Discrimination between ΛCDM, quintessence, and modified gravity models using wide area surveys

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In the past decade or so observations of supernovae, large scale structures, and the cosmic microwave background have confirmed the presence of what is called dark energy—the cause of the accelerating expansion of the Universe. Its density has also been measured as well as the value of other cosmological parameters according to the concordance ΛCDM model with few percent uncertainties. The next generation of surveys should allow to constrain this model with better precision or to distinguish between a ΛCDM and alternative models such as modified gravity and (interacting)-quintessence models. In this work we parametrize both the homogeneous and anisotropic components of matter density in the context of interacting dark energy models with the goal of discriminating between $f(R)$ modified gravity and its generalizations, and interacting dark energy models, for which we also propose a phenomenological description of energy-momentum conservation equations inspired by particle physics. It is based on the fact that the simplest interactions between particles/fields are elastic scattering and decay. The parametrization of the growth rate proposed here is nonetheless general and can be used to constrain other interactions. As an example of applications, we present an order of magnitude estimation of the accuracy of the measurement of these parameters using Euclid and Planck surveys data, and leave a better estimation to a dedicated work.

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I. INTRODUCTION

Today it is a well-established fact that according to Einstein's theory of gravity ~73% of the mass and energy in the Universe is in a strange form with unusual properties inconsistent with any type of matter known to us. It is generally called *dark energy* (see e.g. [1–4] for recent reviews). In the last two decades or so numerous models have been suggested to explain this mysterious and dominant constituent of the Universe. The majority of these theories can be classified in one of the following three categories: (1) Models based on a scalar field, e.g. quintessence [5–8] and its variants such as *k*-essence [9–13] in which the kinetic term in the Lagrangian has a nonstandard form, and varying neutrino mass models [14,15] in which the accelerating expansion of the Universe is generated by the variation of neutrinos mass due to their interaction with a light scalar field; (2) Modified gravity models in which dark energy is explained as the deviation of gravitational interaction from Einstein's theory of gravity. Examples of such models include scalar-tensor [16–18] and f(R) gravity [19–21], chameleon [22–24], and Dvali, Gabadadze, and Porratti [25]; (3) A cosmological constant—introduced by Einstein himself [26] and interpreted by Lemaître as the energy density of vacuum [27]. It is phenomenologically the simplest of three categories, and is still the best fit to all observational data [28–30]. However, naive estimations of vacuum energy are ~42 to 123 orders of magnitude larger than the observed dark energy [31]. For this reason, alternative explanations have been explored even before the observation of the accelerating expansion of the Universe [5,6]. The main task of cosmologists today is discriminating between these models, in particular, distinguishing the first two categories mentioned above from a cosmological constant.

A notable difference between a cosmological constant and some of the alternative models is the presence of a weak interaction between matter and dark energy. Pure quintessence models, in which there is no interaction between the scalar field and matter are somehow pathological because all known fundamental particles, including neutrinos which have very small couplings, interact nongravitationally with some other particles. Even axiomatic weakly coupled particles such as axions [32,33] are expected to interact with gauge bosons such as gluons. The fields in candidate extensions of the standard model are related to each others by symmetries, thus either by gauge interaction or by mass mixing. On the other hand, if a field such as quintessence interacts only with gravity, then naturally it should be considered as belonging to the gravity sector. An example of such fields is dilaton which was first introduced in the context of the Kaluza-Klein model for the unification of gravity and electromagnetic forces [34,35], and is also associated with conformal gravity models, see e.g. [36] and references therein. But gravity is a universal force and interacts with all other particles. Thus, in contradiction with the assumption above, the quintessence field must have an interaction with other particles. In fact, dilaton does have nonminimal interaction with other species, see for instance [37,38]. This makes the

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task of finding a candidate for a noninteracting quintessence field very difficult. A more problematic issue with pure quintessence models is the fact that they do not solve the coincidence problem of dark energy, i.e. why it becomes dominant only at late times and after galaxy formation. Interacting quintessence models in which the quintessence field has a weak interaction with some matter species, in particular, with dark matter, can solve or at least soften the huge fine-tuning of dark energy density with respect to matter in the early Universe [39,40].

In modified gravity models the deviation from Einstein's theory of gravity can be, either written explicitly, or presented by introducing new fields, usually scalars, in the matter sector. The first presentation is called the *Jordan frame* and the second the *Einstein frame*. Because in the latter case the model looks very similar to an interacting quintessence model,[1] it is necessary to find a proper definition that discriminates between what is called *modified gravity* and what is called *interacting quintessence*. In modified gravity, the scalar field is usually related to a dilaton field, thus it has a geometrical origin and arises from a broken conformal symmetry [16]. For this reason, the scalar field always interacts with the trace of the energy-momentum tensor of matter [21]. The situation is not so straightforward for interaction between matter and the scalar field in interacting quintessence models for which various types of interactions are considered in the literature, see for instance [41,42]. In many of these models in analogy with modified gravity, in particular f(R) models, the interaction term is considered to be proportional to the trace of the energy-momentum tensor of matter.[2]

In this work we try to determine the interaction between dark matter and dark energy in a collisional description of interactions inspired by particle physics. Using the Boltzmann equation with a collisional term and some results from studies of the microphysics of dark energy condensate [43], we show that the interaction can be described only approximately by spacetime dependent functions, and in general one needs the distribution in the phase space $f(x, p)$ where p is the 4-momentum, see Sec. III for more details. However, at present and for the foreseeable future, we cannot observe the phase space distribution of dark energy. Considering this fact, we use thermodynamical description of average energy-momentum and velocity to obtain approximate covariant expressions for interactions between matter and a scalar field as dark energy. This leads to a modification of the energy-momentum conservation equation, which explicitly deviates from modified gravity. Their difference can be used as a mean for classifying models and discriminating between these two categories in the data.

On the observational side, one has to find the best way of parametrizing cosmological evolution equations such that they admit discrimination between at least the three major categories of models discussed above. Preferably, they should not depend on the details, which are neither well understood nor can they be targeted with the precision of present and near future surveys. Observations show that dark energy has negligible clustering (see e.g. [44–47] for the latest results). Therefore, its dominant contribution is in the homogeneous component of the Einstein and conservation equations. It also affects the evolution of anisotropies mainly through their dependence on the background cosmology. For this reason, irrespective of the way we measure the equation of state of dark energy—from observations of supernovae that are only sensitive to background cosmology, or from observations of matter perturbations by measuring lensing or galaxy distribution—we must extract the parameters of background cosmology to determine the contribution of dark energy. Consequently, it is crucial to understand how different models affect this component through a proper parametrization that facilitates the discrimination between various models. This is another goal of the present work. Although there are few popular parametrizations [48–51] in the literature, especially for testing modification of Einstein's theory of gravity at large scales, as we will show in this work, they are not suitable for discriminating between the modified gravity and (interacting)-quintessence models. We should remind that for the ΛCDM model the growth rate $\mathbf{f}$ is roughly scale independent. Therefore, observation of the violation of this property would be a clear signature of inconsistency with standard cosmology. But the measurement of $\mathbf{f}$ and the expansion rate H by themselves are not enough for discriminating between modified gravity, quintessence, and interacting quintessence, and a parametrization that does not depend on the details of these models is necessary to highlight their differences. Evidently, one can simply fit the data with these models and compare their goodness of fit. But, this does not take into account the degeneracies and similarities. Therefore, a smart parametrization and better data analyzing methods are necessary. Moreover, the fact that most popular modified gravity models can be formulated as a scalar field theory means that their differences from (interacting)-quintessence must be understood and the parametrization must be performed in a way that it highlights these differences and helps discrimination.

In this work we propose a new set of parameters to describe, in a model-independent way, the effect of an interacting dark energy on the evolution of the expansion rate of the Universe and another set of parameters for the growth rate. These quantities are the most sensitive

[1]Note that when we talk about interacting quintessence models we mean models in which the scalar field interacts with some other constituents of the Universe. All quintessence models have a self-interaction, which is not explicitly considered in the formulation presented in this work

[2]For the reasons described in detail in Sec. III, when we talk about the interaction term, we mean the modification of the energy-momentum conservation equation due to an interaction.

measurables for discriminating between the dark energy models. Consequently, the ultimate goal of various measurement methods is to constrain cosmological and dark energy models by measuring one or both of these quantities. For instance, galaxy distribution and lensing surveys determine the power spectrum of fluctuations for one or multiple redshift bins. Future large surface and sensitive spectroscopic surveys such as Euclid will allow for the determination of the matter power spectrum for a statistically significant number of redshift bands, and thereby extract the growth rate, see e.g. [52] for the methodology applied to The WiggleZ Dark Energy Survey. The baryon acoustic oscillation (BAO) measurements determine the expansion rate and angular diameter distance at one or multiple redshift bins, and can thereby estimate the variation of these quantities. Supernovae data measure the expansion rate directly, and the variation of $H(z)$ can be extracted. Therefore, the parametrizations that we will discuss here are relevant for all measurement methods in cosmology.

In Sec. II we present a new parametrization forthe Friedmann equation in the context of a general interacting dark energy model.[3] In Ref. [54] we defined the quantity $B(z) \propto d\bar{\rho}/dz$ and proposed it for the measurement of the equation of state of dark energy defined as $w \equiv P_{\rm de}/\rho_{\rm de}$, where $P_{\rm de}$ and $\rho_{\rm de}$ are the pressure and energy density of dark energy, respectively. It is especially suitable for measuring the deviation from a cosmological constant, see Appendix A for the definition of $B(z)$ and a review of its properties. In addition, we argued that in what concerns the sign of $\gamma(z)$ [see Eq. (2) below for its definition], this quantity has distinct geometrical properties which make it less sensitive to the uncertainties of other quantities such as H_0 or Ω_m, respectively, the present value of Hubble constant and the density fraction of matter. The sign of $\gamma(z)$ is the discriminator between what is called phantom models which have $w < -1$, and normal scalar fields (quintessence) models and a cosmological constant for which $w \geq -1$.

Using this parametrization and the properties of $B(z)$, we show that in the presence of an interaction between dark energy and other components, one obtains a different estimation for $\gamma_{\rm de}^{\rm eff}$ (see the next section for its definition) from $H(z)$ and from $B(z)$ when the data is analyzed with the null hypothesis of a ΛCDM model as dark energy. In this way one can predict the sensitivity of surveys to interacting dark energy models in a model-independent manner. Then, we discuss the properties of the parameters for each category of models, their differences, and how this information can be used to discriminate between various dark energy models. In Sec. III we describe phenomenological interactions for the interacting quintessence models and compare it with modified gravity. This leads to an approximate description for nongravitational interactions between dark matter and dark energy.

In Sec. IV we present evolution equations of the overdensity and velocity fields in each category of models for the interactions obtained in Sec. III. Then we describe how one can discriminate between the interacting quintessence and modified gravity models by using the matter power spectrum and its evolution, i.e. the growth rate of anisotropies. Because the growth rate plays a special role in discriminating between various dark energy models, in Sec. V we parametrize its evolution, and as an example of an application, we obtain an order of magnitude estimate for the discriminatory ability of the Euclid mission [55] which measures both parameters of the homogeneous component (the background cosmology) and the evolution of the growth rate of matter anisotropies. In addition, we compare our parametrization with other parametrizations that can be found in the literature which are usually designed to test Einstein's theory of gravity. Our conclusions and outlines are summarized in Sec. VI. The properties of the functions $B(z)$ (and $A(z)$) are reviewed in Appendix A. The Fisher matrix for dark energy without the parametrization of its equation of state $w(z)$ is described in Appendix B. A summary of the covariant formulation of a classical scalar field as a perfect fluid is given in Appendix C. In Appendix D we calculate an approximate analytical solution for the growth rate of matter anisotropies.

Here we must emphasize that the predictions for future missions obtained in this work are only representative and order of magnitude estimations of what is expected from future surveys. They should be considered as a *QD* (quick and dirty), handshaking predictions. Their purpose is only to show that it is possible to measure the new parameters with reasonable uncertainties. A proper prediction for future observation projects requires detailed consideration of the instrumental response, the simulation of a data analysis procedure, and an understanding of the sources of systematic and statistical errors. Fulfilling these requests necessitates a dedicated investigation which is beyond the scope of the present work which targets theoretical issues related to the discrimination between various dark energy models. In fact, a number of authors have performed predictions for the uncertainties of various measured quantities by future missions, see for instance [56–59]. They usually consider models that can be classified as modified gravity according to the classification criteria discussed in Sec. III. Nonetheless, some of their parameters can be related to the quantities defined in this work, thus their predictions can be used to obtain a rough estimation of the expected uncertainties for the new parameters.

Throughout this work we use the Einstein frame for modified gravity models unless explicitly specified otherwise. In this way, a unified description can be made for all

[3] After the submission of this paper a similar parametrization for the Friedmann equation was reported independently by [53].

interacting dark energy models based on a scalar field formulation.

II. FRIEDMANN EQUATION IN INTERACTING DARK MATTER-DARK ENERGY MODELS

A priori the measurement of the equation of state of dark energy is simple. It is enough to measure the expansion rate of the Universe $H(z) \equiv \dot{a}(z)/a$, or a quantity related to it such as the luminosity distance $D(z)$ at different redshifts. Then, by modeling known constituents of the Universe as noninteracting perfect fluids, one can fit the data and measure the effective equation of state of dark energy $w_{\rm eff}(z)$, defined as $P_{\rm eff}(z)/\rho_{\rm eff}(z)$. The suffix "eff" is used to remind that pressures and densities obtained in this way can be effective quantities rather than the physical pressure and density of constituents, because we have neglected any interaction between components. Therefore, from now on *effective quantities* mean quantities determined from data by considering a null hypothesis.

In practice, however, this is not so simple. The density of a perfect fluid changes with the redshift as $(1+z)^{3\gamma}$ [γ is defined in (2)]. Therefore, at low redshifts when $z \rightarrow 0$, the total density is not very sensitive to the value of γ or equivalently $w(z)$ and their variation with z, see Appendix A for more details. This statement is independent of the type of data or proxy used for determining $H(z)$ or $D(z)$. On the other hand, at high redshifts where $H(z)$ is more sensitive to the equation of state, dark energy is subdominant. Moreover, it is more difficult to measure $H(z)$ and $D(z)$ at higher redshifts and measurement uncertainties can make the estimation of $w(z)$ and its evolution unusable for discriminating between models.

If the constituents of the Universe do not interact with each other, the Friedmann equation, which determines the evolution of the expansion function $a(t)$, can be written as

$$\frac{H^2}{H_0^2} = \frac{\rho(z)}{\rho_0} = \Omega_m(1+z)^3 + \Omega_h(1+z)^4 + \Omega_K(1+z)^2 + \Omega_{\rm de}(1+z)^{3\gamma(z)}, \qquad (1)$$

$$\rho_c(z) \equiv \frac{3H^2}{8\pi G},$$

$$\gamma(z) = \frac{1}{\ln(1+z)}\int_0^z dz' \frac{1+w(z')}{1+z'}, \qquad P_{\rm de}(z) \equiv w(z)\rho_{\rm de}$$

$$m = \text{cold dark matter}, \quad b = \text{baryons}, \quad h = \text{hot matter}, \quad k = \text{curvature}, \quad \text{and} \quad \text{de} = \text{dark energy}. \qquad (2)$$

In this class of models the matter and radiation densities evolve only due to the expansion. This is a good approximation for all redshifts $z < z_{\rm cmb} \sim 1100$.

