Two-post-Newtonian light propagation in the scalar-tensor theory: An *N*-point mass case

Xue-Mei Deng[1,3,*] and Yi Xie[2,3,†]

[1]*Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China*
[2]*Department of Astronomy, Nanjing University, Nanjing 210093, China*
[3]*Key Laboratory of Modern Astronomy and Astrophysics, Nanjing University, Ministry of Education, Nanjing 210093, China*

Within the framework of the scalar-tensor theory, its second post-Newtonian (2PN) approximation is obtained with Chandrasekhar's approach. By focusing on an *N*-point mass system as the first step, we reduce the metric to its 2PN form for light propagation. Unlike previous works, at 2PN order, we abandon the hierarchized hypothesis and do not assume two parametrized post-Newtonian parameters γ and β to be unity. We find that although there exist γ and β in the 2PN metric, only γ appears in the 2PN equations of light. As a simple example for applications, a gauge-invariant angle between the directions of two incoming photons for a differential measurement is investigated after the light trajectory is solved in a static and spherically symmetric spacetime. It shows the deviation from the general relativity $\delta\theta_{\mathrm{STT}}$ does *not* depend on β even at 2PN level in this circumstance, which is consistent with previous results. A more complicated application is light deflection in a 2-point mass system. We consider a case that the light propagation time is much less than the time scale of its orbital motion and thus treat it as a static system. The 2-body effect at 2PN level originating from relaxing the hierarchized hypothesis is calculated. Our analysis shows the 2PN 2-body effect in the Solar System is one order of magnitude less than future ~1 nano-arcsecond experiments, while this effect could be comparable with first post-Newtonian component of $\delta\theta_{\mathrm{STT}}$ in a binary system with two Sun-like stars and separation by ~0.1 AU if an experiment would be able to measure $\gamma - 1$ down to $\sim10^{-6}$.

I. INTRODUCTION

Some future space missions, such as the laser astrometric test of relativity (LATOR) [1,2], phobos laser ranging [3], the beyond Einstein advanced coherent optical network (BEACON) [4], the Télémétrie InterPlanétaire Optique (TIPO) [5], the astrodynamical space test of relativity using optical devices [6] and the search for anomalous gravitation using atomic sensors [7], will measure distances of laser links and angles among these links with unprecedented precision. As a sensitive and useful tool in gravitational physics, especially for some high-order effects, the propagation of light carries lots of information about the nature of spacetime and plays an important role in high-precision experiments and measurements.

Thus, from the practical and theoretical aspects, it motivates us to investigate a second post-Newtonian (2PN) light propagation model at the c^{-4} level in the framework of the scalar-tensor theory (STT) for a gravitational *N*-point mass system.

A. The necessity of 2PN c^{-4} terms

For the precision of distance measurements, TIPO could achieve millimeter level [5] and BEACON could be even higher, reaching 0.1 nm [4]. As explained in Ref. [8] (in Sec. V), a complete metric of the Solar System up to c^{-4} level is demanded for modeling the light propagation in

those experiments. For the angle measurements, the precision of LATOR could achieve nano-arcsecond (nas) level or higher [1,2]. It would require c^{-4} terms in the metric to calculate the deflection of light ray, because when light grazes solar limb the 2PN deflection contributed by the Sun is $G^2 m_\odot^2/(c^4 R_\odot^2) \approx 10^{-12} \sim$ micro-arcsecond (μas), which is much larger than the threshold of LATOR. The corrections of alternative theories of gravity to this effect at 2PN order would be at least several orders of magnitude less than $\sim\mu$as. However, these corrections perhaps still need to be considered for future experiments.

Several authors have obtained the 2PN metric for the general relativity (GR) and alternative theories of gravity. Chandrasekhar and Nutku first calculated the 2PN metric and equations of hydrodynamics in GR [9]. A field theory approach is also employed to derive the 2PN Lagrangian for the equations of motion of *N* bodies in the multi-scalar-tensor theory without solving the metric [10]. With introducing an intermediate-range gravity term, the 2PN approximation of the scalar-tensor theory was obtained [11] in which the energy-momentum tensor was expressed by using the invariant density [12]. The 2PN approximation of Einstein-aether (ae-theory) theory [13] was deduced in the form of both superpotentials and an *N*-point mass. More recently, the IAU2000 resolutions are extended to include all the c^{-4} terms for the requirements from some space missions [8]. Most of such works express the 2PN metric in terms of superpotentials without definition of the masses and multipole moments of the bodies which is not

*xmd@pmo.ac.cn
†yixie@nju.edu.cn

trivial in the post-Newtonian order especially for the self-gravitating bodies in an N-massive bodies system.

An explicit application of them is to model light propagation in the near zone of a gravitational system into the level of the next leading order. The 2PN light deflection for one body in the standard parametrized post-Newtonian (PPN) formalism [14,15] was studied [16,17] by introducing another parameter (ϵ or Λ) at c^{-4} of g_{ij}, the space-space component of metric, under the isotropic gauge. The 2PN light propagation in the Schwarzschild spacetime was researched in three gauges (standard, harmonic and isotropic) through introducing two coordinate parameters [18]. Reference [19] discussed the significance of some experiments to observe 2PN light effects for one body in the isotropic gauge. A practical relativistic model for the 2PN light propagation was developed in the harmonic gauge of general relativity [20], in which the first post-Newtonian (1PN) contributions from the rectilinear and uniform N-body and the 2PN contributions *only* from the Sun were considered. The light propagation in 2PN framework of a stationary gravitational field for the Schwarzschild metric (one body) was formulated in the harmonic gauge with introducing one parameter ϵ at spatial isotropic and anisotropic terms of c^{-4} for g_{ij} [21]. The 2PN deflection of light in a spherically symmetric body was discussed in GR in three gauges (standard, harmonic and isotropic) [22]. All of above works mentioned are only considering one body in the 2PN order. The 2PN gravitational redshift in GR was derived [23] by superpotentials. Under the $f(R)$ theory, the 2PN weak lensing was explored in the isotropic gauge [24].

B. The reason for a scalar-tensor theory

Although Einstein's general relativity has passed nearly all the tests in the Solar System, alternative theories of gravity are still required for deeper understanding of the nature of spacetime and for testing possible violations of the Einstein equivalence principle (EEP) in the forthcoming more precision level [25]. In order to test and distinguish alternative gravitational theories, the PPN formalism introduces ten parameters in a post-Newtonian metric to include various gravity theories [14,15]. However, the PPN formalism is only restricted to the 1PN approximation. Some authors (e.g. [26–28]) discussed how to parametrize the 2PN metric. To extend the PPN formalism, one possible way might be to derive 2PN metric for various gravity theories and then find the independent superpotentials in the 2PN level. After that, parameters in the 2PN order could be introduced and endowed with some meanings. However, in this paper, we only focus on one gravitational theory and discuss its parameters.

Among these alternative theories, the most eminent case is a STT because it is the simplest and most natural way to modify GR. Many modern theories, such as the extra-dimensional theory, the string theory, the braneworld and the noncommutative geometry theories, which try to unify gravity and microscopic physics or to explain the dark energy in cosmology, demand a scalar field besides the metric tensor (see [29] for a review). Especially, Damour and Nordtvedt proposed [30] that the deviation of the PPN parameter γ from 1, presenting the discrepancy between STT and GR, might range from $\sim 10^{-7}$ to $\sim 10^{-5}$. The contribution of this deviation in light deflection at 2PN order could be as large as $(\gamma - 1)G^2 m_\odot^2/(c^4 R_\odot^2) \approx 10^{-16} \sim 0.02$ nas, which might be important for future detection as well. Therefore, we shall work in this paper with the scalar-tensor theory of gravity.

Motivated by forthcoming space experiments involving propagation of light in the Solar System, some researchers had studied 2PN light propagation in STT. In the case of a one-body system, 2PN light deflection and light propagation were reported in Refs. [31,32]. As a more comprehensive work, the scalar-tensor propagation of light was investigated in Ref. [33], in which it neglected the differences of PPN parameters γ and β from 1 at c^{-4} level according to present experiment results and used the hierarchized hypothesis so that only the Sun's contribution remained at c^{-4} level while influences from Sun and the planets were all included at c^{-2} and c^{-3} terms.

To make our results fully based on STT, we will keep γ and β at c^{-4} level which requires a PPN definition of mass, such as Eq. (33).

C. The extension to an N-point mass system

As mentioned in Ref. [33], the hierarchized hypothesis has its own limitation. In the practical point of view, by abandoning this hypothesis which means extending to an N-body system, the solution of light equations might improve the accuracy which could be achieved [33]. As a rude estimation, one coupling term in 2PN metric $\sum_a \sum_{b \neq a} G^2 m_a m_b/(c^4 r_a r_b)$, where $r_a = |\boldsymbol{x} - \boldsymbol{y}_a|$, $r_b = |\boldsymbol{x} - \boldsymbol{y}_b|$ and the trajectories of the a-th and b-th masses are respectively represented by $\boldsymbol{y}_a(t)$ and $\boldsymbol{y}_b(t)$, could reach $\sim 10^{-16}$ (equivalent to ~ 0.1 nas), if we consider the Solar System (a hierarchic system) that a-th mass is the Sun and the b-th mass is Jupiter when the light grazes Jupiter's limb. If the influence to the background light sources by a binary system with comparable masses are considered, the coupling term in 2PN might raise larger contributions. Extension to an N-body system would also be helpful to determine which terms have to be included for a specific mission and could be a good test bed to evaluate the accuracy of the hierarchized model [33].

In the theoretical point of view, it is a natural development to build a 2PN theory of a gravitational N-body system for its dynamics and light propagation within it which might show some subtle but interesting effects due to the nonlinearity in the 2PN order. In principle, the 2PN light propagation under the STT should include relativistic multipolar moments of each body in the N-body system.

1PN global and local metrics with the definitions of the multipolar moments and spins in STT have been given in Refs. [34,35].

Therefore, in this paper, we consider 2PN light propagation in an N-point mass system under STT as our first step.

In what follows, our conventions and notations generally follow those of Ref. [36]. The metric signature is $(-, +, +, +)$. Greek indices take the values from 0 to 3, while Latin indices take the values from 1 to 3. A comma denotes a partial derivative, and dot over a quantity denotes a derivative with respect to time. Bold letters denote spatial vectors. The plot of this paper is as follows. In Sec. II, the 2PN metric and equation of light in the scalar-tensor theory are given. Subsequently, in Sec. III, we reduce the results of Sec. II to a 2PN metric of an N-point mass system for light propagation by following the method used in Ref. [37]. And the 2PN light equation is obtained in this condition. In Sec. IV, applications will be given. In Sec. IV A, we derive the 2PN light ray trajectory and deflection in a static, spherically symmetric spacetime and study the parameters in 2PN terms by comparison among the scalar-tensor theory and others. In Sec. IV B, we calculate the light deflection angle caused by a 2-point mass system and estimate the magnitudes of effects for different cases. Finally, the conclusion and discussion are outlined in Sec. V.

II. 2PN METRIC IN THE SCALAR-TENSOR THEORY AND EQUATIONS OF LIGHT

A. 2PN metric in the scalar-tensor theory

The action for the scalar-tensor theory based on [34] reads

$$S = \frac{c^3}{16\pi} \int \left(\phi R - \frac{\theta(\phi)}{\phi} \phi^{,\sigma} \phi_{,\sigma} - \frac{16\pi}{c^4} \mathcal{L}_I(g_{\mu\nu}, \Psi) \right) \\ \times \sqrt{-g} d^4 x, \tag{1}$$

where c is the speed of light, $\theta(\phi)$ is an arbitrary coupling function of the scalar field ϕ. R and $g = \det(g_{\mu\nu})$ denote, respectively, the Ricci scalar and the determinant of the metric tensor $g_{\mu\nu}$. the matter field is denoted by Ψ. From Eq. (1), we can see the matter fields Ψ only interact with the metric field (namely, $g_{\mu\nu}$). This means the trajectory of a free fall test particle only depends on the spacetime geometry so that it satisfies EEP. Although violations of EEP at galactic and cosmological scales cannot be ruled out, we still focus on the scalar-tensor theory satisfied EEP in this paper.

