Z_4 flavor model in Randall-Sundrum model 1

Carlos Alvarado,[1,*] Alfredo Aranda,[1,2,†] Olindo Corradini,[3,‡] Alma D. Rojas,[1,§] and Eli Santos-Rodriguez[4,‖]

[1]*Facultad de Ciencias, CUICBAS, Universidad de Colima, Colima, Mexico*
[2]*Dual C-P Institute of High Energy Physics, Mexico*
[3]*Centro de Estudios en Física y Matemáticas Básicas y Aplicadas Universidad Autónoma de Chiapas,*
Tuxtla Gutiérrez, Chiapas, Mexico
[4]*International Centre for Theoretical Physics, Meso-American Institute for Science Universidad Autónoma de Chiapas,*
Tuxtla Gutiérrez, Chiapas, Mexico

Randall-Sundrum models provide a possible explanation of (gauge-gravity) hierarchy, whereas discrete symmetry flavor groups yield a possible description of the texture of Standard Model (SM) fermion masses. We use both these ingredients to propose a five-dimensional extension of the Standard Model where the mass hierarchy of the four-dimensional effective field theory is obtained only using localization parameters of order 1. We consider a bulk custodial gauge symmetry group together with an Abelian Z_4 group: the model turns out to yield a rather minimal extension of the SM as it only requires two brane Higgs fields to provide the desired Yukawa interactions and the required spontaneous symmetry breaking pattern. In fact, the presence of an extra dimension allows the use of the Scherk-Schwarz mechanism to contribute to the breaking of the bulk custodial group down to the SM gauge symmetry. Moreover, no right-handed neutrinos are present and neutrino masses are generated radiatively with the help of a bulk charged scalar field that provides the lepton number violation. Using experimental inputs from the Global Neutrino Analysis and recent Daya Bay results, a numerical analysis is performed and allowed parameter regions are displayed.

I. INTRODUCTION

The Standard Model (SM) gauge group is *a priori* consistent with a large flavor group that is not observed experimentally, and Yukawa couplings and mixings are introduced to comply with such experimental evidence [1]. However, the pattern of fermion masses and mixing angles and the nature of the neutrino mass (whether Dirac or Majorana) is not constrained by the gauge symmetry and it is thus natural to go beyond the SM to look for a theoretical explanation of the observed masses and angles and of their hierarchies. To date a promising direction towards the understanding of flavor physics in the SM— notably of fermion mass matrices and mixing—has been the study of particular textures (in the quark sector), such as the nearest-neighbor interactions (NNI) texture [2–4] and the Fritzsch-type textures [5], and the introduction of flavor symmetry groups that constrain the structure of the fermion mass matrix to reproduce the desired texture. The mixing in the lepton sector has also given strong support to the idea of an underlying flavor symmetry and many flavor symmetry groups have been proposed so far [6–8]; of these, discrete groups have proved to be quite successful and we keep exploring in such a direction in the present work.

Although the aforementioned approach, involving (discrete) flavor groups, provides a possible description of the pattern of fermion masses and mixing angles, it seems to not be able to explain the hierarchy(ies) among the different species of fermionic masses, i.e., the flavor symmetry explains the number and location of the "zeroes" in the textures for the fermion mass matrices, but not the relative size among the nonzero entries. In other words it appears that, in order to obtain experimentally suitable values for such physical quantities, one needs to allow for parameters of quite different order of magnitude. Such hierarchy problems have proved to be quite hard in the framework of four-dimensional nonsupersymmetric theories. On the other hand, in the context of extra dimensions, several models—with flat or warped extra dimensions—were proposed as solutions to hierarchy problems, mostly focusing on the gauge-gravity hierarchy. In particular, the Randall-Sundrum model 1 (RS1) [9] has drawn a great deal of attention as it provides a possible description of the TeV-Planck hierarchy with a small (Planck size) extra dimension and treats the TeV scale as a derived scale. The geometry of such space is a slice of five-dimensional anti–de Sitter (AdS) space, a warped product between four-dimensional space, and an extra dimension with the topology of an interval whose extrema are the fixed points of a Z_2 orbifold symmetry. The space thus "ends" on two flat four-dimensional planes (branes) located at the extrema of the extra dimension. Extra-dimensional models, especially Randall-Sundrum models, have also been explored

*arcarlos00@gmail.com
†fefo@ucol.mx
‡olindo.corradini@gmail.com
§alma.drp@gmail.com
‖eli@unach.mx

036010-1

in the context of the SM physics as fermion masses might be explained in terms of the overlaps between the extra-dimensional profiles of fermions and Higgs(es) [10–14].

It is then interesting to consider the possibility of merging both scenarios—flavor symmetries and hierarchy from extra dimensional settings—in a single setup and to determine if a viable model can be constructed. This work presents one such case. To do this we use a minimally extended version of the SM that incorporates a discrete flavor group, and that reproduces fermion mass patterns as well as mixing angles in both the quark and lepton sectors. The model is minimal in the sense that it contains the SM matter fields only (in particular, it does not add right-handed neutrinos), an extended Higgs sector composed of two Higgs $SU(2)$ doublets, a lepton-number-violating scalar, and the smallest Abelian group that renders the NNI textures for the quark mass matrices, namely the cyclic group Z_4.

The model is detailed in the forthcoming section; however, let us stress here that such an approach was already taken by other groups. For example, in Ref. [15] a realization of the so-called lepton minimal flavor violation scenario was obtained in the framework of RS with right-handed neutrinos. There it was found that the first Kaluza-Klein (KK) mass scale could be as low as 3 TeV. Along similar lines, but for the quark sector and providing a solution to the issue of flavor-changing neutral currents in extra dimensional settings, the work in Ref. [16] extends the strong sector in the bulk with an additional $SU(3)$, broken down to quantum chromodynamics by boundary conditions that render the model safe, even for a KK scale of 2 TeV. Closer to our approach is the work by Kadosh and Pallante [17], where an A_4 flavor symmetry is introduced in an RS setup. The main difference with our model is that, unlike Ref. [17], we use a flavor group that reproduces the mixing matrices in both the quark and lepton sectors automatically, without the need of introducing additional flavon fields to generate nonzero entries and/or further suppressions. This makes our model more economical in terms of additional fields and more tractable, especially in the scalar sector. Another important difference is that in our model neutrino masses are generated radiatively and there is no need to introduce right-handed neutrinos, as mentioned above.

