Vorticity and entropy production in tilted Szekeres spacetimes

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We analyze the properties of the tilted Szekeres spacetime, i.e. the version of such spacetime as seen by a congruence of observers with respect to which the fluid is moving. The imperfect fluid and the kinematical variables associated to the four-velocity of the fluid assigned by tilted observers are studied in detail. As it happens for the case of the Lemaitre-Tolman-Bondi spacetime, the fluid evolves nonreversibly (with nonvanishing entropy production) and is nongeodesic. However, unlike the latter case, the tilted observer detects vorticity in the congruence of the fluid world lines. Also, as for the nontilted congruence, the magnetic part of the Weyl tensor vanishes, reinforcing the nonradiative character of this kind of spacetime. Possible physical implications of these results are discussed.

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I. INTRODUCTION

In any physical theory, part of the description of a specific scenario is tightly related to the congruence of observers carrying out the study [1]. In the case of general relativity, the existing arbitrariness in the choice of the four-velocity in terms of which the energy-momentum tensor is split leads to a variety of physical interpretations for the source of a given spacetime (see [2–17], and references therein).

As it has been shown in the references above, when the two possible interpretations of a given spacetime correspond to two congruences of observers related by a local Lorentz boost, both the general properties of the source and the kinematical properties of the congruence would be different.

Particularly enlightening is the case of tilted Lemaitre-Tolman-Bondi (LTB) spacetime whose fluid source is nongeodesic and evolves nonreversibly, unlike the nontilted version (see [17] for details).

An important (though obvious) remark is in order at this point: we are assuming general relativity to be valid. This implies that special relativity is always valid at the local level. Therefore we can always perform a Lorentz boost (locally), thereby ensuring the existence of tilted observers.

In this work we endeavour to study in detail tilted Szekeres spacetimes [18,19]. The motivations to undertake such a task are many.

Szekeres dust models have no Killing vectors and therefore represent an interesting generalization of LTB spacetimes [20,21]. They have been extensively used as cosmological models (see [22,23], and references therein) and in the discussion of the gravitational arrow of time [24].

Physical and topological properties of Szekeres spacetimes, as well as the problem of structure formation and other cosmological related issues, have been considered in [25–34].

A renewed interest in Szekeres spacetimes has appeared, in relation with the potential role of density inhomogeneity to explain the observed cosmic acceleration, without invoking dark energy (see [35–42], and references therein).

As we shall see below, a heat flux term will appear in the energy-momentum tensor of the tilted congruence. As it happens for LTB [17], such a heat flux vector would be necessarily associated to a "truly" (i.e. entropy producing) dissipative phenomenon, even in the absence of bulk and shear viscosity.

We shall see that as in the nontilted version, the magnetic part of the Weyl tensor vanishes, implying that the super-Poynting vector associated to the Bel-Robinson tensor vanishes, too, which suggests the absence of gravitational radiation. Thus, this confirms the nonradiative character of the Szekeres spacetime already stressed in [20].

Besides the dissipative (irreversible) evolution of the tilted version, and the nongeodesic character of the fluid, another important difference with respect to the nontilted version appears; namely, the vorticity of the fluid lines defined by the tilted four-velocity is now nonvanishing. As it can be expected, this vorticity has important consequences on the physical interpretation of the model. We shall comment on this issue in the last section.

II. SZEKERES SPACETIME

Before considering the tilted version of Szekeres spacetime, we find it useful to provide a very brief review of this spacetime in its standard, nontilted, version.

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Since the publication of the first two original papers by Szekeres [18,19], this metric has been presented in many different forms. Here we shall closely follow (with slight changes in notation) the formulation given in [26].

There are two classes of Szekeres spacetimes, which in Szekeres notation correspond to $\beta' \neq 0$ and $\beta' = 0$; here we shall deal only with the first class, denoted as the LT-type Szekeres metric in [26].

Thus, the line element is given by

$$ds^2 = -dt^2 + \frac{(R'E - RE')^2}{E^2(\epsilon + f)} dr^2 + \frac{R^2}{E^2}(dp^2 + dq^2), \quad (1)$$

where a prime denotes a derivative with respect to r, $R = R(t, r)$, $\epsilon = \pm 1, 0$, and $f = f(r) > -\epsilon$ is an arbitrary function of r. We number the coordinates $x^0 = t$, $x^1 = r$, $x^2 = p$, $x^3 = q$.

