PHYSICAL REVIEW D **86,** 045008 (2012)

Lorentz violation and the Higgs mechanism

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(Received 1 March 2012; published 2 August 2012)

We consider scalar quantum electrodynamics in the Higgs phase and in the presence of Lorentz violation. Spontaneous breaking of the gauge symmetry gives rise to Lorentz-violating gauge field mass terms. These may cause the longitudinal mode of the gauge field to propagate superluminally. The theory may be quantized by the Faddeev-Popov procedure, although the Lagrangian for the ghost fields also needs to be Lorentz violating.

DOI: 10.1103/PhysRevD.86.045008 PACS numbers: 11.30.Cp, 11.15.Ex, 11.30.Er, 11.30.Qc

I. INTRODUCTION

Lorentz violation is currently a topic of significant interest in particle physics and other areas. No particularly strong evidence for a deviation from Lorentz invariance has been found, but experimental Lorentz tests are constantly being refined. The study of Lorentz symmetry remains an active area of research, because if any violation of Lorentz invariance were to be found, that would be a discovery of premier importance.

Violations of Lorentz symmetry may be described in an effective quantum field theory called the standard model extension (SME). The SME contains translation-invariant but Lorentz-violating corrections to the standard model. These are parameterized by small tensor-valued background fields [1,2]. The most frequently considered subset of the SME is the minimal SME, which contains only gauge-invariant, superficially renormalizable forms of Lorentz violation. The minimal SME has become the standard framework used for parameterizing the results of experimental Lorentz tests.

Recent searches for Lorentz violation have included studies of matter-antimatter asymmetries for trapped charged particles [3–5] and bound state systems [6,7], measurements of muon properties [8,9], analyses of the behavior of spin-polarized matter [10], frequency standard comparisons [11–14], Michelson-Morley experiments with cryogenic resonators [15–19], Doppler effect measurements [20,21], measurements of neutral meson oscillations [22–27], polarization measurements on the light from cosmological sources [28–31], high-energy astrophysical tests [32–36], precision tests of gravity [37,38], and others. The results of these experiments set constraints on the various SME coefficients, and up-to-date information about most of these constraints may be found in [39].

The one-loop renormalization of various sectors of the minimal SME has been studied. This has included analyses of Abelian [40], non-Abelian [41], and chiral [42] gauge theories with spinor matter, as well as scalar field theories with Yukawa interactions [43]. Notably absent from this list is a full treatment of gauge theories with charged scalar fields. Such theories play a crucially important role in the standard model, but they are complicated by the possibility of spontaneous gauge symmetry breaking.

This paper represents a first step towards the understanding of Lorentz-violating scalar quantum electrodynamics (SQED). The emphasis will be on the Higgs mechanism and the way that it affects the quantization of the theory. The Higgs mechanism is the most important mechanism for endowing gauge bosons with mass, because it has a straightforward generalization to non-Abelian gauge theories.

In Lorentz-invariant SQED, the mass term produced by the Higgs mechanism resembles a Proca mass term. However, if the dynamics of the scalar field responsible for the gauge symmetry breaking are not Lorentz invariant, it is possible to have mass terms with different structures. Any Lorentz violation in the scalar sector will be transferred to the gauge sector when the Goldstone boson of the spontaneously broken symmetry is ‘‘eaten’’ by the gauge field—becoming the longitudinal component of the massive vector excitation. There have been some previous discussions of spontaneous symmetry breaking in the context of the full electroweak sector of the SME [2,44]. However, earlier work has not focused on how the Lorentz violation affects the gauge boson mass terms or the quantization of the theory. These will be our primary objects of study. Since Lorentz violation is physically a small effect, we shall generally only work to first order in the SME coefficients.

This paper is organized as follows: In Sec. II, we shall introduce the SQED Lagrange density with dimensionless Lorentz-violating coefficients. After including the effects of gauge symmetry breaking, we examine several sectors of the theory, paying particular attention to the structure of the gauge boson mass terms. Section III discusses the quantization of the spontaneously broken gauge theory, including the introduction of interacting Faddeev-Popov ghosts. Section IV recasts these results using a change of coordinates, which can be used to move certain types of Lorentz violation from one sector of the theory to another. Finally, Sec. V summarizes the paper's conclusions.

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II. LORENTZ-VIOLATING LAGRANGIANS

A. Lagrangian structure

The Lagrange density for our study of Lorentz-violating SQED is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}k_F^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + (g^{\mu\nu} + k_\Phi^{\mu\nu})(D_\mu\Phi)^*(D_\nu\Phi) + \mu^2\Phi^*\Phi - \frac{\lambda}{2}(\Phi^*\Phi)^2. \tag{1}$$

$D_\mu = \partial_\mu + ieA_\mu$ is the usual covariant derivative, and $V(\Phi) = -\mu^2\Phi^*\Phi + \frac{\lambda}{2}(\Phi^*\Phi)^2$ is the scalar potential. The Lorentz violation enters through the coefficients k_F in the gauge sector and k_Φ in the scalar sector. Both of these background tensors are dimensionless.