In interacting dark energy models the matter and radiation terms in the right-hand side of the Friedmann equation (1) can contain an additional redshift-dependent factor:

$$\frac{H^2}{H_0^2} = \frac{\rho_c(z)}{\rho_{c0}} = \sum_i \Omega_i \mathcal{F}_i(z)(1+z)^{3\gamma_i} \qquad i = m, b, h, k, \quad \text{and} \quad \text{de}. \qquad (3)$$

Without lack of generality we assume that $\mathcal{F}_{\rm de} = 1$ and all redshift-dependent terms are included in $\gamma(z)$. In quintessence models the coefficient of the curvature term also is constant because it is assumed to be related to geometry/gravity and independent of the behavior of other components. At present observations are consistent with only the gravitational interaction between the various components in (3), thus additional interactions must be very weak. By definition and without lack of generality, we consider $\mathcal{F}_i(z=0) = 1$. Observations also show that $\Omega_k \approx 0$; therefore, throughout this work we assume $\Omega_k = 0$ unless explicitly specified otherwise. Note that in the case of modified gravity models, a parametrization similar to (3) can be defined in both the Einstein and Jordan frames.

A simple example for which an approximate expression for $\mathcal{F}_i(z)$ coefficients can be found is a model with a cosmological constant as dark energy and a slowly decaying dark matter. The decay remnants are assumed to be visible relativistic particles [60]. In this case,

$$\frac{H^2}{H_0^2} \approx \Omega_m(1+z)^3 \exp\left(\frac{\tau_0 - t}{\tau}\right) + \Omega_b(1+z)^3 + \Omega_h(1+z)^4 + \Omega_m(1+z)^4\left(1 - \exp\left(\frac{\tau_0 - t}{\tau}\right)\right) + \Omega_\Lambda, \qquad (4)$$

$$\mathcal{F}_m(t) \approx \exp\left(\frac{\tau_0 - t}{\tau}\right) + (1+z)\left(1 - \exp\left(\frac{t_0 - t}{\tau}\right)\right), \qquad \tau \gg \tau_0, \quad \mathcal{F}_b = \mathcal{F}_h = 1 \quad \gamma(z) = 0, \qquad (5)$$

where τ is the lifetime of dark matter and τ_0 is the age of the Universe. It is demonstrated that in this example, if the decay/interaction of dark matter is not considered in the data analysis, a $w_{\rm eff} < -1$ can be obtained for dark energy, see [61,62] for more details regarding the setup and the proof.

Note that in (5), we have included the contribution of relativistic remnants in $\mathcal{F}_m$. However, as this component has a redshift dependence similar to hot matter, it also makes sense to consider it as hot matter and add it to the hot component. It is even possible to add this term to the dark energy contribution, as long as it is small and induces only a slight deviation from a cosmological constant. In this case, one can show that the *effective dark energy* will have $w_{\rm eff} < -1$ [62]. The reason for such freedom is that we do not measure or take into account the decay remnants.

This example clearly shows that parametrization (3) is not unique when all the components and their interactions are not known. Therefore, one has to be very careful about degeneracies when data are analyzed and interpreted. In particular, prior assumptions such as the stability of matter and the radiation components can affect measurements and conclusions. This example also shows that for ruling out the ΛCDM model, it is enough to prove that at least one of $\mathcal{F}_i \neq 1$, or $\gamma_{de} \neq 0$.[4]

Extension of this example to quintessence models without coupling to matter is straightforward and one simply needs to consider $\gamma(z) \neq 0$. A more interesting extension is to assume that the quintessence scalar field is one of the remnants of the decay of dark matter, which during cosmological time condensates and makes a classical quintessence field. In this case, it has been shown [40,43,63] that coefficients $\mathcal{F}_m$, $\mathcal{F}_h$, and the equation of state of dark energy $w(z)$ [or equivalently $\gamma(z)$] are not independent. However, their relations are too sophisticated and cannot be described in an analytical form and numerical techniques should be employed [40].

According to (5),

$$\mathcal{F}_m(z) > \mathcal{F}_m(z=0), \tag{6}$$

and because $\tau \gg \tau_0$, $\mathcal{F}_i(z)$ coefficients are close to 1 at all redshifts. In general, for an interaction which transfers energy from dark matter to other components, the inequality (6) is applied because at high redshifts one expects a larger contribution of dark matter in the total density than in a noninteracting model. Conversely, if energy is transferred from other components, for instance from dark energy, to dark matter:

$$\mathcal{F}_m(z) < \mathcal{F}_m(z=0). \tag{7}$$

An example of such models is *scaling dark energy* [64,65] in which at early times dark energy has a much larger contribution in the total energy density, but it gradually decays to dark matter and only recently its equation of state approaches $w \sim -1$. Another example is the class of models called *early dark energy*. Although the original model [66–68] is a pure quintessence/k-essence, there are variants of this model in which, there is an interaction in the dark sector [69] or between the dark energy and visible sectors [70].

In models with elastic interaction between two sectors, no energy is transferred between them, and $\mathcal{F}_m(z) = \mathcal{F}_h(z) = 1$. Nonetheless, the phase space of matter and dark energy in these models can change and thereby w_{de} can depend on z.

[4]This statement is true if the baryon pressure is negligible. Future surveys can be sensitive to small baryon pressure. In this case, it must be taken into account before any conclusion regarding the ΛCDM model is made.

For $\mathrm{f}(R)$ modified gravity models, the homogeneous Einstein equations and the energy conservation equation in the Jordan frame are [21][5]:

$$(1+\mathrm{f}_R)H^2 + \frac{1}{6}\mathrm{f} - \frac{a''}{a^3}\mathrm{f}_R + \frac{\mathrm{f}'_R}{a}H = \frac{8\pi G}{3}\sum_i \rho_i, \tag{8}$$

$$\frac{a''}{a^3} = -\frac{4\pi G}{3}\sum_i(\rho_i + 3P_i) + (1+\mathrm{f}_R)H^2 + \frac{\mathrm{f}}{6} - \frac{H\mathrm{f}'_R}{a} - \frac{\mathrm{f}''_R}{2a^2}, \tag{9}$$

$$\dot{\rho}_i + 3H\rho_i = -\frac{\dot{f}_R}{2(1+\mathrm{f}_R)}(\rho_i - 3P_i), \qquad i = m, h, k, \tag{10}$$

where $a' = a\dot{a}$.[6] The dot and prime mean derivation with respect to comoving and conformal time, respectively. Subscript R means derivation with respect to scalar curvature $R \equiv R_{\mu\nu}g^{\mu\nu}$. We remind that at linear order the effect of matter perturbations on R is zero, thus R only depends on z and the effect of $\mathrm{f}(R) \neq 0$ on the evolution of perturbations manifests itself by changing the background cosmology.

After solving the density conservation equation (10), the Friedmann equation (8) can be written as follows:

$$\rho_i(z) = \rho_i(z=0)(1+z)^{3\gamma_i}\left(\frac{1+\mathrm{f}_R(z=0)}{1+\mathrm{f}_R(z)}\right)^{-(1-3w_i/2)}, \tag{11}$$

$$\frac{H^2}{H_0^2} = \frac{\rho_c(z)}{\rho_{c0}} = \sum_i \Omega_i \mathcal{F}_i(z)(1+z)^{3\gamma_i}, \tag{12}$$

$$\rho_{de} = \frac{3}{8\pi G}\frac{1}{1+\mathrm{f}_R}\left(-\frac{\mathrm{f}(R)}{6} + \frac{a''}{a^3}\mathrm{f}_R - H\dot{\mathrm{f}}_R\right), \tag{13}$$

$$\mathcal{F}_i(z) = \left(\frac{1+\mathrm{f}_R(R(z=0))}{1+\mathrm{f}_R(R(z))}\right)^{-(1-3w_i/2)}, \qquad w_m = 0, \qquad w_h = \frac{1}{3}, \qquad w_k = -\frac{1}{3}. \tag{14}$$

Equation (13) is the energy density of *effective dark energy* in $\mathrm{f}(R)$ gravity models. Similar to the quintessence models, we can assume $\mathcal{F}_{de} = 1$. The only explicit difference between (12) and the same equation for an interacting quintessence model is the presence of a nontrivial coefficient for the curvature term if $\Omega_k \neq 0$. Nonetheless, the

[5]When equations apply to both dark matter and baryons, we indicate them collectively with subscript m.

[6]Here we have written the Einstein and conservation equations in the Jordan frame because they lead to the expressions for $\mathcal{F}_i$ coefficients, which are explicitly very different from the quintessence case.

evolution of coefficients $\mathcal{F}_i(z)$ with the redshift is different from their counterparts in interacting quintessence models, in particular, from models in which energy is transferred to dark energy at low redshifts, see Eq. (6). In fact, the function $\mathrm{f}(R)$ is not completely arbitrary and must satisfy a number of constraints. Notably, $\mathrm{f}(R)|_{|R|\gg 0} \to 0$ to make the model consistent with Einstein's theory of gravity in mild or strong gravity fields, and $\mathrm{f}_R > 0$ due to the stability constraint [71]. Under these conditions:

$$\mathcal{F}_i(z) > \mathcal{F}_i(z=0). \tag{15}$$

Comparing (15), (6), and (7) one can immediately conclude that the measurement of $\mathcal{F}_m(z)$ and its evolution with redshift can discriminate between dark energy models in which energy is transferred from dark energy to dark matter such as scaling models, and f_R modified gravity models. But it cannot discriminate modified gravity from models in which energy is transferred from dark matter to dark energy such as the model discussed in [40,43,63]. To discriminate the latter and other models of this category from f_R modified gravity, the coefficient of relativistic (hot) component $\mathcal{F}_h(z)$ and its evolution must be measured. Evidently, such measurements are very difficult. For instance, one has to measure very precisely the temperature of cosmic microwave background (CMB) at high redshifts or $H(z)$ at a large number of redshift bins and fit the data with $\mathcal{F}_h \neq 1$. In the Einstein frame the evolution of matter density is the same as in Eq. (11) [30], but the evolution equation of hot matter is similar to ΛCDM. Therefore, in that concerning the discrimination from interacting quintessence, that discussed for the Jordan frame is applicable.

A. Model-independent discrimination of interacting dark energy models

In this section we show that if ΛCDM or a simple quintessence are considered as the null hypothesis, the measurements of effective dark energy density and effective equation of state from $H(z)$ and the function $A(z)$ defined in Appendix A separately, give different values for these quantities if dark energy interacts with matter. The similarity of $\mathcal{F}_m(z)$, especially if the curvature of the Universe is zero, means that we cannot distinguish between interacting quintessence and modified gravity models in a model-independent manner—except for the cases explained above. For this reason in this section we only study the discrimination between interacting dark energy models parametrized as in Eq. (12) and a cosmological constant and/or noninteracting quintessence.

For analyzing cosmological data, ΛCDM with a stable and noninteracting dark matter is usually used as the null hypothesis. Nonetheless, the methodology explained below is not sensitive to the redshift dependence of γ_{de}, and we can consider the more general case of noninteracting quintessence as the null hypothesis. The expansion of the Universe for such cosmologies is ruled by Eq. (1). Therefore, we rearrange the terms in Eq. (12) such that it looks similar to Eq. (1). Then, we determine the effective quantities which are measured by fitting a ΛCDM or a noninteracting quintessence model to the data:

$$\frac{H^2}{H_0^2} = \sum_i \Omega_i(1+z)^{3\gamma_i} + \sum_i \Omega_i(\mathcal{F}_i(z)-1)(1+z)^{3\gamma_i} + \Omega_{\mathrm{de}}(1+z)^{3\gamma_{\mathrm{de}}(z)}. \tag{16}$$

In a null hypothesis model only γ_{de} is redshift dependent and γ_i, $i = m, h, k$ are constant. By comparing (16) with (1) the *effective* contribution of dark energy is expressed as

$$\Omega_{\mathrm{eff}}^{(H)}(1+z)^{3\gamma_{\mathrm{eff}}^{(H)}(z)} = \sum_i \Omega_i(\mathcal{F}_i(z)-1)(1+z)^{3\gamma_i} + \Omega_{\mathrm{de}}(1+z)^{3\gamma_{\mathrm{de}}(z)}. \tag{17}$$

In both the interacting quintessence and modified gravity models coefficients F_i's are defined such that $F_i(z=0)=1$; therefore, at $z=0$ the first term in (17) is null, and we can separate Ω_{eff} and $\gamma_{\mathrm{de}}(z)$:

$$\Omega_{\mathrm{eff}}^{(H)} = \Omega_{\mathrm{de}}, \qquad \gamma_{\mathrm{eff}}^{(H)}(z=0) = \gamma_{\mathrm{de}}(z=0), \tag{18}$$

$$\gamma_{\mathrm{eff}}^{(H)}(z) = \frac{\log\Big(\sum_i \frac{\Omega_i}{\Omega_{\mathrm{de}}}(\mathcal{F}_i(z)-1)(1+z)^{3\gamma_i} + (1+z)^{3\gamma_{\mathrm{de}}(z)}\Big)}{3\log(1+z)}, \tag{19}$$

where superscript (H) means measured from Hubble constant H.

Suppose we can also measure $A(z)$ defined in (A4), and use it to determine the effective density and equation of state of dark energy. For an interacting dark energy model parametrized according to (16) quantities $B(z)$ and $A(z)$ are

$$\begin{aligned} B(z) &\equiv \frac{1}{3(1+z)^2\rho_0}\frac{d\rho}{dz} \\ &= \sum_{i=m,h,k}\Omega_i\Big(\gamma_i\mathcal{F}_i(z) + (1+z)\frac{d\mathcal{F}_i}{dz}\Big)(1+z)^{3(\gamma_i-1)} \\ &\quad + \Omega_{\mathrm{de}}(w(z)+1)(1+z)^{3(\gamma_{\mathrm{de}}(z)-1)}, \end{aligned} \tag{20}$$

$$\begin{aligned} A(z) &\equiv B(z) - \sum_{i=m,h,k}\Omega_i\gamma_i(1+z)^{3(\gamma_i-1)} \\ &= \sum_{i=m,h,k}\Omega_i\Big(\gamma_i(\mathcal{F}_i(z)-1) + (1+z)\frac{d\mathcal{F}_i}{dz}\Big)(1+z)^{3(\gamma_i-1)} \\ &\quad + \Omega_{\mathrm{de}}(w(z)+1)(1+z)^{3(\gamma_{\mathrm{de}}(z)-1)}. \end{aligned} \tag{21}$$

Using (A4) in Appendix A as the definition of $A(z)$, we find the following expression for its parameters:

$$\Omega_{\rm eff}^{(A)}(w_{\rm eff}^{(A)}(z)+1)(1+z)^{3\gamma_{\rm eff}^{(A)}(z)} = \sum_i \Omega_i\Big(\gamma_i(\mathcal{F}_i(z)-1)+(1+z)\frac{d\mathcal{F}_i}{dz}\Big)(1+z)^{3\gamma_i} + \Omega_{\rm de}(w(z)+1)(1+z)^{3\gamma_{\rm de}(z)} = (1+z)A(z), \quad (22)$$

where superscript (A) means measured from $A(z)$. Equations (17) and (22) are fundamentally different. In particular,

$$\Omega_{\rm eff}^{(A)} = \frac{\sum_i \Omega_i \frac{d\mathcal{F}_i(z=0)}{dz} + \Omega_{\rm de}(w(z=0)+1)}{w_{\rm eff}^{(A)}(z=0)+1}, \quad (23)$$

which in contrast to $\Omega_{\rm eff}^{(H)}$, in general is not equal to $\Omega_{\rm de}$. Equality arises only when $\mathcal{F}_i$ do not vary with the redshift, i.e. $\mathcal{F}_i = 1$ at all redshifts. This condition is satisfied by the null hypothesis ΛCDM and by the noninteracting quintessence models. Therefore, assuming that Ω_m and Ω_k are known (e.g. from CMB), simultaneous measurements of $H(z)$ and $A(z)$ at even one $z > 0$ is *a priori* enough for testing the presence of an interaction between dark matter and dark energy independent of the underlying model. Evidently, in practice the measurements must be performed at many redshift bins to improve the statistics and to compensate for measurement errors.