The function E is given by

$$E(r, p, q) = \frac{S}{2}\left[\left(\frac{p - P}{S}\right)^2 + \left(\frac{q - Q}{S}\right)^2 + \epsilon\right], \quad (2)$$

where $S = S(r)$, $P = P(r)$, and $Q = Q(r)$ are arbitrary functions.

From Einstein equations, it follows that R satisfies the equation

$$\dot{R}^2 = \frac{2M}{R} + f, \quad (3)$$

where a dot denotes a derivative with respect to t, and $M = M(r)$ is an arbitrary function. From the above equation, it follows that

$$\ddot{R} = \frac{-M}{R^2}, \quad (4)$$

from where the meaning of M as an effective gravitational mass becomes evident.

As in the LTB case, there are three subfamilies of solutions depending on the value of f, namely, hyperbolic ($f > 0$), parabolic ($f = 0$), and elliptic ($f < 0$).

On the other hand, the parameter ϵ determines how the two-surfaces, $r = $ constant, foliate the three-dimensional spatial sections of constant t. These may be spheres ($\epsilon = +1$), planes ($\epsilon = 0$), or quasispheres ($\epsilon = -1$).

For the "spherical" case $\epsilon = +1$, we have the following relationship between coordinates q, p and "spherical angle" coordinates (θ, ϕ):

$$\frac{(p - P)}{S} = \cot\frac{\theta}{2} \cos\phi, \quad (5)$$

$$\frac{(q - Q)}{S} = \cot\frac{\theta}{2} \sin\phi. \quad (6)$$

The LTB spacetime is recovered in the case $E' = 0$, $\epsilon = +1$.

For the nontilted (comoving) congruence, the source of Szekeres spacetime is pure dust; therefore the energy-momentum tensor is

$$T_{\mu\nu} = \bar{\mu}(t, r, p, q)v_\mu v_\nu, \quad (7)$$

where the vector v^μ ($v^\mu v_\mu = -1$) is just

$$v^\mu = (1, 0, 0, 0), \quad (8)$$

and $\bar{\mu}$ denotes the energy-momentum density as measured by comoving observers. Then, from the (t, t) component of the Einstein equations, we have

$$8\pi\bar{\mu} = \frac{2(M' - 3ME'/E)}{R^2(R' - RE'/E)}. \quad (9)$$

The two nonvanishing kinematical variables are the expansion scalar

$$\bar{\Theta} = v^\mu_{;\mu} = \frac{\dot{R}'E - \dot{R}E'}{R'E - RE'} + 2\frac{\dot{R}}{R} \quad (10)$$

and the shear tensor, which can be written as

$$\bar{\sigma}_{\mu\nu} = \sigma\left(n_\mu n_\nu - \frac{1}{3}h_{\mu\nu}\right), \quad (11)$$

where $h_{\mu\nu} = g_{\mu\nu} + v_\mu v_\nu$, $\bar{\sigma}^{\mu\nu}\bar{\sigma}_{\mu\nu} = \frac{2}{3}\sigma^2$, and

$$n^\mu = \left(0, \frac{E(\epsilon + f)^{1/2}}{R'E - RE'}, 0, 0\right) \quad (12)$$

($n^\mu n_\mu = 1$), with

$$\sigma = \frac{E}{R}\left(\frac{R\dot{R}' - \dot{R}R'}{R'E - RE'}\right). \quad (13)$$

Next, the electric part of the Weyl tensor

$$E_{\mu\nu} = C_{\mu\gamma\nu\delta}v^\gamma v^\delta \quad (14)$$

can be written as

$$E_{\mu\nu} = \mathcal{E}\left(n_\mu n_\nu - \frac{1}{3}h_{\mu\nu}\right), \quad (15)$$

where

$$\mathcal{E} = 3\frac{\ddot{R}}{R} + 4\pi\bar{\mu}. \quad (16)$$

The magnetic parts of the Weyl and the Riemann tensors vanish identically.