The gauge and Yukawa sectors of our model are presented in Sec. II, where the expressions for the mass matrices in both the lepton and quark sectors are described and explicitly shown. Section III shows the numerical analysis and results for both sectors and some remarks about the scalar potential and flavor-changing neutral currents. Finally, we present the conclusions. An Appendix has been included in order to keep the discussion in the paper self-contained.

II. MODEL

The five-dimensional field theory we consider is a higher-dimensional extension of the SM and lives in the RS1 background. Such space is a slice of AdS_5 space of Planck-size curvature, where the fifth dimension y is taken to be an orbifolded circle S_1/Z_2 of radius R, and fields are odd or even under the action of the orbifold, i.e., under reflection $y \rightarrow -y$. In other words the extra dimension has the topology of a line that stretches between $y = 0$ and $y = \pi R$ and the latter are fixed points of the orbifold, where two flat 3-branes, one with positive and the other with negative tension, are accommodated. The brane located at $y = 0$ is referred to as the "UV brane", whereas the brane located at $y = \pi R$ is referred to as the "IR brane". The bulk line element is thus given by

$$ds^2 = dy^2 + e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu, \quad -\pi R \leq y \leq \pi R, \quad (1)$$

with $k \sim M_{\text{Planck}}$ being (proportional to) the AdS curvature, and the TeV-Planck hierarchy is realized provided $kR \approx 10$. Such geometry is realized with a negative bulk cosmological constant and with the aforementioned brane tensions: all three are of Planck size. Moreover, all the fields we deal with have vacuum expectation values (vevs) of order at most TeV, and we may thus neglect their back-reaction to the geometry (1).

The five-dimensional model we consider has a gauge-symmetry group that includes the Standard Model custodial symmetry $G_{CS} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in order to suppress excessive contributions to the Peskin-Takeuchi T parameter [18,19], augmented with a discrete Abelian group Z_4; hence, $G = G_{CS} \times Z_4$. All the fields of our model are charged under Z_4 and, with the exception of the Higgs fields, are all bulk fields, along the lines of what is done in Ref. [19]. This setup thus enjoys a very nice interpretation in terms of AdS/CFT correspondence as only the Higgs fields are seen as TeV-scale composites of the strongly coupled sector of the four-dimensional theory and, since all other fields are bulk fields, all the phenomenology of the model is addressable from the weakly coupled five-dimensional model.

The matter content of the model involves (three families of) left-handed quark doublets Q_{Li} and lepton doublets L_{Li} along with two copies of right-handed quark doublets,

$$U_{Ri} = \begin{pmatrix} u_{Ri} \\ \tilde{d}_i \end{pmatrix}$$

and

$$D_{Ri} = \begin{pmatrix} \tilde{u}_i \\ d_{Ri} \end{pmatrix},$$

and one copy of right-handed lepton doublets,

$$E_{Ri} = \begin{pmatrix} \tilde{\nu}_i \\ l_{Ri} \end{pmatrix}.$$

The zero-modes of "tilded" fields will be projected out from the IR brane by Scherk-Schwarz gauge-symmetry breaking [19] and only their KK modes are eventually nonzero on the brane. We then use two Higgs fields Φ_1

and Φ_2 that are bi-doublets of $SU(2)_L \times SU(2)_R$ and are confined to the IR brane. Lastly, we introduce a bulk singlet scalar field h charged under hypercharge and lepton number that participates in neutrino mass generation. The Z_4 charge assignment for all such fields is similar to the one given in Ref. [20], namely

$$(q_1, q_2, q_3) = (2, 0, 3), \qquad (2)$$

$$(u_1, u_2, u_3) = (d_1, d_2, d_3) = (3, 1, 2), \qquad (3)$$

for the quark sector, with $q_i = Q(Q_{Li})$, $u_i = Q(U_{Ri})$ and $d_i = Q(D_{Ri})$,

$$(\alpha_1, \alpha_2, \alpha_3) = (2, 0, 3), \qquad (4)$$

$$(e_1, e_2, e_3) = (3, 1, 2), \qquad (5)$$

for the lepton sector, with $\alpha_i = Q(L_{Li})$ and $e_i = Q(E_{Ri})$, and

$$(\phi_1, \phi_2) = (1, 2), \qquad (\chi) = 1, \qquad (6)$$

for the scalars with $\phi_i = Q(\Phi_i)$, and $\chi = Q(h) = 1$. The main difference with Ref. [20] is that the left-handed up-type and down-type quarks have the same Z_4 charges: the reason is dictated by the fact that we are using Higgs bi-doublets confined on the IR brane. These fields transform as $\Phi' = e^{i\lambda_{aL}\tau_{aL}} \Phi e^{-i\lambda_{aR}\tau_{aR}}$ under $SU(2)_L \times SU(2)_R$ gauge transformations ($\tau_{aR} = \tau_{aL} = \frac{1}{2}\sigma_a$, where σ_a are the Pauli matrices) and, as explained later, are responsible for the spontaneous breaking of $SU(2)_L \times SU(2)_R \to SU(2)_D$ on the IR brane. In other words the vevs of Φ_i are $SU(2)_D$ singlets,

$$\langle\Phi\rangle = \frac{v}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes(1\ 0) + \begin{pmatrix}0\\1\end{pmatrix}\otimes(0\ 1)\right], \qquad (7)$$

so that $\langle\tilde{\Phi}_i\rangle = \langle\Phi_i\rangle$, with $\tilde{\Phi} \equiv (i\sigma_2)\Phi^*(-i\sigma_2)$, and U_R and D_R thus enter on equal footing in the Yukawa Lagrangian and must then have the same Z_4 charges.

It is easy to check that (7) is invariant under the diagonal part of $SU(2)_L \times SU(2)_R$, namely

$$\delta\langle\Phi\rangle = i\lambda_a(\tau_{aL}\langle\Phi\rangle - \langle\Phi\rangle\tau_{aR}) = 0, \qquad (8)$$

and thus preserves $SU(2)_D$.

