^2x(1-x)}\Big)^{\frac{\epsilon}{2}} \times 2^{1-\delta_{n,0}}(-4)x(1-x)\Big(\eta^{\mu\nu}p^2 - p^\mu p^\nu\Big)\,, \tag{A13}$$

that gives rise to the following contribution to the β function:

$$\beta(g_4, ER) = \frac{g_4^3}{4\pi^2}\sum_{n=-\infty}^{\infty}T(r)\int_0^1 dx\frac{x^2(1-x)^2E^2}{M_n^2+E^2x(1-x)}. \tag{A14}$$

[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998).

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); Phys. Rev. Lett. **83**, 4690 (1999); W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999).

[3] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).

[4] N. Arkani-Hamed, M. Porrati, and L. Randall, J. High Energy Phys. 08 (2001) 017; R. Rattazzi and A. Zaffaroni, J. High Energy Phys. 04 (2001) 021.

[5] R. S. Chivukula, D. A. Dicus, and H.-J. He, Phys. Lett. B **525**, 175 (2002).
[6] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Rev. Lett. **86**, 4757 (2001).
[7] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Lett. B **513**, 232 (2001).
[8] R. S. Chivukula and H.-J. He, Phys. Lett. B **532**, 121 (2002).
[9] J. Alexandre, Int. J. Mod. Phys. A **26**, 4523 (2011).
[10] R. Iengo and M. Serone, Phys. Rev. D **81**, 125005 (2010).
[11] R. Iengo, J. G. Russo, and M. Serone, J. High Energy Phys. 11 (2009) 020.
[12] J. Collins, A. Perez, D. Sudarsky, L. Urrutia, and H. Vucetich, Phys. Rev. Lett. **93**, 191301 (2004).
[13] X.-G. He, S. S. C. Law, and R. R. Volkas, Phys. Rev. D **84**, 125017 (2011).
[14] H. Hatanaka, M. Sakamoto, and K. Takenaga, Phys. Rev. D **84**, 025018 (2011).
[15] C. Csaki, C. Grojean, H. Murayama, L. Pilo, and J. Terning, Phys. Rev. D **69**, 055006 (2004).
[16] J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Phys. Rev. D **10**, 1145 (1974); **11**, 972E (1975); B. W. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. D **16**, 1519 (1977).
[17] Y. Kahn and J. Thaler, J. High Energy Phys. 07 (2012) 007.
[18] D. Anselmi and M. Halat, Phys. Rev. D **76**, 125011 (2007).
[19] M. Regis, M. Serone, and P. Ullio, J. High Energy Phys. 03 (2007) 084.