The fact that both the shear tensor and the electric part of the Weyl tensor have only one nonvanishing independent component, as well as the vanishing of the magnetic part of the Weyl tensor, illustrates how close this spacetime is to the spherical symmetry. This similarity is further reinforced by the fact that the specific forms (11) and (15) are typical of spherical symmetry. Thus the qualification of "quasispherical" assigned by Szekeres himself to his solution [18] looks well justified, in spite of the fact that it has no Killing vectors.

We shall next consider the tilted version of Szekeres spacetime.

III. TILTED SZEKERES SPACETIME

In order to obtain a tilted congruence, let us perform a Lorentz boost from the locally comoving Minkowskian frame (associated to v^μ) to the locally Minkowskian frame with respect to which a fluid element has "radial" (in the r direction) velocity ω. For simplicity we shall consider a boost only in the "radial" direction and ω to be a function of t and r alone.

The corresponding tilted congruence is characterized by the four-velocity vector field

$$V^\mu = \left(\frac{1}{(1-\omega^2)^{1/2}}, \frac{E(\epsilon+f)^{1/2}}{R'E - RE'}\frac{\omega}{(1-\omega^2)^{1/2}}, 0, 0\right). \quad (17)$$

For the tilted observer, the matter distribution described by (7) will be that given by

$$T_{\mu\nu} = \mu V_\mu V_\nu + P h_{\mu\nu} + \Pi_{\mu\nu} + q_\mu V_\nu + V_\mu q_\nu, \quad (18)$$

where μ is the energy density, P the isotropic pressure, q^μ the heat flux vector, and $\Pi_{\mu\nu}$ the anisotropic pressure tensor, that this observer measures (of course the projector tensor $h_{\mu\nu}$ is now defined in terms of V^μ instead of v^μ).

The above is the canonical, algebraic decomposition of a second order symmetric tensor with respect to a unit time-like vector, which has the above indicated standard physical meaning when $T_{\alpha\beta}$ is the energy-momentum tensor describing some energy distribution, and V^μ the four-velocity assigned by a certain observer.

It is immediate to see that

$$\mu = T_{\alpha\beta} V^\alpha V^\beta, \qquad q_\alpha = -\mu V_\alpha - T_{\alpha\beta} V^\beta, \quad (19)$$

$$P = \frac{1}{3}h^{\alpha\beta} T_{\alpha\beta}, \qquad \Pi_{\alpha\beta} = h_\alpha^\mu h_\beta^\nu (T_{\mu\nu} - P h_{\mu\nu}). \quad (20)$$

A quick calculation taking into account the above renders

$$\mu = \frac{\bar\mu}{1-\omega^2}, \qquad P = \frac{\bar\mu\omega^2}{3(1-\omega^2)}, \quad (21)$$

$$q^\alpha = q N^\alpha, \qquad q = -\frac{\bar\mu\omega}{1-\omega^2}, \quad (22)$$

$$N^\alpha = \left(\frac{\omega}{(1-\omega^2)^{1/2}}, \frac{E(\epsilon+f)^{1/2}}{R'E - RE'}\frac{1}{(1-\omega^2)^{1/2}}, 0, 0\right), \quad (23)$$

$$\Pi_{\alpha\beta} = \frac{\bar\mu\omega^2}{1-\omega^2}\left(N_\alpha N_\beta - \frac{1}{3}h_{\alpha\beta}\right), \quad (24)$$

where use has been made of

$$v_\alpha = \frac{1}{(1-\omega^2)^{1/2}} V_\alpha - \frac{\omega}{(1-\omega^2)^{1/2}} N_\alpha. \quad (25)$$

Alternatively, it may be useful to define two auxiliary variables (P_r and $P_\perp$) as

$$P_r = N^\alpha N^\beta T_{\alpha\beta}, \qquad P_\perp = K^\alpha K^\beta T_{\alpha\beta}, \quad (26)$$

where K^α is a unit spacelike vector (orthogonal to V^α) in the p direction.

