Searching for new heavy chiral quark pairs via their annihilation to multiple vector bosons

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Drawing the analogy of replacing the nucleon by heavy chiral quark Q, the pion by Goldstone boson G, and πNN coupling by GQQ coupling, we construct a statistical model for $Q\bar{Q} \to nG$ annihilation, i.e., into n longitudinal weak bosons. This analogy is becoming prescient since the LHC direct bound $m_Q > 611$ GeV implies strong Yukawa coupling. Taking $m_Q \in (1, 2)$ TeV, the mean number $\langle n_G \rangle$ ranges from 6 to over 10, with negligible two or three boson production. With individual t' or b' decays suppressed either by phase space or quark mixing, and given the strong Yukawa coupling, $Q\bar{Q} \to nV_L$ is the likely outcome for very heavy $Q\bar{Q}$ production at the LHC.

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I. INTRODUCTION

Despite the hint for a light Higgs boson at 125 GeV [1], there has been keen interest in the search of new heavy chiral quarks at the Large Hadron Collider (LHC), resulting in the stringent limit of $m_Q > 611$ GeV/c^2 [2]. This is already above the perturbative, tree-level partial wave unitarity bound (UB) that is nominally around 550 GeV/c^2 [3]. Thus, if such heavy quarks exist, their Yukawa couplings would already be in the strong coupling regime. With TeV scale heavy quark masses, the actual UB violation (UBV) in the high energy limit for $Q\bar{Q}$ scattering may be out of reach. Instead, the question to ask is: *Should the current search strategy for ultraheavy quark Q at the LHC be modified?* In this article we draw on the analogy of the proton to argue that $Q\bar{Q} \to nG$ ($G \equiv V_L$ is the longitudinal component of the vector boson) may be the new signature at the LHC.

The πNN coupling $g^2_{\pi NN}/4\pi \simeq 14$ [4] gives $g_{\pi NN} \simeq 13$, which is very large and quite close to the πNN "Yukawa coupling," $\lambda_{\pi NN} \equiv \sqrt{2} m_N/f_\pi \simeq 14$. Although $\lambda_Q \equiv \sqrt{2} m_Q/v \gtrsim 3.5$ ($v \cong 246$ GeV is the electroweak (EW) symmetry breaking scale) from the current m_Q bound is not yet as large, drawing analogy with $p\bar{p}$ annihilation, we expect that $Q\bar{Q} \to nG$ may be the dominant process for $m_Q \in (1, 2)$ GeV.

II. PHENOMENOLOGY OF $p\bar{p} \to n\pi$

Let us briefly review the observed phenomena regarding $p\bar{p}$ annihilation, which is well known [5,6] to go mainly via a "fireball" into n pions. The salient features of the annihilation "fireball" are (see Fig. 1):

(i) Size of order $1/m_\pi$;
(ii) Temperature $T \simeq 120$ MeV;
(iii) Average number of emitted pions $\langle n_\pi \rangle \simeq 5$;
(iv) A soft-pion p_π^2/E_π^2 factor modulates the Maxwell–Boltzman distribution for the pions.

It is worthwhile to elucidate these features a little further. The size $1/m_\pi$ means that the $p\bar{p}$ annihilation system, destined to shed the p and $\bar{p}$ content, extends over a region $\sim 1/m_\pi$. The system seems to thermalize to a temperature of order 120 MeV, hence "loses memory" of its origins, and the emitted pions carry momenta that satisfy a thermal distribution. This rapid thermalization probably takes place due to the rather large πNN (as well as $\pi\pi$) coupling, while the p_π^2/E_π^2 suppression [7] (satisfied rather well by data; see Fig. 12 of Ref. [5]) for low pion momentum from the thermal distribution reflects the Goldstone nature of pion couplings. That is, the π as Goldstone boson couples derivatively, hence cannot get emitted at zero momentum. This seems to explain the enhancement factor of 1.3 for the mean kinetic energy $\langle K_\pi \rangle \equiv \langle E_\pi \rangle - m_\pi$ beyond equipartition expectation of $\frac{3}{2}T$. Thus, the relatively high $\langle E_\pi \rangle \sim 370$ MeV gives rise to $\langle n_\pi \rangle \simeq 5.1$, as compared to the maximal allowed number of pions, $2m_N/m_\pi \simeq 13.4$.