Variation of the action (1) with respect to $g^{\alpha\beta}$ has

$$R_{\mu\nu} = \frac{8\pi}{\phi c^2} \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) + \frac{\theta(\phi)}{\phi^2} \phi_{,\mu} \phi_{,\nu} \\ + \frac{1}{\phi} \left(\phi_{;\mu\nu} + \frac{1}{2} g_{\mu\nu} \Box_g \phi \right), \tag{2}$$

where $\Box_g(\cdot) = (\cdot)_{;\mu\nu} g^{\mu\nu}$, $T_{\mu\nu}$ is the stress-energy-momentum tensor of matter which is defined by

$$\frac{c^2}{2} \sqrt{-g} T_{\mu\nu} \equiv \frac{\partial(\sqrt{-g} \mathcal{L}_I)}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\alpha} \frac{\partial(\sqrt{-g} \mathcal{L}_I)}{\partial g_{,\alpha}^{\mu\nu}}, \tag{3}$$

and T is the trace of $T^{\mu\nu}$. Following Refs. [38,39], the mass, current, and stress densities can be defined as

$$\sigma \equiv T^{00} + T^{ii}, \tag{4}$$

$$\sigma_i \equiv c T^{0i}, \tag{5}$$

$$\sigma_{ij} \equiv c^2 T^{ij}. \tag{6}$$

Variation of the action (1) with respect to ϕ yields

$$\Box_g \phi = \frac{1}{3 + 2\theta(\phi)} \left(\frac{8\pi}{c^2} T - \phi_{,\alpha} \phi^{,\alpha} \frac{d\theta}{d\phi} \right). \tag{7}$$

Based on Ref. [34], the scalar field is expanded as its background value ϕ_0 as follows:

$$\phi = \phi_0(1 + \zeta), \tag{8}$$

where ζ is a dimensionless perturbation of the background value ϕ_0. Especially, decomposition of the coupling function $\theta(\phi)$ reads

$$\theta(\phi) = \omega_0 + \omega_1 \zeta + \frac{1}{2} \omega_2 \zeta^2 + \dots, \tag{9}$$

where $\omega_0 \equiv \theta(\phi_0)$, $\omega_1 \equiv (d\theta/d\zeta)_{\phi=\phi_0}, \dots, \omega_n \equiv (d^n\theta/d\zeta^n)_{\phi=\phi_0}$, which can lead to the gauge-invariant definitions of PPN parameters γ and β [see Eqs. (A3) and (A10) in Appendix A]

$$\gamma \equiv \frac{\omega_0 + 1}{\omega_0 + 2}, \qquad \beta \equiv 1 + \frac{\omega_1}{(2\omega_0 + 3)(2\omega_0 + 4)^2}.$$

By using Chandrasekhar's approach [9,40], we deal with the theory in the form of a Taylor expansion in the parameter $\varepsilon \equiv 1/c$. The expansions of the metric $g_{\mu\nu}$ and the scalar field to the second order have the forms of

$$g_{00} = -1 + \varepsilon^2 N + \varepsilon^4 L + \varepsilon^6 Q + \mathcal{O}(7), \tag{10}$$

$$g_{0i} = \varepsilon^3 L_i + \varepsilon^5 Q_i + \mathcal{O}(6), \tag{11}$$

$$g_{ij} = \delta_{ij} + \varepsilon^2 H_{ij} + \varepsilon^4 Q_{ij} + \mathcal{O}(5), \tag{12}$$

$$\zeta = \varepsilon^2 \overset{(2)}{\zeta} + \varepsilon^4 \overset{(4)}{\zeta} + \varepsilon^6 \overset{(6)}{\zeta} + \mathcal{O}(7), \tag{13}$$

where $O(n)$ means of the order ε^n.

According to Eqs. (8), (9), and (13), we derive

$$\theta(\phi) = \omega_0 + \varepsilon^2 \omega_1 \overset{(2)}{\zeta} + \varepsilon^4 \left(\frac{1}{2} \omega_2 \overset{(2)}{\zeta}^2 + \omega_1 \overset{(4)}{\zeta} \right) + \mathcal{O}(5), \tag{14}$$

$$\frac{d\theta}{d\phi} = \frac{1}{\phi_0} (\omega_1 + \varepsilon^2 \omega_2 \overset{(2)}{\zeta} + \mathcal{O}(4)). \tag{15}$$

We use the gauge condition imposed on the component of the metric tensor proposed by Kopeikin and Vlasov [34] as follows:

$$\left(\frac{\phi}{\phi_0}\sqrt{-g}g^{\mu\nu}\right)_{,\nu} = 0. \tag{16}$$

Based on fields equations of Eqs. (2) and (7), we obtain the evolution equations of the metric coefficients at 2PN order by using the gauge conditions Eq. (16) (see Appendix A for detail).

B. 2PN equations of light

Generally, for a photon propagating in a spacetime in which EEP is valid, the basic equations of light based on Ref. [41] are

$$0 = g_{\mu\nu}\frac{dx^{\mu}}{dt}\frac{dx^{\nu}}{dt}, \tag{17}$$

$$\frac{d^2x^i}{dt^2} = \left(\varepsilon\Gamma^0{}_{\nu\sigma}\frac{dx^i}{dt} - \Gamma^i{}_{\nu\sigma}\right)\frac{dx^{\nu}}{dt}\frac{dx^{\sigma}}{dt}, \tag{18}$$

where we have replaced the affine parameter with coordinate time t. Following Ref. [21], assuming $\dot{\boldsymbol{x}} = cs\boldsymbol{\mu}$ and $\boldsymbol{\mu}\cdot\boldsymbol{\mu} = 1$, we find the expression for s from Eq. (17)

$$
\begin{aligned}
s = {}& 1 - \frac{1}{2}\varepsilon^2(N + H_{ij}\mu^i\mu^j) - \varepsilon^3 L_k\mu^k \\
& + \frac{1}{2}\varepsilon^4\Big[-Q_{ij}\mu^i\mu^j - L + \frac{1}{2}NH_{ij}\mu^i\mu^j \\
& - \frac{1}{4}N^2 + \frac{3}{4}H_{ij}\mu^i\mu^j H_{kl}\mu^k\mu^l\Big].
\end{aligned}
\tag{19}
$$

Then, by substituting the metric Eqs. (10)–(12) into Eq. (18), we obtain the equations of light propagation based on Eq. (18) as follows:

$$
\begin{aligned}
\ddot{x}^i = {}& \frac{1}{2}N_{,i} + \frac{1}{2}H_{jk,i}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c} - H_{ij,k}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c} - N_{,j}\frac{\dot{x}^j}{c}\frac{\dot{x}^i}{c} \\
& + \varepsilon\Big\{-L_{i,j}\frac{\dot{x}^j}{c} + L_{j,i}\frac{\dot{x}^j}{c} - \frac{1}{2}N_{,t}\frac{\dot{x}^i}{c} + \frac{1}{2}H_{jk,t}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c}\frac{\dot{x}^i}{c} - H_{ij,t}\frac{\dot{x}^j}{c} - L_{k,j}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c}\frac{\dot{x}^i}{c}\Big\} \\
& + \varepsilon^2\Big\{-\frac{1}{2}H_{ik}N_{,k} - NN_{,j}\frac{\dot{x}^i}{c}\frac{\dot{x}^j}{c} + H_{il}H_{lj,k}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c} - \frac{1}{2}H_{il}H_{jk,l}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c} - L_{i,t} \\
& + \frac{1}{2}L_{,i} - L_{,j}\frac{\dot{x}^i}{c}\frac{\dot{x}^j}{c} - Q_{ij,k}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c} + \frac{1}{2}Q_{jk,i}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c}\Big\} + \mathcal{O}(3).
\end{aligned}
\tag{20}
$$

From Eq. (20), we can see that the 2PN metric for light propagation could be cut off to

$$g_{00} = -1 + \varepsilon^2 N + \varepsilon^4 L + \mathcal{O}(5), \tag{21}$$

$$g_{0i} = \varepsilon^3 L_i + \mathcal{O}(5), \tag{22}$$

$$g_{ij} = \delta_{ij} + \varepsilon^2 H_{ij} + \varepsilon^4 Q_{ij} + \mathcal{O}(5). \tag{23}$$

Furthermore, with using the relationship $H_{ij} = \delta_{ij}\gamma N$ based on Eqs. (A5) and (A6) in Appendix A and by substituting $\dot{\boldsymbol{x}}\cdot\dot{\boldsymbol{x}} = c^2 s^2$ and Eq. (19) into Eq. (20), the equation of light is simplified as

$$
\begin{aligned}
\ddot{x}^i = {}& \frac{1}{2}(1 + \gamma)N_{,i} - (1 + \gamma)N_{,k}\frac{\dot{x}^k}{c}\frac{\dot{x}^i}{c} \\
& + \varepsilon\Big\{-L_{i,j}\frac{\dot{x}^j}{c} + L_{j,i}\frac{\dot{x}^j}{c} - \frac{1}{2}(1 + \gamma)N_{,t}\frac{\dot{x}^i}{c} - \frac{1}{2}L_{j,k}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c}\frac{\dot{x}^i}{c} - \frac{1}{2}L_{k,j}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c}\frac{\dot{x}^i}{c}\Big\} \\
& + \varepsilon^2\Big\{-\gamma(1 + \gamma)NN_{,i} - L_{i,t} + \frac{1}{2}L_{,i} + (\gamma^2 - 1)NN_{,k}\frac{\dot{x}^k}{c}\frac{\dot{x}^i}{c} \\
& - \frac{1}{2}Q_{ij,k}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c} - \frac{1}{2}Q_{ik,j}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c} + \frac{1}{2}Q_{jk,i}\frac{\dot{x}^j}{c}\frac{\dot{x}^k}{c} - L_{,j}\frac{\dot{x}^j}{c}\frac{\dot{x}^i}{c}\Big\} + \mathcal{O}(3).
\end{aligned}
$$

III. 2PN LIGHT RAY PROPAGATION IN A SYSTEM OF N-POINT MASSES

A. Solving the metric for 2PN light propagation

Considering an N-body system of nonspinning point masses as our first move, we follow the notation adopted by Ref. [37] and use the matter stress-energy tensor as follows:

$$c^2 T^{\mu\nu}(\boldsymbol{x}, t) = \sum_a \mu_a(t)v_a^{\mu}v_a^{\nu}\delta(\boldsymbol{x} - \boldsymbol{y}_a(t)), \tag{24}$$

where δ denotes the three-dimensional Dirac distribution, the trajectory of the a-th mass is represented by $\boldsymbol{y}_a(t)$, the coordinate velocity of the a-th body is $\boldsymbol{v}_a = d\boldsymbol{y}_a(t)/dt$ and $v_a^{\mu} \equiv (c, \boldsymbol{v}_a)$, and μ_a denotes an effective time-dependent mass of the a-th body defined by

$$\mu_a(t) = \left(\frac{m_a}{\sqrt{g g_{\rho\sigma} \frac{v_a^\rho v_a^\sigma}{c^2}}}\right)_a, \qquad (25)$$

where subscript a denotes evaluation at the a-th body and m_a is the constant Schwarzschild mass. Another useful notation is

$$\tilde{\mu}_a(t) = \mu_a(t)\left[1 + \frac{v_a^2}{c^2}\right], \qquad (26)$$

where $v_a^2 = \boldsymbol{v}_a \cdot \boldsymbol{v}_a$. Both μ_a and $\tilde{\mu}_a$ reduce to the Schwarzschild mass at Newtonian order: $\mu_a = m_a + \mathcal{O}(2)$ and $\tilde{\mu}_a = m_a + \mathcal{O}(2)$. Then the mass, current, and stress densities in Eqs. (4)–(6) for the N-point masses read

$$\sigma = \sum_a \tilde{\mu}_a \delta(\boldsymbol{x} - \boldsymbol{y}_a(t)), \qquad (27)$$

$$\sigma_i = \sum_a \mu_a v_a^i \delta(\boldsymbol{x} - \boldsymbol{y}_a(t)), \qquad (28)$$

$$\sigma_{ij} = \sum_a \mu_a v_a^i v_a^j \delta(\boldsymbol{x} - \boldsymbol{y}_a(t)). \qquad (29)$$

The next step is to work out N, $\overset{(2)}{\zeta}$ and H_{ij} in 1PN approximation as