Then we may write

$$\Pi_{\alpha\beta} = \Pi\left(N_\alpha N_\beta - \frac{1}{3}h_{\alpha\beta}\right), \quad (27)$$

with

$$\Pi = P_r - P_\perp, \qquad P = \frac{P_r + 2P_\perp}{3}, \quad (28)$$

from where the physical meaning of P_r and $P_\perp$ becomes evident.

Using the above definitions, the relationships linking tilted and nontilted variables, besides (21) and (22), read

$$P_\perp = 0, \qquad 3P = \Pi = P_r = \mu\omega^2. \quad (29)$$

We have next calculated the kinematical variables for the tilted congruence. These were obtained with MAPLE 13.

The four-acceleration is now nonvanishing, and its components are

$$a^0 = \frac{\omega^2\omega'}{(1-\omega^2)^2}\frac{E\sqrt{\epsilon+f}}{R'E - RE'} + \frac{\omega^2}{1-\omega^2}\frac{E\dot R' - E'\dot R}{R'E - RE'} + \frac{\omega\dot\omega}{(1-\omega^2)^2}, \quad (30)$$

$$a^1 = \frac{1}{\omega}\frac{E\sqrt{\epsilon+f}}{R'E - RE'}a^0, \quad (31)$$

$$a^2 = \frac{\omega^2\Delta}{1-\omega^2}\frac{E}{R} \qquad a^3 = \frac{\omega^2\Gamma}{1-\omega^2}\frac{E}{R}, \quad (32)$$

where subscripts p, q denote a derivative with respect to these coordinates and

$$\Delta = \frac{E_p'E - E'E_p}{R'E - RE'}, \qquad \Gamma = \frac{E_q'E - E'E_q}{R'E - RE'}. \quad (33)$$

The expansion scalar now becomes

$$\begin{aligned}\Theta &= \frac{1}{\sqrt{1-\omega^2}}\frac{ER\dot R' + \dot R(2R'E - 3RE')}{R(R'E - RE')} + \frac{\omega\dot\omega}{(1-\omega^2)^{3/2}} + \frac{2\omega\sqrt{\epsilon+f}}{\sqrt{1-\omega^2}R} + \frac{\omega'}{(1-\omega^2)^{3/2}}\frac{E\sqrt{\epsilon+f}}{(R'E - RE')} = \\ &= \frac{1}{\sqrt{1-\omega^2}}\bar\Theta + \frac{\omega\dot\omega}{(1-\omega^2)^{3/2}} + \frac{2\omega\sqrt{\epsilon+f}}{\sqrt{1-\omega^2}R} + \frac{\omega'}{(1-\omega^2)^{3/2}}\frac{E\sqrt{\epsilon+f}}{(R'E - RE')}.\end{aligned} \quad (34)$$

Next, for the vorticity tensor

$$\Omega_{\mu\nu} = \frac{1}{2}(V_{\mu;\nu} - V_{\nu;\mu}) + \frac{1}{2}(a_\mu V_\nu - a_\nu V_\mu), \quad (35)$$

we find the following nonvanishing components:

$$\Omega_{02} = \frac{\omega^2 \Delta}{(1-\omega^2)^{3/2}} \frac{R}{E}, \qquad \Omega_{03} = \frac{\omega^2 \Gamma}{(1-\omega^2)^{3/2}} \frac{R}{E}, \quad (36)$$

$$\Omega_{12} = -\frac{\omega}{(1-\omega^2)^{3/2}} \frac{R}{E} \sqrt{\epsilon + f}(E_p' E - E' E_p), \quad (37)$$

$$\Omega_{13} = -\frac{\omega}{(1-\omega^2)^{3/2}} \frac{R}{E} \sqrt{\epsilon + f}(E_q' E - E' E_q), \quad (38)$$

whereas for the vorticity vector,

$$\Omega_\mu = \eta_{\mu\nu\rho\sigma} V_{\alpha;\beta} V^\sigma g^{\nu\alpha} g^{\rho\beta}, \quad \eta_{0123} = -\sqrt{-g}, \quad (39)$$

we find

$$\Omega_0 = \Omega_1 = 0, \quad (40)$$

$$\Omega_2 = \frac{\omega\Gamma}{1-\omega^2} \frac{R}{E} \qquad \Omega_3 = \frac{-\omega\Delta}{1-\omega^2} \frac{R}{E}. \quad (41)$$