A priori one can use other quantities such as the angular diameter distance D_A or the luminosity distance D_L which are easier to measure than $A(z)$. However, both quantities arefunctionals of $H(z)$—through the integration of $1/H^{1/2}(z)$. Thus, in general they do not have an analytical expression. Besides, their derivatives depend on $\mathcal{F}_i$'s only, in contrast to (22) which depends on both $\mathcal{F}_i$'s and their derivatives. Therefore, $\Omega_{\rm eff}$ and $\gamma_{\rm eff}$ obtained from dD_A/dz or dD_L/dz will be equal to those determined from $H(z)$ irrespective of the underlying cosmology. This shows that the function $A(z)$ (or equivalently $B(z)$) introduced in [54] has special properties and is well suited for discriminating between dark energy models. It can be measured from the supernovae data, see [54] for the methodology. As for the LSS data, one needs to determine both $H(z)$ and its evolution $dH(z)/dz$ to be able to calculate $A(z)$, for instance from the BAO and the power spectrum of matter fluctuations [72]. This is not an easy task. As an example, consider supernovae observations that measure the luminosity distance D_L to a supernova from its standardized apparent magnitude. The angular luminosity distance D_A is related to the luminosity distance, see (A9). To determine dD_A/dz *a priori* one can use the measured D_A, and determine its derivative (slope). However, due to the scattering and discreteness of the data, such a measurement will have large uncertainties. The same problem arises for $dH(z)/dz$ or $A(z)$ because they depend on derivatives of D_L, see Eqs. (A5) and (A8). Nonetheless, there are various methods such as binning of data, using a fit in place of discrete data, etc. that allow to improve the estimation. Near future large area surveys such as such as HETDEX [73,74], Euclid [55], BigBOSS [75], and LSST [76] will be able to determine these quantities with relatively good precision, see also Sec. V for the measurement methodology. In particular, large surface spectroscopic and lensing surveys such as Euclid are able to determine the variation of total density with redshift $d\rho/dz \propto B(z)$ with good precision. In Appendix B we obtain the Fisher matrix for dark energy parameters without considering a specific parametrization for the equation of state $w(z)$.

B. Discrimination precision

Measurements of cosmological parameters show that $w_{\rm de}^{\rm obs} \sim -1$ irrespective of which proxy or measurement method—supernovae, CMB, or LSS—has been used. This means that $|\mathcal{F}_i(z) - 1| \approx 0$ and $d\mathcal{F}_i(z)/dz \approx 0$. Moreover, the addition of $\mathcal{F}_i(z)$ to the model increases the number of parameters. Given the fact that we have essentially two observables: $H(z)$ and one of $D_A(z)$, $D_L(z)$ or $B(z)$, a greater number of parameters also means greater degeneracy, and thus more uncertainty for discrimination between ΛCDM, noninteracting quintessence, and interacting dark energy models.

One way of measuring the presence of interaction without having to fit data to the large number of parameters in Eqs. (16) and (21), is to measure how different $\Omega_{\rm eff}^{(H)}$, $\Omega_{\rm eff}^{(A)}$, $\gamma_{\rm eff}^{(H)}$, and $\gamma_{\rm eff}^{(A)}(z)$ are, because as we discussed in the previous section, when $\mathcal{F}_i \neq 1$ these effective quantities are not the same. To this end, a natural criteria is

$$\Theta(z) \equiv \frac{\Omega_{\rm eff}^{(A)}(w_{\rm eff}^{(A)}(z)+1)(1+z)^{3\gamma_{\rm eff}^{(A)}(z)} - \Omega_{\rm eff}^{(H)}(w_{\rm eff}^{(H)}(z)+1)(1+z)^{3\gamma_{\rm eff}^{(H)}(z)}}{\Omega_{\rm eff}^{(H)}(w_{\rm eff}^{(H)}(z)+1)(1+z)^{3\gamma_{\rm eff}^{(H)}(z)}}. \quad (24)$$

This quantity can be explained explicitly as a function of Ω_i, $\mathcal{F}_i$, γ_i, and is zero when $F_i = 1$, $dF_i/dz = 0$. Note that we have chosen expression (22) for comparison rather than (17) because it is not possible to determine $\Omega_{\rm eff}^{(A)}$ in a model-independent manner, see Eq. (23). By contrast $\Omega_{\rm eff}^{(H)} = \Omega_{\rm de}$, thus $\gamma_{\rm eff}^{(H)}$ and thereby $w_{\rm eff}^{(H)}$ can be determined without any reference to the $\mathcal{F}_i$ coefficients. In [54] we suggested to use the sign and evolution of $A(z)$ to discriminate between dark energy with $\gamma(z) \neq 0$ and a cosmological constant. Here $\Theta(z)$ plays a similar role in discriminating between interacting and noninteracting dark energy.

Assuming that Ω_m and Ω_h are determined independently and with very good precision, for instance from

CMB anisotropies with marginalization over γ_{de}, Θ can be determined from the measurement of $H(z)$ and $B(z)$. The latter can be measured from whole sky or wide area spectroscopic survey data such as Euclid, or multiband photometric surveys such as DES. Evidently the determination of $B(z)$ which depends on dH/dz is very difficult. However, it is easy to see that there is no other quantity that can be measured more easily and discriminates between the ΛCDM and dynamical dark energy models with a better precision. For instance, the BAO method determines $H(z)$ and $D_A(z)$ directly. But, $D_A(z)$ depends on $w(z)$ or equivalently $\gamma(z)$ through an integral, see Eq. (A9). Therefore, it is less sensitive to the variation of $\gamma(z)$ with the redshift. This is analogous to binning a data. Evidently, a binned data is less noisy and has a smaller uncertainty. But, if the goal is to measure the variation of data, the binning can completely smear out small variations. Therefore, irrespective of methods and measured proxies, we are limited by the inherent properties of the physical system. In this respect, the precision with which $\Theta(z)$ can be measured gives the ultimate sensitivity of an observation/data set to deviation from ΛCDM.

III. INTERACTIONS

In the previous section we used the Friedmann equation for parametrizing the interaction between matter and dark energy. Evolution of their densities is ruled by energy-momentum conservation. But, in the presence of nongravitational interactions between the constituents the energy-momentum tensor of each component $T_i^{\mu\nu}$ is not separately conserved, and the conservation equation can only be written for the total energy-momentum tensor $T^{\mu\nu}$ defined as

$$T^{\mu\nu} \equiv \sum_i T^{\mu\nu}_{i(\text{free})} + T^{\mu\nu}_{\text{int}}, \tag{25}$$

$$T^{\mu\nu}_{;\nu} = \sum_i T^{\mu\nu}_{i(\text{free});\nu} + T^{\mu\nu}_{\text{int}\ ;\nu} = 0, \tag{26}$$

where $T^{\mu\nu}_{i(\text{free})}$ is the energy-momentum tensor of the component i in the absence of the interaction with other components, i.e. $T^{\mu\nu}_{i(\text{free});\nu} = 0$, and $T^{\mu\nu}_{\text{int}}$ is the energy-momentum tensor of the interaction,[7] and $T^{\mu\nu}_{\text{int};\nu} = 0$. In the literature on interacting dark energy models (see e.g. [39]) when only two constituents—matter and dark energy—are considered, the energy-momentum conservation equations are usually written as

$$T_m^{\mu\nu}{}_{;\nu} = Q^\mu, \qquad T_\varphi^{\mu\nu}{}_{;\nu} = -Q^\mu \tag{27}$$

for an interaction current Q^μ. Comparing (26) and with (27), it is clear that the tensors in the left-hand side of equations in (27) do not correspond to free energy-momentum tensors, and along with Q^μ they are obtained somewhat arbitrarily by the division of (26). In fact, the equations in (27) are inspired by the perturbation theory in which for each perturbative order, the right-hand sides of these equations are estimated by using quantities from one perturbative order lower. Thus, they constitute an iterative set of equations from the zero order (free) model in which $Q^\mu = 0$, up to higher orders. This approach is not suitable for dark energy where we ignore, not only interactions but also the free model. Therefore, a more general expression should be used:

$$T_m^{\mu\nu}{}_{;\nu} = -Q^\mu_m, \qquad T_\varphi^{\mu\nu}{}_{;\nu} = -Q^\mu_\varphi,$$
$$T^{\mu\nu}_{\text{int}\ ;\nu} = Q^\mu_m + Q^\mu_\varphi. \tag{28}$$

In these equations the matter and dark energy tensors $T_m^{\mu\nu}$ and $T_\varphi^{\mu\nu}$ have the same expression as in the absence of interaction, but with respect to fields which are not free. These expressions can be justified by considering the Lagrangian of the model. In the Einstein frame the Lagrangian for a weakly interacting system can be divided into the free and interaction parts:

$$\mathcal{L} = \sum_i \mathcal{L}_i + \mathcal{L}_{\text{int}}. \tag{29}$$

Considering only local interactions, in the dynamics equations for the fields partial derivative of $\mathcal{L}_{\text{int}}$ with respect to each field determines the interaction term. Dynamic equations can be related to energy-momentum conservation equations (27) [40]. Therefore, interaction currents Q^μ_m and Q^μ_φ are generated by partial derivatives of $\mathcal{L}_{\text{int}}$ with respect to the corresponding field.

In the previous section we explained that the scalar field in scalar-tensor modified gravity models is related to a dilaton. Consequently, the interaction term is proportional to the trace of matter, see Eq. (10) for an explicit example of f(R) models. In this case there is no interaction between the scalar field and relativistic particles, and it can be shown that $Q^\mu_m = -Q^\mu_\varphi$ [21], i.e. $T^{\mu\nu}_{\text{int}\ ;\nu} = 0$ and the conservation equations in (27) can be used. The interaction current Q^μ for these models can be written as

$$Q^\mu = \mathcal{C}(\varphi) T_m \partial^\mu \varphi, \tag{30}$$

where $T_m = g_{\mu\nu} T_m^{\mu\nu}$. In the $f(R)$ models the coupling $\mathcal{C}$ is a constant. Here we consider φ dependence to cover the more general cases. Some authors have also considered $Q^\mu \propto T_m u^\mu_m$ for the interacting quintessence models [41]. In fact, the interaction current of the interacting dark energy models in the literature is usually considered to be $Q^0 \propto \rho_m = T_m$ for cold dark matter, i.e. similar to what is obtained for the $f(R)$ modified gravity models [21]. However, given the fact that these models share some

[7] Nongravitational interactions between cosmological constituents must be weak. Therefore, separation of the energy-momentum tensor to the free and interaction components is allowed.

important properties with the modified gravity models, such as the absence of interaction between relativistic matter and the scalar field, we classify them in the modified gravity category. In fact, interactions in interacting quintessence models can be more diverse than in this simple case. In the rest of this section we describe how they can be formulated without considering their details.

In the context of quantum field theory, the Lagrangian $\mathcal{L}$ can be easily written for various types of fields and their interactions, see e.g. [43]. But these formulations are usually complicated, and are necessary if the microphysics of dark energy models is studied. There are various ways to write $\mathcal{L}$ and/or $T^{\mu\nu}$ with respect to macroscopic quantities which are *a priori* measurable from cosmological observations. For instance, one can use a fluid description for components. The Lagrangian of a fluid is defined as [77]

$$\mathcal{L}_f = \frac{1}{2}(P+\rho)g_{\mu\nu}u^\mu u^\nu + \frac{1}{4}(P-\rho)g_{\mu\nu}g^{\mu\nu} + \frac{1}{2}g_{\mu\nu}\Pi^{\mu\nu}, \tag{31}$$

$$\rho \equiv K + V, \qquad P \equiv K - V, \tag{32}$$

where K and V are, respectively, kinetic and potential energy, and $\Pi^{\mu\nu}$ is the traceless shear tensor. Note that if we impose the traceless condition on the Lagrangian, the last term in the right-hand side of Eq. (31) becomes zero. Therefore, this term must be considered as a Lagrange multiplier, and the traceless condition is imposed after determination of $T_f^{\mu\nu}$ [77]. It is easy to check that the Lagrangian $\mathcal{L}_f$ leads to the familiar expression for the energy-momentum tensor of a fluid:

$$T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\left[\frac{\partial(\sqrt{-g}\mathcal{L})}{\partial g_{\mu\nu}} - \partial_\rho\left(\frac{\partial(\sqrt{-g}\mathcal{L})}{\partial(\partial_\rho g_{\mu\nu})}\right)\right],$$
$$T_f^{\mu\nu} = (\rho + P)u^\mu u^\nu - g^{\mu\nu}P + \Pi^{\mu\nu}. \tag{33}$$

Transformation of a Lagrangian written with respect to fields to a fluid description is easy, and one can determine the energy-momentum of the interaction $T_{\rm int}^{\mu\nu}$ and the current Q^μ defined in (28) directly and without any ambiguity, see Appendix C. However, their descriptions as a function of the density and pressure of the fluid depend on the self-interaction potential $V(\varphi)$. For instance, a Higgs-like interaction between a scalar and a fermion $\propto \varphi\bar{\psi}\psi$ is described as $\propto (\rho_\psi - P_\psi)(\rho_\varphi - P_\varphi)^{1/2}$ if $V(\varphi) \propto \varphi^2$, and as $\propto (\rho_\psi - P_\psi)(\rho_\varphi - P_\varphi)^{1/4}$ if $V(\varphi) \propto \varphi^4$. Therefore, when the objective is a general parametrization of interactions without considering the details of the underlying model, this type of description is not very suitable.