$N = 2\square^{-1}\{-4\pi G\sigma\}$

$$= 2\sum_a G\left\{\frac{\tilde{\mu}_a}{r_a} - \varepsilon \partial_t(\tilde{\mu}_a) + \varepsilon^2 \frac{1}{2}\partial_t^2(\tilde{\mu}_a r_a)\right\} + \mathcal{O}(3)$$

$$= 2\sum_a \frac{Gm_a}{r_a} + \varepsilon^2\left\{\sum_a \frac{Gm_a}{r_a}[4v_a^2 - (n_a v_a)^2]\right.$$

$$+ 2(2 - 3\gamma)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_{ab}}$$

$$\left. + \sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_{ab}^2}(n_a n_{ab})\right\} + \mathcal{O}(3), \qquad (30)$$

$$\overset{(2)}{\zeta} = (1 - \gamma)\sum_a \frac{Gm_a}{r_a} + \varepsilon^2\left\{\frac{1}{2}(1 - \gamma)\sum_a \frac{Gm_a}{r_a}[4v_a^2 - (n_a v_a)^2]\right.$$

$$+ (1 - \gamma)(2 - 3\gamma)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_{ab}}$$

$$\left. + \frac{1}{2}(1 - \gamma)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_{ab}^2}(n_a n_{ab})\right\} + \mathcal{O}(3), \qquad (31)$$

$$H_{ij} = 2\delta_{ij}\gamma\sum_a \frac{Gm_a}{r_a} + \varepsilon^2\delta_{ij}\gamma\left\{\sum_a \frac{Gm_a}{r_a}[4v_a^2 - (n_a v_a)^2]\right.$$

$$+ 2(2 - 3\gamma)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_{ab}}$$

$$\left. + \sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_{ab}^2}(n_a n_{ab})\right\} + \mathcal{O}(3), \qquad (32)$$

by the relation of $\tilde{\mu}_a$ that

$$\tilde{\mu}_a = m_a\left\{1 + \varepsilon^2\left[\left(N - \frac{1}{2}\delta_{ij}H_{ij}\right)_a + \frac{3}{2}v_a^2\right] + \mathcal{O}(4)\right\}$$

$$= m_a\left\{1 + \varepsilon^2\left[(2 - 3\gamma)\sum_{b \neq a} \frac{Gm_b}{r_{ab}} + \frac{3}{2}v_a^2\right] + \mathcal{O}(4)\right\}, \qquad (33)$$

where $r_a = |\boldsymbol{x} - \boldsymbol{y}_a|$ and $r_{ab} = |\boldsymbol{y}_a - \boldsymbol{y}_b|$. Scalar products are denoted with parentheses such as $(n_a v_a) = \boldsymbol{n}_a \cdot \boldsymbol{v}_a$, $\boldsymbol{n}_a = (\boldsymbol{x} - \boldsymbol{y}_a)/r_a$ and $\boldsymbol{n}_{ab} = (\boldsymbol{y}_a - \boldsymbol{y}_b)/r_{ab}$. For L_i,

$$L_i = -2(1 + \gamma)\square^{-1}\{-4\pi G\sigma_i\}$$

$$= \sum_a \int \frac{Gm_a v_a^i \delta(z - y_a)}{|\boldsymbol{x} - \boldsymbol{z}|}d^3z = \sum_a \frac{Gm_a}{r_a}v_a^i + \mathcal{O}(2). \qquad (34)$$

Based on Eq. (A8), L can be simplified as

$$L = (3\gamma - 2\beta - 1)\square^{-1}\{-4\pi G\sigma\} + 2(\gamma - 1)\square^{-1}\{-4\pi G\sigma_{kk}\} - \frac{1}{2}\beta N^2$$

$$= (3\gamma - 2\beta - 1)\sum_a G\left\{\frac{\tilde{\mu}_a}{r_a}(N)_a\right\} + 2(\gamma - 1)\sum_a G\left\{\frac{\mu_a}{r_a}v_a^2\right\} - \frac{1}{2}\beta N^2$$

$$= 2(\gamma - 1)\sum_a \frac{Gm_a}{r_a}v_a^2 - 2\beta\sum_a \frac{G^2 m_a^2}{r_a^2} + 2(3\gamma - 2\beta - 1)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_{ab}} - 2\beta\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_b} + \mathcal{O}(1), \qquad (35)$$

with the help of

$$\sum_a \frac{G\tilde{\mu}_a}{r_a}(N)_a = 2\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_{ab}} + \mathcal{O}(2), \qquad (36)$$

$$\sum_a \frac{G\mu_a}{r_a}v_a^2 = \sum_a \frac{Gm_a}{r_a}v_a^2 + \mathcal{O}(2), \qquad (37)$$

$$-\frac{1}{2}\beta N^2 = -2\beta\sum_a \frac{G^2 m_a^2}{r_a^2} - 2\beta\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_b} + \mathcal{O}(2). \qquad (38)$$

And

$$\overset{(4)}{\zeta} = 2(\gamma - 1)\sum_a \frac{Gm_a}{r_a} v_a^2$$

$$+ (1 + 6\gamma - 3\gamma^2 - 4\beta)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_{ab}}$$

$$+ \left(\frac{5}{2} - \gamma + \frac{1}{2}\gamma^2 - 2\beta\right)\sum_a \frac{G^2 m_a^2}{r_a^2}$$

$$+ \left(\frac{5}{2} - \gamma + \frac{1}{2}\gamma^2 - 2\beta\right)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_b} + \mathcal{O}(1) \quad (39)$$

It follows that we derive the metric Q_{ij} as follows:

$$\Box Q_{ij} = -8(1 + \gamma)\pi G \sigma_{ij} - \frac{1}{2}(1 + \gamma)N_{,i}N_{,j}$$

$$+ \delta_{ij}\left\{8\pi G\sigma\left(\frac{3}{2}\gamma - \frac{3}{2}\gamma^2 - \beta + 1\right)N\right.$$

$$\left. + 16\gamma\pi G\sigma_{kk} + \frac{1}{2}(\gamma^2 + \beta - 1)\nabla^2(N^2)\right\}, \quad (40)$$

where the quadratic part of potentials in Q_{ij} can be rewritten as

$$N_{,i}N_{,j} = 4\sum_a G^2 m_a^2 \left(\frac{1}{r_a}\right)_{,i}\left(\frac{1}{r_a}\right)_{,j}$$

$$+ 4\sum_a \sum_{b \neq a} G^2 m_a m_b \left(\frac{1}{r_a}\right)_{,i}\left(\frac{1}{r_b}\right)_{,j}$$

$$= \frac{1}{2}\sum_a G^2 m_a^2 (\partial_{ij}^2 + \delta_{ij}\Delta)\left(\frac{1}{r_a^2}\right)$$

$$+ 4\sum_a \sum_{b \neq a} G^2 m_a m_b \partial_{ai}\partial_{bj}\left(\frac{1}{r_a r_b}\right), \quad (41)$$

where ∂_{ai} denotes the partial derivative with respect to y_a. The integral of the self terms can be readily deduced from $\Delta \ln r_a = 1/r_a^2$ [37]. On the other hand, the interaction terms are obtained by

$$\Delta \ln S_{ab} = \frac{1}{r_a r_b}, \quad (42)$$

where $S_{ab} = r_a + r_b + r_{ab}$ [12]. Consequently we solve

$$-\frac{1}{2}(1 + \gamma)\Box^{-1}[N_{,i}N_{,j}] = -\frac{1}{4}(1 + \gamma)\sum_a G^2 m_a^2\left(\partial_{ij}^2 \ln r_a + \frac{\delta_{ij}}{r_a^2}\right) - 2(1 + \gamma)\sum_a \sum_{b \neq a} G^2 m_a m_b \partial_{ai}\partial_{bj}\ln S_{ab}$$

$$= \frac{1}{2}(1 + \gamma)\sum_a G^2 m_a^2\left(\frac{n_a^i n_b^j}{r_a^2} - \frac{\delta_{ij}}{r_a^2}\right) - 2(1 + \gamma)\sum_a \sum_{b \neq a} G^2 m_a m_b\left[\frac{n_{ab}^i n_{ab}^j - \delta_{ij}}{r_{ab}S_{ab}} + \frac{(n_{ab}^i - n_a^i)(n_{ab}^j + n_b^j)}{S_{ab}^2}\right],$$

$$(43)$$

where two relations that

$$\partial_{ij}^2 \ln r_a = \frac{\delta_{ij} - 2n_a^i n_a^j}{r_a^2}, \quad (44)$$

and

$$\partial_{ai}\partial_{bj}\ln S_{ab} = \frac{n_{ab}^i n_{ab}^j - \delta_{ij}}{r_{ab}S_{ab}} + \frac{(n_{ab}^i - n_a^i)(n_{ab}^j + n_b^j)}{S_{ab}^2}, \quad (45)$$

are used. Then, Q_{ij} can be worked out as

$$Q_{ij} = 2(1 + \gamma)\sum_a \frac{Gm_a}{r_a} v_a^i v_a^j + \frac{1}{2}(1 + \gamma)\sum_a \frac{G^2 m_a^2}{r_a^2} n_a^i n_a^j - 2(1 + \gamma)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_{ab}S_{ab}} n_{ab}^i n_{ab}^j$$

$$- 2(1 + \gamma)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{S_{ab}^2}(n_{ab}^i - n_a^i)(n_{ab}^j + n_b^j)$$

$$+ \delta_{ij}\left[-4\gamma\sum_a \frac{Gm_a}{r_a} v_a^2 - 2(3\gamma - 3\gamma^2 - 2\beta + 2)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_{ab}}\right.$$

$$+ \left(2\gamma^2 - \frac{1}{2}\gamma + 2\beta - \frac{5}{2}\right)\sum_a \frac{G^2 m_a^2}{r_a^2} + 2(1 + \gamma)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_{ab}S_{ab}}$$

$$\left. + 2(\gamma^2 + \beta - 1)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_b}\right] + \mathcal{O}(1). \quad (46)$$

B. The 2PN metric of light ray propagation for an *N*-point mass case

Collecting all these results together, we have the 2PN metric for light propagation in the scalar-tensor theory as

$$g_{00} = -1 + \varepsilon^2 2 \sum_a \frac{Gm_a}{r_a} + \varepsilon^4 \left\{ -2\beta \sum_a \frac{G^2 m_a^2}{r_a^2} + 2(1+\gamma) \sum_a \frac{Gm_a}{r_a} v_a^2 \right.$$

$$- \sum_a \frac{Gm_a}{r_a}(n_a v_a)^2 + 2(2-2\beta-1)\sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_{ab}} + \sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_{ab}^2}(n_{ab} n_a)$$

$$\left. - 2\beta \sum_a \sum_{b \neq a} \frac{G^2 m_a m_b}{r_a r_b} \right\} + \mathcal{O}(5), \tag{47}$$

$$g_{0i} = -\varepsilon^3 2(1+\gamma) \sum_a \frac{Gm_a}{r_a} v_a^i + \mathcal{O}(5), \tag{48}$$

$$g_{ij} = \delta_{ij} + \varepsilon^2 2\gamma \sum_a \frac{Gm_a}{r_a} \delta_{ij} + \varepsilon^4 \delta_{ij} \left\{ \left(2\gamma^2 - \frac{1}{2}\gamma + 2\beta - \frac{5}{2} \right) \sum_a \frac{G^2 m_a^2}{r_a^2} - \gamma \sum_a \frac{Gm_a}{r_a}(n_a v_a)^2 \right.$$

$$\left. + \sum_a \sum_{b \neq a} G^2 m_a m_b \left[2(\gamma^2 + \beta - 1)\frac{1}{r_a r_b} + 2(2\beta - \gamma - 2)\frac{1}{r_a r_{ab}} + \gamma \frac{(n_a n_{ab})}{r_{ab}^2} + 2(1+\gamma)\frac{1}{r_{ab} S_{ab}} \right] \right\}$$

$$+ \varepsilon^4 \left\{ \frac{1}{2}(1+\gamma) \sum_a \frac{G^2 m_a^2}{r_a^2} n_a^i n_a^j + 2(1+\gamma) \sum_a \frac{Gm_a}{r_a} v_a^i v_a^j \right.$$

$$\left. - 2(1+\gamma) \sum_a \sum_{b \neq a} G^2 m_a m_b \left[\frac{n_{ab}^i n_{ab}^j}{r_{ab} S_{ab}} + \frac{(n_{ab}^i - n_a^i)(n_{ab}^j + n_b^j)}{S_{ab}^2} \right] \right\} + \mathcal{O}(5). \tag{49}$$

If the above result returns to GR ($\gamma = \beta = 1$) and two-body case, it coincides with the result of Ref. [37]. It is worth noting that the parameters at the spatial isotropic ($\delta_{ij} \sum_a \frac{G^2 m_a^2}{r_a^2}$) and anisotropic ($\sum_a \frac{G^2 m_a^2}{r_a^4} r_a^i r_a^j$) parts of ε^4 for g_{ij} are respectively $(2\gamma^2 - \frac{1}{2}\gamma + 2\beta - \frac{5}{2})$ and $\frac{1}{2}(1+\gamma)$ in the STT. For the one-body case, different theories impose different values on the above parameters (see Table I). When we take Einstein's GR, the parameters coming from two parts are totally unity [20]. Reference [13] shows, for a single body, the ae-theory has two unequal coefficients of the spatial isotropic and anisotropic part in the 2PN g_{ij} but only one parameter c_{14} presents in them. Recently, Ref. [21] introduces only one parameter ϵ to parametrize these two parts. From our calculations, it shows that the parametrization with single parameter ϵ in Ref. [21] is *not* valid for the STT. Parametrized 2PN issues are more complicated and were researched to a certain extent by the previous works [26–28], but it is beyond the territory of this paper.