Finally, for the shear tensor the following nonvanishing components appear:

$$\sigma_{00} = \frac{2}{3} \frac{\omega^2}{1-\omega^2} \sigma_I, \qquad \sigma_{01} = -\frac{1}{\omega} \sqrt{g_{11}} \sigma_{00}, \quad (42)$$

$$\sigma_{02} = -\frac{1}{2} \frac{\omega^2 \Delta}{(1-\omega^2)^{3/2}} \frac{R}{E}, \qquad \sigma_{03} = -\frac{1}{2} \frac{\omega^2 \Gamma}{(1-\omega^2)^{3/2}} \frac{R}{E}, \quad (43)$$

$$\sigma_{11} = \frac{1}{\omega^2} g_{11} \sigma_{00}, \qquad \sigma_{12} = -\frac{1}{\omega} \sqrt{g_{11}} \sigma_{02}, \quad (44)$$

$$\sigma_{13} = -\frac{1}{\omega} \sqrt{g_{11}} \sigma_{03}, \qquad \sigma_{22} = -\frac{1}{2} \frac{1-\omega^2}{\omega^2} g_{22} \sigma_{00}, \quad (45)$$

$$\sigma_{33} = \sigma_{22}, \quad (46)$$

where

$$\sigma_I = -\frac{\omega}{(1-\omega^2)^{3/2}} \frac{\sqrt{\epsilon+f}}{R} + \frac{1}{(1-\omega^2)^{1/2}} \frac{E}{R} \frac{R\dot{R}' - R'\dot{R}}{R'E - RE'}$$
$$+ \frac{\omega\dot\omega}{(1-\omega^2)^{3/2}} + \frac{\omega'}{(1-\omega^2)^{3/2}} \frac{E\sqrt{\epsilon+f}}{R'E - RE'}. \quad (47)$$

It is a simple matter to check that the shear tensor may be written as

$$\sigma_{\alpha\beta} = \sigma_I \left(N_\alpha N_\beta - \frac{1}{3} h_{\alpha\beta}\right) + \sigma_{II}(N_\alpha K_\beta + N_\beta K_\alpha)$$
$$+ \sigma_{III}(N_\alpha L_\beta + N_\beta L_\alpha), \quad (48)$$

where

$$\sigma_{II} = \frac{\omega\Delta}{2(1-\omega^2)}, \qquad \sigma_{III} = \frac{\omega\Gamma}{2(1-\omega^2)}, \quad (49)$$

and L^α is a unit spacelike vector in the q direction.

Observe that comparing (24) with (48), it follows that the anisotropy described by $\Pi_{\alpha\beta}$ cannot be associated exclusively to shear viscosity.

It is worth stressing that the appearance of vorticity is unavoidable for a generic tilted Szekeres spacetime. Indeed, vanishing of vorticity implies, because of (41),

$$\Delta = \Gamma = 0, \quad (50)$$

producing

$$\frac{E_p}{E_q} = \psi(p, q), \qu(51)$$

where ψ is an arbitrary function of its arguments. Then using (2) in (51), we obtain

$$p - P(r) = q\psi(p, q) - Q(r)\psi(p, q), \quad (52)$$

which implies $P(r) = Q(r) = 0$, where the regularity condition $P(0) = Q(0) = 0$ has been used. Conditions (50) can also be satisfied for some axially symmetric Szekeres models [43].

Thus for a generic Szekeres spacetime there is a non-vanishing vorticity according to the tilted observer. Furthermore, as it follows from (49) the shear tensor has the form (48), implying three independent components instead of one for the nontilted observer.