There are potentially also CPT-odd operators in both the scalar and vector sectors. However, the CPT-odd scalar coefficients a_Φ^μ are unobservable in a theory with only a single species of charged matter; they can be eliminated by a redefinition of the phase of the matter field. The gauge coefficients k_{AF}^μ are not so trivial; they generate birefringence in the gauge field propagation and may actually destabilize the theory. However, this birefringence effect is extremely tightly constrained; moreover, the k_{AF} does not interact with the Higgs mechanism in any particularly interesting fashion, and so k_{AF} will be neglected.

Any other Lorentz-violating terms constructed from the gauge and scalar fields would need to possess at least one of the following undesirable features [2]: explicit spacetime dependence (with an accompanying violation of energy-momentum conservation), nonrenormalizability (leading to suppression at low energies), gauge noninvariance (and nonconservation of the gauge charges), or nonlocality (threatening the unitarity of the theory). We shall not consider such terms, and this ensures that the scalar self-interaction term must take its usual, Lorentz-invariant form.

k_F has the symmetries of the Riemann tensor and a vanishing double trace, but the structure of of k_Φ is more subtle. Reality of the action requires that $k_\Phi^{\mu\nu} = k_S^{\mu\nu} + ik_A^{\mu\nu}$, where $k_S^{\mu\nu} = k_S^{\nu\mu}$ is symmetric and traceless in its Lorentz indices, while $k_A^{\mu\nu} = -k_A^{\nu\mu}$ is antisymmetric. The discrete symmetries of k_S are quite similar to those of k_F. In a Lorentz-violating theory, the three spatial reflections that together constitute P are generally inequivalent. Components of the tensors k_F and k_S are odd under a reversal of a specific space-time coordinate if that coordinate appears as a Lorentz index an odd number of times. Overall, a particular coefficient $k_F^{\mu\nu\rho\sigma}$ acquires a sign $(-1)^\mu(-1)^\nu(-1)^\rho(-1)^\sigma$ under either a P or T transformation, where $(-1)^\mu = 1$ if $\mu = 0$ and $(-1)^\mu = -1$ if $\mu = 1$, 2, or 3. The transformation of $k_S^{\mu\nu}$ is similarly associated with the sign $(-1)^\mu(-1)^\nu$. Both k_F and k_S are even under charge conjugation (C) and the combined operation CPT.

However, k_A has a different symmetry structure. $k_A^{\mu\nu}$ transforms as $(-1)^\mu(-1)^\nu$ under P, but the additional factor of i changes the transformation under C and T. Under T it transforms as $-(-1)^\mu(-1)^\nu$, because T is antilinear; k_A is also odd under C. Through integration by parts, the k_A term in $\mathcal{L}$ is equivalent to $\frac{1}{2}ek_A^{\mu\nu}\Phi^*\Phi F_{\mu\nu}$; since $\Phi^*\Phi$ is even under C, P, and T separately, this shows that $k_A^{\mu\nu}$ must have the same symmetries as $F_{\mu\nu}$. The fact that the k_A term in a minimally coupled but Lorentz-violating $\mathcal{L}$ can be written in this form means that any additional, nonminimal, dimension-four couplings between the scalar and gauge fields are redundant; their effects are already completely contained in k_A. This redundancy was not recognized in [44], which studied $ik_1^{\mu\nu}[(D_\mu\Phi)^*(D_\nu\Phi) - (D_\mu\Phi)(D_\nu\Phi)^*]$ and $k_2^{\mu\nu}\Phi^*\Phi F_{\mu\nu}$ forms of Lorentz violation separately.

B. Spontaneous Lorentz violation

The focus of this paper is primarily on spontaneous *gauge* symmetry breaking. However, the Lorentz-violating terms k_F, k_S, and k_A could also arise from spontaneous breaking of Lorentz symmetry. Spontaneous Lorentz breaking has many advantageous features as a way of introducing Lorentz violation into a theory. In particular, theories with spontaneous Lorentz violation are consistent with a pseudo-Riemannian geometric interpretation of gravitation, while explicitly Lorentz-violating theories generally are not.

The different Lorentz-violating coefficients might be generated by different forms of Lorentz symmetry breaking. If a symmetric two-index tensor field $X^{\mu\nu}$ has Lagrange density

$$\mathcal{L}_X = K_X(\partial^\alpha X^{\mu\nu}) - V_X(X^{\mu\nu}X_{\mu\nu}), \tag{2}$$

with kinetic term K_X and potential V_X, Lorentz symmetry will be broken if $V_X(\zeta)$ has a minimum for either $\zeta > 0$ or $\zeta < 0$ [45,46]. The Lorentz-violating vacuum expectation value of $X^{\mu\nu}$ is $x^{\mu\nu}$, and if $X^{\mu\nu}$ is coupled to other fields, $x^{\mu\nu}$ can generate either a k_S or k_F term. An interaction Lagrange density $g_SX^{\mu\nu}(D_\mu\Phi)^*(D_\nu\Phi)$ produces a k_S, and $g_FX^{\mu\nu}F_{\alpha\mu}F^\alpha{}_\nu$ a k_F. If there is only a single Lorentz-violating vacuum expectation value $x^{\mu\nu}$, the k_S and k_F terms may still have different magnitudes, if the couplings g_S and g_F differ.