C. The light ray propagation equations

Substituting metric coefficients N for Eq. (30), H_{ij} for Eq. (32), L_i for Eq. (34), L for Eq. (35) and Q_{ij} for Eq. (46) into Eq. (20), we have

$$\ddot{x}^i = F_{1PN}^i + F_{2PN}^i + F_v^i, \tag{50}$$

where the 1PN monopole component is

$$F_{1PN}^i = -(1+\gamma) \sum_a \frac{Gm_a}{r_a^2} \left[n_a^i - 2\frac{(n_a \dot{x})\dot{x}^i}{c^2} \right], \tag{51}$$

the 2PN monopole component is

TABLE I. Parameters at the spatial isotropic and anisotropic parts of ε^4 for g_{ij} in different theories.

Parameter at	GR [20]	ae-theory [13]	Parameter in [21]	STT (our work)
$\delta_{ij} \frac{G^2 m^2}{r^2}$	1	$1 + \frac{1}{2} c_{14}$	ϵ	$2\gamma^2 - \frac{1}{2}\gamma + 2\beta - \frac{5}{2}$
$\frac{G^2 m^2}{r^4} r^i r^j$	1	$1 - \frac{1}{2} c_{14}$	ϵ	$\frac{1}{2}(1+\gamma)$

$$F_{2PN}^i = \varepsilon^2(1+\gamma)\sum_a \frac{G^2 m_a^2}{r_a^3}\left\{2(1+\gamma)n_a^i + \frac{(n_a\dot{x})^2}{c^2}n_a^i - \frac{(n_a\dot{x})\dot{x}^i}{c^2}\right\} + \varepsilon^2(1+\gamma)\sum_a\sum_{b\neq a}\frac{G^2 m_a m_b}{r_a}\left\{\frac{1}{r_a r_{ab}}n_a^i - \frac{1}{2r_{ab}^2}(n_a n_{ab})n_a^i\right.$$

$$+ (1-\gamma)\frac{1}{r_a r_b}n_a^i + (1+3\gamma)\frac{1}{r_b^2}n_b^i - \frac{3}{2r_{ab}^2}n_{ab}^i - \frac{2}{r_a r_{ab}}\frac{(n_a\dot{x})\dot{x}^i}{c^2} + \frac{(n_a n_{ab})}{r_{ab}^2}\frac{(n_a\dot{x})\dot{x}^i}{c^2} - \frac{1}{r_{ab}^2}\frac{(n_{ab}\dot{x})\dot{x}^i}{c^2} + 2(1-\gamma)\frac{1}{r_b^2}\frac{(n_b\dot{x})\dot{x}^i}{c^2}$$

$$\left. - 2(1-\gamma)\frac{1}{r_a r_b}\frac{(n_a\dot{x})\dot{x}^i}{c^2}\right\} + \varepsilon^2(1+\gamma)\sum_a\sum_{b\neq a}\frac{G^2 m_a m_b}{S_{ab}^2}\left\{-2\frac{1}{r_{ab}}\frac{(n_{ab}\dot{x})(n_a\dot{x})+(n_{ab}\dot{x})(n_b\dot{x})}{c^2}n_{ab}^i\right.$$

$$- 4\frac{1}{S_{ab}}\frac{[(n_{ab}\dot{x})+(n_b\dot{x})][(n_a\dot{x})+(n_b\dot{x})]}{c^2}(n_{ab}^i - n_a^i) + \frac{1}{r_a}\frac{(n_{ab}\dot{x})+(n_b\dot{x})}{c}\left[\frac{(n_a\dot{x})}{c}n_a^i - \frac{\dot{x}^i}{c}\right]$$

$$- 2\frac{1}{r_b}\left[\frac{(n_b\dot{x})^2}{c^2} - 1\right](n_{ab}^i - n_a^i) + 2\frac{1}{r_{ab}}\left[\frac{(n_a\dot{x})\dot{x}^i}{c^2} + \frac{(n_b\dot{x})\dot{x}^i}{c^2}\right] + \frac{1}{r_{ab}}\frac{(n_{ab}\dot{x})^2}{c^2}[n_a^i + n_b^i]$$

$$\left. + 2\frac{1}{S_{ab}}\frac{[(n_{ab}\dot{x})-(n_a\dot{x})][(n_{ab}\dot{x})+(n_b\dot{x})]}{c^2}(n_a^i + n_b^i) + \frac{1}{r_b}\frac{(n_{ab}\dot{x})-(n_a\dot{x})}{c}\left[\frac{(n_b\dot{x})}{c}n_b^i - \frac{\dot{x}^i}{c}\right] - \frac{1}{r_{ab}}(n_a^i + n_b^i)\right\}, \tag{52}$$

the influence of the bodies' orbital motions is

$$F_v^i = -\varepsilon 2(1+\gamma)\sum_a \frac{Gm_a}{r_a^2}\left\{\frac{(n_a\dot{x})}{c}v_a^i\right.$$

$$- \frac{(v_a\dot{x})}{c}n_a^i + \left[\frac{1}{2}(n_a v_a) + \frac{(v_a\dot{x})(n_a\dot{x})}{c^2}\right]\frac{\dot{x}^i}{c}\right\}$$

$$+ \varepsilon^2(1+\gamma)\sum_a \frac{Gm_a}{r_a^2}\left\{-\left[v_a^2 - \frac{3}{2}(n_a v_a)^2 + \frac{(v_a\dot{x})^2}{c^2}\right]n_a^i\right.$$

$$+ \left[(n_a v_a) + 2\frac{(v_a\dot{x})(n_a\dot{x})}{c^2}\right]v_a^i + \left[2v_a^2\frac{(n_a\dot{x})}{c}\right.$$

$$\left.\left. - 3(n_a v_a)^2\frac{(n_a\dot{x})}{c} + 2(n_a v_a)\frac{(v_a\dot{x})}{c}\right]\frac{\dot{x}^i}{c}\right\}. \tag{53}$$

In the case of a one-body system studied by many previous works, we always can construct a reference frame within which the body is static so that the velocity-dependent terms vanish.

IV. APPLICATIONS IN AN N-POINT MASS SYSTEM: N = 1 AND N = 2

After obtaining the light ray propagation equations, we will apply them into a 1-point mass system ($N = 1$) and a 2-point mass system ($N = 2$) to calculate the deflection angles for differential measurements.

A. Deflection angle by a 1-point mass system

For a 1-point mass system, it has a static and spherically symmetric spacetime. The 2PN light deflection angle caused by the central body has been computed in some previous works, such as Ref. [16–22]. Here, we leave the details of calculation in Appendix B and show the final result of the deflection angle $\Delta\theta \equiv \theta - \vartheta_0$ as

$$\Delta\theta = \left(\frac{1+\gamma}{2}\right)\frac{4Gm_a}{c^2 d_a}$$

$$- (1+\gamma)\left[2(1+\gamma) - \frac{1}{8}(7+8\gamma)\pi\right]\frac{G^2 m_a^2}{c^4 d_a^2}. \tag{54}$$

where $d_a = k \times (r_a \times k)$ is an impact parameter to represent the closest distance between the unperturbed light ray and body a, $d_a = |d_a|$. When $\gamma = 1$, Eq. (54) will reduce to the result of GR [41]. And it is identical with the deflection angle reported in Ref. [32], which considers 2PN light propagation in a one-body system under the framework of STT.

Table II lists the summary of 2PN parameters appearing in the metric, light ray trajectory and deflection angle in (1) GR, (2) ae-theory, (3) Ref. [21], and (4) STT (our work). In GR, there is not any parameter. For the ae-theory, there is only one parameter c_{14}. From Ref. [21], three parameters, γ, β and ϵ, show up. It is very interesting that only one parameter γ appears in 2PN light ray trajectory and light deflection for the scalar-tensor theory although there are two parameters γ and β in 2PN metric for the theory. If we let $\bar{\gamma} \equiv \gamma - 1$, Eq. (54) returns to

$$\Delta\theta = \delta\theta_{GR} + \delta\theta_{STT}, \tag{55}$$

where

$$\delta\theta_{GR} = \frac{4Gm_a}{c^2 d_a} - \left(8 - \frac{15}{4}\pi\right)\frac{G^2 m_a^2}{c^4 d_a^2}$$

$$= 1.75\left(\frac{m_a}{M_\odot}\right)\left(\frac{R_\odot}{d_a}\right) \text{arcsecond}$$

$$+ 3.50\left(\frac{m_a}{M_\odot}\right)^2\left(\frac{R_\odot}{d_a}\right)^2 \mu\text{as}, \tag{56}$$

$$\delta\theta_{STT} = \bar{\gamma}\frac{2Gm_a}{c^2 d_a} - \bar{\gamma}\left(8 - \frac{31}{8}\pi\right)\frac{G^2 m_a^2}{c^4 d_a^2} + \mathcal{O}(\bar{\gamma}^2)$$

$$= 18.3\left(\frac{\bar{\gamma}}{2.1\times 10^{-5}}\right)\left(\frac{m_a}{M_\odot}\right)\left(\frac{R_\odot}{d_a}\right) \mu\text{as}$$

$$+ 81.1\left(\frac{\bar{\gamma}}{2.1\times 10^{-5}}\right)\left(\frac{m_a}{M_\odot}\right)^2\left(\frac{R_\odot}{d_a}\right)^2 \text{pas} + \mathcal{O}(\bar{\gamma}^2). \tag{57}$$

Here, $\delta\theta_{GR}$ is the light deflection equivalently caused by GR and its 1PN and 2PN effects are ~1.75 as and ~3.50 μas, respectively, both of which are much greater

TABLE II. Summary of 2PN parameters in different theories.

Theories	Parameters in metric	Parameters in light ray trajectory	Parameters in 2PN deflection
GR [22]	None	None	$-8 + \frac{15}{4}\pi$
ae-theory [13]	c_{14}	c_{14}	$-8 + (\frac{15}{4} + \frac{1}{8}c_{14})\pi$
From Ref. [21]	γ, β, ϵ	γ, β, ϵ	$-2(1+\gamma)^2 + [2(1+\gamma) - \beta + \frac{3}{4}\epsilon]\pi$
STT (our work)	γ, β	γ	$-2(1+\gamma)^2 + \frac{1}{8}(1+\gamma)(7 + 8\gamma)\pi$

than the thresholds of future ~ 1 nas space missions. $\delta\theta_{\text{STT}}$ represents the discrepancy between STT and GR in the deflection angle. For the effect of the deviation from GR in STT in the deflection angle which would be measured by experiments conducted in the solar system, the leading effect is 18.3 μas at the 1PN level. As Damour and Nordtvedt mentioned in Ref. [30], the deviation of γ from unity at the levels of $\sim 10^{-7}$ to $\sim 10^{-5}$ might be detected by some new space projects (e.g. LATOR). The 2PN term in $\delta\theta_{\text{STT}}$ might also be important because it could reach ~ 0.1 nas, close to the thresholds of future space-born experiments.

B. Deflection angle by a 2-point mass system

From a 1-point mass system to an N-point mass system, the spacetime will no longer be spherical symmetric and static. This complicates the problem significantly. We suppose that there are three characteristic length scales: the length scale of the N-point mass system $\mathfrak{L}$, the distance between the light source and the system $\mathfrak{R}_\lambda$ and the distance from the observer and the system $\mathfrak{R}_{\text{obs}}$. And there are two characteristic time scales: the time scale of orbital motions of the N-point mass system $\mathfrak{T}$ and the time scale of the light crossing the system $\mathfrak{T}_{\text{cross}}$. In general, $\mathfrak{L}$, $\mathfrak{R}_\lambda$ and $\mathfrak{R}_{\text{obs}}$ could be arbitrary, while $\mathfrak{T}$ is determined by the total mass of the system $\mathfrak{M}$ and $\mathfrak{L}$, as well as $\mathfrak{T}_{\text{cross}} \leq \mathfrak{L}/c$.