It is worth mentioning that the magnetic part of the Weyl tensor vanishes, like for the nontilted case. However the magnetic part of the Riemann tensor

$$Z_{\mu\nu} = \frac{1}{2} \eta_{\mu\rho\alpha\beta} R_{\nu\sigma}{}^{\alpha\beta} V^\rho V^\sigma \quad (53)$$

now has nonvanishing components which are associated to the dissipative flux q^α. They are

$$Z_{23} = -Z_{32}$$
$$= -\frac{\omega}{1-\omega^2} \frac{R}{E^2} \frac{1}{R'E - RE'} [E(R\ddot{R}' + 2\ddot{R}R')$$
$$+ E'(2\dot{R}^2 + R\ddot{R})]. \quad (54)$$

Finally, the electric part of the Weyl tensor,

$$E_{\mu\nu} = C_{\mu\alpha\nu\beta} V^\alpha V^\beta, \quad (55)$$

now has the following components:

$$E_{00} = \frac{2}{3} \frac{\omega^2}{1-\omega^2} \mathcal{E}, \qquad E_{01} = -\frac{2}{3} \frac{\omega}{1-\omega^2} \sqrt{g_{11}} \mathcal{E}, \quad (56)$$

$$E_{22} = -\frac{1}{3} g_{22} \mathcal{E}, \qquad E_{33} = E_{22}, \quad (57)$$

where

$$\mathcal{E} = \frac{E}{R}\frac{R'\ddot{R} - R\ddot{R}'}{R'E - RE'}. \tag{58}$$

From the above it follows that we may write the electric part of the Weyl tensor as

$$E_{\alpha\beta} = \mathcal{E}\left(N_\alpha N_\beta - \frac{1}{3}h_{\alpha\beta}\right), \tag{59}$$

which is similar to the expression (15) for the nontilted case.

IV. TRANSPORT EQUATION AND ENTROPY PRODUCTION

In order to evaluate the possibility of entropy production we need a transport equation for the heat conduction.

Due to the well-known pathologies of the Eckart [44] and Landau [45] approaches we shall resort to causal dissipative theories. Reasons for doing that have been extensively discussed in recent years (see [46–59], and references therein).

Thus for a second order phenomenological theory for dissipative fluids we obtain from the Gibbs equation and conservation equations (see [53,58] for details)

$$TS^\alpha_{;\alpha} = -q^\alpha\left[h^\mu_\alpha(\ln T)_{,\mu} + V_{\alpha;\mu}V^\mu\right.$$
$$\left. + \beta_1 q_{\alpha;\mu}V^\mu + \frac{T}{2}\left(\frac{\beta_1}{T}V^\mu\right)_{;\mu}q_\alpha\right], \tag{60}$$

where S^α is the entropy four-current, T is temperature and $\beta_1 = \frac{\tau}{\kappa T}$, where τ and κ denote thermal relaxation time and the thermal conductivity coefficient, respectively.

Then from the second law of thermodynamics

$$S^\alpha_{;\alpha} \geq 0, \tag{61}$$

the following transport equation is obtained (see [53,58] for details):

$$\tau h^{\alpha\beta}V^\gamma q_{\beta;\gamma} + q^\alpha = -\kappa h^{\alpha\beta}(T_{,\beta} + Ta_\beta)$$
$$- \frac{1}{2}\kappa T^2\left(\frac{\tau V^\beta}{\kappa T^2}\right)_{;\beta}q^\alpha. \tag{62}$$

For simplicity, in the above equations we have not included thermodynamic viscous/heat coupling coefficients, nor have we included couplings of heat flux to the vorticity. Also, because of the comment below (49) and for simplicity we have omitted the contributions of shear viscosity (see [53] for details).

Let us first consider the situation within the context of the standard (Eckart-Landau) irreversible thermodynamics, in which case $\tau = 0$. Then after simple manipulations we obtain

$$S^\alpha_{;\alpha} = -\frac{1}{2}T^{\alpha\beta}_{\rm dis}\mathcal{L}_\chi g_{\alpha\beta}, \tag{63}$$

where $\mathcal{L}_\chi$ denotes the Lie derivative with respect to the vector field $\chi^\alpha = \frac{V^\alpha}{T}$, and $T^{\alpha\beta}_{\rm dis} = V^\alpha q^\beta + V^\beta q^\alpha$. From the above it is evident that if χ defines a conformal Killing vector, i.e.

$$\mathcal{L}_\chi g_{\alpha\beta} = \Phi g_{\alpha\beta}, \tag{64}$$

for an arbitrary function Φ, then

$$S^\alpha_{;\alpha} = 0, \tag{65}$$

a well-known result [60].