A more serious problem of the fluid description of the interaction Lagrangian is the fact that the conservation equations in (28) are equivalent to the field equations and can be obtained from them [40]. Therefore, they do not contain quantum processes such as decay and scattering. It is well known that the Boltzmann equation plays the role of an intermediate between quantum and classical descriptions of interacting systems [78–82]. In this case, components are defined by their phase space distribution $f(p, x)$, where p and x are, respectively, momentum and spacetime coordinates. Interactions are included as collision terms in the right-hand side of the Boltzmann equation [83–85], from which one can obtain energy-momentum and number conservation equations directly:

$$p^\mu \partial_\mu f_i(p,x) - \Gamma^\mu_{\nu\rho}p^\nu p^\rho \frac{\partial f_i}{\partial p^\mu} \equiv L[f_i] = C_i(p,x), \tag{34}$$

$$n^\mu_{;\mu} = \int d\bar{p}\, C_i(p,x),$$
$$d\bar{p} \equiv \frac{\mathbf{g}}{(2\pi)^3} d^4p\,\delta(E^2 - \vec{p}^2 - m_i^2), \tag{35}$$

$$T_i^{\mu\nu}{}_{;\nu} = \int d\bar{p}\, p^\mu C_i(p,x), \tag{36}$$

where $\mathbf{g}$ is the number of internal degrees of freedom (e.g. spin) of species i. The conservation equations (35) and (36) are obtained by using the following property of the Boltzmann operator L defined in (34), see e.g. [84]:

$$\left[\int d\bar{p}\, p^\mu p^{\mu_1} p^{\mu_2} \dots p^{\mu_n} f(p,x)\right]_{;\mu} = \left[\int d\bar{p}\, p^{\mu_1} p^{\mu_2} \dots p^{\mu_n} L[f(p,x)]\right]. \tag{37}$$

Collisional terms can be written by using the cross sections of the interactions which can be determined separately from the quantum formulation of the model [60,83]. In the context of interacting dark energy models, the simplest examples of collisional terms are elastic scattering between dark matter and dark energy and slowly decaying dark matter with a small branching ratio to dark energy [40].[8] Note that we assume no interaction between dark energy and visible matter and radiation. For these interactions the collisional terms are

$$C_m(p,x) = -\Gamma_m m_m f_m(p,x) - f_m(p,x) \times \int d\bar{p}_\varphi f_\varphi(p_\varphi, x) A_k(p, p_\varphi)\sigma_{m\varphi}(p, p_\varphi) + \int d\bar{p}_m d\bar{p}_\varphi f_m(p_m, x) f_\varphi(p_\varphi, x) A_k(p_m, p_\varphi) \times \frac{d\sigma_{m\varphi}(p_m, p_\varphi, p)}{d\bar{p}}, \tag{38}$$

[8] In models where energy is transferred from dark energy to dark matter, the interaction must be nonlinear and very sophisticated such that a very light quintessence field must be able to produce massive dark matter particles. At present, no fundamental description for such models is available.

$$C_\varphi(p,x) = \Gamma_m m_m \int d\bar{p}_m f_m(p_m,x)\frac{d\mathcal{M}(p_m,p)}{d\bar{p}} - f_\varphi(p,x) \times \int d\bar{p}_m f_m(p_m,x) A_k(p,p_m)\sigma_{m\varphi}(p_m,p) + \int d\bar{p}_m d\bar{p}_\varphi f_m(p_m,x) f_\varphi(p_\varphi,x) A_k(p_m,p_\varphi) \times \frac{d\sigma_{m\varphi}(p_m,p_\varphi,p)}{d\bar{p}}, \quad (39)$$

$$A_k(p_1,p_2) \equiv [(p_1.p_2)^2 - m_1^2 m_2^2]^{(1/2)}, \quad (40)$$

where Γ_m is the total decay width of dark matter, $\mathcal{M}(p_m, p)$ is the multiplicity of φ with momentum p in the decay remnants of dark matter particles with momentum p_m, and $\sigma_{m\varphi}(p_m, p_\varphi)$ is the total cross section of the interaction between dark matter and dark energy with momentum p_m and p_φ, respectively.[9]

The disadvantage of this approach is that it needs phase space distribution of components which is not always available, especially for dark energy. Moreover, the absence of an explicit description for the Lagrangian means that the total energy-momentum tensor needed for determining Einstein equations and metric evolution, can be obtained only by solving Eq. (36) for all components. These equations are differentio-integral and usually do not have an analytical solution. Thus, in practice interacting models can be studied only numerically, otherwise one needs to consider some approximation. For instance, dark energy interaction with matter must be very weak. Thus, $|T^{\mu\nu}_{\rm int}| \ll |\sum_i T^{\mu\nu}_i|$. Therefore, we can neglect its contribution in the total energy-momentum tensor and Einstein equations.[10] As for the integration of the collision term in Eqs. (35) and (36), under some physically motivated assumptions they can be simplified and integrated. For instance, when dark matter is assumed to be a scalar, the expression for the scattering cross section is very simple, see e.g. [40]. It is simply proportional to the coupling constant and delta functions for energy-momentum conservation. It is expected that the mass of the quintessence field be very small, especially much smaller than the mass of dark matter. The momentums of both components are also expected to be small. In this case, their distribution at large momentums is strongly suppressed, and the cross section around the peak of distribution can be considered to be approximately constant. Under these simplifications, it is easy to see that scattering term in the right-hand side of (36) is proportional to integrals of the form:

$$\int d\bar{p}_1 d\bar{p}_2 P_1^\mu f_1(p_1,x) f_2(p_2,x) = n_1^\mu \int d\bar{p}_2 f_2(p_2,x) \approx \frac{n_1^\mu u_\rho}{m_2}\int d\bar{p}_2 p_2^\rho f_2(p_2,x) = \frac{n_1^\mu u_{2\rho} n_2^\rho}{m_2}, \quad (41)$$

$$u_i^\mu \equiv \frac{n_i^\mu}{|n_i|}, \qquad n_i^\mu \approx \frac{u_{i\nu}T_i^{\mu\nu}}{m_i} = \frac{\rho u_i^\mu}{m_i}, \quad (42)$$

where n_i^μ and u_i^μ are number density and velocity of species i, respectively. The approximate expression for n^μ in (42) is valid when the distribution in momentum space is concentrated around a peak. Using similar approximations, the decay terms in the right-hand side of (36) can be also described as a function of the velocity and number vectors. Finally, after grouping all the constant or approximately constant factors together, the energy-momentum conservation equations for dark matter and dark energy can be written as

$$T_m^{\mu\nu}{}_{;\nu} \approx -L_m n_m^\mu + A_{ms} n_m^\mu u_{\varphi\rho} n_\varphi^\rho \equiv Q_m^\mu, \quad (43)$$

$$T_\varphi^{\mu\nu}{}_{;\nu} \approx L_\varphi n_m^\mu + A_{\varphi s} n_\varphi^\mu u_{m\rho} n_m^\rho \equiv Q_\varphi^\mu, \quad (44)$$

where constants L_i and $A_{\rm is}$ are the decay width and scattering amplitude for species i. In the rest of this work we use these equations as an approximation for the energy-momentum conservation equations irrespective of dark matter type (spin) and the details of the interaction between two dark components. They affect the constants L_i and $A_{\rm is}$ which are used as parameters. One can also add a dark matter self-annihilation term to (43). But, it is easy to show that self-annihilation is proportional to $|n_m|^2$. Thus, it is only significant in dense regions, i.e. at small spatial scales such as the central region of dark matter halos, which are in the nonlinear regime and are not studied in the present work. Here we only consider homogeneous and linear perturbations. Therefore, the effect of annihilation is negligible. We remind that Eq. (43) is not restricted to cold dark matter and can be also used for relativistic matter, for instance neutrinos in the early Universe, or a hot component at low redshifts.

Although in the rest of this work we consider the interaction terms described in this section, for what concerns the study of differences between the modified gravity and interacting quintessence models, the formulation of anisotropies and discrimination methods explained in the next

[9]Note that although dark energy is a condensate, i.e. its *particles* have the same energy, presumably zero momentum, a general condensate state can contain a very large number of *particles* in different energy levels, see [43] for more details.

[10]In some dark energy models, such as *early dark energy*, it is assumed that the density of dark energy at high redshifts is much larger, and only at low redshifts it is reduced. Although at redshifts relevant for dark energy surveys the cosmology must be very close to ΛCDM, one must be aware that in many models of this type, the approximation of weak interaction can be applied only at low redshifts. It is also expected that these models leave a detectable signature on the CMB spectrum [66,67].

two sections can be applied to other choices of interactions. It is enough to find an interaction current similar to what we have found for the decay and scattering above and add them to the right-hand side of Eqs. (43) and (44).

IV. COSMOLOGY AND EVOLUTION OF ANISOTROPIES

In this section we first determine the $\mathcal{F}_i$ coefficients defined in Sec. II for both the modified gravity and quintessence models according to the interaction currents and energy-momentum conservation equations obtained in the previous section. Then, we consider the effect of the interactions on the evolution of the anisotropies, and describe how the interaction parameters can be extracted from the data.

A. Interaction coefficients in the Friedmann equation

1. Modified gravity

Using the energy-momentum conservation equation (28) and the interaction current for the modified gravity models, the scalar field equation and the evolution equation of the homogeneous matter density can be determined as the following [21]:

$$\bar{\varphi}'' + 2\mathcal{H}\bar{\varphi}' + a^2 V_\varphi(\bar{\varphi}) = a^2 \mathcal{C}(\bar{\varphi}) \sum_i (\bar{\rho}_i - 3\bar{P}_i), \quad \mathcal{H} = \frac{a'}{a}, \tag{45}$$

$$\bar{\rho}_i' + 3\mathcal{H}(\bar{\rho}_i + \bar{P}_i) = \mathcal{C}(\bar{\varphi})\bar{\varphi}'(\bar{\rho}_i - 3\bar{P}_i), \quad i = m, b, h, \tag{46}$$

where barred quantities are homogeneous components, the subscriptφ means the derivative with respect to φ. Note that here we have generalized the original calculation in [21] by considering a φ-dependent $\mathcal{C}(\bar{\varphi})$ coefficient in the right-hand side of these equations to cover a larger class of modified gravity models, see e.g. [39]. For f(R) models $\mathcal{C} = \sqrt{4\pi G/3}$ [21]. Equations (45) and (46) are coupled and an analytical solution cannot be found without considering an explicitly $V(\varphi)$. Therefore, to solve the equation for $\bar{\rho}$, which is in fact the only directly observable quantity, we simply consider the right-hand side of the equation as a time-dependent source. The solution of Eq. (46) can be written as

$$\bar{\rho}_i(z) = \bar{\rho}_i(z_0)(1+z)^{3(1+w_i)} e^{(1-3w_i)F(\bar{\varphi})}, \quad F(\varphi) \equiv \int \mathcal{C}(\bar{\varphi}) d\varphi, \qquad i = m, b, h, \tag{47}$$

where $w_i \equiv \bar{P}_i/\bar{\rho}_i$ for all species except dark energy are assumed to be constant and are given in Eq. (14). Comparing this solution with (3), we find:

$$\mathcal{F}_i(z) = e^{(1-3w_i)F(\bar{\varphi}(z))} \approx 1 + (1-3w_i)F(\bar{\varphi}(z)). \tag{48}$$

In the f(R) models, $\mathcal{C}(\bar{\varphi}) = \sqrt{4\pi G/3} \equiv C$ [21] is a constant, thus

$$F(\bar{\varphi}) = C\bar{\varphi}(z). \tag{49}$$

Using the transformation from the Jordan frame to the Einstein frame $\bar{\varphi}(z) = \ln(\mathrm{f}_R(z)+1)/2C$ [21], one can relate $\mathcal{F}_i(z)$ to f_R:

$$\mathcal{F}_i(z) \approx 1 + \frac{(1-3w_i)}{2}\ln(\mathrm{f}_R + 1) \approx (1+\mathrm{f}_R)^{-(1-3w_i/2)}. \tag{50}$$

The approximate expression in (50) is the same as Eq. (14). Note that in (47) all constant coefficients, including $(1+\mathrm{f}_R(z_0))^{-(1-3w_i/2)}$, are included in $\bar{\rho}_i(z_0)$. *A priori* one can test the presence of a f(R) modified gravity by measuring simultaneously $\mathcal{F}_m(z)$, $\mathcal{F}_h(z)$, and the equation of state of dark energy from Eq. (13). In fact, in this equation if we neglect the last term that depends on the time derivative, the effective dark energy density becomes

$$\rho_{\mathrm{de}} \approx \frac{3}{8\pi G}\frac{\mathrm{f}_R}{1+\mathrm{f}_R}\left(-\frac{d\ln \mathrm{f}(R)}{6dR} - \frac{R}{6}\right). \tag{51}$$

To be consistent with observations, f(R) cannot be a fast varying function of R. Therefore, the dominant term in (51) is the term proportional to R which makes the relation between ρ_{de}, R, and f(R) very simple. Other $\mathcal{F}_i$'s and the evolution of the corresponding densities have also known expressions, notably $\mathcal{F}_h(z) = 1$. Therefore, *a priori* simultaneous fitting of these quantities can test the f(R) modified gravity models. More generally, in modified gravity models the dark energy term in the Friedmann equation is an effective contribution generated from nonconventional interaction between matter and gravity. Therefore, it is more correlated to matter than in the ΛCDM or (interacting)-quintessence models. In the former *a priori* there is no correlation in the dark sector, and in the latter case the interaction can be very small and is only necessary for reducing fine-tunings and making the model more natural. Similar correlation tests can be performed for other modified gravity models too. Evidently, giving the small deviation of dark energy from a cosmological constant, the measurements and calculation of correlations are not trivial tasks. Furthermore, the discrimination must be cross-checked by using anisotropies for distinguishing between dark energy models, explained in Sec. IV B.

2. Interacting quintessence

In the same way, we can determine $\mathcal{F}_i$ coefficients for (interacting)-quintessence using Eq. (43). We replace n^μ with approximation (42) and include the $1/m$ factors in the L and A_S coefficients. After these simplifications, the evolution equation for the density of interacting quintessence models becomes

$$\bar{\rho}'_i + 3\mathcal{H}(\bar{\rho}_i + \bar{P}_i) = -L_i a\bar{\rho}_i + A_{\rm si} a\bar{\rho}_i\bar{\rho}_\varphi, \quad (52)$$

where i indicates any cold matter or relativistic species that interact with quintessence field.[11] A clear difference between the interaction term in (52) and (46) is that the former does not explicitly depend on the scalar field, and therefore we do not need to know and solve a field equation similar to (45).[12] The solutions of this equation and corresponding $\mathcal{F}_i$'s are

$$\bar{\rho}_i(z) = \bar{\rho}_i(z_0)(1+z)^{3(1+w_i)} \times \exp\left(L_i(\tau(z) - \tau(z0)) + A_{si}\int dz \frac{\bar{\rho}_\varphi(z)}{(1+z)H(z)}\right), \quad (53)$$

$$\mathcal{F}_i(z) = \exp\left(-L_i(\tau(z) - \tau(z_0)) + A_{\rm si}\int dz \frac{\bar{\rho}_\varphi(z)}{(1+z)H(z)}\right) \approx 1 + L_i(\tau(z_0) - \tau(z)) + A_{\rm si}\int_{z_0}^{z} dz \frac{\bar{\rho}_\varphi(z)}{(1+z)H(z)}, \quad (54)$$

where $\tau(z)$ is the age of the Universe at redshift z. Note that even in the absence of expansion, the density of dark matter at high redshifts is higher if $L_i > 0$.

Along with the consistency relation explained above for the modified gravity models, the explicit dependence of (54) on measurable quantities $\bar{\rho}_\varphi(z)$ and $H(z)$ *a priori* allows to discriminate between interacting quintessence and modified gravity models. Note that the prior knowledge about the evolution of these quantities are mandatory for distinguishing the underlying model and without such information one cannot single out any of these models.

B. Matter perturbations in interacting dark energy cosmologies

Although dark energy influences the evolution of perturbations mainly through quantities related to the homogeneous component—background cosmology—the study of anisotropies can be a powerful means both for measuring the equation of state and for discriminating between candidate models. Standard candles, such as supernovae type Ia, allow direct measurements of distances, and thereby cosmological parameters. However, they are rare events, can deviate from being standard due to absorption or late detection [86], subtypes, and dependence of their light curve on other properties such as metallicity, mass, and magnetic field of progenitors [87]. Determination of dark energy properties from the evolution of perturbations provides additional information and a means for cross-checking the two methods.

Matter perturbations in presence of an interacting dark energy [88] and in the f(R) modified gravity models [19–21,56] have been calculated by various authors, thus here we do not repeat them and simply use their results. Our main objective is to find and discuss features that can be used for discriminating between dark energy models.

Considering only scalar perturbations, we define the first-order metric in conformal gauge as follows:

$$ds^2 = a^2(\eta)[(1 + 2\psi(\mathbf{x}))d\eta^2 - (1 - 2\phi(\mathbf{x}))\delta_{ij}dx^i dx^j]. \quad (55)$$

As we mentioned in the Introduction, for modified gravity models we write the evolution equations in the Einstein frame. Thus, here only their interaction terms distinguish them from the quintessence models.