The novel interactions above also include couplings of Φ and A to the part of $X^{\mu\nu}$ that represents fluctuations around the vacuum value $x^{\mu\nu}$. However, these interactions do not affect any of this paper's results concerning the propagation modes of the gauge and Higgs fields. Moreover, the interaction vertices involved are higher dimensional and thus generically suppressed.

A k_A term may also be generated if an antisymmetric two-index field $Y^{\mu\nu}$ with Lagrange density

$$\mathcal{L}_Y = K_Y(\partial^\alpha Y^{\mu\nu}) - V_Y(Y^{\mu\nu}Y_{\mu\nu}) \tag{3}$$

gets a vacuum expectation $y^{\mu\nu}$ [47]. A coupling $ig_A Y^{\mu\nu}[(D_\mu\Phi)^*(D_\nu\Phi) - (D_\mu\Phi)(D_\nu\Phi)^*]$ then produces the k_A term. (In fact, the antisymmetric $y^{\mu\nu}$ could also generate a $k_S^{\mu\nu}$ proportional to $y^{\mu\alpha}y^{\nu}{}_{\alpha}$, if the theory includes higher-order couplings.)

C. Spontaneous gauge symmetry breaking

Whether the Lorentz violation derives from spontaneous Lorentz symmetry breaking or some other mechanism is an important question. However, the answer has little direct bearing on the structure of the Higgs sector of Lorentz-violating SQED. We shall now turn our attention to the systematics of the Lorentz-violating Higgs mechanism.

Because of the "wrong sign" scalar mass term in Eq. (1), there are static solutions to the field equations with nonzero values of Φ. The Lorentz violation, which appears only in the kinetic terms, does not affect these solutions, which are derived from

$$\left.\frac{\delta\mathcal{L}}{\delta\Phi^*}\right|_{\text{static}} = \mu^2\Phi - \lambda(\Phi^*\Phi)\Phi = 0. \tag{4}$$

The static solutions Φ_0 must satisfy $|\Phi_0| = \frac{\mu}{\sqrt{\lambda}} \equiv v$. Such solutions obviously break the $U(1)$ gauge invariance associated with the gauge transformation

$$\Phi \to \Phi' = e^{i\alpha}\Phi, \tag{5}$$

$$\Phi^* \to \Phi'^* = e^{-i\alpha}\Phi^*, \tag{6}$$

$$A_\mu \to A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\alpha. \tag{7}$$

However, it is possible to make Φ_0 real by a gauge rotation and then decompose the field into its vacuum expectation value and excitations:

$$\Phi = v + \frac{1}{\sqrt{2}}(h + i\varphi), \tag{8}$$

where h is the Higgs field and φ represents the Goldstone boson.

The original Lagrange density $\mathcal{L}$ may be expanded in terms of these new variables, giving

$$\begin{aligned}\mathcal{L} = {} & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}k_F^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}(g^{\mu\nu} + k_\Phi^{\mu\nu})\{(\partial_\mu h)(\partial_\nu h) + (\partial_\mu\varphi)(\partial_\nu\varphi) + e^2h^2A_\mu A_\nu + e^2\varphi^2A_\mu A_\nu \\ & + 2\sqrt{2}e^2vhA_\mu A_\nu + 2e^2v^2A_\mu A_\nu + i[(\partial_\mu h)(\partial_\nu\varphi) - (\partial_\mu\varphi)(\partial_\nu h)] + i\sqrt{2}ev[(\partial_\mu h)A_\nu - A_\mu(\partial_\nu h)] \\ & + ie[(\partial_\mu h)(A_\nu h) - (A_\mu h)(\partial_\nu h)] + ie[(\partial_\mu\varphi)(A_\nu\varphi) - (A_\mu\varphi)(\partial_\nu\varphi)] - e[(\partial_\mu h)(A_\nu\varphi) + (A_\mu\varphi)(\partial_\nu h)] \\ & + \sqrt{2}ev[A_\mu(\partial_\nu\varphi) + (\partial_\mu\varphi)A_\nu] + e[(A_\mu h)(\partial_\nu\varphi) + (\partial_\mu\varphi)(A_\nu h)]\} - V(h,\varphi).\end{aligned} \tag{9}$$

The expansion of the potential around v takes the standard form,

$$V(h,\varphi) = -\frac{\mu^4}{\lambda} + \mu^2h^2 + \mu\sqrt{\frac{\lambda}{2}}h(h^2+\varphi^2) + \frac{\lambda}{8}(h^2+\varphi^2)^2. \tag{10}$$

The physical excitations of the theory are the (massive) gauge field A and the Higgs h. Note that because h and A are, respectively, even and odd under C, mixing between propagation states of these fields must involve the C-odd coefficient k_A.