A. Gauge symmetry breaking

The bulk (custodial) gauge symmetry must be broken down to the SM gauge group on the four-dimensional effective action at the TeV scale and the usual electroweak spontaneous symmetry breaking must also be accounted for. We realize this in the same way as in Refs. [17,19]: by orbifold Scherk-Schwarz projection, i.e., by assigning independent orbifold charges to a field at the two different ends of the interval, and with canonical spontaneous breaking.

We can impose on the bulk fields a $Z_2 \times Z'_2$ charge assignment. For scalar fields this corresponds to the field transformations

$$\phi(x, -y) = Z\phi(x, y), \qquad (9)$$

$$\phi(x, \pi R - y) = Z'\phi(x, \pi R + y), \qquad (10)$$

with $Z, Z' = \pm$ and $y = 0, \pi R$ being the fixed points of $Z_2 \times Z'_2$. For 5D spinors, single valuedness of the Lagrangian upon the action of $Z_2 \times Z'_2$ requires

$$\psi(x, y) = Z\gamma_5\psi(x, -y),$$
$$\psi(x, \pi R + y) = Z'\gamma_5\psi(x, \pi R - y). \qquad (11)$$

We can fix

$$\gamma_5 = \begin{pmatrix}\mathbb{1} & 0\\0 & -\mathbb{1}\end{pmatrix}$$

and decompose the 5D 4-spinor as

$$\psi = \begin{pmatrix}\xi\\\eta\end{pmatrix},$$

with ξ (η) being left-handed (right-handed) Weyl spinors, in the 4D sense. At the fixed points, i.e., setting $y = 0$ in (11), we thus have that the left-handed Weyl spinor ξ has charges (Z, Z') and the corresponding right-handed Weyl spinor η has charges $(-Z, -Z')$.[1] Below, we mostly only care about the extra-dimensional zero modes, whose profiles are summarized in the Appendix. We thus have that the fermion left-handed zero mode (A12) only exists for charges $(Z, Z') = (+, +)$, whereas the right-handed zero mode exists for $(Z, Z') = (-, -)$. In other words, other nontrivial orbifold projections lift the mass of the zero mode. For vector fields, whose would-be zero modes are constant, we again have that only $(Z, Z') = (+, +)$ allows for massless zero modes, and other charge assignments lift the mass and thus realize the low-energy symmetry breaking.

In our model we break $SU(2)_R$ gauge symmetry down to $U(1)_R$ on the UV brane via the Scherk-Schwarz mechanism, i.e., we assign $(Z, Z') = (-, +)$ to the electrically charged vector bosons of $SU(2)_R$, and $(Z, Z') = (+, +)$ to the neutral gauge boson. Since the former couple the upper and lower parts of the right-handed fermion doublets, single-valuedness of the bulk Lagrangian requires that if the upper part is even on the UV brane, the lower part must be odd and vice versa. Hence, in order to have zero modes for both the upper and lower parts of the right-handed quarks, we need to double the number of right-handed fields [19] in the quark sector, as already mentioned above. The charge assignment for such fields thus reads

---

[1]Henceforth, when referring to the fermionic $Z_2 \times Z'_2$ charges, we will indicate the charges associated to the left-handed part of the doublet.

$$Q_{Li}[+,+], \qquad U_{Ri} = \begin{pmatrix} u_{Ri}[-,-] \\ \tilde{d}_i[+,-] \end{pmatrix},$$

$$D_{Ri} = \begin{pmatrix} \tilde{u}_i[+,-] \\ d_{Ri}[-,-] \end{pmatrix}, \qquad (12)$$

for the quark sector, and

$$L_{Li}[+,+], \qquad E_{Ri} = \begin{pmatrix} \tilde{\nu}_i[+,-] \\ e_{Ri}[-,-] \end{pmatrix}, \qquad (13)$$

for the leptonic sector. A vev on the UV brane then provides the breaking $U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ [19].

On the IR brane the vevs of the Higgs bi-doublets provide the spontaneous breaking $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow$

$SU(2)_D \times U(1)_{B-L}$. Hence, finally the superposition of all such breakings only leaves $U(1)_{\text{em}}$ untouched. In fact,

$$\underbrace{\tau_{3L} + \tau_{3R}}_{\tau_{3D}} + \frac{1}{2}(B\text{-}L) = \tau_{3L} + Y = Q_{\text{em}}, \qquad (14)$$

with $\tau_{aD} = \tau_{aL} \otimes \mathbb{1}_R + \mathbb{1}_L \otimes \tau_{aR}$ being the generators of $SU(2)_D$ (above, the tensor product is left implied.)

B. Yukawa sector

1. Quarks

The mass terms for the quarks come from the following Yukawa interactions on the IR brane:

$$-\mathcal{L}^q_{\text{Yukawa}} = \int dy \frac{\sqrt{-g}\,\delta(y-\pi R)}{\Lambda} [(\gamma^1_u)_{ij}\bar{Q}_{Li}(x^\mu,y)\Phi_1(x^\mu)U_{Rj}(x^\mu,y) + (\gamma^2_u)_{ij}\bar{Q}_{Li}(x^\mu,y)\Phi_2(x^\mu)U_{Rj}(x^\mu,y)$$

$$+ (\gamma^1_d)_{ij}\bar{Q}_{Li}(x^\mu,y)\Phi_1(x^\mu)D_{Rj}(x^\mu,y) + .(\gamma^2_d)_{ij}\bar{Q}_{Li}(x^\mu,y)\Phi_2(x^\mu)D_{Rj}(x^\mu,y)] + \text{H.c.}, \qquad (15)$$

where $\Lambda \equiv M_{\text{Pl}}$ is the Planck-scale and $(\gamma^{1,2}_{u,d})_{ij}$ are dimensionless parameters assumed of $\mathcal{O}(1)$ that, together with the quark extra-dimensional profiles, generate the effective four-dimensional Yukawa couplings. The Z_4 charge assignments for the quark and scalar fields then induce the desired Yukawa NNI textures (writing each scalar contribution separately)