In what follows, we will consider a specific case that the gravitational system contains only two point masses with

body a and b ($N = 2$) and $\mathfrak{L} \gg \max(\mathfrak{R}_\lambda, \mathfrak{R}_{\text{obs}})$, and then calculate the resulting deflection angle.

1. Light equations

For this 2-point mass system with the configuration $\mathfrak{L} \gg \max(\mathfrak{R}_\lambda, \mathfrak{R}_{\text{obs}})$, it indicates $\mathfrak{T}_{\text{cross}} \ll \mathfrak{T}$. However, this does not mean that the impact of the orbital motion can be neglected, if this effect firstly emerges at less than or equal to 2PN level. Nevertheless, the calculation including this motion demands a huge amount of work and, thus, we would leave it for our next steps. In this paper, we firstly consider a case of this binary system that the effects of velocity-dependent terms are smaller than those of 2PN static effects, i.e. $F_v^i = 0$ in Eq. (50), so that the light equations are

$$\ddot{x}^i = F_{1\text{PN}}^i + F_{2\text{PN}}^i|_{1\text{BE}} + F_{2\text{PN}}^i|_{2\text{BE}}, \quad (58)$$

where the contribution at 1PN is

$$F_{1\text{PN}}^i = -(1+\gamma)\frac{Gm_a}{r_a^2}\left[n_a^i - 2\frac{(n_a\dot{x})\dot{x}^i}{c^2}\right] + a \leftrightarrow b, \quad (59)$$

the contribution of one-body effect at 2PN is

$$F_{2\text{PN}}^i|_{1\text{BE}} = \epsilon^2(1+\gamma)\frac{G^2m_a^2}{r_a^3}\left\{2(1+\gamma)n_a^i \right. $$
$$\left. + \frac{(n_a\dot{x})^2}{c^2}n_a^i - \frac{(n_a\dot{x})\dot{x}^i}{c^2}\right\} + a \leftrightarrow b, \quad (60)$$

and the contribution of two-body effect at 2PN is

$$F_{2\text{PN}}^i|_{2\text{BE}} = \epsilon^2(1+\gamma)\frac{G^2m_am_b}{r_a}\left\{\frac{1}{r_ar_{ab}}n_a^i - \frac{1}{2r_{ab}^2}(n_an_{ab})n_a^i + (1-\gamma)\frac{1}{r_ar_b}n_a^i + (1+3\gamma)\frac{1}{r_b^2}n_b^i - \frac{3}{2r_{ab}^2}n_{ab}^i\right.$$
$$- \frac{2}{r_ar_{ab}}\frac{(n_a\dot{x})\dot{x}^i}{c^2} + \frac{(n_an_{ab})}{r_{ab}^2}\frac{(n_a\dot{x})\dot{x}^i}{c^2} - \frac{1}{r_{ab}^2}\frac{(n_{ab}\dot{x})\dot{x}^i}{c^2} + 2(1-\gamma)\frac{1}{r_b^2}\frac{(n_b\dot{x})\dot{x}^i}{c^2} - 2(1-\gamma)\frac{1}{r_ar_b}\frac{(n_a\dot{x})\dot{x}^i}{c^2}\right\}$$
$$+ \epsilon^2(1+\gamma)\frac{G^2m_am_b}{S_{ab}^2}\left\{-2\frac{1}{r_{ab}}\frac{(n_{ab}\dot{x})(n_a\dot{x}) + (n_{ab}\dot{x})(n_b\dot{x})}{c^2}n_{ab}^i - 4\frac{1}{S_{ab}}\frac{[(n_{ab}\dot{x}) + (n_b\dot{x})][(n_a\dot{x}) + (n_b\dot{x})]}{c^2}(n_{ab}^i - n_a^i)\right.$$
$$+ \frac{1}{r_a}\frac{(n_{ab}\dot{x}) + (n_b\dot{x})}{c}\left[\frac{(n_a\dot{x})}{c}n_a^i - \frac{\dot{x}^i}{c}\right] - 2\frac{1}{r_b}\left[\frac{(n_b\dot{x})^2}{c^2} - 1\right](n_{ab}^i - n_a^i) + 2\frac{1}{r_{ab}}\left[\frac{(n_a\dot{x})\dot{x}^i}{c^2} + \frac{(n_b\dot{x})\dot{x}^i}{c^2}\right]$$
$$+ \frac{1}{r_{ab}}\frac{(n_{ab}\dot{x})^2}{c^2}[n_a^i + n_b^i] + 2\frac{1}{S_{ab}}\frac{[(n_{ab}\dot{x}) - (n_a\dot{x})][(n_{ab}\dot{x}) + (n_b\dot{x})]}{c^2}(n_a^i + n_b^i)$$
$$\left. + \frac{1}{r_b}\frac{(n_{ab}\dot{x}) - (n_a\dot{x})}{c}\left[\frac{(n_b\dot{x})}{c}n_b^i - \frac{\dot{x}^i}{c}\right] - \frac{1}{r_{ab}}(n_a^i + n_b^i)\right\} + a \leftrightarrow b, \quad (61)$$

where the symbol $a \leftrightarrow b$ means the same terms but with the labels a and b exchanged.

2. Light ray trajectory

Following the procedure mentioned in Appendix B, we could work out the time derivative of light ray trajectory $\frac{1}{c}\delta\dot{x}(t)$ and compute the deflection angle caused by these two point masses. For F^i_{1PN} and $F^i_{2PN}|_{1BE}$, we could have obtained the corresponding trajectory of the light ray by adopting the iterative method used in Ref. [18]. However, in order to integrate $F^i_{2PN}|_{2BE}$, we assume that the light source, light ray trajectory and the observer are all closer to the body a than to the body b, i.e. $r_a < r_{ab}$ along the light path from end to end. Thus, we could obtain

$$\frac{1}{c}\dot{x}(t) = k + \frac{1}{c}\delta\dot{x}_{1PN}(x_N) + \frac{1}{c}\delta\dot{x}_{2PN}(x_N)|_{1BE} + \frac{1}{c}\delta\dot{x}_{2PN}(x_N)|_{2BE}, \tag{62}$$

where

$$\frac{1}{c}\delta\dot{x}_{1PN}(x) = -(1+\gamma)\frac{Gm_a}{c^2 r_a}\left\{\frac{k \times (\mathbf{r_a} \times k)}{r_a - k \cdot r_a} + k\right\} - (1+\gamma)\frac{Gm_b}{c^2 r_b}\left\{\frac{k \times (\mathbf{r_b} \times k)}{r_b - k \cdot r_b} + k\right\}, \tag{63}$$

$$\frac{1}{c}\delta\dot{x}_{2PN}(x)|_{1BE} = -\frac{1}{4}(1+\gamma)\frac{G^2 m_a^2}{c^4 r_a^4}(k \cdot r_a)r_a - \frac{1}{4}(1+\gamma)\frac{G^2 m_b^2}{c^4 r_b^4}(k \cdot r_b)r_b$$
$$+ \frac{G^2 m_a^2}{c^4}d_a\left\{(1+\gamma)^2\frac{1}{r_a(r_a - k \cdot r_a)}\left(\frac{2}{r_a} + \frac{1}{r_a - k \cdot r_a}\right) - \frac{1}{8}(1+\gamma)(7+8\gamma)\frac{1}{d_a^2}\left[\frac{k \cdot r_a}{r_a^2} + \frac{1}{d_a}\left(\frac{\pi}{2} + \arctan\frac{k \cdot r_a}{d_a}\right)\right]\right\}$$
$$+ \frac{G^2 m_a^2}{c^4 r_a}k\left[\frac{1}{4}(1+\gamma)(5+4\gamma)\frac{1}{r_a} - (1+\gamma)^2\frac{1}{r_a - k \cdot r_a}\right] + \frac{G^2 m_b^2}{c^4}d_b\left\{(1+\gamma)^2\frac{1}{r_b(r_b - k \cdot r_b)}\left(\frac{2}{r_b} + \frac{1}{r_b - k \cdot r_b}\right)\right.$$
$$\left. - \frac{1}{8}(1+\gamma)(7+8\gamma)\frac{1}{d_b^2}\left[\frac{k \cdot r_b}{r_b^2} + \frac{1}{d_b}\left(\frac{\pi}{2} + \arctan\frac{k \cdot r_b}{d_b}\right)\right]\right\} + \frac{G^2 m_b^2}{c^4 r_b}k\left[\frac{1}{4}(1+\gamma)(5+4\gamma)\frac{1}{r_b} - (1+\gamma)^2\frac{1}{r_b - k \cdot r_b}\right], \tag{64}$$

$$\frac{1}{c}\delta\dot{x}_{2PN}(x)|_{2BE} = (1+\gamma)\frac{G^2 m_a m_b}{c^4 r_a r_{ab}}k\left\{-(1+\gamma)\frac{r_{ab} \cdot d_a}{(r_a - k \cdot r_a)(r_{ab} - k \cdot r_{ab})} + (2+\gamma)\right\} + (1+\gamma)\frac{G^2 m_a m_b}{c^4 r_{ab}^3}$$
$$\times k\left\{-(1+\gamma)\frac{r_{ab} \cdot d_a}{r_a - k \cdot r_a} - \frac{r_a \cdot r_{ab}}{2r_a} - \frac{(k \cdot r_a)(k \cdot r_{ab})}{r_{ab}} + 3\frac{(k \cdot r_{ab})^2}{r_{ab}^2}r_a - \frac{1}{2}(7+6\gamma)(k \cdot r_{ab})\right.$$
$$\times \ln\frac{r_a + k \cdot r_a}{r_{0a} + k \cdot r_{0a}}\right\} + (1+\gamma)\frac{G^2 m_a m_b}{c^4 r_a r_{ab}}d_a\left\{(1+\gamma)\frac{1}{(r_a - k \cdot r_a)^2(r_{ab} - k \cdot r_{ab})}\right.$$
$$\times \left[(r_{ab} \cdot d_a) - (k \cdot r_a)(k \cdot r_{ab}) + (k \cdot r_{ab})r_{ab} + \frac{1}{2}(k \cdot r_{ab})r_a\right] + (3+2\gamma)\frac{1}{r_a - k \cdot r_a}$$
$$\left. - \frac{1}{2}(1+\gamma)\frac{1}{(r_a - k \cdot r_a)^2}r_a - (1+\gamma)\frac{1}{(r_a - k \cdot r_a)(r_{ab} - k \cdot r_{ab})}r_a\right\} + (1+\gamma)\frac{G^2 m_a m_b}{c^4 r_{ab}^3}$$
$$\times d_a\left\{-\frac{1}{2}(3+4\gamma)\frac{k \cdot r_{ab}}{r_a - k \cdot r_a} + \frac{1}{3}(1+\gamma)\frac{(k \cdot r_a)^2}{(r_a - k \cdot r_a)(r_{ab} - k \cdot r_{ab})^2}r_{ab}\right.$$
$$\left. + \frac{1}{3}(1+\gamma)\frac{(k \cdot r_a)^2}{(r_a - k \cdot r_a)(r_{ab} - k \cdot r_{ab})} - \frac{1}{2}\frac{r_a \cdot r_{ab}}{r_a(r_a - k \cdot r_a)} + \left[3\frac{(k \cdot r_{ab})^2}{r_{ab}^2} - 1\right]\ln\frac{r_a + k \cdot r_a}{r_{0a} + k \cdot r_{0a}}\right\}$$
$$+ (1+\gamma)^2\frac{G^2 m_a m_b}{c^4 r_a r_{ab}}\frac{1}{r_{ab} - k \cdot r_{ab}}d_b\left[2 - \frac{r_a}{r_a - k \cdot r_a} - \frac{r_a}{r_{ab} - k \cdot r_{ab}}\ln\frac{r_a + k \cdot r_a}{r_{0a} + k \cdot r_{0a}}\right]$$
$$+ (1+\gamma)^2\frac{G^2 m_a m_b}{c^4 r_{ab}^2}\frac{1}{(r_{ab} - k \cdot r_{ab})^2}d_b\left[\frac{r_a \cdot r_{ab}}{r_a - k \cdot r_a} - \frac{3}{2}\frac{(k \cdot r_a)(k \cdot r_{ab})}{r_a - k \cdot r_a} + \frac{1}{3}r_a\right.$$
$$\left. + \frac{1}{2}(k \cdot r_{ab})\ln\frac{r_a + k \cdot r_a}{r_{0a} + k \cdot r_{0a}}\right] + (1+\gamma)^2\frac{G^2 m_a m_b}{c^4 r_{ab}^3}\frac{1}{r_{ab} - k \cdot r_{ab}}d_b\left[\frac{r_a \cdot r_{ab}}{r_a - k \cdot r_a} - \frac{3}{2}\frac{(k \cdot r_a)(k \cdot r_{ab})}{r_a - k \cdot r_a}\right.$$
$$\left. + \frac{1}{3}r_a + \frac{1}{2}(k \cdot r_{ab})\ln\frac{r_a + k \cdot r_a}{r_{0a} + k \cdot r_{0a}}\right] + (1+\gamma)\frac{G^2 m_a m_b}{c^4 r_{ab}^3}r_{ab}\left\{4\frac{k \cdot r_a}{r_{ab}} - \frac{k \cdot r_a}{r_a} - 3\frac{k \cdot r_{ab}}{r_{ab}^2}r_a\right.$$
$$\left. - 2\frac{(k \cdot r_a)(k \cdot r_{ab})^2}{r_{ab}^3} - \frac{d_a}{r_{ab}}\left(\frac{\pi}{2} + \arctan\frac{k \cdot r_a}{d_a}\right) + \frac{1}{2}(1+4\gamma)\ln\frac{r_a + k \cdot r_a}{r_{0a} + k \cdot r_{0a}}\right\} + O\left(\frac{1}{c^4}\frac{r_a}{r_{ab}}\right), \tag{65}$$

where $\boldsymbol{r}_{0a} = \boldsymbol{r}_a(t = t_0)$ and $r_{0a} = |\boldsymbol{r}_{0a}|$. For GR ($\gamma = \beta = 1$) and the one-body case, Eq. (62) will reduce to the results of Ref. [18].