The Goldstone boson field φ does not have physical excitations. By working in the unitarity gauge, we may choose the gauge parameter α so as to make Φ everywhere real (at the classical level). This eliminates φ from external states. However, quantum fluctuations in the φ field cannot be entirely eliminated, and the Goldstone boson field will appear as a virtual intermediary in loop calculations; this is actually crucial to the renormalizability and unitarity of SQED.

D. Propagation and interactions

Propagation of physical fields is governed by the portion $\mathcal{L}'_2 = \mathcal{L}_{2,Ah}$ of $\mathcal{L}$ that is bilinear in just A and h. This is

$$\begin{aligned}\mathcal{L}'_2 = {} & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}k_F^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \\ & + \frac{1}{2}(g^{\mu\nu} + k_S^{\mu\nu})(\sqrt{2}ev)^2A_\mu A_\nu \\ & + \frac{1}{2}(g^{\mu\nu} + k_S^{\mu\nu})(\partial_\mu h)(\partial_\nu h) \\ & - \frac{1}{2}(\sqrt{2}\mu)^2h^2 + \frac{1}{\sqrt{2}}evk_A^{\mu\nu}hF_{\mu\nu}.\end{aligned} \tag{11}$$

To the extent that the longitudinal component of the massive gauge field A is really the Goldstone boson of the broken symmetry, we should expect that the longitudinal A should propagate like Φ. As we shall see at the end of this section, the longitudinal part of A does indeed propagate like Φ at high energies, although this phenomenon is not evident from a naive inspection of the Lagrange density [Eq. (11)].

However, before addressing this point, we shall show how the mass and kinetic parts of the full bilinear Lagrange density $\mathcal{L}_2$, which includes the Goldstone bosons, combine to preserve the transversality of the gauge propagator. The gauge part of $\mathcal{L}_2$,

$$\mathcal{L}_{2,A} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}k_F^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}(g^{\mu\nu} + k_S^{\mu\nu})(\sqrt{2}ev)^2 A_\mu A_\nu, \tag{12}$$

is manifestly transverse, except for the term with the gauge boson mass $m_A = \sqrt{2}ev$, which is certainly not. However, there is also a vertex that mixes the gauge and Goldstone boson propagators; it comes from

$$\mathcal{L}_{2,A\varphi} = \mathcal{L}_{2,A} + \frac{1}{2}(g^{\mu\nu} + k_S^{\mu\nu})\{(\partial_\mu\varphi)(\partial_\nu\varphi) + m_A[A_\mu(\partial_\nu\varphi) + (\partial_\mu\varphi)A_\nu]\}. \tag{13}$$

To second order in m_A and first order in k_S, there are two possible insertions from $\mathcal{L}_{2,A\varphi}$ that contribute to the polarization tensor $i\Pi^{\mu\nu}(q)$. The first is the photon mass insertion, which contributes $im_A^2(g^{\mu\nu} + k_S^{\mu\nu})$. The second insertion involves two A-φ vertices, with a φ propagator between them. This propagator is

$$D_\varphi(q) = \frac{i}{q^2}\left(1 - k_S^{\gamma\delta}\frac{q_\gamma q_\delta}{q^2}\right), \tag{14}$$

making the polarization tensor

$$i\Pi^{\mu\nu}(q) = im_A^2(g^{\mu\nu} + k_S^{\mu\nu}) + [m_A(g^{\mu\alpha} + k_S^{\mu\alpha})q_\alpha] \times\left[\frac{i}{q^2}\left(1 - k_S^{\gamma\delta}\frac{q_\gamma q_\delta}{q^2}\right)\right][m_A(g^{\beta\nu} + k_S^{\beta\nu})(-q_\beta)] \tag{15}$$

$$= im_A^2\left[g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} + k_S^{\mu\nu} - k_S^{\mu\alpha}\frac{q_\alpha q^\nu}{q^2} - k_S^{\beta\nu}\frac{q^\mu q_\beta}{q^2} + k_S^{\gamma\delta}\frac{q^\mu q^\nu q_\gamma q_\delta}{(q^2)^2}\right]. \tag{16}$$

Although its structure is rather complicated, this tensor is transverse, $q_\mu\Pi^{\mu\nu} = 0$. This is a key consistency condition for the theory; it ensures the conservation of the total charge (including the charge present in the vacuum).

There are also terms in $\mathcal{L}_2$ that mix h with the other fields. However, they are less important, for two separate reasons. An insertion with an intermediate Higgs involves a massive propagator; without a pole at $q^2 = 0$, this cannot affect the pole structure of the gauge propagator. Moreover, any mixing of h with A or φ violates C. Since k_A is the only source of C violation in the theory, any modification of the A or φ propagator by a virtual h insertion will necessarily be second order in the Lorentz violation.