At a more refined level, it is found that $\langle n_{\pi^\pm} \rangle \simeq 3.1$ and $\langle n_{\pi^0} \rangle \simeq 2.1$, with $2\langle n_{\pi^0} \rangle / \langle n_{\pi^\pm} \rangle > 1$, i.e., more neutral pions are emitted than charged ones. Furthermore, the pion multiplicity distribution appears Gaussian,

$$P(n_\pi) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(n_\pi - \langle n_\pi \rangle)^2/2\sigma^2}, \tag{1}$$

with $\sigma \sim 1$. More specifically [8],

$$\sigma \simeq \frac{1}{2}\sqrt{\langle n_\pi \rangle}, \tag{2}$$

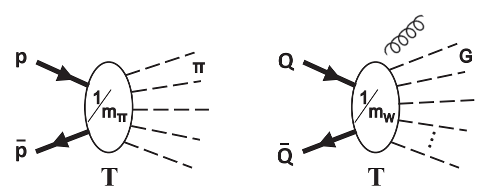

FIG. 1. Illustration for $p\bar{p} \to n\pi$ for $n = \langle n_\pi \rangle \cong 5$, and the analog of $Q\bar{Q} \to nG$, where $G = V_L$ is the Goldstone boson of EW symmetry breaking. Depending on $m_Q = 1$–2 GeV, $\langle n_G \rangle$ could go from 6 to 12 [Eq. (6)]. The gluon line is to indicate the relatively soft shedding of color.

TABLE I. Sample multiplicity distributions: $P_{p\bar{p}}(n)$ is the observed distribution for $p\bar{p} \to n\pi$ [5], while $P_{Q\bar{Q}_{1(2)}}(n)$ is the $Q\bar{Q} \to nG$ distribution for $m_Q = 1(2)$ TeV according to Eq. (6), where $G \equiv V_L$ is the EW Goldstone boson.

$P(n)\backslash n$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$P_{p\bar{p}}$	0.4%	8%	18%	46%	22%	6%	0.3%	⋯									
$P_{Q\bar{Q}_1}$	0.1%	1%	6%	19%	31%	27%	12%	3%	0.4%								
$P_{Q\bar{Q}_2}$					⋯	0.2%	0.9%	3%	8%	16%	22%	22%	16%	8%	3%	0.9%	0.2%

is argued from statistical models [9]. Thus, $\sigma \simeq 1.13$ gives a good fit to data [5], which is given in Table I. Note the rather small $p\bar{p} \to \pi\pi$ 2-body fraction, and the cutoff of pion multiplicity above 8.

This successful "statistical model" which accounts for gross features of $p\bar{p} \to n\pi$ annihilation goes back to Fermi [10], who considered a system of noninteracting pions. It has been refined through the years, and the strong interactions of the pions do play a role. One final aspect is a focusing of incoming waves by attractive potential that leads to strong absorption in a smaller region than originally suggested.

III. $Q\bar{Q} \to nV_L$ ANALOG

We mean by Q a left-handed chiral doublet (with corresponding right-handed weak singlets) that is degenerate in mass, thereby possessing a *heavy isospin* symmetry I_Q, much like the nucleon N. This is nothing but the 4th generation [11]. To draw true analogy with the πNN case, the GQQ Yukawa coupling λ_Q should be of order 13–14, i.e., $m_Q \gtrsim 2$ TeV. However, we will assume that analogous phenomena already appear for 1 TeV, hence we will consider $m_Q \in (1, 2)$ TeV.

With the Higgs mechanism already established, the Goldstone boson $G \equiv V_L$ carries a length $1/M_W$ [12], which defines the *size* of the $Q\bar{Q}$ annihilation fireball. Besides the dynamical mechanism for m_p and m_Q generation, comparing $m_\pi \propto m_u + m_d$ with $M_W \propto g$, where g is the weak gauge coupling, the size of the fireball is in part determined by unrelated, "random" parameters.

The fireball temperature $T \equiv T_{Q\bar{Q}\to nG}$ is harder to assess. Noting that $T_{p\bar{p}\to n\pi} \sim 120\,\text{MeV} < T_c^{\text{QCD}} \sim 170\,\text{MeV}$, likely $T < T_c^{\text{EW}}$, where T_c^{EW} is the EW transition temperature. By this analogy, however, one notes that T_c^{QCD} arises from the detailed underlying theory for hadron phenomena (which includes $p\bar{p} \to n\pi$), QCD. Even though we believe EW symmetry breaking probably [13] arises from strong Yukawa coupling, λ_Q, we do not yet have an underlying theory for λ_Q itself. Thus, we do not have a good handle on T, except that it is in the 100 GeV scale, of order v. We shall therefore take as nominal