3. Light deflection

With the definition of a gauge-invariant angle θ between the directions of two incoming photons (light signals 1 and 2) and we use that the position of the photon at the moment t of observation coincides with the position of the observer so that

$$\begin{aligned} \boldsymbol{x}_{\text{obs}} &= \boldsymbol{x}_{01} + c(t - t_{01})\boldsymbol{k}_1 + \delta\boldsymbol{x}_1 \\ &= \boldsymbol{x}_{02} + c(t - t_{02})\boldsymbol{k}_2 + \delta\boldsymbol{x}_2, \end{aligned} \tag{66}$$

where $(t_{01}, \boldsymbol{x}_{01})$ denotes the moment and position of the light signal 1 of emission and $(t_{02}, \boldsymbol{x}_{02})$ for the light signal 2, respectively.

By assuming that the initial unit vector of light ray 1 $\boldsymbol{k}_1$ is perpendicular to the line connecting the body a and the b, $\boldsymbol{r}_{ab}$, and light ray 2 moves along the line connecting the body a and the observer, which lead to that geometrical relationships that

$$\frac{\boldsymbol{k}_1 \cdot \boldsymbol{r}_a}{r_a} \approx \cos\vartheta_0, \qquad \frac{\boldsymbol{k}_2 \cdot \boldsymbol{r}_a}{r_a} = 1,$$

$$\frac{\boldsymbol{k}_1 \cdot \boldsymbol{r}_{ab}}{r_{ab}} = 0, \qquad \frac{d_{1a}}{r_a} \approx \sin\vartheta_0 \tag{67}$$

$$d_{2a} = 0, \qquad \frac{\boldsymbol{k}_2 \cdot \boldsymbol{d}_{1a}}{d_{1a}} = \sin\vartheta_0,$$

$$\frac{\boldsymbol{k}_2 \cdot \boldsymbol{r}_{ab}}{r_{ab}} = \sin\vartheta_0, \qquad \boldsymbol{k}_2 = \frac{\boldsymbol{r}_a}{r_a}. \tag{68}$$

and letting $d_{1a} \equiv d_a$, we could obtain the deflection angle for a static observer as

$$\begin{aligned} \Delta\theta &\equiv \theta - \vartheta_0 \\ &= \left(\frac{1+\gamma}{2}\right)\frac{4Gm_a}{c^2 d_a}\left(\frac{1+\cos\vartheta_0}{2}\right) + (1+\gamma)\frac{Gm_a}{c^2 r_{ab}}\left(1 - \frac{\cos\vartheta_0}{1 - \sin\vartheta_0}\right) \\ &\quad - 2(1+\gamma)^2\frac{G^2 m_a^2}{c^4 d_a^2}\left(\frac{1+\cos\vartheta_0}{2}\right) + (1+\gamma)^2\frac{G^2 m_b^2}{c^4 r_{ab}^2}\frac{\sin\vartheta_0(1+\sin\vartheta_0)}{\cos^2\vartheta_0} - \frac{1}{8}(1+\gamma)\frac{G^2 m_a^2}{c^4 d_a^2}\sin\vartheta_0\cos\vartheta_0 \\ &\quad - \frac{1}{8}(1+\gamma)\frac{G^2 m_a^2}{c^4 r_{ab}^2}\frac{\sin\vartheta_0}{\cos\vartheta_0} + \frac{1}{8}(7+8\gamma)(1+\gamma)\frac{G^2 m_a^2}{c^4 d_a^2}(\pi - \vartheta_0) \\ &\quad - \frac{1}{8}(7+8\gamma)(1+\gamma)\frac{G^2 m_a^2}{c^4 r_{ab}^2}\left[\frac{1}{\cos^2\vartheta_0}\left(\frac{\pi}{2} + \vartheta_0\right) - \frac{\pi}{2}\right] \\ &\quad - 2(1+\gamma)\frac{G^2 m_a m_b}{c^4 r_{ab}^2}\sin\vartheta_0\cos\vartheta_0 - (1+\gamma)^2\frac{G^2 m_a m_b}{c^4 r_{ab}^2}\sin\vartheta_0 \\ &\quad - 2(1+\gamma)^2\frac{G^2 m_a m_b}{c^4 r_{ab} d_a}\left(\frac{1+\cos\vartheta_0}{2}\right)\left(1 - \frac{\cos\vartheta_0}{1 - \sin\vartheta_0}\right)\frac{\cos\vartheta_0}{\sin\vartheta_0} \\ &\quad - 2(1+\gamma)(2+\gamma)\frac{G^2 m_a m_b}{c^4 r_{ab} d_a}\left(\frac{1+\cos\vartheta_0}{2}\right) + (1+\gamma)^2\frac{G^2 m_a m_b}{c^4 r_{ab} d_a}\frac{\sin\vartheta_0\cos\vartheta_0}{1 - \sin\vartheta_0} + \mathcal{O}\left(\frac{1}{c^4}\frac{r_a}{r_{ab}}\right). \end{aligned} \tag{69}$$

Similar to the case of 1-point mass system, the 2PN light deflection in a 2-point mass system with static approximation does *not* depend on PPN parameter β either. One possible reason for this is that β usually associates with motion of bodies so that when the system could be treated as a static system due to $\mathfrak{T}_{\text{cross}} \ll \mathfrak{T}$, it would disappear in the deflection angle. However, when $\mathfrak{T}_{\text{cross}} \sim \mathfrak{T}$, we suppose the time evolutions of the gravitational fields, the motion of bodies and PPN parameter β would explicitly show up in the expression of deflection angle, and this more complicated issue will be the next move of our investigation.

4. Quantitative examples

If the light ray just grazes the limb of the body a, the angle between light ray 1 and 2 at the observer ϑ_0 could be very close to 0 and be neglected so that

$$\Delta\theta = \left(\frac{1+\gamma}{2}\right)\frac{4Gm_a}{c^2 d_a} - (1+\gamma)\left[2(1+\gamma) - \frac{1}{8}(7+8\gamma)\pi\right]\frac{G^2 m_a^2}{c^4 d_a^2} - 2(1+\gamma)(2+\gamma)\frac{G^2 m_a m_b}{c^4 r_{ab} d_a}, \tag{70}$$

and it could be written in a more practical form

$$\Delta\theta = \delta\theta_{\text{GR}} + \delta\theta_{\text{STT}}, \tag{71}$$

where

$$\delta\theta_{GR} = \frac{4Gm_a}{c^2 d_a} - \left(8 - \frac{15}{4}\pi\right)\frac{G^2 m_a^2}{c^4 d_a^2} - 12\frac{G^2 m_a m_b}{c^4 r_{ab} d_a}$$

$$= 1.75\left(\frac{m_a}{M_\odot}\right)\left(\frac{R_\odot}{d_a}\right) \text{arcsecond} + 3.50\left(\frac{m_a}{M_\odot}\right)^2\left(\frac{R_\odot}{d_a}\right)^2 \mu\text{as} - 51.5\left(\frac{m_b}{m_a}\right)\left(\frac{m_a}{M_\odot}\right)^2\left(\frac{R_\odot}{d_a}\right)\left(\frac{1\text{ AU}}{r_{ab}}\right) \text{nas} \quad (72)$$

$$\delta\theta_{STT} = \bar{\gamma}\frac{2Gm_a}{c^2 d_a} - \bar{\gamma}\left(8 - \frac{31}{8}\pi\right)\frac{G^2 m_a^2}{c^4 d_a^2} - 10\bar{\gamma}\frac{G^2 m_a m_b}{c^4 r_{ab} d_a} + \mathcal{O}(\bar{\gamma}^2)$$

$$= 18.3\left(\frac{\bar{\gamma}}{2.1\times 10^{-5}}\right)\left(\frac{m_a}{M_\odot}\right)\left(\frac{R_\odot}{d_a}\right) \mu\text{as} + 81.1\left(\frac{\bar{\gamma}}{2.1\times 10^{-5}}\right)\left(\frac{m_a}{M_\odot}\right)^2\left(\frac{R_\odot}{d_a}\right)^2 \text{pas}$$

$$- 0.902\left(\frac{\bar{\gamma}}{2.1\times 10^{-5}}\right)\left(\frac{m_b}{m_a}\right)\left(\frac{m_a}{M_\odot}\right)^2\left(\frac{R_\odot}{d_a}\right)\left(\frac{1\text{ AU}}{r_{ab}}\right) \text{pas} + \mathcal{O}(\bar{\gamma}^2). \quad (73)$$

If we ignore the mass of body b, the results of Eqs. (72) and (73) could return to the 1-point mass case Eqs. (56) and (57).

For experiments conducted in the Solar System, these results could be applied to some cases where only the Sun and Jupiter need to be considered. If we suppose a light ray passing the limb of the Sun, letting the body a be the Sun in Eqs. (72) and (73), then the 2PN deflection angle of $\delta\theta_{GR}$ contains 3.50 μas caused by the Sun itself and -9.46 pico-arcsecond (pas) due to the coupling term of the Sun and Jupiter. For the effect of the deviation from GR at the 2PN level in $\delta\theta_{STT}$, it consists of 81.1 pas from the Sun itself and -0.166 femto-arcsecond (fas) caused by the coupling effect of the Sun and the Jupiter, both of which are below the thresholds of future space missions.

In another case that a light ray just grazes the limb of Jupiter, letting the body a be Jupiter in Eqs. (72) and (73), the deflection angle at 2PN level of $\delta\theta_{GR}$ consists of 0.302 nas by Jupiter and -0.0921 nas by the coupling of the Sun and Jupiter. The 2PN contributions in $\delta\theta_{STT}$ are made of 7.00 fas by Jupiter and -1.61 fas by the coupling of the Sun and Jupiter.

One of the major differences of this work from previous works is abandoning the hierarchized hypothesis, which leads to the 2-body effects in the light deflection at 2PN level. Although these effects in the 2PN light deflection is at least 1 order of magnitude lower than the thresholds of future $\sim$nas experiments in the Solar System, their contributions in $\delta\theta_{GR}$ and $\delta\theta_{STT}$ could reach $-0.515\,\mu$as and -9.02 pas, respectively, for an imaginary observer in a binary system with two $\sim 1M_\odot$ Sun-like stars and separation by ~ 0.1 AU, both of which increase 3 orders of magnitude more than the system with mass ratio $\sim 10^{-3}$ (such as $M_{Jupiter}/M_\odot$) and the same separation. Hence, if this observer would be able to measure $\bar{\gamma}$ down to $\sim 10^{-6}$, the 2-body effect in $\delta\theta_{GR}$ had to be considered, because the third term in $\delta\theta_{GR}$ and the first term in $\delta\theta_{STT}$ are comparable with the level of $\sim 1\,\mu$as.

An even more extreme case would be replacing one Sun-like star in above hypothetical example with a neutron star with radius ~ 10 km, which could make the 2PN 2-body effects in $\delta\theta_{GR}$ and $\delta\theta_{STT}$ raise to -35.9 mas and $-0.627\,\mu$as.