$$\gamma^1_{u,d} = \begin{pmatrix} 0 & * & 0 \\ * & 0 & 0 \\ 0 & 0 & * \end{pmatrix}, \qquad \gamma^2_{u,d} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix}, \qquad (16)$$

which after electroweak symmetry breaking lead to the following quark mass matrices:

$$M_u = \begin{pmatrix} 0 & \Gamma^u_{12}\langle\Phi_1\rangle & 0 \\ \Gamma^u_{21}\langle\Phi_1\rangle & 0 & \Gamma^u_{23}\langle\Phi_2\rangle \\ 0 & \Gamma^u_{32}\langle\Phi_2\rangle & \Gamma^u_{33}\langle\Phi_1\rangle \end{pmatrix},$$

$$M_d = \begin{pmatrix} 0 & \Gamma^d_{12}\langle\Phi_1\rangle & 0 \\ \Gamma^d_{21}\langle\Phi_1\rangle & 0 & \Gamma^d_{23}\langle\Phi_2\rangle \\ 0 & \Gamma^d_{32}\langle\Phi_2\rangle & \Gamma^d_{33}\langle\Phi_1\rangle \end{pmatrix}, \qquad (17)$$

with $\langle\Phi_i\rangle$ denoting the vevs. The $\Gamma^{u,d}_{ij}$ are the effective four-dimensional Yukawa couplings that depend on the fermion extra-dimensional profiles overlap with the Higgs bi-doublets at the IR boundary. Thus, under the zero mode approximation (ZMA) (see Appendix for the KK decomposition and the explicit fermion profiles), each of the Yukawa terms above looks like

$$-\mathcal{L}_q \supset \int_{-\pi R}^{\pi R} dy \frac{e^{-4k|y|}\delta(y-\pi R)}{\Lambda} \gamma_{ij} \frac{\bar{Q}^{(0)}_{Li}(x)f^{(0)}_{QLi}(y)}{\sqrt{2\pi R}} \Phi(x) \frac{Q_{Rj}(x)f^{(0)}_{QRj}(y)}{\sqrt{2\pi R}} e^{\pi kR}$$

$$= \left\{ \gamma_{ij} \frac{k}{\Lambda} \sqrt{\frac{(1/2 - c^Q_{Li})(1/2 - c^q_{Rj})}{[e^{2\pi kR(1/2-c^Q_{Li})} - 1][e^{2\pi kR(1/2-c^q_{Rj})} - 1]}} e^{(1-c^Q_{Li}-c^q_{Rj})\pi kR} \right\} \bar{Q}^{(0)}_{Li}(x)\Phi(x)Q^{(0)}_{Rj}(x), \qquad (18)$$

where $q = U, D$, $\Phi = \Phi_{1,2}$, and i, j are family indices chosen accordingly (note also the inclusion of a canonically normalizing factor for the Higgs bi-doublet). The effective Yukawa coupling, given by the product inside the curly brackets, depends on two quark localization c-parameters,

$$\Gamma_{ij}^{u,d} = (\gamma_{u,d})_{i,j} \frac{k}{\Lambda} \sqrt{\frac{(1/2 - c_{Li}^Q)(1/2 - c_{Rj}^{u,d})}{[e^{2\pi kR(1/2 - c_{Li}^Q)} - 1][e^{2\pi kR(1/2 - c_{Rj}^{u,d})} - 1]}}$$
$$\times e^{(1 - c_{Li}^Q - c_{Rj}^{u,d})\pi kR}, \qquad (19)$$

where $c_{Li}^u = c_{Li}^d \equiv c_{Li}^Q$ since the left-handed components of the u and d quarks form an $SU(2)_L$ doublet. In order to extract the c-parameters that lead to experimentally allowed observables, we follow the Harayama parametrization in Ref. [3], which is a transformation of the up- and down-type quark mass matrices to a basis such that these display the NNI form without modifying the mass eigenvalues nor the Cabibbo-Kobayashi-Maskawa (CKM) matrix entries. Once parametrized in that form and following [20], we make the assumption that the 1–2 and $2-1$ entries in $M_{u,d}$ are equal, leading to

$$\hat{M}_{u,d} = m_{t,b} \begin{pmatrix} 0 & q_{u,d}/y_{u,d} & 0 \\ q_{u,d}/y_{u,d} & 0 & b_{u,d}(y_{u,d}) \\ 0 & d_{u,d}(y_{u,d}) & y_{u,d}^2 \end{pmatrix}, \qquad (20)$$

where $y_{u,d}$ are free parameters and

$$p_{u,d} = \frac{m_{u,d}^2 + m_{c,s}^2}{m_{t,b}^2}, \qquad (21)$$

$$q_{u,d} = \frac{m_{u,d}m_{c,s}}{m_{t,b}^2}, \qquad (22)$$

$$b_{u,d}(y_{u,d}) = \sqrt{\frac{p_{u,d} + 1 - y_{u,d}^4 - R_{u,d}(y_{u,d})}{2} - \left(\frac{q_{u,d}}{y_{u,d}}\right)^2}, \qquad (23)$$

$$d_{u,d}(y_{u,d}) = \sqrt{\frac{p_{u,d} + 1 - y_{u,d}^4 + R_{u,d}(y_{u,d})}{2} - \left(\frac{q_{u,d}}{y_{u,d}}\right)^2}, \qquad (24)$$

with

$$R_{u,d}(y_{u,d}) = ((1 + p_{u,d} - y_{u,d}^4)^2 - 4(p_{u,d} + q_{u,d}^4) + 8q_{u,d}^2 y_{u,d}^2)^{1/2}. \qquad (25)$$

$\hat{M}_{u,d}$ are real matrices arising from the phase factorization of $M_{u,d}$,

$$M_{u,d} = P_{u,d}^* \hat{M}_{u,d} P_{u,d}, \qquad (26)$$

with $P_{u,d}$ being diagonal phase matrices such that $P = P_u P_d^* = \text{diag}(1, e^{i\beta_{ud}}, e^{i\alpha_{ud}})$, $\beta_{ud} = \beta_u - \beta_d$ and $\alpha_{ud} = \alpha_u - \alpha_d$. Therefore, four parameters $y_{u,d}$, β_{ud} and α_{ud} have to be chosen to fit the CKM matrix,

$$V_{\text{CKM}} = \mathcal{O}_u^T P \mathcal{O}_d, \qquad (27)$$

where $\mathcal{O}_{u,d}$ diagonalize $\hat{M}_{u,d}\hat{M}_{u,d}^T$,

$$\mathcal{O}_{u,d}^T \hat{M}_{u,d}\hat{M}_{u,d}^T \mathcal{O}_{u,d} = \text{diag}(m_{u,d}^2, m_{c,s}^2, m_{t,b}^2). \qquad (28)$$

We show the numerical results that reproduce the experimental values in Sec. III.