Special examples of Lorentz-violating mass terms of the general form

$$M^{\mu\nu}A_\mu A_\nu = \frac{1}{2}(g^{\mu\nu} + k_S^{\mu\nu})m_A^2 A_\mu A_\nu \tag{17}$$

have previously been studied. These mass terms were not considered in the context of the Higgs mechanism, but the earlier studies' conclusions about photon propagation remain valid even in the Higgs phase. Mass terms considered have included an isotropic but boost-invariance-violating $-\frac{1}{2}m_\gamma^2 A_j A_j$, as an alternative to the Proca mass term [48,49]; or $-\frac{e^2}{24\pi^2}(b^2 g^{\mu\nu} + 2b^\mu b^\nu)$, which could be generated by unusual radiative corrections [50]. (Note however, that while the gauge boson mass in these situations is assumed to be small, the Lorentz-violating and Lorentz-invariant parts of the mass term are of comparable size.) Most recently, Lorentz-violating Stueckelberg mass terms have also been considered [51]. The previous analyses of these models have demonstrated an interesting interplay between Lorentz-violating mass terms and the kinetic part of the gauge-sector Lagrangian. In the concrete examples that were considered in [48–50], there were only two distinct eigenvalues in the mass-squared matrix $M^\mu{}_\nu$. If the eigenvalue $\frac{1}{2}m_0^2$ corresponding to the timelike direction is smaller in magnitude than a spacelike eigenvalue $\frac{1}{2}m_1^2$, there could be propagation with signal and group velocities greater than 1 and as large as $\frac{m_1}{m_0}$. However, this superluminal propagation is limited to modes that are approximately longitudinal.

This shows that the existence of a Lorentz-violating mass term can have profound effects on the propagation of gauge bosons, even when the bosons' momenta are far above the mass scale m_A. The mass term (which might be expected to be important only in the infrared) affects the ultraviolet behavior of the theory through its influence on the gauge. Requiring charge conservation forces A to obey a gauge condition $M^{\mu\nu}\partial_\mu A_\nu = 0$. The relative sizes of the elements of the mass matrix $M^\mu{}_\nu$ determine the required gauge. However, the absolute magnitudes of the matrix components are irrelevant; the gauge condition produced by a mass matrix $\zeta M^\mu{}_\nu$ is independent of ζ.

When a Lorentz-violating gauge field mass term arises through the Higgs mechanism, there is a clear physical mechanism underlying superluminal propagation. If the timelike eigenvalue of $k_S^\mu{}_\nu$ is λ_0 and the largest spacelike eigenvalue is $\lambda_1 > \lambda_0$, the free Φ field has a kinetic term that supports propagation up to speeds of $\sqrt{\frac{\lambda_1}{\lambda_0}}$. When the Goldstone boson is eaten by the gauge field, this possibility for superluminal propagation is transferred to the gauge field, although the Lorentz-violating term that makes this possible is part of the mass term in $\mathcal{L}_A$, rather than the kinetic term.

In addition to the propagation governed by $\mathcal{L}_2$, there are also interaction vertices in the theory. For tree-level calculations, only those vertices involving A and h are needed. These vertices are given by the interaction Lagrange density

$$\mathcal{L}_I' = \frac{1}{2}(g^{\mu\nu} + k_S^{\mu\nu})e^2(h^2 + 2vh)A_\mu A_\nu - \frac{1}{2}ek_A^{\mu\nu}[h(\partial_\mu h)A_\nu - h(\partial_\nu h)A_\mu] - \mu\sqrt{\frac{\lambda}{2}}h^3 - \frac{\lambda}{8}h^4. \tag{18}$$

This includes the usual Higgs self-interaction terms from Φ^4 theory, as well as a seagull vertex (involving two Higgs and two gauge fields) and a related three-particle vertex with one of the Higgs fields replaced by the vacuum expectation value v. The seagull and three-field vertices have their Lorentz structures modified by k_S in precisely the same way as the gauge boson mass term.

The remaining terms in $\mathcal{L}_2'$ and $\mathcal{L}_I'$ are the C-violating terms involving k_A. These can be expressed in terms of the gauge field strength. The three-field interaction is equivalent to $\frac{1}{4}ek_A^{\mu\nu}h^2F_{\mu\nu}$, and there is no Lorentz-invariant analogue for such a term. Terms involving k_S are similar to Lorentz-invariant terms, in that they involve replacing the Minkowski metric tensor $g^{\mu\nu}$ with an arbitrary symmetric $k_S^{\mu\nu}$. In contrast, there is no Lorentz-invariant, antisymmetric, two-index tensor to be contracted with $F^{\mu\nu}$, so the $k_A^{\mu\nu}F_{\mu\nu}$ interactions have a uniquely Lorentz-violating structure.