V. CONCLUSIONS AND DISCUSSION

In this paper, we mainly focus on the scalar-tensor theory and obtain its 2PN metric. To investigate its next to the leading order contributions in the light propagation and related measurements in the near zone of a gravitational system, we reduce the metric to its 2PN form for light by supposing an N-point mass system. In doing so, at the 2PN level, we abandon the hierarchized hypothesis and do not assume two PPN parameters γ and β to be in unity, which makes our work differ from previous works. It is found that although there exist γ and β in the 2PN metric, only γ appears in the 2PN equations of light.

As one simple example of applications for future experiments, a gauge-invariant angle between the directions of two incoming photons for a differential measurement is investigated after the light trajectory is obtained in a static and spherically symmetric spacetime. It shows the deviation from GR $\delta\theta_{STT}$ does *not* depend on β even at 2PN level in this circumstance, which is consistent with previous results.

A more complicated application is light deflection in a 2-point mass system. We consider a case that the light crossing time is much less than the time scale of its orbital motion, $\mathfrak{T}_{cross} \ll \mathfrak{T}$, and thus treat it as a "static" system. The 2-body effect at 2PN level originating from relaxing the hierarchized hypothesis is calculated. Our analysis shows the 2PN 2-body effect in the Solar System is 1 order of magnitude less than future $\sim$1 nas experiments, while this effect could be comparable with 1PN component of $\delta\theta_{STT}$ in a binary system with two Sun-like stars and separation by ~ 0.1 AU if an experiment would be able to measure $\bar{\gamma}$ down to $\sim 10^{-6}$.

Our next move is to relax the static approximation for the 2-point mass system with $\mathfrak{T}_{cross} \sim \mathfrak{T}$ and investigate the time evolution pattern of the 2PN light deflection. We suppose the PPN parameter β would appear in the result.

ACKNOWLEDGMENTS

APPENDIX A: EVOLUTION EQUATIONS OF METRIC COEFFICIENTS OF SECOND ORDER POST-NEWTONIAN APPROXIMATION

In this appendix, we will give the metric coefficients of the scalar-tensor theory at the 2PN order as follows.

1. N and $\overset{(2)}{\zeta}$

The equations for N and $\overset{(2)}{\zeta}$ are

$$\Box N = -8\pi G\sigma, \qquad (A1)$$

$$\Box \overset{(2)}{\zeta} = -4(1-\gamma)\pi G\sigma, \qquad (A2)$$

where $\Box$ is the D'Alembert operator in the Minkowski spacetime and

$$\gamma \equiv \frac{\omega_0 + 1}{\omega_0 + 2}, \qquad (A3)$$

$$G \equiv \frac{2}{\phi_0(1+\gamma)}. \qquad (A4)$$

And we can see $G = 1/\phi_0$ when $\gamma = 1$.

2. H_{ij}

The equation for H_{ij} yields

$$\Box H_{ij} = -8\gamma\pi G\sigma\delta_{ij}, \qquad (A5)$$

which gives $H_{ij} \equiv V\delta_{ij}$. Then, we obtain

$$\Box V = -8\gamma\pi G\sigma. \qquad (A6)$$

Furthermore, we have $V = \gamma N$ by comparison with Eqs. (A1) and (A6).

3. L_i

The equation for L_i is

$$\Box L_i = 8(\gamma + 1)\pi G\sigma_i. \qquad (A7)$$

4. L and $\overset{(4)}{\zeta}$

The equations for L and $\overset{(4)}{\zeta}$ are

$$
\begin{aligned}
\Box L = &-8\pi G\sigma\left(V - 2N + \frac{4\beta - \gamma - 3}{\gamma - 1}\overset{(2)}{\zeta}\right) \\
&- 8(\gamma - 1)\pi G\sigma_{kk} - \frac{1}{2}N_{,k}N_{,k} - \frac{1}{2}N_{,k}V_{,k} \\
&- \overset{(2)}{\zeta}_{,k}N_{,k} - \frac{4(\beta-1)}{(\gamma-1)^2}\overset{(2)}{\zeta}_{,k}\overset{(2)}{\zeta}_{,k},
\end{aligned} \qquad (A8)
$$

$$
\begin{aligned}
\Box \overset{(4)}{\zeta} = &-4\pi G\sigma\left[(\gamma-1)(N-V) + \frac{8(\beta-1)}{\gamma-1}\overset{(2)}{\zeta}\right] \\
&- 8(\gamma - 1)\pi G\sigma_{kk} + \frac{1}{2}\overset{(2)}{\zeta}_{,k}N_{,k} \\
&- \frac{1}{2}\overset{(2)}{\zeta}_{,k}V_{,k} + \frac{4(1-\beta)}{(\gamma-1)^2}\overset{(2)}{\zeta}_{,k}\overset{(2)}{\zeta}_{,k},
\end{aligned} \qquad (A9)
$$

where

$$\beta \equiv 1 + \frac{\omega_1}{(2\omega_0 + 3)(2\omega_0 + 4)^2}. \qquad (A10)$$

5. Q_{ij}

The equation for Q_{ij} reads as

$$\Box Q_{ij} = -8(1+\gamma)\pi G\sigma_{ij} - \frac{1}{2}N_{,i}N_{,j} + VN_{,ij} - VV_{,ij} - \frac{1}{2}V_{,i}V_{,j} - \overset{(2)}{\zeta}N_{,ij}$$
$$- \frac{1}{2}\overset{(2)}{\zeta}_{,i}N_{,j} - \frac{1}{2}\overset{(2)}{\zeta}_{,j}N_{,i} + \frac{1}{2}\overset{(2)}{\zeta}_{,i}V_{,j} + \frac{1}{2}\overset{(2)}{\zeta}_{,j}V_{,i} - 2V\overset{(2)}{\zeta}_{,ij} + \overset{(2)}{\zeta}V_{,ij} + 2\overset{(2)}{\zeta}\overset{(2)}{\zeta}_{,ij}$$
$$+ \frac{2(2\gamma-1)}{\gamma-1}\overset{(2)}{\zeta}_{,i}\overset{(2)}{\zeta}_{,j} + \delta_{ij}\Bigg\{ +8\pi G\sigma\Bigg[+\gamma(N-2V) + \frac{4\beta-4+\gamma^2-\gamma}{\gamma-1}\overset{(2)}{\zeta}\Bigg]$$
$$+ 16\gamma\pi G\sigma_{kk} + \frac{1}{2}N_{,k}V_{,k} + \frac{1}{2}V_{,k}V_{,k} - \overset{(2)}{\zeta}_{,k}V_{,k} + \frac{4(\beta-1)}{(\gamma-1)^2}\overset{(2)}{\zeta}_{,k}\overset{(2)}{\zeta}_{,k} \Bigg\}. \tag{A11}$$

6. Q_i

The equation for Q_i is

$$\Box Q_i = +8(\gamma+1)\pi G\sigma_i(2V-N-\overset{(2)}{\zeta}) + 8\pi G\sigma L_i + \frac{1}{2}N_{,i}N_{,t} + N_{,i}V_{,t} + \frac{1}{2}V_{,i}N_{,t} + \frac{1}{2}VN_{,it} + V_{,i}V_{,t}$$
$$- \frac{1}{2}VV_{,it} - \frac{1}{2}N_{,k}L_{i,k} + \frac{1}{2}N_{,ik}L_k - N_{,k}L_{k,i} + \frac{1}{2}V_{,k}L_{i,k} - \frac{1}{2}V_{,ik}L_k - V_{,k}L_{k,i} - \frac{1}{2}\overset{(2)}{\zeta}_{,t}N_{,i}$$
$$- \frac{1}{2}\overset{(2)}{\zeta}N_{,it} - \overset{(2)}{\zeta}_{,it}V + \frac{3}{2}\overset{(2)}{\zeta}_{,t}V_{,i} + \frac{1}{2}\overset{(2)}{\zeta}V_{,it} - L_{i,k}\overset{(2)}{\zeta}_{,k} - \overset{(2)}{\zeta}_{,ik}L_k + \frac{(3\gamma-1)}{\gamma-1}\overset{(2)}{\zeta}_{,i}\overset{(2)}{\zeta}_{,t} + \overset{(2)}{\zeta}\overset{(2)}{\zeta}_{,it}. \tag{A12}$$

7. Q and $\overset{(6)}{\zeta}$

The equations for Q and $\overset{(6)}{\zeta}$ are

$$\Box Q = -8\pi G\sigma\Bigg\{ +N^2 - 2NV - 2L - \frac{4\beta-\gamma-3}{\gamma-1}(2N-V)\overset{(2)}{\zeta}$$
$$+ \Bigg[1 - \frac{\iota}{(\gamma-1)^2} - \frac{4(\beta-1)(\gamma^2+8\beta-2\gamma-7)}{(\gamma-1)^3}\Bigg]\overset{(2)}{\zeta}^2 + \frac{4\beta-\gamma-3}{\gamma-1}\overset{(4)}{\zeta}\Bigg\}$$
$$- 8\pi G\sigma_{kk}\Bigg[(2\gamma-1)V + (2-\gamma)N - \frac{\gamma^2+8\beta-2\gamma-7}{\gamma-1}\overset{(2)}{\zeta}\Bigg] + 16\pi G\sigma_k L_k$$
$$- \frac{1}{2}NN_{,k}N_{,k} + \frac{1}{2}VN_{,k}V_{,k} + \frac{3}{2}N_{,t}N_{,t} + \frac{3}{2}V_{,t}V_{,t} + N_{,t}V_{,t} + NN_{,tt} + VN_{,tt} - N_{,k}L_{k,t} - L_k N_{,kt} - V_{,kt}L_k - N_{,k}L_{,k}$$
$$- \frac{1}{2}V_{,k}L_{,k} + N_{,k}Q_{kl,l} + N_{,kl}Q_{kl} - \frac{1}{2}N_{,l}Q_{kk,l} - L_{k,l}L_{l,k} + L_{l,k}L_{l,k} + 2\overset{(2)}{\zeta}_{,t}V_{,t} - 2\overset{(2)}{\zeta}_{,kt}L_k - \overset{(2)}{\zeta}_{,k}L_{,k}$$
$$+ \frac{2(\gamma^2-\gamma+2\beta-2)}{(\gamma-1)^2}\overset{(2)}{\zeta}_{,t}\overset{(2)}{\zeta}_{,t} + \frac{4(\beta-1)}{(\gamma-1)^2}N\overset{(2)}{\zeta}_{,k}\overset{(2)}{\zeta}_{,k} + \overset{(2)}{\zeta}\overset{(2)}{\zeta}_{,k}N_{,k} + \frac{2\iota}{(\gamma-1)^3}\overset{(2)}{\zeta}\overset{(2)}{\zeta}_{,k}\overset{(2)}{\zeta}_{,k}$$
$$+ \frac{4(\beta-1)(\gamma^2+8\beta-2\gamma-7)}{(\gamma-1)^4}\overset{(2)}{\zeta}\overset{(2)}{\zeta}_{,k}\overset{(2)}{\zeta}_{,k} - \overset{(4)}{\zeta}_{,k}N_{,k} - \frac{8(\beta-1)}{(\gamma-1)^2}\overset{(2)}{\zeta}_{,k}\overset{(4)}{\zeta}_{,k}, \tag{A13}$$

and

$$\Box\overset{(6)}{\zeta} = -4\pi G\sigma\left[+(\gamma-1)(NV+L) - \frac{8(\beta-1)}{\gamma-1}\overset{(2)}{\zeta}(N-V) - \frac{2(32\beta^2+\iota\gamma-64\beta-\iota+32)}{(\gamma-1)^3}\overset{(2)^2}{\zeta} + \frac{8(\beta-1)}{\gamma-1}\overset{(4)}{\zeta}\right]$$

$$-4\pi G\sigma_{kk}\left[(\gamma-1)(3V-N) - \frac{16(\beta-1)}{\gamma-1}\overset{(2)}{\zeta}\right] - 8\pi G(\gamma-1)\sigma_k L_k + \frac{1}{2}\overset{(2)}{N}\overset{(2)}{\zeta}_{,k}N_{,k} + \frac{1}{2}\overset{(2)}{V}\overset{(2)}{\zeta}_{,k}V_{,k} - 2\overset{(2)}{\zeta}_{,kl}L_k - \overset{(2)}{\zeta}_{,k}L_{k,t}$$

$$+\frac{1}{2}\overset{(2)}{\zeta}_{,k}L_{,k} + \overset{(2)}{\zeta}_{,k}Q_{kl,l} + \overset{(2)}{\zeta}_{,lk}Q_{lk} - \frac{1}{2}\overset{(2)}{\zeta}_{,l}Q_{kk,l} + (N+V)\overset{(2)}{\zeta}_{,tt} + \frac{4\beta+2\gamma-\gamma^2-5}{(\gamma-1)^2}\overset{(2)}{\zeta}_{,t}\overset{(2)}{\zeta}_{,t}$$

$$+\frac{32(\beta-1)^2+2\iota(\gamma-1)}{(\gamma-1)^4}\overset{(2)}{\zeta}\overset{(2)}{\zeta}_{,k}\overset{(2)}{\zeta}_{,k} - \frac{8(\beta-1)}{(\gamma-1)^2}\overset{(2)}{\zeta}_{,k}\overset{(4)}{\zeta}_{,k} + \frac{1}{2}\overset{(4)}{\zeta}_{,k}N_{,k} - \frac{1}{2}\overset{(4)}{\zeta}_{,k}V_{,k}, \tag{A14}$$

where

$$\iota \equiv \frac{1}{2}\frac{(\gamma-1)^4}{\gamma+1}\omega_2. \tag{A15}$$

In general relativity, $\iota = 0$ [13].