2. Charged leptons

The charged lepton masses are similarly obtained from the Yukawa interactions on the IR brane,

$$-\mathcal{L}_{\text{Yukawa}}^e = \int dy \frac{\sqrt{-g}\delta(y - \pi R)}{\Lambda}$$
$$\times [(\gamma_e^1)_{ij}\bar{L}_{Li}(x^\mu, y)\Phi_1(x^\mu)E_{Rj}(x^\mu, y)$$
$$+ (\gamma_e^2)_{ij}\bar{L}_{Li}(x^\mu, y)\Phi_2(x^\mu)E_{Rj}(x^\mu, y)]. \qquad (29)$$

After electroweak symmetry breaking we obtain

$$M_e = m_\tau \begin{pmatrix} 0 & \Gamma_{12}^e\langle\Phi_1\rangle & 0 \\ \Gamma_{21}^e\langle\Phi_1\rangle & 0 & \Gamma_{23}^e\langle\Phi_2\rangle \\ 0 & \Gamma_{32}^e\langle\Phi_2\rangle & \Gamma_{33}^e\langle\Phi_1\rangle \end{pmatrix}. \qquad (30)$$

The analog of Eq. (19) for charged leptons is

$$\Gamma_{ij}^e = (\gamma_e)_{i,j} \frac{k}{\Lambda} \sqrt{\frac{(1/2 - c_{Li}^L)(1/2 - c_{Rj}^e)}{[e^{2\pi kR(1/2 - c_{Li}^L)} - 1][e^{2\pi kR(1/2 - c_{Rj}^e)} - 1]}}$$
$$\times e^{(1 - c_{Li}^L - c_{Rj}^e)\pi kR}. \qquad (31)$$

We can parametrize the M_e matrix[2] following again [3], and so M_e has dependence on both dimensionless parameters y_e and z_e:

$$\hat{M}_e = m_\tau \begin{pmatrix} 0 & q_e z_e/y_e & 0 \\ q_e/(y_e z_e) & 0 & \sqrt{B_e(y_e, z_e)} \\ 0 & \sqrt{D_e(y_e, z_e)} & y_e^2 \end{pmatrix}, \qquad (32)$$

where p_e and q_e are the analogs of $p_{u,d}$ and $q_{u,d}$ for charged leptons and

$$B_e(y_e, z_e) = \frac{1}{2}\{p_e + 1 - y_e^4 \pm \sqrt{(1 - p_e + y_e^4)^2 - 4(q_e^2 - y_e^2 z_e^2)(q_e^2 - y_e^2/z_e^2)}\} - \frac{q_e^2}{y_e^2 z_e^2},$$
$$D_e(y_e, z_e) = \frac{1}{2}\{p_e + 1 - y_e^4 \mp \sqrt{(1 - p_e + y_e^4)^2 - 4(q_e^2 - y_e^2 z_e^2)(q_e^2 - y_e^2/z_e^2)}\} - \frac{q_e^2 z_e^2}{y_e^2}. \qquad (33)$$

---

[2]We do not assume $(\hat{M}_e)_{12} = (\hat{M}_e)_{21}$ as we did in the quark sector.

Observe that we can have two different solutions: the plus (minus) case is obtained by taking the plus (minus) and minus (plus) signs in B_e and D_e, respectively. Depending on which solution we choose we have two different regions for the mathematically allowed values of z_e and y_e: Region I (plus case) and II (minus case), respectively. Each region is constrained by the nonnegative real values of B_e an D_e.

The matrix $\hat{M}_e$ allows us to obtain the lepton *c*-parameters by comparison to Eq. (30). Under the assumption that the phases in the charged lepton sector are zero, there are only two parameters left, y_e and z_e, whose actual values will be set by the neutrino sector results.

3. Neutrino sector

Since there are no right-handed neutrinos present in the model, neutrino masses are generated radiatively[3] as in Ref. [20]. In the present extra-dimensional setting this mechanism is going to set two of the lepton c_{Li}^L-parameters. As can be noted in the next section, their values turn out to be $\mathcal{O}(1)$, as any localization *c*-parameter must be.

The radiative mechanism introduces a cubic lepton-number-violating scalar interaction among the two $SU(2)_L$ doublets and the charged scalar h, as well as the Zee operator coupling the left-handed lepton doublet to the singlet charged scalar [21,22]. The Randall-Sundrum geometry enhances this cubic scalar interaction as well as the Zee operator with factors proportional to the extra-dimensional profiles of the fermions and, if allowed to propagate through the bulk, those of the scalars. In the scenario where both scalar Higgs doublets are confined to the IR brane and the scalar singlet h is a bulk field, the cubic scalar operator is an interaction on the TeV brane which looks like

$$-\mathcal{L}_{\Phi\Phi h} = \int dy \sqrt{-g}\,\delta(y - \pi R)\tilde{\lambda}_{\alpha\beta}\epsilon_{ij}\Phi_i^\alpha(x)\Phi_j^\beta(x)h(x, y),$$

(34)

where i, j are SU(2) indices and $\tilde{\lambda}_{\alpha\beta}$ is antisymmetric and with mass dimension $+1/2$. Under the ZMA approach this term acquires the form

$$-\mathcal{L}_{\Phi\Phi h} = \int_{-\pi R}^{+\pi R} dy\, e^{-4k|y|}\delta(y - \pi R)\lambda_{\alpha\beta}\sqrt{\Lambda}\epsilon_{ij}\Phi_i^\alpha(x)\Phi_j^\beta(x)\frac{h^{(0)}(x)f_h^{(0)}(y)}{\sqrt{2\pi R}}(e^{\pi kR})^2,$$