We use a fluid description for both matter and dark energy. After linearizing the energy-momentum conservation equations and taking their Fourier transform with respect to spatial coordinates, the evolution equations for density and velocity perturbations of the matter component i and dark energy can be written as

$$\delta\rho'_{(i)} + 3\mathcal{H}\delta\rho_{(i)}(1 + C^2_{s(i)}) + (1 + w_{(i)})\bar{\rho}_{(i)}(3\phi' - ik_j v^j_{(i)}) = \delta Q_{(i)0}, \quad (56)$$

$$((1 + w_{(i)})\bar{\rho}_{(i)}v_{(i)j})' + 4\mathcal{H}(1 + w_{(i)})\bar{\rho}_{(i)}v_{(i)j} - ik_{(i)}C^2_{s(i)}\delta\rho_{(i)} - ik_l\Pi^l_{(i)j} - ik_j(1 + w_{(i)})\bar{\rho}_{(i)}\psi = \delta Q_{(i)j}, \quad (57)$$

$$\delta\rho'_\varphi + 3\mathcal{H}\delta\rho_\varphi(1 + C^2_{s\varphi}) - (1 + w_\varphi)\bar{\rho}_\varphi(3\phi' - ik_j v^j_\varphi) = \delta Q_{\varphi 0}, \quad (58)$$

$$((1 + w_{(i)})\bar{\rho}_\varphi v_{\varphi j})' + 4\mathcal{H}(1 + w_\varphi)\bar{\rho}_\varphi v_{\varphi j} - ik_j C^2_{s\varphi}\delta\rho_\varphi - ik_l\Pi^l_{\varphi j} - ik_j(1 + w_\varphi)\bar{\rho}_\varphi\psi = \delta Q_{\varphi j}, \quad (59)$$

where $C^2_{s(i)} \equiv \delta P_{(i)}/\delta\rho_{(i)}$ is the speed of sound for species i,[13] $v_{(i)}$ is its velocity, and $\Pi^l_{(i)j}$ is its anisotropic shear. The perturbation of the interaction current for the modified gravity and quintessence models derived from (30), (43), and (44) are the following:

[11] If species i has an interaction with another component, for instance is scattered by another species, we can add a second scattering term to (53). The best example is the scattering of photons or neutrinos by baryons. Here for the sake of simplicity we neglect such interactions which are not the main concern of this work. However, in a full formulation of the problem they should be considered, especially if they can mimic an interaction with dark energy.

[12] For $f(R)$ modified gravity in which $\mathcal{C}$ is constant $\bar{\varphi}'$ in (46) can be replaced by an expression depending on density and pressure, and there is no need for solving the field equation of the scalar field either.

[13] To prevent confusion between spacetime indices and indices indicating the species, when there is a risk of confusion we put the latter inside brackets

Modified gravity:

$$\delta Q_{(i)0} = \rho_{(i)}[(1-3w_{(i)})\mathcal{C}_{\varphi}(\bar{\varphi})\bar{\varphi}'\delta\varphi + \mathcal{C}(\bar{\varphi})((1-3w_{(i)})\delta\varphi' + (1-3C^2_{s(i)})\bar{\varphi}'\delta_{(i)})] = -\delta Q_{\varphi 0}, \tag{60}$$

$$\delta Q_{(i)j} = ik_j\mathcal{C}(\bar{\varphi})\bar{\rho}_{(i)}(1-3w_{(i)})\delta\varphi = -\delta Q_{\varphi j}, \quad i = m, b, h. \tag{61}$$

Interacting quintessence:

$$\delta Q_{(i)0} = -aL_{(i)}(\delta\rho_{(i)} + \bar{\rho}_{(i)}\psi) + aA_{s(i)}[\bar{\rho}_{\varphi}\delta\rho_{(i)} + \bar{\rho}_{(i)}(\delta\rho_{\varphi} + \bar{\rho}_{\varphi}\psi)], \tag{62}$$

$$\delta Q_{(i)j} = av_{(i)j}(-L_{(i)}\bar{\rho}_{(i)} + A_{s(i)}\bar{\rho}_{(i)}\bar{\rho}_{\varphi}), \tag{63}$$

$$\delta Q_{\varphi 0} = aL_{\varphi}(\delta\rho_{(i)} + \psi\bar{\rho_{(i)}}) + aA_{\varphi s}\left[\delta\rho_{\varphi}\bar{\rho}_{(i)} + \bar{\rho}_{\varphi}\delta\rho_{(i)} - \bar{\rho}_{(i)}\delta\rho_{\varphi}\left(\frac{1+C^2_{s\varphi}}{1+w_{\varphi}} + \psi\right)\right], \tag{64}$$

$$\delta Q_{\varphi j} = av_{(i)j}(L_{\varphi}\bar{\rho}_{(i)} + A_{s(i)}\bar{\rho}_{(i)}\bar{\rho}_{\varphi}), \quad i = \text{all matter interacting with } \varphi \ldots \tag{65}$$

In (60), $\mathcal{C}_{\varphi}$ is the derivative of $\mathcal{C}(\varphi)$ with respect to φ. To obtain these equations we have used the following definition and properties:

$$\rho_{\varphi} \equiv u_{(\varphi)\mu}u_{(\varphi)\nu}T^{\mu\nu}_{\varphi} = \frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi + V(\varphi), \tag{66}$$

$$P_{\varphi} \equiv \frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi - V(\varphi), \tag{67}$$

$$u_{(\varphi)^{\mu}} \equiv \frac{\partial^{\mu}\varphi}{\partial_{\nu}\varphi\partial^{\nu}\varphi} = \frac{\partial^{\mu}\varphi}{(\rho_{\varphi} + P_{\varphi})^{(1/2)}}, \tag{68}$$

$$\frac{\delta\rho_{\varphi} + \delta P_{\varphi}}{\rho_{\varphi} + P_{\varphi}} = 2\left[\frac{a\partial^0(\delta\varphi)}{(\bar{\rho}_{\varphi} + \bar{P}_{\varphi})^{(1/2)}} + \psi\right]. \tag{69}$$

Evidently, these equations are valid for both modified gravity and interacting quintessence. They are also highly coupled; thus, it is impossible or very difficult to find an analytical solution for them. To complete the evolution equations for modified gravity, we also need the evolution of $\delta\varphi$. This can be obtained by expanding the field $\varphi = \bar{\varphi} + \delta\varphi$ and using the covariant field equation, see e.g. [43]:

$$\frac{1}{\sqrt{-g}}\partial_{\mu}\left(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\varphi\right) + V_{\varphi}(\varphi) = \mathcal{C}(\varphi)T_m, \tag{70}$$

$$\delta\varphi'' + 2\mathcal{H}\delta\varphi' + \psi'(\bar{\varphi}' + 2\mathcal{H}\bar{\varphi}) + \psi\left(\bar{\varphi}'' + 2\mathcal{H}\bar{\varphi}' + 2\frac{a''}{a}\bar{\varphi}\right) - \left(k^2 - \frac{a''}{a} + V_{\varphi\varphi}\right)\delta\varphi = \mathcal{C}(\varphi)\delta T_m + C_{\varphi}\bar{T}\delta\varphi. \tag{71}$$

As we mentioned in previous sections, solving these equations is not the main aim of the present work. Our goal is to single out the differences in these models that can be used for discriminating them from other models. For instance, in modified gravity models the perturbation of interaction current does not depend on the metric perturbations ψ and ϕ. By contrast, in interacting dark energy the current perturbation depends on the metric perturbation and it is easy to that:

$$\frac{\text{term} \propto \psi}{\text{term} \propto \delta} = 1. \tag{72}$$

Because according to observations $\delta\varphi$, φ', and $\delta\varphi'$ are very small, in both models the terms proportional to $\delta_i \equiv \delta\rho_i/\rho_i$ are dominant. In this case, it is easy to see that for modified gravity $\delta Q_{(i)0} \propto \mathcal{C}(\varphi)$ and for interacting quintessence $\delta Q_{(i)0} \propto (-L_{\varphi} + A_{s(i)}\bar{\rho}_{\varphi})$. Although *a priori* these quantities evolve differently, both of them are expected to vary very slowly. Thus, it is not possible to distinguish them, especially in a model-independent way. Other properties such as (72) cannot be used directly either. Nonetheless, they influence the growth rate $\propto \delta'_i/\delta_i$, density power spectrum, and density-velocity correlations, etc. In the next section we discuss how these measurable quantities can be related to the interaction current, and thereby allow to discriminate between modified gravity and quintessence models.

Perturbation equations (62)–(65) depend on metric perturbations ψ and ϕ, and their time derivatives. These quantities can be determined from the Einstein equations for perturbations (see e.g. [89]):

$$k^2\phi + 3\mathcal{H}(\phi' + \mathcal{H}\psi) = 4\pi Ga^2\sum_i \delta\rho_i, \tag{73}$$

$$k^2(\phi' + \mathcal{H}\psi) = -4\pi Ga^2\sum_i ik_jv^j_{(i)}(\bar{\rho}_{(i)} + \bar{P}_{(i)}), \tag{74}$$

$$\phi'' + \mathcal{H}(\psi' + 2\phi') + \left(\frac{2a''}{a} - \frac{a'^2}{a^2}\right)\psi + \frac{k^2}{3} = -4\pi Ga^2\sum_i \delta P_i, \tag{75}$$

$$k^2(\phi - \psi) = -12\pi Ga^2\sum_i\left(k_jk^l - \frac{1}{3}\delta^l_j\right)\Pi^j_{(i)l}. \tag{76}$$

Note that in these equations the interaction energy is neglected. The reason is that we need $T^{\mu\nu}_{\text{int}}$, which in the phenomenological description of interactions is not known. Nonetheless, its omission in Eqs. (73)–(76) should not induce large errors because present observations show that any nongravitational interaction between various constituents of the Universe—if any—must be very small, and therefore this approximation is justified. Metric perturbations ψ and ϕ cannot be directly observed, except through lensing. Otherwise, they can be extracted from these equations when density-density and density-velocity correlations, and induced anisotropic shear $\Pi^j_{(i)l}$ are determined from the LSS data.

Although phenomenological interaction currents (43) and (44) are inspired from the well-understood scattering of particles, one cannot rule out other types of interaction. Even for these cases *a priori* one should be able to write equations similar to (62)–(65) and (72). The fact that the latter relations are independent of the strength of the coupling between dark energy and matter proves that finding a different proportionality between the ψ and δ terms would be a clear signature of an unusual quintessence model, e.g. one with a nonminimal interaction with gravity. Evidently, such measurements are not easy. Nonetheless, with the huge amount of data expected from near future surveys and their better precision, more accurate measurements of the parameters should be possible, and the precision analysis necessary for detailed examination of dark energy models should be achievable.

V. ESTIMATION OF FORECAST PRECISION FOR SURVEYS

In this section we first describe how in practice the background cosmology parameters defined in Sec. II are calculated. Their uncertainties determine how well a survey can discriminate between modified gravity and (interacting)-quintessence models, independent of the data type or observation method. Then, we calculate and parametrize the evolution equation of the growth rate of matter anisotropies and discuss its measurement uncertainty. As an example we make an order of magnitude estimate for the expected uncertainty of these quantities for the Euclid mission [55]. As we mentioned in the Introduction, a proper forecast requires a detailed study of the observational effects and uncertainties which is beyond the scope of the present work.

A. Discriminating between a cosmological constant and other models

As we discussed in Sec. II B, the discrimination ability of surveys between a cosmological constant and a redshift-dependent dark energy can be evaluated by using the function $\Theta(z)$ defined in (24). To calculate the quantity Θ and its uncertainty, we need to know uncertainties of the estimation of the effective background cosmological parameters. The function Θ depends on $\Omega^{(H)}_{\text{eff}}$, $w^{(H)}_{\text{eff}}(z)$, $\Omega^{(A)}_{\text{eff}}$, and $w^{(A)}_{\text{eff}}(z)$, the effective dark energy fractional density and equation of state dark energy determined, by fitting $H(z)$ and $A(z)$, respectively. By measuring $H(z)$, from either supernovae or BAO data, one can determine $w^{(H)}_{\text{eff}}(z)$ and $\Omega^{(H)}_{\text{eff}}$ relatively easily. On the other hand, the measurements of $w^{(A)}_{\text{eff}}(z)$ and $\Omega^{(A)}_{\text{eff}}$ are less straightforward, because one has to determine dH/dz, or equivalently dD_A/dz and d^2D_A/dz^2 (see Appendix A for the relation between these quantities). For this reason, the uncertainty of Θ is dominated by the uncertainties of $w^{(A)}_{\text{eff}}(z)$ and $\Omega^{(A)}_{\text{eff}}$. Finally, the coefficients $\mathcal{F}_i$'s that present the evolution of the equation of state of various constituents, are determined by fitting the deviation of $H(z)$ from the null hypothesis of a ΛCDM cosmology. However, as we argued in Sec. II, there are strong degeneracies between $\mathcal{F}_i$'s and $\gamma(z)$ which can be resolved only by using other types of data, in particular, matter anisotropies, see Sec. V B for more details.

As an example, we estimate the uncertainty of Θ for the Euclid mission. For the parametrization $w_{\text{eff}}(z) = w_p + w_a z/(1+z)$, according to the Euclid Red Book [90], the standard deviation for these coefficients are expected to be $\sigma_{w_p} \sim 0.015$ and $\sigma_{w_a} \sim 0.15$ for Euclid data alone, and $\sigma_{w_p} \sim 0.007$ and $\sigma_{w_a} \sim 0.035$ for Euclid + Planck data. No forecast for the expected uncertainty of dH/dz is yet available. For this reason, we simply use error propagation rules to determine a rough estimation for $\sigma_{dH/dz}$ from the available forecasts. We approximate dH/dz with its definition as a difference ratio: $dH/dz \approx \Delta H/\Delta z$, then we use the general uncertainty propagation rule to a function of n variables $f(x_1, \ldots, x_n)$:

$$\sigma_f^2 = \sum_{i,j=1,\ldots,n} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} C_{ij}, \tag{77}$$

where C_{ij} is the covariance matrix for random variables $x_1, \ldots, x_n$. Assuming $\sigma_H/H \sim 1\%$, negligible error for z, and $\mathcal{F}_i \sim 1$, the dominant source of error in dH/dz from $w(z)$. Because the coefficients of derivatives with respect to these parameters in (77) is roughly of the order of 1, we estimate $\sigma_{dH/dz}/(dH/dz) \sim 10$–$15\%$. Functions $A(z)$ and $B(z)$ are related to dH/dz, see (A3), and when the uncertainties of the H, density fractions Ω_i's and redshift z are much smaller, $\sigma_{w^{(A)}_{\text{eff}}}/w^{(A)}_{\text{eff}}(z) \sim \sigma_{\Omega^{(A)}_{\text{eff}}}/\Omega^{(A)}_{\text{eff}} \sim \sigma_{\sigma_B} \sim \sigma_{dH/dz} \sim 10\%$ around an optimal redshift of $z \sim 0.5$. Measurement precisions of $\mathcal{F}_i$'s also are of the order of precision of dH/dz, i.e. $\sigma_{\mathcal{F}_i}/\mathcal{F}_i \sim \sigma_{dH/dz}/(dH/dz) \sim 10$–$15\%$.

Evidently, uncertainties obtained here are very rough estimations. The aim of these exercises is just to show what level of error we expect from near future surveys. A proper prediction requires a detailed simulation of measurements and data analyzing methods, instrumental effects, and systematic and statistical errors. They need a dedicated study which we leave to a future work.