$$T \sim \frac{2}{3} v \sim 160 \text{ GeV}, \tag{3}$$

which can be interpreted as either $1.3 \times \frac{1}{2} v$ (here 1.3 corresponds to $T_{p\bar{p}\to n\pi}/f_\pi$), or $v \times T_{p\bar{p}\to n\pi}/T_c^{\text{QCD}}$. The latter would give 170 GeV, which is not so different from Eq. (3). We stress, however, that the fireball temperature could be 1.5, even twice as high, and should be determined eventually by experiment. The Goldstone p_G^2/E_G^2 factor should still modulate the thermal p_G distribution. But because of the smallness of M_W^2 compared with $4m_Q^2$, the modulation is considerably milder than the $p\bar{p} \to n\pi$ case, so $\langle K_G \rangle$ should be closer to $\frac{3}{2} T$.

Assuming Eq. (3) but without applying the 1.3 enhancement factor over equipartition (as is the case for $p\bar{p} \to n\pi$), we take $\langle K_G \rangle \sim \frac{3}{2} T \sim 240$ GeV, hence $\langle E_G \rangle \sim 320$ GeV, or

$$\langle |p_G| \rangle \sim 310 \text{ GeV}, \tag{4}$$

with $\gamma_G \sim 4$. For $m_Q = 1(2)$ TeV, or $2m_Q = 2(4)$ TeV, this corresponds to

$$\langle n_G \rangle \sim 6.25(12.5), \tag{5}$$

where we artificially keep three digits of significance for generating a "realistic" multiplicity distribution. Assuming Eqs. (1) and (2), we have $\sigma \simeq \sqrt{\langle n_G \rangle}/2 \sim 1.25(1.77)$, and the multiplicity distribution is

$$P(n_G) \simeq 0.319 e^{-((n_G - 6.25)^2/3.13)} (0.226 e^{-((n_G - 12.5)^2/6.25)}), \tag{6}$$

for $m_Q = 1(2)$ TeV. We note that a higher fireball temperature T would result in lower $\langle n_G \rangle$, higher $\langle |p_G| \rangle$ and a narrower distribution (controlled by σ).

We illustrate the $Q\bar{Q} \to nG$ process in Fig. 1 (gluon emission discussed later), and tabulate the multiplicity distributions in Table I. For $m_Q = 1$ TeV, about 90% of $Q\bar{Q}$ annihilations go into 5–8 prongs of $V_L \equiv G$. Several V_Ls should be considerably above 300 GeV momentum, while 4-prong events (at 6%) are in general composed of V_Ls with momentum $\sim$500 GeV. Therefore, W-tagged "fat" jets, j_W, should become a useful tool for identifying these multi-V_L events. For $m_Q = 2$ TeV, again over 90% of $Q\bar{Q}$ annihilations go into 10–15 prong V_Ls, which is a rather large number. For 9–12 prong events (at $\sim$50%), a significant number of V_Ls would have momentum above 400 GeV, while for higher multiplicity, many should still carry momentum higher than the mean, Eq. (4). These high multiplicity nV_L events would be possible hallmarks for heavy $Q\bar{Q}$ production.

IV. PRODUCTION AND COMPETING MODES

If our analogy with $p\bar{p}$ annihilation is already realized for $m_Q = 1$ TeV, then even at 8 TeV running of LHC, where of order 15 fb^{-1} data is expected in 2012, one could already get a hint. The cross section is of order a couple fb, so one might observe some number of 4 or more W-tagged jet (j_W) events, with additional jet multiplicity that are less well W-tagged. The competing modes would be regular $Q\bar{Q}$ production, followed by "free quark decay," e.g. (assuming $m_{b'} > m_{t'}$) $b'\bar{b}' \to t\bar{t}W^+W^- \to b\bar{b}WWWW$, or $t'\bar{t}' \to b\bar{b}W^+W^-$ [14]; we shall assume Cabibbo-Kobayashi-Maskawa quark-mixing matrix (CKM) hierarchy for simplicity. We see that the W-jet multiplicity is lower, associated with isolated high p_T b-jets, and practically no Z-jets [15] or $Z \to \ell^+\ell^-$. In contrast, $Q\bar{Q} \to nV_L$ does not have isolated b-jets (b-jets would come in pairs at lower fraction, to form a j_Z from $Z \to b\bar{b}$), W-jets multiplicity is higher, and tend to have Z-jets [16]. We expect the $Q\bar{Q} \to nV_L$ fireball process would dominate over the $Q\bar{Q} \to b\bar{b}WW(WW)$ free quark decay process, as we would argue shortly.