III. QUANTIZATION AND GHOST FIELDS

Calculation of quantum corrections for a theory with spontaneously broken gauge symmetry requires the introduction of a gauge-fixing term in the action (which leads naturally to the inclusion of Faddeev-Popov ghosts). The gauge-fixing term serves two purposes. It can eliminate the zero modes in the gauge field action, which is necessary for the derivation of a well-defined propagator; the gauge-fixing term fulfills this function in all gauge theories, whether or not they involve spontaneous symmetry breaking. However, when the gauge symmetry is broken, the gauge-fixing function may also be chosen to remove any terms that mix the gauge and Goldstone boson fields.

To quantize the gauge field according to the Faddeev-Popov procedure [52], we begin with the gauge-invariant functional integral for the theory and insert the identity, in the form

$$\mathbb{1} = \int \mathcal{D}\alpha(x)\delta[G(A', h', \varphi') - \omega]\det\left[\frac{\delta G(A', h', \varphi')}{\delta\alpha}\right], \tag{19}$$

where $A_\mu' = A_\mu - \frac{1}{e}\partial_\mu\alpha$, $h' = h - \alpha\varphi$, and $\varphi' = \varphi + \alpha(\sqrt{2}v + h)$ are the infinitesimally gauge-transformed fields from Eqs. (5)–(7). The gauge-fixing function $[G(A, h, \varphi) - \omega]$ is then integrated over a Gaussian distribution of ω values. We take

$$G = \frac{1}{\sqrt{\xi}}[(g^{\mu\nu} + k_G^{\mu\nu})\partial_\mu A_\nu - \sqrt{2}\xi ev\varphi]. \tag{20}$$

The Lorentz-invariant terms in Eq. (20) are identical to those in the gauge-fixing function for the R_ξ gauge, and k_G is an (as yet undetermined) Lorentz-violating tensor coefficient. It is not possible to include Lorentz violation in the φ part of G without introducing higher derivatives into the final ghost action.

The Faddeev-Popov procedure introduces two new sets of terms into the Lagrange density. The first set is the result of the integration over ω,

$$-\frac{1}{2}G^2 = -\frac{1}{2\xi}[(g^{\mu\nu} + k_G^{\mu\nu})\partial_\mu A_\nu]^2 - \sqrt{2}ev(g^{\mu\nu} + k_G^{\mu\nu})(\partial_\mu\varphi)A_\nu - \xi e^2v^2\varphi^2. \tag{21}$$

The Lorentz violation k_G in the gauge-fixing function should be chosen to eliminate the A-φ mixing term in $\mathcal{L}_2 - \frac{1}{2}G^2$. This requires $k_G^{\mu\nu} = k_S^{\mu\nu}$, and the gauge part of $\mathcal{L}_2$ becomes

$$\begin{aligned}\mathcal{L}_{2,A} &- \frac{1}{2\xi}[(g^{\mu\nu} + k_G^{\mu\nu})\partial_\mu A_\nu]^2 \\ &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 + \frac{1}{2}m_A^2A^\mu A_\mu \\ &\quad - \frac{1}{4}k_F^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{\xi}k_S^{\mu\nu}(\partial_\mu A_\nu)(\partial^\rho A_\rho) \\ &\quad + \frac{1}{2}k_S^{\mu\nu}m_A^2A_\mu A_\nu.\end{aligned} \tag{22}$$

The Lorentz-violating kinetic terms can be recast as $-k_\xi^{\mu\nu\rho\sigma}(\partial_\mu A_\nu)(\partial_\rho A_\sigma)$, where $k_\xi^{\mu\nu\rho\sigma} = \frac{1}{4}k_F^{\mu\nu\rho\sigma} + \frac{1}{\xi}g^{\mu\nu}k_S^{\rho\sigma}$. The $(\partial^\mu A_\mu)^2$ term combines with the Maxwell and Proca terms to produce the usual propagator

$$D_A^{\mu\nu}(q) = \frac{-i}{q^2 - m_A^2}\left[g^{\mu\nu} - (1 - \xi)\frac{q^\mu q^\nu}{q^2 - \xi m_A^2}\right], \tag{23}$$

and the Lorentz-violating terms may be treated as vertices. The ξ-dependent part of the k_ξ vertex is superficially similar in structure to the k_F part. However, while the k_F term only involves the physical degrees of freedom contained in $F^{\mu\nu}$, the gauge-fixing part only involves purely gauge degrees of freedom, since k_G couples to the symmetric part of $\partial_\mu A_\nu$.