Finally, we want to make a remark about the redshift dependence of $w(z)$, which in the literature is usually parametrized [72]. In Appendix A we show that for the same value of w at two different redshifts, different parametrizations lead to a very different evolution for $A(z)$. Conversely, if we estimate $w(z)$ from the measurement of $A(z)$, the parametrization of $w(z)$ can lead to a very different evolution for this function, despite the employment of the same data for $A(z)$. Therefore, we must estimate w at each redshift without parametrizing it. As for the estimation of uncertainties, for instance from the Fisher matrix, they can be determined from the set of $\{w(z), \gamma(z), z\}$ at every redshift bin rather than from a parametrization, see Appendix B

B. Discrimination between modified gravity and interacting quintessence models

If we observe a nonzero Θ, then we must use the power spectrum and growth rate of perturbations to investigate the nature and origin of deviation from a cosmological constant. The comparison between the evolution equation of modified gravity and interacting quintessence models in Sec. IV B showed that their interaction currents are very different, and thereby the evolution of matter anisotropies and dark energy density in these models are not the same. In fact, if we could decompose the interaction current to terms proportional to scalar metric perturbations and matter density fluctuations, it would be possible to distinguish between these models. However, in practice measured quantities are matter power spectrum and its growth rate $\mathbf{f}(z, k)$ is defined as

$$\mathbf{f}(z, k) \equiv \frac{d\ln D}{d\ln a} = \frac{\delta_m'}{\mathcal{H}\delta_m}, \qquad D \equiv \frac{\delta_m(z,k)}{\delta_m(z=0,k)}. \tag{78}$$

The function $\mathbf{f}(z, k)$ is usually extracted from the power spectrum using a model [91–95], for instance, a power law for the primordial spectrum, including its modification by the Kaiser effect [96–99] and redshift distortion due to the velocity dispersion [100].

To obtain the evolution equation of $\mathbf{f}(z, k)$, we replace potentials ψ and ϕ by expressions depending only on $\delta_m \equiv \delta\rho_m/\bar{\rho}_m$ and $\theta_m \equiv ik_j v^j_{(m)}$. Assuming a negligible anisotropic shear at $z \lesssim \mathcal{O}(1)$ which concerns galaxy surveys, scalar metric perturbations—gravitational potentials—ψ and ϕ can be determined from the Einstein equations (73)–(76):[14]

$$\phi = \psi = \frac{4\pi G\bar{\rho}_m}{k^2}\left(\delta_m + 3(1+w_m)\frac{\mathcal{H}\theta_m}{k^2}\right) + \Delta\psi, \tag{79}$$

[14]In this section for the sake of simplicity of notation we consider that $\mathcal{F}_i$'s factors for species are included in w_i's, i.e. $(1+z)^{3\gamma_i}\mathcal{F}_i$ is redefined as $(1+z)^{3\gamma_i(z)}$ and w_i is obtained from (2) using this redefined γ_i. Therefore, for interacting dark energy models w_m is nonzero and in general depends on redshift.

$$\Delta\psi = \frac{4\pi G}{k^2}(\delta\rho_\varphi - 3\mathcal{H}\delta\varphi(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)}), \tag{80}$$

$$\phi' = -\frac{4\pi G\bar{\rho}_m\mathcal{H}}{k^2}\left(\delta_m + \left(3 + \frac{k^2}{\mathcal{H}^2}\right)(1+w_m)\frac{\mathcal{H}\theta_m}{k^2}\right) + \Delta\phi', \tag{81}$$

$$\Delta\phi' = -\mathcal{H}\Delta\psi + 4\pi Ga^2\delta\varphi(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)}. \tag{82}$$

Note that in (79) and (81) we have separated terms which vanish for the ΛCDM model and written them as $\Delta\psi$ and $\Delta\phi'$. As observations show that dark energy behaves very similar to a cosmological constant—at least for $z \lesssim \mathcal{O}(1)$, both these quantities are expected to be very small. It is why we write them as a variation of ψ and ϕ'. For future use it is also better to redefine them as follows:

$$\epsilon_0 \equiv \frac{\delta\rho_\varphi}{\bar{\rho}_m}, \qquad \epsilon_1 \equiv \frac{\mathcal{H}(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)}\delta\varphi}{\bar{\rho}_m}, \tag{83}$$

$$\Delta\psi = \frac{4\pi G\bar{\rho}_m}{k^2}(\epsilon_0 - 3\epsilon_1), \tag{84}$$

$$\Delta\phi' = -\frac{4\pi G\bar{\rho}_m\mathcal{H}}{k^2}\left(\epsilon_0 - \left(3 + \frac{k^2}{\mathcal{H}^2}\right)\epsilon_1\right). \tag{85}$$

After replacing ϕ' and ψ in (56) and (57) with (81) and (79), respectively, the evolution equation of matter and velocity perturbations can be written as

$$\delta_m' + \frac{\bar{\rho}_m'}{\bar{\rho}_m} + 3\mathcal{H}\left\{(1+C_{\rm sm}^2)\delta_m + (1+w_m)\frac{3\Omega_m\mathcal{H}^2}{2k^2} \times\left[\delta_m + \left(3+\frac{k^2}{\mathcal{H}^2}\right)(1+w_m)\frac{\mathcal{H}\theta_m}{k^2}\right] + \left(\epsilon_0 - \left(3 + \frac{k^2}{\mathcal{H}^2}\right)\right)\right\} + (1+w_m)\theta_m = \delta Q_{m0}, \tag{86}$$

$$\theta_m' + \frac{w_m'}{1+w_m}\theta_m + \frac{\bar{\rho}_m'}{\bar{\rho}_m}\theta_m + 4\mathcal{H}\theta_m - \frac{C_{\rm sm}^2 k^2}{1+w_m}\delta_m - 3\Omega_m\mathcal{H}^2\left(\delta_m + 3(1+w_m)\frac{\mathcal{H}\theta_m}{k^2} + \epsilon_0 - 3\epsilon_1\right) = ik_i\delta Q^i_{(m)}, \tag{87}$$

where δQ_{m0} and $ik_i\delta Q^i_{(m)}$ are interaction currents and $\Omega_m \equiv 8\pi Ga^2\bar{\rho}_m/3\mathcal{H}^2$. Moreover, in present and near future wide area surveys such as DES and Euclid the value of $\mathcal{H}/ck \ll 1$.[15] For instance, for Euclid $\mathcal{H}/ck \lesssim 0.01$. Therefore, we can neglect terms proportional to $\mathcal{H}/k$. Under these approximations, the evolution equations of density and velocity become:

[15]Note that the speed of light $c = 1$ is assumed in metric (55), and therefore it does not explicitly appear in our calculations.

Modified gravity:

$$\delta'_m + 3\mathcal{H}(C^2_{\rm sm} - w_m)\delta_m + (1+w_m)\theta_m = \frac{3\Omega_m(1-3w_m)\mathcal{C}_\varphi(\bar{\varphi})}{8\pi G}a\mathcal{H}\epsilon_1 + \mathcal{C}(\bar{\varphi})\left(\frac{3\Omega_m(1-3w_m)}{8\pi G}\right)^{(1/2)}\frac{\Omega_m(1+C^2_{s\varphi})}{2\Omega_\varphi(1+w_\varphi)}a\mathcal{H}\epsilon_0, \tag{88}$$

$$\theta'_m + \mathcal{H}\theta_m - \frac{C^2_{\rm sm}k^2}{1+w_m}\delta_m - \frac{3\Omega_m}{2}\mathcal{H}^2(\delta_m + \epsilon_0 - 3\epsilon_1) = -\frac{\sqrt{3}k^2(1-3w_m)\Omega_m}{(8\pi G(1+w_\varphi)\Omega_\varphi)^{(1/2)}}\mathcal{C}(\bar{\varphi})\epsilon_1. \tag{89}$$

Interacting quintessence:

$$\delta'_m + 3\mathcal{H}(C^2_{\rm sm} - w_m)\delta_m + (1+w_m)\theta_m = aA_{\rm sm}\epsilon_0, \tag{90}$$

$$\theta'_m + \mathcal{H}\theta_m - \frac{C^2_{\rm sm}k^2}{1+w_m}\delta_m - \frac{3\Omega_m}{2}\mathcal{H}^2(\delta_m + \epsilon_0 - 3\epsilon_1) = -\frac{w_m}{1+w_m}(-L_m + A_{\rm sm}\bar{\rho}_\varphi)a\theta_m. \tag{91}$$

Now that we have the evolution equations for δ_m and θ_m, we can determine the evolution of the growth rate. The procedure for calculating $d\mathbf{f}(z,k)/dz$ is straightforward. We replace θ_m in (89) and (91) with its value obtained from (88) and (91), respectively, for the modified gravity and interacting quintessence models. Then, we replace δ'_m with its value from Eq. (78). The final equation has the following general form:

$$\mathbf{f}'\mathcal{H} + \mathbf{f}(\mathcal{H}' + \mathcal{H}^2) + \mathbf{f}^2\mathcal{H}^2 + 3(C^2_{\rm sm} - w_m)(\mathcal{H}' + \mathbf{f}\mathcal{H}^2) + 3(C^2_{\rm sm} - w_m)\mathcal{H}^2 + \frac{3}{2}\Omega_m(1+w_m)^2\mathcal{H}^2 + k^2C^2_{\rm sm} + E_0\mathbf{f}\mathcal{H} + E_1k^2 + E_2\mathcal{H} + E_3\mathcal{H}^2 + E_4 = 0. \tag{92}$$

Coefficients E_0, E_1, E_2, E_3, E_4 depend on z and k, and have the following values for the two models discussed here:

Modified gravity:

$$E_0 \equiv \mathcal{C}(\bar{\varphi})a(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)}(3(C^2_{\rm sm} - w_m) + 1 - 3w_m), \tag{93}$$

$$E_1 \equiv \mathcal{C}(\bar{\varphi})\frac{(1+w_m)(1-3w_m)\bar{\rho}_m\epsilon_1}{\mathcal{H}(\bar{\varphi})a(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)}}, \tag{94}$$

$$E_2 \equiv 3(1+w_m)(C^2_{\rm sm} - w_m)\mathcal{C}(\bar{\varphi})a(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)} - \mathcal{C}_\varphi(\bar{\varphi})(1-3w_m)\frac{a\bar{\rho}_m\epsilon_1}{\mathcal{H}\delta_m} - \mathcal{C}(\bar{\varphi})a(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)} \times \left(\frac{(1-3w_m)(1+C^2_{s\varphi})\bar{\rho}_m\epsilon_0}{2(1+w_\varphi)\bar{\rho}_\varphi\delta_m} + 3(C^2_{\rm sm} - w_m)\right), \tag{95}$$

$$E_3 \equiv \frac{3\Omega_m(1+w_m)^2}{2}\left(\frac{\epsilon_0}{\delta_m} - \frac{3\epsilon_1}{\delta_m}\right), \tag{96}$$

$$E_4 \equiv -\frac{1}{\delta_m}\left(\mathcal{C}_\varphi(\bar{\varphi})(1-3w_m)\frac{a\bar{\rho}_m\epsilon_1}{\mathcal{H}} + \mathcal{C}(\bar{\varphi})a(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)}\frac{(1-3w_m)(1+C^2_{s\varphi})\bar{\rho}_m\epsilon_0}{2(1+w_\varphi)\bar{\rho}_\varphi}\right)' + (3\mathcal{C}(\bar{\varphi})a(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)}(C^2_{\rm sm} - w_m))' - \mathcal{C}(\bar{\varphi})a(1-3w_m)(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)} \times \left[\mathcal{C}_\varphi(\bar{\varphi})(1-3w_m)\frac{a\bar{\rho}_m\epsilon_1}{\mathcal{H}\delta_m} + \mathcal{C}(\bar{\varphi})a(\bar{\rho}_\varphi + \bar{P}_\varphi)^{(1/2)}\left(\frac{(1-3w_m)(1+C^2_{s\varphi})\bar{\rho}_m\epsilon_0}{2(1+w_\varphi)\bar{\rho}_\varphi\delta_m} + 3(C^2_{sm} - w_m)\right)\right]. \tag{97}$$

Interacting quintessence:

$$E_0 \equiv w_m a(-L_m + A_{sm}\bar{\rho}_\varphi), \tag{98}$$

$$E_1 \equiv 0, \tag{99}$$

$$E_2 \equiv 3w_m a(C^2_{\rm sm} - w_m)(-L_m + A_{\rm sm}\bar{\rho}_\varphi) + A_{\rm sm}a\bar{\rho}_m(1+3w_m)\frac{\epsilon_0}{\delta_m}, \tag{100}$$

$$E_3 \equiv \frac{3\Omega_m(1+w_m)^2}{2}\left(\frac{\epsilon_0}{\delta_m} - \frac{3\epsilon_1}{\delta_m}\right), \tag{101}$$

$$E_4 \equiv -A_{\rm sm}a\bar{\rho}_m\left(\frac{\epsilon'_0}{\delta_m} + \frac{2a\epsilon_0}{\delta_m}(-L_m + A_{\rm sm}\bar{\rho}_\varphi)\right). \tag{102}$$

In the calculation of (92)–(102), we have neglected terms proportional to $\mathcal{H}/ck$.

For the ΛCDM model $E_i = 0, i = 0, \cdots, 4$. For a non-interacting quintessence model all E_i coefficients are zero except E_3. A notable difference between the modified gravity and interacting quintessence models is the coefficient E_1 which is strictly zero for the interacting dark energy models and nonzero for modified gravity that leaves an additional scale dependent signature on the evolution of matter anisotropies. The other explicitly scale dependent term is common for all models and is expected to be very small because it is proportional to the square of sound speed which is very small for cold matter. In addition, in contrast to the rest of the E_i coefficients, E_1 and E_3 are dimensionless. Evidently, the contribution of the $E_1 k^2$ term with respect to other terms in Eq. (92) increases for larger k, i.e. at short distances. But, the effect of nonlinearities, i.e. mode coupling also increases at large k, see e.g. [101]. They can imitate interactions and lead to the misinterpretation of data. For this reason, it is suggested that observation of galaxy clusters is a good discriminator between dark energy models [102,103], because clusters are still close to the linear regime, but have relatively large k.

Discriminating a power of a survey can be estimated by the precision of the E_1 and E_3 measurements. However, one expects some degeneracies when Eq. (92) is fitted to determine E_i's. Moreover, in galaxy surveys, $\mathbf{f}$ and $\mathbf{f}'$ (or more exactly $d\mathbf{f}/dz$) are determined from the measurement of the power spectrum from the galaxy distribution, and $\mathcal{H}$ and $\mathcal{H}'$ from the BAO effect on the spectrum. Thus, these measurements are not completely independent. An independent measurement of $\mathcal{H}$ and $\mathcal{H}'$, e.g. using supernovae will help to reduce degeneracies and error propagation from measured quantities to the estimation of E_i's. The relation between $\mathcal{H}'$ and $B(z)$ defined in (A5) shows the logical connection of parametrization of the homogeneous component—background cosmology—and the evolution of fluctuations, especially in what concerns the discrimination between dark energy models. In fact, anisotropies depend on the equation of state of matter, which in the context of interacting dark energy models, is modified by its interaction with dark energy. Thus, their independent measurements optimize their employment in distinguishing between various models.

Although *a priori* $d\mathbf{f}/dz$ can be determined directly from the data by differentiating $\mathbf{f}$, usually due to shot noise the errors would be very large unless we extensively rebin the data. However, rebinning smears the redshift dependence, which is the most important information for discriminating between models. Another approach is to solve Eq. (92) analytically. It does not have an analytical solution for the general case, but as we show in Appendix D, when w_m and $C^2_{\rm sm}$ are approximated by constant values, and the cosmology is matter, the radiation or cosmological constant dominated, i.e. up to the desired precision only one component determines its evolution, an approximate solution can be found. At present the epoch where matter and dark energy have comparable contributions, the coefficients in (92) even for ΛCDM vary with the redshift. Nonetheless, their variation arrives very quickly to saturation. Therefore, the true solution is not very different from the approximate analytical one under the explained conditions, and it is possible to determine perturbations around the analytical solution by linearizing Eq. (92), see Appendix D for more details.