There are arguments that, if the heavy chiral quarks Q themselves are responsible [13] for EW symmetry breaking, then $m_Q > 1$ TeV is likely [17]. Our earlier analogy with the πNN "Yukawa" coupling suggests $m_Q \sim 2$ TeV. If so, the prospect for the 2012 LHC run at 8 TeV is not good, and one would have to wait for the 13–14 TeV run, expected by late 2014. Running the HATHOR code [18] for $Q\bar{Q}$ production, we estimate the 14 TeV cross sections to be of order 50–60 fb for $m_Q = 1$ TeV, dropping to ~3 fb for $m_Q = 1.5$ TeV, and 0.2–0.3 fb for $m_Q = 2$ TeV. From $2m_Q = 2$ to 4 TeV, one quickly runs out of parton luminosity. Note that $q\bar{q} \to Q\bar{Q}$ production dominates over $gg \to Q\bar{Q}$ production, as the valence quark supplies the needed large parton momentum fraction.

From the cross section and expected LHC luminosities, for $m_Q \lesssim 1.5$ TeV, again we do not foresee a problem for discovery. Note that, assuming I_Q symmetry, i.e., near degeneracy of t' and b', then

(i) $t'(b') \to b'(t') + W^*$ decay: Suppressed by both phase space and small Goldstone momentum;

(ii) $t'(b') \to b(t) + W$: Suppressed by CKM element $|V_{t'b}|$ ($|V_{tb'}|$). With no sign of new physics in $B_s \to J/\psi\phi$, $B_s \to \mu^+\mu^-$, and $B_d \to K^{*0}\mu^+\mu^-$, one expects [19] such CKM elements to be less than 0.1.

In contrast, once $q\bar{q} \to Q\bar{Q}$ pulls the heavy quark pair out of the vacuum, the $Q\bar{Q}$ pair "sees" a cross section of order $1/M_W^2$, which is at the μb level. With $q\bar{q} \to Q\bar{Q}$ production, there is Yukawa attraction [20] between $Q\bar{Q}$ that mimics the focusing attraction for $p\bar{p} \to n\pi$. Thus, there is good likelihood that $Q\bar{Q} \to nG$, i.e., nV_L, would dominate over free quark decay.

We comment that the produced $Q\bar{Q}$ is likely in a color-octet state, hence in general it would need to shed color. However, gluons have no way to sense the $T \sim 160$ GeV (or higher) of the fireball, which is of EW nature. Instead, the heaviness of Q means gluon radiation is $1/m_Q$ suppressed (heavy quark symmetry). We illustrate gluon radiation in Fig. 1, but expect the associated gluon-jet to be soft and does not provide a discriminant. Since the fireball is viewed as a nonperturbative process, it is hard to assess how this gluon is actually radiated.

V. DISCUSSION AND CONCLUSION

A natural question is whether the annihilation gets modified by heavy Q motion. Here, $p\bar{p}$ annihilation data again provide a guide. Figure 1 of Ref. [6] illustrates, for example, that for $p_{\bar{p}} < m_p/4$ (lab frame), the annihilation cross section σ_{ANN} predominates the total cross section σ_{TOT}. In fact, σ_{ANN} always dominate over the elastic cross section σ_{EL} even for $p_{\bar{p}}$ greater than several times m_p. Only for $p_{\bar{p}} \gtrsim 3m_p$ does the inelastic σ_{PROD} become significant, dominating beyond $5m_p$ or so. For our purpose, we expect annihilation, in the way we discussed, to be dominant before the motion turns rather relativistic. If one really has a collider of much higher energy, then $Q\bar{Q}$ scattering becomes an issue related to UBV. If the analog to $\sigma_{\mathrm{EL}}^{\bar{p}p}$ implies subsequent free Q decay, then one might still observe free quark decay. The question of how Q decays in large Yukawa coupling limit needs to be investigated nonperturbatively.

In case $m_Q \gtrsim 1.5$ TeV, one quickly runs out of parton luminosities (higher energy would be preferred), hence one would need high luminosity running of LHC at 14 TeV. However, the situation need not be so pessimistic: the very large Yukawa coupling suggests the existence of bound states below $2m_Q$. For example, as discussed in Ref. [20], there is likely an isosinglet, color-octet ω_8 resonance that can be produced via $q\bar{q} \to \omega_8$. How ω_8 decays would depend on more details of the $Q\bar{Q}$ bound state spectrum and properties. The beauty of our analogy with $p\bar{p} \to n\pi$ annihilation is precisely the thermal nature of this fireball process [5,6], with little "remembrance," either of the initial $p\bar{p}$ state, or detailed resonances in the hadron spectrum. Thus, we make no assertion on ω_8 decay properties here, except that it offers hope for an enhanced production cross section.