The other terms that the Faddeev-Popov procedure adds to $\mathcal{L}$ come from the functional determinant in Eq. (19). Since

$$\frac{\delta G}{\delta\alpha} = \frac{\delta G}{\delta A_\mu}\left(\frac{1}{e}\partial_\mu\right) + \frac{\delta G}{\delta\varphi}(v + h) \tag{24}$$

$$= \frac{1}{\sqrt{\xi}}\left[(g^{\mu\nu} + k_S^{\mu\nu})\left(-\frac{1}{e}\partial_\mu\partial_\nu\right) - \xi m_A(\sqrt{2}v + h)\right], \tag{25}$$

$\det[\delta G/\delta\alpha]$ may be exponentiated as a part of the action by introducing ghost fields c and $\bar{c}$ with Lagrange density

$$\mathcal{L}_c = (g^{\mu\nu} + k_S^{\mu\nu})(\partial_\mu \bar{c})(\partial_\nu c) - \xi m_A^2 \left(1 + \frac{h}{\sqrt{2}v}\right)\bar{c}c. \quad (26)$$

The (gauge-dependent) mass term for the Faddeev-Popov ghosts is unaffected by the Lorentz violation; the interaction vertex with the Higgs field is also unmodified. However, the ghosts do acquire a modification to their kinetic term, equivalent to the k_S for the original scalar field Φ. For each of the spinless fields (h, φ, and c), the Lorentz violation may be treated as a vertex to be inserted along propagation lines. Several loop diagrams involving the k_S in the ghost sector have already been evaluated [53].

IV. COORDINATE REDEFINITIONS

The appearance of the same Lorentz-violating coefficients k_S in the Higgs, Goldstone boson, and ghost sectors may be unsurprising, because of the structure of the k_S term. If k_F vanishes in the original Lagrange density $\mathcal{L}$, then k_S describes a mismatch between the natural coordinates for describing the gauge and matter fields. Having a vanishing k_F means that the chosen coordinates are natural for the gauge field. However, redefining coordinates according to

$$x^\mu \rightarrow x'^\mu = x^\mu - \frac{1}{2}k_S{}^\mu{}_\nu x^\nu \quad (27)$$

will transform the Lorentz violation coefficients in $\mathcal{L}$ to

$$k_\Phi^{\mu\nu} \rightarrow k_\Phi'^{\mu\nu} = ik_A^{\mu\nu} \quad (28)$$

$$k_F^{\mu\nu\rho\sigma} \rightarrow k_F'^{\mu\nu\rho\sigma} = k_F^{\mu\nu\rho\sigma} - \frac{1}{2}(g^{\mu\rho}k_S^{\nu\sigma} - g^{\mu\sigma}k_S^{\nu\rho} - g^{\nu\rho}k_S^{\mu\sigma} + g^{\nu\sigma}k_S^{\mu\rho}). \quad (29)$$

If this transformation is made prior to the calculations, many of the Lorentz-violating terms that could appear after spontaneous symmetry breaking are actually absent. By eliminating k_S prior to quantization and spontaneous symmetry breaking, we can ensure that there is no Lorentz-violating modification of the gauge field mass term, nor is any Lorentz violation required in the ghost sector.

In fact, it is straightforward to see how a transformation that eliminates k_S from the kinetic term for Φ likewise eliminates k_S from the ghost kinetic term. Both terms have the same basic scalar kinetic structure, and a transformation that carries $(g^{\mu\nu} + k_S^{\mu\nu})\partial_\mu\partial_\nu \rightarrow \partial^\mu\partial_\mu$ will have the same effect in either sector. Accompanying the redefinition of the coordinates in Eq. (27) must be a similar linear reshuffling of the gauge fields; the transformed A'_μ must be exactly what enters in conjunction with $\partial'_\mu \equiv \frac{\partial}{\partial x'^\mu}$ in the covariant derivative. So Eq. (27) simply takes $(g^{\mu\nu} + k_S^{\mu\nu})A_\mu A_\nu \rightarrow A^\mu A_\mu$.

Since it is possible to define away the k_S Lorentz violation, we might be tempted to dismiss analyses that include k_S as entirely unnecessary. However, since the transformation that eliminates k_S is a global redefinition of the coordinates, it can only be used to eliminate this type of Lorentz violation from a single sector. This is already evident from the fact that removing k_S from the matter sector introduces it into the k'_F of the gauge sector. Ultimately, physical observables that depend on k_S need to involve differences between SME coefficients across different sectors. In pure SQED, the only observable difference is $k_S^{\mu\nu} - k_{F\alpha}{}^{\mu\alpha\nu}$.

If both the Lorentz violation and the mass are small enough to be treated as perturbations, it is straightforward to determine the dispersion relations for the gauge field modes in the coordinate system with all Lorentz violation moved into the gauge sector. For the transverse polarization states with wave vector $\vec{q}$, the frequencies are [54]

$$q_0^\pm = |\vec{q}|[1 + \rho(\hat{q}) \pm \sigma(\hat{q})] + \frac{m_A^2}{2|\vec{q}|}, \quad (30)$$

where $\rho(\hat{q}) = -\frac{1}{2}\tilde{k}^\alpha{}_\alpha$, and $\sigma^2(\hat{q}) = \frac{1}{2}\tilde{k}^{\alpha\beta}\tilde{k}_{\alpha\beta} - \rho^2(\hat{q})$, with $\tilde{k}^{\alpha\beta} = k_F'^{\alpha\mu\beta\nu}\hat{q}_\mu\hat{q}_\nu$ and $\hat{q}^\mu = (1, \vec{q}/|\vec{q}|)$. The result in Eq. (30) simply represents the conventional dispersion relation, plus the usual perturbations due to the k'_F Lorentz violation and the mass $m_A \ll |\vec{q}|$.