A rough estimation of the uncertainties of E_i's measured by Euclid can be performed in the same manner as that presented in Sec. V A for Θ and $\mathcal{F}_i$'s. It is expected that the growth rate $\mathbf{f}$ can be reconstructed from the Euclid + Planck data with an uncertainty of $\sigma_{\mathbf{f}}/\mathbf{f} \lesssim 3\%$ [85]. Considering Eq. (92) and the estimation of the uncertainty of $\mathcal{H}'$ obtained in Sec. V A, the uncertainty of $\sigma_{\mathbf{f}'}/\mathbf{f}'$ must be ~10%–15%0. This limits our ability to distinguish between a ΛCDM model where $E_i = w_m = 0$, and the quintessence or interacting dark energy models where these quantities are not zero. Considering the linear equation obtained in Appendix D from the expansion of $\mathbf{f}$ around its solution for ΛCDM, the total uncertainty of the deviation from this model is roughly the same as that obtained for $\mathbf{f}'$, i.e. $\sim 10\% - 15\%$. But, the uncertainty in the estimation of each E_i is expected to be larger because of the degeneracy of these parameters. Evidently, the determination of $\mathbf{f}$ and $\mathbf{f}'$ at multiple redshifts should somewhat help reduce degeneracies and improve discrimination between models. More precise estimations as well as the estimation of the effect of nonlinearities and the optimal choice of the scale range need detailed simulation of surveys. We leave these tasks for future works.

C. Interpretation and comparison with other parametrizations

It would be useful to have better insight into the physical meaning of the parameters defined in the previous section, and to compare them with that used in the literature for parametrizing dark energy models.

We begin with ϵ_0 and ϵ_1 defined in (83). Their definitions show that the former depends only on dark energy density anisotropy and the latter only on the peculiar velocity of dark energy field, i.e. on its kinematics, see (C3). They follow each other closely and approach zero when the field approaches its minimum value. However, their exponent close to the minimum depends on the interaction. Therefore, their measurements give us information about the potential and interactions of the scalar field. Moreover, the difference in the dependence of the evolution equation of anisotropies and the growth factor to ϵ_0 and ϵ_1 shows that only by the separation of the kinematics and dynamics of dark energy—scalar field—would it be possible to distinguish between modified gravity and other scalar field models.

The deviation of gravity potentials ϕ and ψ from their value in ΛCDM $\Delta\psi$ is the quantity that can be measured directly from the lensing data [104]. For this reason various authors have used $\Delta\psi$ to parametrize the deviation from

ΛCDM [48–51]. However, Eqs. (79) and (80) show that although $\Delta\psi \neq 0$ is by definition a signature of the deviation from ΛCDM, in contrast to claims in the literature, it is not necessarily the signature of a modified gravity model because quintessence models, both interacting and noninteracting, also induce $\Delta\psi \neq 0$. This is also another manifestation of the difference between the kinematics and dynamical effects of the interacting dark energy models described above.

Because we have used the Einstein frame for both the quintessence and modified gravity models, in the absence of an anisotropic shear, $\phi = \psi$ even in non-ΛCDM models. In linear approximation the gravitational lensing effect depends on the total potential $\Phi \equiv \phi + \psi$ (see e.g. [104] for a review). Therefore, in the Einstein frame

$$\Phi = 2\phi = 2\psi = \Phi_{\Lambda\mathrm{CDM}} + 2\Delta\psi,$$
$$\Phi_{\Lambda\mathrm{CDM}} \equiv \frac{4\pi G\bar{\rho}_m}{k^2}\left(\delta_m + 3(1+w_m)\frac{\mathcal{H}\theta_m}{k^2}\right) \equiv \frac{4\pi G\bar{\rho}_m}{k^2\Delta_m}. \tag{103}$$

In the notation of [48] $\Phi = 2\Sigma\Phi_{\Lambda\mathrm{CDM}}$, thus,

$$\Sigma = 1 + \frac{\Delta\psi}{\Phi_{\Lambda\mathrm{CDM}}} = \frac{\epsilon_0 - 3\epsilon_1}{k^2\Delta_m}. \tag{104}$$

The other quantity that affects the evolution of lensing and directly depends on cosmology is the growth factor of matter anisotropies which determines the evolution of Δ_m defined in (103). This quantity can be obtained from integration of the growth rate **f** defined in (78) and is usually parametrized as Ω_m^γ. For ΛCDM $\gamma \approx 0.55$ [105]. In this respect there is no difference between our formulation and that used in the literature. Evidently, this simple parametrization cannot distinguish between various dark energy models. By contrast, the more sophisticated decomposition proposed in Sec. V B is able to distinguish between quintessence and modified gravity. Note that in the Jordan frame there are two other parameters: $\eta \equiv (\psi - \phi)/\phi$ and $Q = \phi/\phi_{\Lambda\mathrm{CDM}}$. The parameter $\Sigma = Q(1 + \eta/2)$, thus it is not independent. In the Einstein frame $\eta = 1$ unless there is an anisotropic shear. At first glance it seems that there is less information in the Einstein frame about modified gravity than in the Jordan frame. However, one should notice that in the Einstein frame the fundamental parameters are ϵ_0 and ϵ_1, and other quantities such as $\Delta\psi$ and **f** can be explained as a function of these parameters. Therefore, the amount of information in the Einstein and Jordan frames about modified gravity—if it is what we call dark energy—is the same. The advantage of formulation in the Einstein frame and the definition of ϵ_0 and ϵ_1 is that they can be used for both major categories of models. Moreover, they have explicit physical interpretations that can be easily related to the underlying model of dark energy.

More recently based on an original work by Skordis [106], two groups [107,108] have suggested new parametrizations which are basically only for discriminating modified gravity models from ΛCDM. Both groups use the following approximate description for the Einstein equation:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} + U_{\mu\nu}. \tag{105}$$

The tensor $U_{\mu\nu}$ is called the *energy-momentum tensor of dark energy* [106], and originally its definition has been for formulation of all modifications of Einstein's theory of gravity. In [107] this tensor is expanded with respect to potentials ψ and ϕ, and the coefficients of this expansion are used for parametrizing the underlying modified gravity model.

Note that Eq. (105) is at all scales an approximation because the right-hand side is explicitly proportional to the Newton coupling constant. Considering the $f(R)$ models which are the simplest modification of Einstein's theory of gravity, in contrast to (105), the coupling to matter is modified in both frames, see Eqs. (8)–(10) for the Jordan frame, and the formulation of the $f(R)$ model in the Einstein frame in [21]. In fact, in the Einstein frame the modification is explicit in the energy-momentum conservation equation. This means that if a deviation from ΛCDM is observed, it would be very difficult to verify the consistency of the model at short distances because the deviation of coupling from the Newton constant G is put by hand to zero. Moreover, this formulation and parametrization by definition does not help to detect the interaction between dark energy and matter, because it depends only on the total variation of metric potentials. In addition, in this formulation $U_{\mu\nu}$ is assumed to be a conserved component, which as we discussed in Sec. III, is not consistent because in contrast, e.g. to perturbative quantum field theories, we never measure the *free component*. Furthermore, Eq. (105) has exactly the same form for the quintessence models; thus, in this framework it is not possible to discriminate between this class and modified gravity models without knowing the underlying model in detail.

The formulation in [108] uses a Lagrangian formalism with quadratic and higher order deviations from Einstein's theory of gravity. The energy-momentum tensor of dark energy $U_{\mu\nu}$ is obtained be using variational methods from this Lagrangian. It is a function of $g_{\mu\nu}$ or the set $\{g_{\mu\nu}, \varphi, \partial_\mu\varphi\}$ when the dark-(energy) sector also includes a scalar field. Then, they use 3 + 1 spacetime decomposition; thus, all coefficients of the above expansion depend only on time, and apply variational methods to determine perturbations $\delta U_{\mu\nu}$ around an arbitrary background. Their formulation is technically and theoretically interesting, especially for studying various modified gravity models, but there is neither a model-independent parametrization for dark energy nor for observables.

VI. OUTLINE

We have parametrized the interaction between dark energy and matter for the modified gravity and interacting quintessence models as modifications of the evolution of matter and radiation background and perturbations densities, and the equation of state of dark energy. We have shown that when the interaction is ignored in the data analysis, the effective value of the parameters are not the same if we calculate them from the Friedmann equation or from a function proportional to the derivative with respect to the redshift of the total mean energy density of the Universe. We have also defined a single quantity that evaluates the strength of the interaction. Its observational uncertainty can be used to estimate the discrimination of the power of a cosmological survey.

We have obtained a phenomenological description for the interaction current in the context of interacting quintessence models motivated by particle physics. Based on these results, we have suggested to distinguish between modified gravity and (interacting)-quintessence dark energy models of nongravitational origin by the way they modify the energy-momentum conservation equation. If the interaction current is proportional to the trace of the energy-momentum tensor of matter, we classify the model as *modified gravity*, otherwise, as *(interacting)-quintessence* and its variants, such as k-essence, quintom, cosmon, etc.

We have determined the modification of the evolution equation of density and velocity perturbations in the context of the modified gravity and interacting quintessence models discussed above, and used them to obtain a parametrized description of the evolution equation of the growth factor that can be used for both of these models as well as a simple ΛCDM model, which has been considered as the null hypothesis in our discussions. The difference between the value of these parameters can distinguish between the aforementioned models. We have also obtained order of magnitude estimations for uncertainties on these quantities measured with the Euclid mission. A better forecast for these uncertainties needs simulations of the survey and the data analysis which we have left to future works.

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APPENDIX A: PROPERTIES OF $A(z)$

One of the principle aims of LSS surveys is the measurement of the Hubble constant $H(z)$, angular diameter distance D_A, and luminosity distance D_L, mainly by measuring baryon acoustic oscillations which play the role of a reference distance scale [109]. The maximum effect of the BAO on the power spectrum is at redshift ~0.3 [109]. However, as we mentioned in the Introduction a direct determination of $\gamma(z)$ from the Hubble constant, D_A, or D_L when $z \to 0$ is not possible. In fact, using Eq. (1) and the definition of the angular diameter distance, it is easy to see that

$$\ln\left[\left(\frac{d}{dz}((1+z)D_A)\right)^{-1} - \Omega_m(1+z)^3 - \Omega_h(1+z)^4 - \Omega_K(1+z)^2\right] = \ln\Omega_{\rm de} + 3\gamma(z)\log(1+z). \quad \text{(A1)}$$

At small redshifts the last term on the right-hand side of (A1) which contains $\gamma(z)$ approaches zero, and the effect of the latter becomes negligibly small. Now, consider the following quantities:

$$H^2(z) = \frac{8\pi G}{3}\rho(z), \quad \text{(A2)}$$

$$B(z) \equiv \frac{1}{3(1+z)^2\rho_0}\frac{d\rho}{dz} = \frac{2H(z)}{3H_0^2(1+z)^2}\frac{dH}{dz} = \frac{2\mathcal{H}(z)}{3(1+z)\mathcal{H}_0}\left(\frac{(1+z)d\mathcal{H}}{dz} + \mathcal{H}\right), \quad \text{(A3)}$$

$$\begin{aligned} A(z) &\equiv B(z) - \Omega_m - \frac{4}{3}\Omega_h(1+z) - \frac{2\Omega_K}{3(1+z)} \\ &= \Omega_{\rm de}\left(\gamma + (1+z)\ln(1+z)\frac{d\gamma}{dz}\right)(1+z)^{3(\gamma-1)} \\ &= \Omega_{\rm de}(w(z)+1)(1+z)^{3(\gamma-1)}, \end{aligned} \quad \text{(A4)}$$

where $H(z) = \dot{a}/a$ is the expansion rate of the Universe and $\rho(z)$ is the total density at redshift z. It is clear that $A(z)$ is proportional to the deviation of dark energy from a cosmological constant at any redshift including $z = 0$. In addition, its sign determines whether dark energy has normal or phantomlike equation of state at a given redshift. It can be shown [54] that when $dw/dz \ll 3w(z)(w(z)+1)/(1+z)$, the sign of dA/dz is opposite to the sign of $w(z)+1$. This condition is satisfied at low redshifts—see examples of the models in Fig. 1. It means that $A(z)$ is a concave or convex function of the redshift, respectively, for positive or negative $w(z)+1$. Observations show that the contribution of Ω_k and Ω_h at low redshifts is much smaller than the uncertainty of Ω_m. The function dA/dz does not depend on Ω_m. Thus, the uncertainty on the value of Ω_m can shift the value of $A(z)$ but it does not change its slope and its shape, i.e. its concavity or convexity will be preserved.

The function $B(z)$ can be easily related to directly measurable quantities:

$$B(z) \equiv \frac{1}{3(1+z)^2\rho_0}\frac{d\rho}{dz} = \frac{\frac{2}{1+z}\left(\frac{dD_l}{dz} - \frac{D_l}{1+z}\right) - \frac{d^2D_l}{dz^2}}{\frac{3}{2}\left(\frac{dD_l}{dz} - \frac{D_l}{1+z}\right)^3}, \quad \text{(A5)}$$

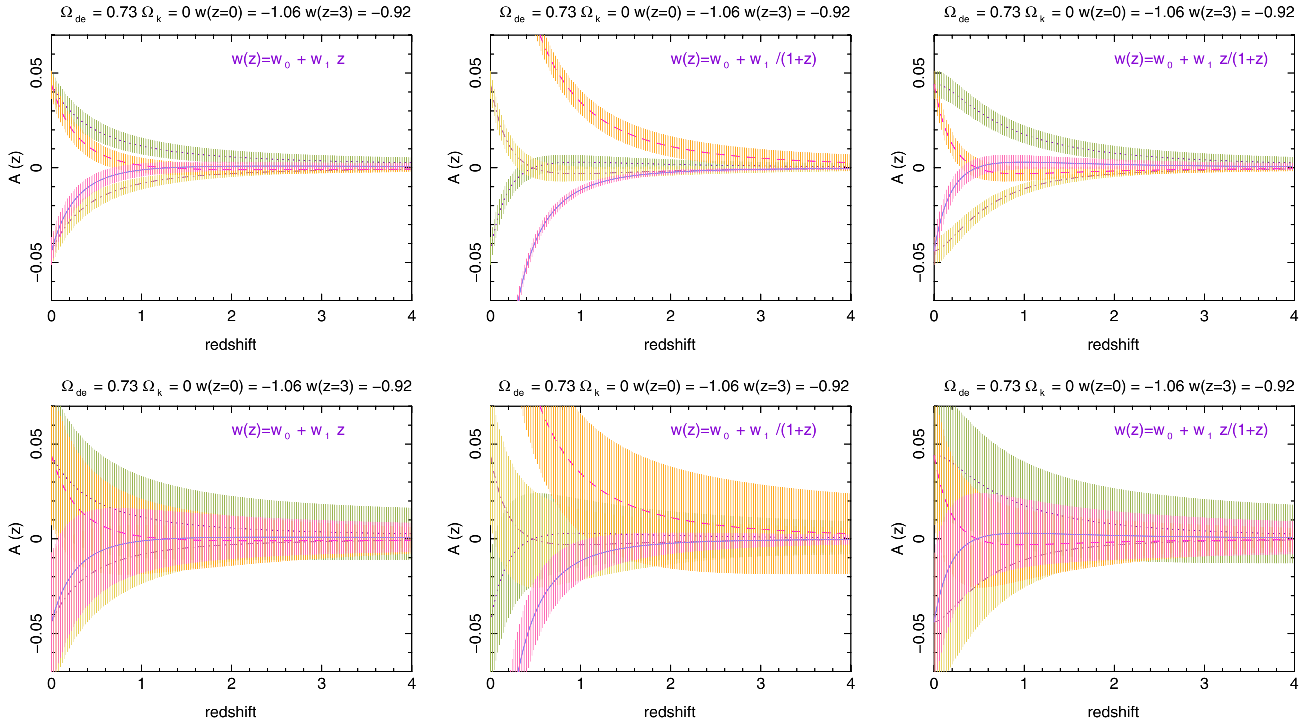

FIG. 1 (color online). $A(z)$ as a function of the redshift. To see how well $A(z)$ can distinguish between various models and how systematic and statistical errors as well as the parametrization affect the reconstructed model, we consider 3 parametrizations as written on the plot above. Note that parametrizations for the plot in the center and on the right are equivalent up to a redefinition of coefficients w_0 and w_1. We first consider a given value for $w(z)$ at $z = 0$ and $z = 3$, determine corresponding coefficients w_{0i} and w_{1i} where index i is for initial. Then to simulate systematic errors we plot the following models: $w_0 = -1 + |w_{0i} + 1|$, $w_1 = w_{1i}$ (dotted line), $w_0 = -1 - |w_{0i} + 1|$, $w_1 = -w_{1i}$ (dot-dash), $w_0 = -1 + |w_{0i} + 1|$, $w_1 = -w_{1i}$ (dashed) and $w_0 = w_{0i}$ and $w_1 = w_{1i}$ (full line). The shaded regions present statistical errors. The uncertainty of az is $1\sigma_{A(z=0)} = 0.01$ (top row) and $1\sigma_{A(z=0)} = 0.05$ (bottom row) at $z = 0$ and evolves with the redshift as $\sigma_A(z) = \sigma_A(z = 0)(1 + z)^2$. It seems to be possible to distinguish between normal and phantom dark energy models easily, if uncertainties are limited to few percents. Evidently, achieving such a precision is challenging even for space missions such as Euclid.