APPENDIX B: APPLICATION IN A STATIC, SPHERICALLY SYMMETRIC SPACETIME

1. Light ray trajectory

In this appendix, we will consider the gravitational field outside a static, spherically symmetric body and mainly focus on the parameters in 2PN level in several gravity theories. The trajectory of the light ray can be obtained through integrating the following equation:

$$\ddot{x} = -(1+\gamma)\frac{Gm_a}{r_a^2}\left[n_a - 2\frac{(n_a\dot{x})\dot{x}}{c^2}\right] + \varepsilon^2(1+\gamma)\frac{G^2m_a^2}{r_a^3}$$

$$\times\left\{\left[2(1+\gamma) + \frac{(n_a\dot{x})^2}{c^2}\right]n_a - \frac{(n_a\dot{x})\dot{x}}{c^2}\right\}, \tag{B1}$$

by adopting the iterative method used in Ref. [18]. We assume the unperturbed light ray as follows:

$$x_N = x_0 + c(t-t_0)k, \tag{B2}$$

where k^i is a unit vector representing the light direction at $t = -\infty$, t_0 is an instant on the light path and x_0^i is the position of photon at t_0. The instant t_0 can be arbitrarily

chosen, for example the emission or the observation instant. The photon's coordinates can be written as sum of perturbations on x_N^i:

$$x(t) = x_N + \delta x \equiv x_N + \delta x_{1PN} + \delta x_{2PN}. \tag{B3}$$

And we use the following assumption for motion of the bodies:

$$r_A(t) = x(t) - y_a = x(t) - y_a(t_a), \tag{B4}$$

where t_a is the moment of the closest approach between the body a and the unperturbed light ray. $y_a(t_a)$ means the position of a-th body (y_a) is evaluated at the time t_a. After these, we obtain the results with the method as [18]

$$\frac{1}{c}\dot{x}(t) = k + \frac{1}{c}\delta\dot{x}_{1PN}(x_N) + \frac{1}{c}\delta\dot{x}_{2PN}(x_N), \tag{B5}$$

$$x(t) = x_N(t) + [\delta x_{1PN}(x_N) - \delta x_{1PN}(x_0)] + [\delta x_{2PN}(x_N) - \delta x_{2PN}(x_0)]. \tag{B6}$$

It is worth noting that the 2PN terms in our solution actually have two sources: direct and indirect. The direct part comes from the 2PN order itself. The indirect part comes from the 1PN terms when the 1PN solution is iterated into itself in order to attain a 2PN accuracy, namely, we substitute $x_N + \delta x_{1PN}$ into the trajectory of the light ray in the 1PN approximation. And

$$\frac{1}{c}\delta\dot{x}_{1PN}(x) = -(1+\gamma)\frac{Gm_a}{c^2r_a}\left\{\frac{k\times(r_a\times k)}{r_a - k\cdot r_a} + k\right\}, \tag{B7}$$

$$\frac{1}{c}\delta\dot{x}_{2PN}(x) = -\frac{1}{4}(1+\gamma)\frac{G^2m_a^2}{c^4r_a^4}(k\cdot r_a)r_a + \frac{G^2m_a^2}{c^4}d_a\left\{(1+\gamma)^2\frac{1}{r_a(r_a-k\cdot r_a)}\left(\frac{2}{r_a} + \frac{1}{r_a-k\cdot r_a}\right)\right.$$

$$\left. -\frac{1}{8}(1+\gamma)(7+8\gamma)\frac{1}{d_a^2}\left[\frac{k\cdot r_a}{r_a^2} + \frac{1}{d_a}\left(\frac{\pi}{2} + \arctan\frac{k\cdot r_a}{d_a}\right)\right]\right\}$$

$$+\frac{G^2m_a^2}{c^4r_a}k\left[\frac{1}{4}(1+\gamma)(5+4\gamma)\frac{1}{r_a} - (1+\gamma)^2\frac{1}{r_a-k\cdot r_a}\right], \tag{B8}$$

and

$$\delta \boldsymbol{x}_{1\text{PN}}(\boldsymbol{x}) = -(1+\gamma)\frac{Gm_a}{c^2}\left\{\frac{\boldsymbol{k} \times (\boldsymbol{r}_a \times \boldsymbol{k})}{r_a - \boldsymbol{k} \cdot \boldsymbol{r}_a} + \boldsymbol{k}\ln(r_a + \boldsymbol{k} \cdot \boldsymbol{r}_a)\right\}, \tag{B9}$$

$$\delta \boldsymbol{x}_{2\text{PN}}(\boldsymbol{x}) = \frac{1}{8}(1+\gamma)\frac{G^2 m_a^2}{c^4 r_a^2}\boldsymbol{r}_a + \frac{G^2 m_a^2}{c^4}\boldsymbol{k}\left[-\frac{1}{8}(1+\gamma)(7+8\gamma)\frac{1}{d_a}\arctan\frac{\boldsymbol{k} \cdot \boldsymbol{r}_a}{d_a} - (1+\gamma)^2\frac{1}{r_a - \boldsymbol{k} \cdot \boldsymbol{r}_a}\right]$$
$$+ \frac{G^2 m_a^2}{c^4}\boldsymbol{d}_a\left\{(1+\gamma)^2\frac{1}{(r_a - \boldsymbol{k} \cdot \boldsymbol{r}_a)^2} - \frac{1}{8}(1+\gamma)(7+8\gamma)\frac{\boldsymbol{k} \cdot \boldsymbol{r}_a}{d_a^3}\left[\frac{\pi}{2} + \arctan\frac{\boldsymbol{k} \cdot \boldsymbol{r}_a}{d_a}\right]\right\}, \tag{B10}$$

where $\boldsymbol{d}_a = \boldsymbol{k} \times (\boldsymbol{r}_a \times \boldsymbol{k})$ is an impact parameter to represent the closest distance between the unperturbed light ray and body a, $d_a = |\boldsymbol{d}_a|$. For GR ($\gamma = \beta = 1$), Eqs. (B5) and (B6) will reduce to the results of Ref. [18].

2. Light deflection

In some practical astronomical measurements, a differential measurement is more powerful. This concept is employed by LATOR mission [1,2] through a skinny triangle formed by two spacecrafts and the International Space Station. Thus, we construct a gauge-invariant angle θ between the directions of two incoming photons based on [18,41]. It reads

$$\cos\theta = \frac{h_{\alpha\beta}K_1^\alpha K_2^\beta}{\sqrt{h_{\alpha\beta}K_1^\alpha K_1^\beta}\sqrt{h_{\alpha\beta}K_2^\alpha K_2^\beta}}, \tag{B11}$$

where the spatial projection operator is

$$h_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta, \tag{B12}$$

which projects the two incoming photons onto the hypersurface orthogonal to the observer's four-velocity $u^\alpha \equiv dx^\alpha/cd\tau$ and $K_1^\alpha \equiv dx_1^\alpha(t)/dt$, and $K_2^\beta \equiv dx_2^\beta(t)/dt$ are the tangent vectors of the paths $x_1^\alpha(t)$ and $x_2^\beta(t)$ of the two incoming photons. Then, for a static observer, we obtain

$$\theta = \vartheta_0 + (1+\gamma)\frac{Gm_a}{c^2 r_a \sin\vartheta_0}\left\{\frac{\boldsymbol{k}_1 \cdot [\boldsymbol{k}_2 \times (\boldsymbol{r}_a \times \boldsymbol{k}_2)]}{r_a - \boldsymbol{k}_2 \cdot \boldsymbol{r}_a} + \frac{\boldsymbol{k}_2 \cdot [\boldsymbol{k}_1 \times (\boldsymbol{r}_a \times \boldsymbol{k}_1)]}{r_a - \boldsymbol{k}_1 \cdot \boldsymbol{r}_a}\right\}$$
$$- (1+\gamma)^2\frac{G^2 m_a^2}{c^4 \sin\vartheta_0}\left[\frac{\boldsymbol{k}_2 \cdot \boldsymbol{d}_{1a}}{d_{1a}^3}\left(1 + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{r}_a}{r_a}\right) + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{d}_{2a}}{d_{2a}^3}\left(1 + \frac{\boldsymbol{k}_2 \cdot \boldsymbol{r}_a}{r_a}\right)\right] - \frac{1}{8}(1+\gamma)\frac{G^2 m_a^2}{c^4 r_a^2}\left[\frac{\boldsymbol{k}_1 \cdot \boldsymbol{r}_a}{d_{1a}} + \frac{\boldsymbol{k}_2 \cdot \boldsymbol{r}_a}{d_{2a}}\right]$$
$$+ \frac{1}{8}(7+8\gamma)(1+\gamma)\frac{G^2 m_a^2}{c^4 \sin\vartheta_0}\left\{\frac{\boldsymbol{k}_1 \cdot \boldsymbol{d}_{2a}}{d_{2a}^3}\left[\frac{\pi}{2} + \arctan\left(\frac{\boldsymbol{k}_2 \cdot \boldsymbol{r}_a}{d_{2a}}\right)\right] + \frac{\boldsymbol{k}_2 \cdot \boldsymbol{d}_{1a}}{d_{1a}^3}\left[\frac{\pi}{2} + \arctan\left(\frac{\boldsymbol{k}_1 \cdot \boldsymbol{r}_a}{d_{1a}}\right)\right]\right\}, \tag{B13}$$

where

$$\vartheta_0 = \arccos(\boldsymbol{k}_1 \cdot \boldsymbol{k}_2), \tag{B14}$$

and we use that the position of the photon at the moment t of observation coincides with the position of the observer so that

$$\boldsymbol{x}_{\text{obs}} = \boldsymbol{x}_{01} + c(t - t_{01})\boldsymbol{k}_1 + \delta\boldsymbol{x}_1$$
$$= \boldsymbol{x}_{02} + c(t - t_{02})\boldsymbol{k}_2 + \delta\boldsymbol{x}_2, \tag{B15}$$

where $(t_{01}, \boldsymbol{x}_{01})$ denotes the moment and position of the light signal 1 of emission and $(t_{02}, \boldsymbol{x}_{02})$ for the light signal 2, respectively. In Eq. (B13), $\boldsymbol{d}_{1a} = \boldsymbol{k}_1 \times (\boldsymbol{r}_a \times \boldsymbol{k}_1)$ and is an impact parameter to represent the closest distance between the unperturbed light ray 1 and a-th body. Similarly, $\boldsymbol{d}_{2a} = \boldsymbol{k}_2 \times (\boldsymbol{r}_a \times \boldsymbol{k}_2)$ and is an impact parameter to represent the closest distance between the unperturbed light ray 2 and a-th body.

Furthermore, we assume that one of the two light rays moves along the line connecting the body a and the observer, namely source 2. This means the impact parameter for source 2 is zero so that $d_{2a} = |\boldsymbol{k}_2 \times (\boldsymbol{r}_a \times \boldsymbol{k}_2)| = 0$, $\boldsymbol{k}_2 \cdot \boldsymbol{d}_{1a}/d_{1a} = \sin\vartheta_0$, $\boldsymbol{k}_1 \cdot \boldsymbol{r}_a/r_a \simeq \cos\vartheta_0$, $d_{1a}^2 = r_a^2 - (\boldsymbol{k}_1 \cdot \boldsymbol{r}_a)^2$, $\boldsymbol{k}_1 \cdot \boldsymbol{r}_a/d_{1a} \approx \pi/2$ and $d_{1a}/r_a = 0$. Then, we obtain Eq. (54).

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