Since a generic Szekeres spacetime does not admit a conformal Killing vector [61], then the heat flux q^α has to be associated to an irreversible process.

In the case of the causal thermodynamics the following expression emerges from (60):

$$S^\alpha_{;\alpha} = -\frac{1}{2}T^{\alpha\beta}_{\rm dis}\mathcal{L}_\chi g_{\alpha\beta} - \frac{1}{2}\left(\frac{q^2 V^\mu \tau}{\kappa T^2}\right)_{;\mu}. \tag{66}$$

Due to the presence of different phenomenological parameters in different terms in (66), it becomes evident that for a generic Szekeres spacetime $S^\alpha_{;\alpha} \neq 0$.

We recall that in the above expression we have neglected terms involving couplings of heat flux to the vorticity. These couplings give rise to terms of the form $\kappa T \gamma_1 \Omega_{\alpha\beta} q^\beta$, where γ_1 is the thermodynamic coupling coefficient, which have to be added to the right-hand side of (62). Also, the omitted shear viscosity contribution in (60) would give rise to a term of the form $2\eta\gamma_2 \pi^\mu_{\langle\alpha}\omega_{\beta\rangle\mu}$, which has to be added to the transport equation for the shear viscosity, where γ_2 is a coupling coefficient, η denotes the shear viscosity coefficient and $\langle\rangle$ is the spatial tracefree part of the tensor.

Thus, as in the LTB case [17], tilted observers in Szekeres spacetime detect a real (entropy producing) dissipative process, while for the nontilted observer the evolution proceeds adiabatically. In this case however it is worth noticing that vorticity contributes to entropy production, too.

Also, as in the LTB case we might speculate that the origin of such an important difference in the pictures between both sets of observers may be found in the fact that while the fluid is geodesic for nontilted observers, it is not for tilted ones. Then invoking the equivalence between collisional terms and force terms in the Boltzmann equation established in [62], it could be possible that the force term associated with the nonvanishing four-acceleration plays the role of a collisional term, leading to entropy production.

V. CONCLUSIONS

We have analyzed the hydrodynamic and thermodynamic properties of the tilted Szekeres spacetime and compared them with the nontilted case.

Two main differences emerge from our study: first, the fluid as seen by the tilted observer evolves irreversibly in contrast with the picture described by the nontilted observers according to which the system evolves adiabatically; furthermore in the former case the fluid is no longer

geodesic. These differences between both sets of observers also appear in the FRW and LTB spacetimes.

Second and more interesting, the fluid lines as seen by the tilted observers have nonvanishing vorticity. As mentioned before this is true for any generic Szekeres spacetime.

From a cosmological point of view this might have interesting consequences. Indeed, the possibility of rotating universes (besides the well-known case of the Gödel model) and eventual restrictions on the magnitude of the associated vorticity have been studied in the past (see [63–66], and references therein). Also different mechanisms for vorticity generation in a cosmological background have been presented (see [67–71], and references therein). Here we have seen that the appearance of vorticity may also be an observer related phenomenon.

We would like to conclude with the following remarks:

(i) We have considered the simplest possible case of tilted congruence; it is obvious that a boost in all possible directions would produce a much richer physical picture from the point of view of a tilted observer. In the same line of arguments we have considered the simplest dissipative scenario and have not considered the possible presence of an electromagnetic field. The reasons for doing that are, on the one hand, the cumbersome resulting expressions and, on the other, the fact that even at this level of simplicity very interesting physical results emerge.

(ii) It is worth noticing that, as it follows from (29), if $\bar{\mu}$ has a good physical behavior, it also has the physical tilted variables (at least for small values of ω).

(iii) In spite of the differences, the tilted version conserves some of the ''structural'' properties of the nontilted one; e.g. the magnetic part of the Weyl tensor vanishes and the electric part has only one nonvanishing independent component.

(iv) For a generic Szekeres spacetime with no symmetries and nonbarotropic equation of state, it is not obvious that the Gibbs framework would be applicable (see [72]). This point deserves more attention.

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