(35)

with the dimensionless antisymmetric coupling $\lambda_{\alpha\beta}$ of $\mathcal{O}(1)$ (note also the inclusion of a canonically normalizing factor for each one of the Higgs doublets). Thus, for a UV-peaked bulk h field (whose approximated profile and normalization factor are shown in the Appendix) the effective cubic operator acquires the form

$$-\mathcal{L}_{\Phi\Phi h} = \lambda_{\alpha\beta}\epsilon_{ij}\Phi_i^\alpha(x)\Phi_j^\beta(x)\cdot\eta_\lambda^{(UV)},$$

(36)

where

$$\eta_\lambda^{(UV)} = \sqrt{\Lambda}\sqrt{k|b_- - 1|}e^{(b_- - 2)\pi kR}$$

(37)

has the correct mass dimension: $[\eta_\lambda^{(UV)}] = +1$ and $b_- < 1$. A UV-peaked bulk h (again in the ZMA) also modifies the Zee operator,

$$\mathcal{L}_{LLh} = \int dy \sqrt{-g}\,\tilde{\kappa}^{ab}\epsilon_{ij}\overline{(L_{Li}^a)}^c(x, y)L_{Lj}^b(x, y)h^*(x, y),$$

(38)

(where $[\tilde{\kappa}^{ab}] = -1/2$ and is antisymmetric by the Pauli principle) and leaves it as

$$\mathcal{L}_{LLh} = \int_{-\pi R}^{+\pi R} dy\, e^{-4k|y|}\frac{\kappa^{ab}}{\sqrt{\Lambda}}\epsilon_{ij}\overline{(L_{Li}^{a(0)})}^c(x)L_{Lj}^{b(0)}(x)h^{(0)*}(x)\frac{f_{Li}^{a(0)}(y)}{\sqrt{2\pi R}}\frac{f_{Lj}^{b(0)}(y)}{\sqrt{2\pi R}}\frac{f_h^{(0)*}(y)}{\sqrt{2\pi R}},$$

(39)

where κ^{ab} is now dimensionless (and antisymmetric). The effective Zee operator then looks like

$$-\mathcal{L}_{LLh} = \kappa^{ab}\epsilon_{ij}\overline{(L_{Li}^{a(0)})}^c(x)L_{Lj}^{b(0)}(x)h^{*(0)}(x)\cdot\eta_\kappa^{(UV)},$$

(40)

where $\eta_\kappa^{(UV)}$ is dimensionless and given by

---

[3]Using the well-known dimension-five Weinberg operator leads to unacceptable large values for the neutrino masses. This could be remedied by allowing unnatural small values of the dimensionless γ parameters, in contradiction to the philosophy of the general scenario. We therefore stick to the 4D renormalizable argument in Ref. [20] and consider the radiative mechanism.

$$\eta_\kappa^{(UV)} = \frac{k^{3/2}}{\sqrt{\Lambda}} \sqrt{(\tfrac{1}{2} - c_{La}^L)(\tfrac{1}{2} - c_{Lb}^L)|b_- - 1|[e^{2\pi k R(1/2 - c_{La}^L)} - 1][e^{2\pi k R(1/2 - c_{Lb}^L)} - 1]} \int_{-\pi R}^{+\pi R} dy\, e^{(b_- - c_{La}^L - c_{Lb}^L)k|y|}. \quad (41)$$

Following the expressions in Ref. [20] for the 4D Majorana neutrino mass matrix and replacing the effective Yukawa and scalar couplings, we find that the Majorana neutrino mass matrix entries are

$$m_{\nu_e \nu_e} = 2a_e \kappa^{31} \eta_\kappa^{(UV)} m_{\tau\mu} \lambda_{12} \eta_\lambda^{(UV)} \langle \Phi_2 \rangle F(m_\Phi^2, m_h^2), \quad (42)$$

$$m_{\nu_\mu \nu_\mu} = 0, \quad (43)$$

$$m_{\nu_\tau \nu_\tau} = 2b_e' \kappa^{13} \eta_\kappa^{(UV)} m_{e\mu} \lambda_{21} \eta_\lambda^{(UV)} \langle \Phi_1 \rangle F(m_\Phi^2, m_h^2), \quad (44)$$

$$\begin{aligned} m_{\nu_e \nu_\mu} &= m_{\nu_\mu \nu_e} \\ &= 2b_e \kappa^{31} \eta_\kappa^{(UV)} m_{\tau\tau}[\lambda_{21} \eta_\lambda^{(UV)} \langle \Phi_1 \rangle F(m_\Phi^2, m_h^2), \end{aligned} \quad (45)$$

$$\begin{aligned} m_{\nu_e \nu_\tau} &= m_{\nu_\tau \nu_e} \\ &= 2(a_e \kappa^{13} \eta_\kappa^{(UV)} m_{e\mu} \lambda_{12} \eta_\lambda^{(UV)} \langle \Phi_2 \rangle \\ &\quad + b_e' \kappa^{31} \eta_\kappa m_{\tau\mu} \lambda_{21} \eta_\lambda^{(UV)} \langle \Phi_1 \rangle \end{aligned} \quad (46)$$

$$+ c_e \kappa^{31} \eta_\kappa^{(UV)} m_{\tau\tau} \lambda_{12} \eta_\lambda^{(UV)} \langle \Phi_2 \rangle) F(m_\Phi^2, m_h^2), \quad (47)$$

$$m_{\nu_\mu \nu_\tau} = m_{\nu_\tau \nu_\mu} = 0, \quad (48)$$

where $F(m_\Phi^2, m_h^2)$ (accounting for the scalar loop factor) is a function depending on the scalar masses of the charged Higgs, m_Φ, and of the singlet scalar m_h through

$$F(m_\Phi^2, m_h^2) = \frac{1}{16\pi^2} \frac{1}{m_\Phi^2 - m_h^2} \log \frac{m_\Phi^2}{m_h^2}. \quad (49)$$

The parameters a_e, b_e, b_e', and c_e belong to the Yukawa matrix for charged leptons,

$$Y_e = \begin{pmatrix} 0 & \Gamma_{12}^e & 0 \\ \Gamma_{21}^e & 0 & \Gamma_{23}^e \\ 0 & \Gamma_{32}^e & \Gamma_{33}^e \end{pmatrix} \equiv \begin{pmatrix} 0 & a_e & 0 \\ a_e' & 0 & b_e \\ 0 & b_e' & c_e \end{pmatrix},$$

and the entries m_{ij}, i, $j = e$, μ, τ are those of the matrix M_e.