If the decay of the ω_8 is analogous to the fireball picture, then by $m_{\omega_8} < 2m_Q$ and the resonance production nature, there is good hope for earlier discovery. If ω_8 decays through similar chains as discussed in Ref. [20], then it might lead to the discovery of several resonances. The study of Ref. [20] was done with 500 GeV $< m_Q <$ 700 GeV in mind, to avoid issues of bound state collapse [13]. But since this region is now close to being ruled out, a numerical update, in particular also on obtaining the spectrum, is certainly called for. This would require nonperturbative solutions for strong Yukawa coupling.

An offshoot study of Ref. [20] provides an interesting contrast. If free quark decay is suppressed by very small

$V_{t'b}$, and some kinematic selections are operative, it is argued that $\omega_8 \to \pi_8 + W$ (π_8 is some isotriplet, color-octet Yukawa-bound "meson"), followed by $\pi_8 \to W_T + g$, where W_T is transverse, with the upshot of $\omega_8 \to WWg$. This is an exception to our fireball discussion, in that (i) it is effectively 2-body in vector bosons (WW); (ii) the gluon is energetic. If reconstructed [21], one could discover *two* resonances. These signatures arise from special conditions that are unlikely to hold in general.

Multiple weak boson production has been considered below $Q\bar{Q}$ threshold [22]. There is a rise of high multiplicities as one starts to approach threshold of high m_Q. But that would be out of the range of validity for the $gg \to nG$ amplitude via virtual Q loop considered. We remark that our multi-V_L signature is in principle quite distinct from micro-blackhole production [23]. Micro-blackholes in essence emit all types of particles democratically. In contrast, our fireball is heated in the EW sense, and by far prefers emitting the strongly coupled weak Goldstone bosons $V_L \equiv G$. However, since searches so far are based only on the simplified signature of high jet multiplicities, a refined search is needed to separate micro-blackholes from $Q\bar{Q}$ fireballs.

A final remark is in regard to V_LV_L scattering. If a very heavy chiral quark doublet Q exists above the TeV scale, it would not be easily compatible with a light Higgs because of the very large quadratic corrections to the light Higgs mass. A corollary of our argument would then suggest that the traditional $V_LV_L \to V_LV_L$ scattering study for heavy Higgs case may be the wrong place to search for new physics enhancement. Instead, one should again watch out for V_LV_L scattering to high(er) multiplicity of V_Ls [24]. In general, one should treat the UBV in V_LV_L and $Q\bar{Q}$ scattering as one single problem.

In conclusion, the ever-increasing mass bound on heavy sequential chiral quark Q, and the associated question of whether there might be a need to change the search strategy, prompt us to draw analogy with the observed $p\bar{p} \to n\pi$ "fireball" annihilation. We suggest that, in the range of $m_Q \in (1, 2)$ TeV, $Q\bar{Q}$ might annihilate via an analogous fireball into n Goldstone, or longitudinal vector bosons, with mean multiplicity $\langle n_G \rangle$ ranging from ~6 to over 10. The mean multiplicity $\langle n_G \rangle$ is correlated with the fireball "temperature" T, which are the key parameters for experiment to measure. This process would at least dilute the usual free quark decay picture for on-shell $Q\bar{Q}$ production. Strong Yukawa bound states below $2m_Q$ should further aid the discovery of new heavy chiral quarks, and the prospect appears optimistic beyond the UB.

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Note added in proof.—On July 4th, 2012, the ATLAS and CMS experiments announced the observation of a Higgs-like particle. If proven to be the Higgs boson, it puts strongly coupled new chiral quarks in doubt. But if a light Higgs boson and heavy chiral quarks can be reconciled, then the Higgs boson itself can also be emitted in the fireball. However, the Higgs-like particle could turn out to be, for example, a dilaton [17], with weaker couplings in general than the Higgs boson. If this is the case, then this new particle emission from the fireball would be insignificant.

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efficiency and purity, j_Z might be selectable. This would enrich the rare $Z \rightarrow e^+e^-$ and $Z \rightarrow \mu^+\mu^-$. On a separate note, if all j_W are hadronically selected, in principle a recoiling nonobserved "missing-Z" (considering the 20% fraction of $Z \rightarrow \nu\bar{\nu}$) could perhaps be pursued.

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