$$D_l = (1 + z)H_0 \int_0^z \frac{dz}{H(z)}, \tag{A6}$$

or equivalently with respect to normalized angular distance:

$$B(z) = \frac{-\left(2\frac{dD_A}{dz} + (1 + z)\frac{d^2 D_A}{dz^2}\right)}{\frac{2}{3}(1 + z)^2\left(D_A + (1 + z)\frac{dD_A}{dz}\right)^3} \tag{A8}$$

$$D_A = \frac{H_0}{1 + z}\int_0^z \frac{dz}{H(z)} = \frac{D_l}{(1 + z)^2}. \tag{A9}$$

Note that these equations are written for a flat universe, but can be easily extended to the cases where $\Omega_k \neq 0$.

APPENDIX B: FISHER MATRIX FOR EQUATION OF STATE OF DARK ENERGY

The Fisher matrix evaluates the sensitivity—information content—of a measured quantity to variables and parameters that define the underlying model [110]. Under special conditions, e.g. Gaussianity of distributions, the Fisher matrix can be related to the covariance matrix of measurements. In the LSS surveys the main measured quantity is the power spectrum of matter density anisotropies. Application of the Fisher matrix to CMB [111] and galaxy surveys [91,92] is well studied and widely used. In what concerns the measurement of dark energy density, its variation, and its equation of state from galaxy surveys, one has to extract $H(z)$ and $D_A(z)$ either from BAO [29,112,113] or by fitting the complete power spectrum [72]. The Fisher matrix for the two-dimensional power spectrum is determined by Seo and Eisenstein [93–95] with $H(z)$ and $D_A(z)$ as parameters. A transformation from these quantities to coefficients of a parametrized equation of state, for instance $w(z) = w_0 + w_a z/(1 + z)$ allow to determine the covariant matrix for the measurement of w_0 and w_a [72].

Although *a priori* the value of these quantities can be determined at any redshift, in practice the limited volume and deepness of the surveys allow to determine the power spectrum at the average redshift of the survey or for some bins of redshift in the case of large deep surveys. In the latter case, the estimation of $w(z)$ as a function of redshift depends strongly on its parametrization. Figure 1 shows the plot of $A(z)$ for examples in which w is measured at two redshifts. It is evident that this quantity and thereby the underlying dark energy models depend strongly on the parametrization of w, notably when systematic and statistical errors are added.

Simpson and Peacock [114] use $\{w_0, w_a, \Omega_\Lambda, \Omega_k, \Omega_m h^2, \Omega_b h^2, n_s, A_s, \beta, \gamma', \sigma_p\}$ as independent parameters for estimating cosmological parameters from the measurement of the galaxy power spectrum. Here $w_a \equiv -dw/dz$, $\beta(z) \equiv f(z)/b(z)$ where $\mathbf{f}(z)$ is the growth rate of the scalar fluctuations and $b(z)$ is the linear bias, and γ' is the parameter that defines an approximate parametrization for $\mathbf{f}(z) \approx \Omega_m^{\gamma'}(z)$ for ΛCDM [105]. It can be also shown that in what concerns the determination of the Fisher matrix for dark energy, $w(z)$ and dw/dz alone lead to a singularity.[16]

In place of parametrizing $w(z)$, we suggest using $w(z)$, $\gamma(z)$ and z to determine the Fisher matrix for dark energy parameters. It can be easily shown that the Fisher matrix becomes singular if the first two quantities are considered [72], because $w(z)$ and $\gamma(z)$ are not independent—if one knows $w(z)$, then $\gamma(z)$ can be determined from (2). This problem does not arise when w is parametrized because the expansion parameters are explicitly independent. The relationship of $w(z)$ and $\gamma(z)$ is very similar to the relation between $H(z)$ and $D_A(z)$. The Fisher matrix for the $\{H(z), D_A(z), z\}$ set of parameters is calculated in [72]. Using this formulation, a parameter transformation gives the Fisher matrix for $\{w(z), \gamma(z), z\}$. The relation between Fisher matrices with 2 sets of parameters p_i and q_m is [111]

$$\bar{F}_{ij} = \sum_{mn} \frac{\partial q_m}{\partial p_i} F_{mn} \frac{\partial q_n}{\partial p_j}. \tag{B1}$$

For the parameter sets discussed above, the components of the Jacobian matrix are

$$\frac{\partial H(z)}{\partial w(z)} = \frac{3H_0^2 \Omega_{\rm de}}{2H(z)} (1+z)^{3\gamma(z)}, \tag{B2}$$

$$\frac{\partial H(z)}{\partial \gamma(z)} = \frac{3H_0^2 \Omega_{\rm de}}{2H(z)} (1+z)^{3\gamma(z)} \ln(1+z), \tag{B3}$$

$$\frac{\partial D_A(z)}{\partial w(z)} = -\frac{1}{H^2(z)} \frac{\partial H(z)}{\partial w(z)}, \tag{B4}$$

$$\frac{\partial D_A(z)}{\partial \gamma(z)} = -\frac{1}{H^2(z)} \frac{\partial H(z)}{\partial \gamma(z)}, \tag{B5}$$

$$\frac{\partial H(z)}{\partial z} = \frac{H_0^2}{2H(z)}\left(3\Omega_m(1+z)^2 + 2\Omega_k(1+z) + \frac{1+w(z)}{1+z}\right), \tag{B6}$$

$$\frac{\partial D_A(z)}{\partial z} = -\frac{1}{1+z}\left(D_A(z) + \frac{1}{H(z)}\right). \tag{B7}$$

Alternatively, one of the $w(z)$ or $\gamma(z)$ parameters can be replaced by $A(z) = \Omega_{\rm de}(w(z)+1)(1+z)^{3(\gamma-1)}$. In fact, it is preferable to replace $\gamma(z)$ with $A(z)$, because at low redshifts the $\gamma(z)$-dependent term has a very small effect on the evolution $H(z)$ and D_A. By contrast, the deviation of $A(z)$ from its value in the ΛCDM model is maximized for $z \to 0$, see Fig. 1.

APPENDIX C: FLUID DESCRIPTION OF A SCALAR FIELD

The energy-momentum tensor of a scalar field is

$$T_\varphi^{\mu\nu} = -\frac{1}{2} g^{\mu\nu} g^{\rho\sigma} \partial_\rho \varphi \partial_\sigma \varphi + g^{\mu\nu} V(\varphi) + \partial^\mu \varphi \partial^\nu \varphi. \tag{C1}$$

Using definition (33) of a perfect fluid, the density and pressure are defined as

$$\rho_\varphi \equiv u_\mu u_\nu T_\varphi^{\mu\nu} = \frac{1}{2}\partial^\mu \varphi \partial_\mu \varphi + V(\varphi),$$
$$P_\varphi \equiv \frac{1}{2}\partial^\mu \varphi \partial_\mu \varphi - V(\varphi). \tag{C2}$$

u^μ is the velocity vector and $u^\mu u_\mu = 1$. It is easy to verify that with the above definitions for ρ_φ and P_φ:

$$u_\mu = \frac{\partial_\mu \varphi}{(\rho_\varphi + P_\varphi)^{(1/2)}}. \tag{C3}$$

APPENDIX D: SOLUTION OF EVOLUTION EQUATION OF GROWTH RATE

For the ΛCDM cosmology, $E_i = 0, i = 0, \ldots, 4$. We also consider $w_m = C_{\rm sm}^2 = 0$. In this case after dividing Eq. (92) by $\mathcal{H}^2$, the evolution equation of the growth rate becomes

$$\frac{\mathbf{f}'}{\mathcal{H}} + \mathbf{f}\left(\frac{\mathcal{H}'}{\mathcal{H}^2} + 1\right) + \mathbf{f}^2 + \frac{3}{2}\Omega_m = 0. \tag{D1}$$

After changing the variable from η to $\ln a$, this equation changes to

$$\frac{d\mathbf{f}}{dx} + \left(\frac{x''}{x'^2} + 1\right)\mathbf{f} + \mathbf{f}^2 + \frac{3}{2}\Omega_m = 0, \qquad x \equiv \ln\frac{a(\eta)}{a_0(\eta)}. \tag{D2}$$

[16] For the sake of simplicity in the discussion of the Fisher matrix here, we neglect other cosmological parameters, i.e. we assume that dark energy parameters can be factorized from other quantities. In practice, one has to consider a single matrix Fisher matrix containing all parameters. Thus, there would be one single covariant matrix that includes correlation of all uncertainties.

By integrating the Friedmann equation for flat ΛCDM one obtains:

$$\mathcal{H} = \frac{d\ln(\frac{a}{a_0})}{d\eta} = x' = \frac{\mathcal{H}_0 a}{a_0}\sqrt{\Omega_m(a_0)\left(\frac{a_0^3}{a^3}\right) + \Omega_\Lambda}, \tag{D3}$$

$$\mathcal{E} \equiv \frac{\mathcal{H}'}{\mathcal{H}^2} = \frac{x''}{x'^2} = \frac{\Omega_\Lambda - \Omega_m(a_0)\frac{a_0^3}{a^3}}{\Omega_\Lambda + \Omega_m(a_0)\frac{a_0^3}{a^3}} = \frac{\Omega_\Lambda - \Omega_m(a_0)e^{-3x}}{\Omega_\Lambda + \Omega_m(a_0)e^{-3x}}. \tag{D4}$$

For $z = 0$, $\mathcal{E} = -1$ and for $z \to \infty$, $\mathcal{E} = \Omega_\Lambda(a) - \Omega_m(a)$. To be able to solve (D2) analytically we must assume $\mathcal{E}$ is a constant. This is a good approximation if we are interested in only a small range of redshifts. Under this assumption, the solution of (D2) can be obtained by integration:

$$\mathbf{f}_{\Lambda\mathrm{CDM}}(z) \approx \frac{-(\mathcal{E} + 1 - \frac{\alpha_1}{2}) + (\mathcal{E} + 1 + \frac{\alpha_1}{2})(1 + z)^{\alpha_1}}{1 - (1 + z)^{\alpha_1}}, \quad \alpha_1 = \sqrt{(\mathcal{E} + 1)^2 - 6\Omega_m}. \tag{D5}$$

For $-\sqrt{6\Omega_m} - 1 < \mathcal{E} < \sqrt{6\Omega_m} - 1$, α_1 is imaginary and according to this approximation solution $\mathbf{f}(z)$ has an oscillating component. A simple attempt to make (D5) more precise is to take into account that $\mathcal{E}$ depends on the redshift.

To obtain an approximate solution for interacting dark energy models parametrized by coefficients $E_i = 0$, $i = 0, \ldots, 4$ in (92), under the assumption that these corrections are small, we can linearize this equation around $\mathbf{f}_{\Lambda\mathrm{CDM}}$. Note that in general it is expected that in interacting dark energy models w_m and C^2_{sm} are not zero. Therefore, we add also their contribution to the linearized model:

$$\mathbf{f} = \mathbf{f}_{\Lambda\mathrm{CDM}} + \Delta\mathbf{f}, \tag{D6}$$

$$\Delta\mathbf{f}' + \left[\frac{\mathcal{H}'}{\mathcal{H}} + E_0 + \mathcal{H}(1 + 3(C^2_{\mathrm{sm}} - w_m) + 2\mathbf{f}_{\Lambda\mathrm{CDM}})\right]\Delta\mathbf{f} + 3(C^2_{\mathrm{sm}} - w_m)\frac{\mathcal{H}'}{\mathcal{H}} + 3\mathcal{H}\left(C^2_{\mathrm{sm}} - w_m + \frac{\Omega_m}{2}w_m(2 + w_m)\right) + (C^2_{\mathrm{sm}} + E_1)\frac{k^2}{\mathcal{H}} + E_2 + E_3\mathcal{H} + \frac{E_4}{\mathcal{H}} = 0. \tag{D7}$$

The solution of this linearized equation is straightforward and can be formally written as follows:

$$\Delta\mathbf{f}(z) = \frac{\mathcal{H}}{(1 + z)(1 + 3(C^2_{\mathrm{sm}} - w_m))}\exp\left[\int\frac{dz}{1 + z}\left(\frac{E_0}{\mathcal{H}} + 2\mathbf{f}_{\Lambda\mathrm{CDM}}\right)\right]\left\{1 + \int dz\frac{(1 + z)(1 + 3(C^2_{\mathrm{sm}} - w_m))}{(1 + z)\mathcal{H}^2} \times \exp\left[-\int\frac{dz}{1 + z}\left(\frac{E_0}{\mathcal{H}} + 2\mathbf{f}_{\Lambda\mathrm{CDM}}\right)\right]\left[3(C^2_{\mathrm{sm}} - w_m)\frac{\mathcal{H}'}{\mathcal{H}} + 3\mathcal{H}((C^2_{\mathrm{sm}} - w_m) + \frac{\Omega_m}{2}w_m(2 + w_m)) + \frac{k^2}{\mathcal{H}}(C^2_{\mathrm{sm}} + E_1) + E_2 + E_3\mathcal{H} + \frac{E_4}{\mathcal{H}}\right]\right\}. \tag{D8}$$

The determination of the integrals in (D8) requires the details of the redshift dependence of the coefficients E_i's which are model dependent. Nonetheless, they depend on the scalar field which must vary very slowly with the redshift. Therefore, at zero order, they can be considered as constant. Although even with this simplification it is difficult to determine (D8) analytically, a numerical determination allows to write it as an expansion with respect to the E_i coefficient. This expansion would be suitable for the compression with data and the determination of E_i.

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