Now, since κ_{ij} and λ_{ij} are antisymmetric we can rewrite the neutrino mass matrix entries as

$$m_{\nu_e \nu_e} = -(m_{e\mu} m_{\tau\mu} \tan\beta)\mathcal{C}, \quad (50)$$

$$m_{\nu_\mu \nu_\mu} = 0, \quad (51)$$

$$m_{\nu_\tau \nu_\tau} = -\frac{m_{\tau\mu} m_{e\mu}}{\tan\beta}\mathcal{C}, \quad (52)$$

$$m_{\nu_e \nu_\mu} = m_{\nu_\mu \nu_e} = \frac{m_{\mu\tau} m_{\tau\tau}}{\tan\beta}\mathcal{C}, \quad (53)$$

$$m_{\nu_e \nu_\tau} = m_{\nu_\tau \nu_e} = \left(m_{e\mu}^2 \tan\beta + \frac{m_{\tau\mu}^2}{\tan\beta} - m_{\tau\tau}^2 \tan\beta\right)\mathcal{C}, \quad (54)$$

$$m_{\nu_\mu \nu_\tau} = m_{\nu_\tau \nu_\mu} = 0, \quad (55)$$

where $\mathcal{C}$ is a common factor with dimension of the inverse of mass,

$$\mathcal{C}\,1 = 2\kappa^{13} \eta_\kappa^{UV} \lambda_{12} \eta_\lambda^{UV} F(m_\Phi^2, m_h^2), \quad (56)$$

and

$$\tan\beta = \frac{\langle \Phi_2 \rangle}{\langle \Phi_1 \rangle}. \quad (57)$$

III. NUMERICAL ANALYSIS

A. Lepton sector

To perform the numerical analysis in the lepton sector we used the experimental data at 3σ from the global neutrino data analysis in Ref. [23],

	Best Fit Value	3σ range
$\sin^2\theta_{12}$	0.312	$0.27 - 0.36$
$\sin^2\theta_{23}$	0.52	$0.39 - 0.64$
$\Delta m_{21}^2 [10^{-5}\ \mathrm{eV}^2]$	7.59	$7.09 - 8.19$
$\Delta m_{32}^2 [10^{-3}\ \mathrm{eV}^2]$	2.50	$2.14 - 2.76$
	$-(2.40)$	$-(2.13 - 2.67)$

$$(58)$$

with $\delta_{CP} = 0$ and normal (inverted) hierarchy and the recently Daya Bay results (confirmed at 5σ) [24],

$$\sin^2 2\theta_{13} = 0.092 \pm 0.017, \quad (59)$$

which can be rewritten as

$$\sin^2 \theta_{13} = 0.0235 \pm 0.0045. \quad (60)$$

By convenience in the analysis we also define the following range for the neutrino squared mass differences ratio (at 3σ):

$$\text{NH(IH):}\ 0.027(0.028) < \left| \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \right| < 0.035(0.036), \quad (61)$$

which were obtained by summing in quadrature the relative errors of Δm_{21}^2 and $|\Delta m_{32}^2|$. The constraint over the sum of neutrino masses, $\sum m_{\nu_i} < 0.29$ eV, presented recently in Ref. [25] is also considered.

The charged leptons masses used in the analysis are those given by the central values in Ref. [26]:

$$m_e = 0.511 \times 10^{-3}\ \mathrm{GeV}, \quad (62)$$

$$m_\mu = 0.1056 \text{ GeV}, \tag{63}$$

$$m_\tau = 1.776 \text{ GeV}. \tag{64}$$

As in Ref. [17] we take the 5D scale to be $\Lambda = k = M_{\text{Planck}}$ where $M_{\text{Planck}} = 2.44 \times 10^{15}$ TeV is the reduced Planck mass, and the effective scale $ke^{-\pi kR} \approx 6.89$ TeV generated by $\pi kR = 33.5$. For the other parameters involved in the neutrino mass sector we take $m_\Phi = 0.5$ TeV (recall this is the mass of the *charged* Higgses in the loop), $m_h = M_{\text{Planck}}$, $\kappa^{13} = 1 = -\kappa^{31}$, and $\lambda^{12} = 1 = -\lambda^{21}$. Note that since h is UV-peaked its mass is taken to its natural value of M_{Planck}. We note however that the model can reproduce the neutrino sector even in the case of a very light h-field (or even an IR-peaked h-field) due to the fact that m_h only enters through the common factor $\mathcal{C}$ in Eq. (56), and thus does not affect the diagonalization. Its only possible effect is in the absolute size of the neutrino mass matrix entries and it corresponds to acceptable changes of $O(1)$.

$$U_{\text{PMNS}} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta_{CP}} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta_{CP}} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta_{CP}} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}, \tag{66}$$

with $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ and where δ_{CP} is the Dirac-CP-violating phase in the neutrino sector which we assume is zero. $P = \text{diag}(1, e^{i\alpha_{21}}, e^{i\alpha_{31}})$ is a diagonal phase matrix containing the two Majorana CP-violating phases.

The first step in our analysis consists of a scan over the mathematically allowed regions for the z_e and y_e parameters of the charged lepton mass matrix M_e that lead to positive values for B_e and D_e in Eq. (33). The next step is to scan over values for $\tan\beta$ that lead to the allowed neutrino mass differences ratio and mixing angles. Finally, once the ranges for z_e, y_e and $\tan\beta$ values that give the right angles and ratios are determined, we fit the a_- values (or b_-) required to obtain the neutrino masses satisfying the constraint over the sum of neutrino masses [25].