However, there is also a longitudinal polarization state, whose energy is not affected by k'_F at leading order,

$$q_0 = |\vec{q}| + \frac{m_A^2}{2|\vec{q}|}. \quad (31)$$

The reason that k'_F does not affect this dispersion relation is that the presence of the mass term forces A to obey the Lorenz gauge condition $q^\mu A_\mu = 0$ (plus Lorentz-violating corrections that may be neglected at this order). This makes the k'_F term in the equation of motion for the longitudinal mode vanish identically. The lack of any dependence on k'_F might initially seem puzzling, but it is actually quite natural. Since m_A^2 was treated as a perturbation, Eq. (31) applies only in the high-energy regime, when the momentum $|\vec{q}|$ is large compared with the Higgs mass scale. In that regime, the longitudinal component of the gauge field essentially becomes indistinguishable from the uneaten Goldstone boson. The propagation of the longitudinal mode should therefore be governed only by the Lorentz-violating tensor k'_S in the Higgs sector, and in the transformed coordinates used to derive Eq. (31), k'_S vanishes.

Of course, these dispersion relations may be transformed back into the original coordinates with nonzero k_S simply by inverting the coordinate redefinition [Eq. (27)], so that $q^\mu \rightarrow q^\mu - \frac{1}{2}k_S{}^\mu{}_\nu q^\nu$. The result for the longitudinal mode is

$$q_0 = |\vec{q}|\left[1 - \frac{1}{2}k_S^{00} - 2k_S{}^0{}_j\hat{q}_j + \frac{1}{2}k_S{}^j{}_l\hat{q}_j\hat{q}_l\right] + \frac{m_A^2}{2|\vec{q}|}. \quad (32)$$

This exhibits exactly the same kind of potentially superluminal behavior for the longitudinal mode as was discussed in Sec. II D, with the limiting speed controlled by the relative sizes of the spacelike and timelike eigenvalues of $k_S{}^{\mu}{}_{\nu}$.

This analysis also provides insight into another feature of the non-Higgs mass models discussed in [50]. The normal modes of propagation in the presence of the Lorentz-violating mass term do not involve orthogonal polarization vectors. This is related to the nonorthogonal nature of the transformation [Eq. (27)]; a coordinate redefinition that moves Lorentz violation from the gauge field kinetic term to the mass term changes an orthogonal basis of polarization states into a nonorthogonal one. In fact, the transformation required to turn a Lorentz-invariant Proca mass term into the term $-\frac{e^2}{24\pi^2}(b^2 g^{\mu\nu} + 2b^{\mu}b^{\nu})$ from [50] would produce extremely skewed coordinates. This is a reminder that, while the gauge boson mass parameters in [48–50] may be small, the Lorentz violation for the theories involved is, in a meaningful sense, quite large—with the equivalent of k_S being $\mathcal{O}(1)$.

V. CONCLUSION

The focus of this paper has been on SQED with Lorentz violation. With a single scalar field Φ and a single gauge field A, all possible forms of renormalizable, CPT-even Lorentz violation are captured in the coefficients k_{Φ} and k_F, with minimal coupling of the gauge and matter fields through the covariant derivative D_{μ}. In standard SQED, spontaneous breaking of the $U(1)$ gauge symmetry makes the gauge boson massive. We have shown that the Lorentz-violating theory includes an analogous mass term, with a Lorentz-violating generalization of the Proca form.

We have displayed the full Lagrange density for this theory and for the first time introduced the Faddeev-Popov ghosts that are a necessary part of the quantization procedure. The effects of Lorentz violation on the ghosts has already been studied for non-Abelian gauge theories [41], but not for theories with a broken gauge symmetry. Knowledge of the full Faddeev-Popov Lagrange density will make it possible to perform Feynman diagram calculations in the present theory.

We have also shown how Lorentz violation in the scalar and gauge sectors affects the propagation of the physical gauge and Higgs modes. Even with a conventional kinetic term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ for the gauge field, it may be possible for the longitudinal mode to propagate superluminally. This can happen because the longitudinal mode is really an "eaten" Goldstone boson, whose behavior is primarily governed by the structure of the scalar sector.

Quantum field theories involving gauge interactions with charged scalar matter are important; in the standard model, the Higgs sector is responsible for the existence of particle masses. The full treatment of quantum corrections in the SME is an interesting theoretical problem, and this work presents an important step toward a complete understanding of the SME.

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