Performing the scan over $\tan\beta$ values from 0.1 to 1.0 (in steps of 0.1) and from 1 to 20 (in steps of 1) we find that in Region I (plus case) the only value that works is $\tan\beta = 2.0$, while in Region II (minus case) $\tan\beta = 0.3$ and $\tan\beta = 0.6$ give acceptable values.

Figure 1 shows the parameter space obtained in this process. Note that in all cases there is a large region in the z_e-y_e plane consistent with positive values for B_e and D_e. However, the allowed region, consistent with all experimental information, is considerably reduced to small ranges in this parameter space. In the plots we label by *allowed region* the region of parameter space consistent with all experimental data, including the constraint on the sum of neutrino masses [25].

We note that, as found in Ref. [20], only inverted hierarchy for the neutrino masses is obtained in this model

From the expressions for the neutrino mass matrix entries we observe that the only c_{Li}^L coefficients involved are c_{L1}^L and c_{L3}^L, and are taken to be $c_{L1}^L = 0.8$ and $c_{L3}^L = 0.55$ (this is only a choice and corresponds to similar values used in Ref. [17]). We also observe that all these parameters are contained in the $\mathcal{C}$ factor, together with the parameter $b_- = 2 - \sqrt{4 + a_-}$, and in consequence they do not affect the diagonalization matrix. Thus, the lepton mixing matrix U_{PMNS} defined by $U_{\text{PMNS}} = U_L^\dagger U_\nu$, where U_L and U_ν are the diagonalization matrices in the charged lepton and neutrino sectors, respectively, only depends on $\tan\beta$ and the charged leptons mass matrix entries.

We use the standard parametrization of the U_{PMNS} given in Ref. [26],

$$V = U_{\text{PMNS}}P, \tag{65}$$

where

and that the resulting Majorana phases are found to be $\alpha_{21} = \alpha_{31} = \pi$.

It is important to note that throughout the analysis all the γ_{ij}^e have been taken to be of $\mathcal{O}(1)$. In fact, except for γ_{12}^e and γ_{32}^e which are used in the fit and take values of $\mathcal{O}(1)$ in all cases, all other are set equal to 1. The same is true for all the profile c-coefficients: all are of $\mathcal{O}(1)$.

B. Quark sector

In Ref. [27] the values for the parameters $y_{u,d}$, β_{ud} and α_{ud} leading to a correct CKM matrix $|V_{\text{CKM}}|$ were found to be $y_u = 0.9964$, $y_d = 0.9623$, $\beta_{ud} = 1.4675$ and $\alpha_{ud} = 1.9560$. The corresponding mass matrices (in TeV) read

$$\hat{M}_u = \begin{pmatrix} 0 & 0.0000552118 & 0 \\ 0.0000552118 & 0 & 0.0118527 \\ 0 & 0.0183451 & 0.17152 \end{pmatrix}, \tag{67}$$

$$\hat{M}_d = \begin{pmatrix} 0 & 0.0000235694 & 0 \\ 0.0000235694 & 0 & 0.000268666 \\ 0 & 0.00158321 & 0.0039362 \end{pmatrix}. \tag{68}$$

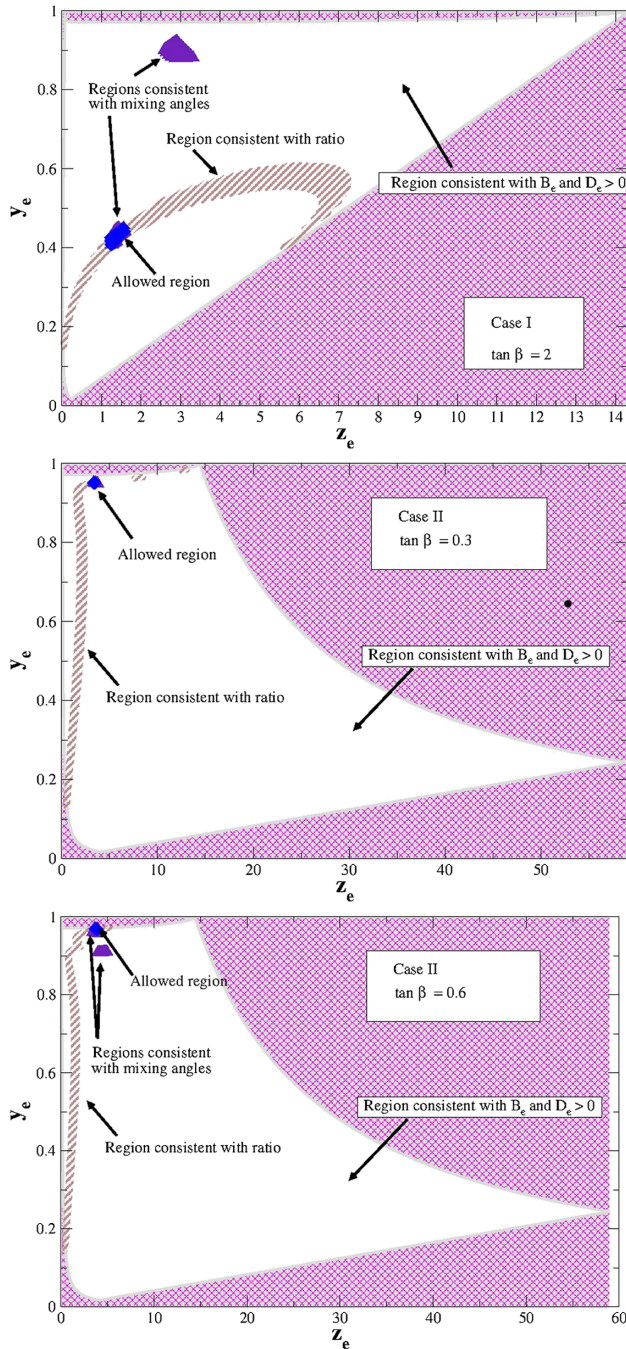Comparing these matrices with Eq. (17) and using the values for $\tan\beta$ found to work in the neutrino sector, we

FIG. 1 (color online). Consistent regions of parameter space in the z_e-y_e plane for Case I and $\tan\beta = 2$ (upper plot), Case II and $\tan\beta = 0.3$ (middle), and Case II and $\tan\beta = 0.6$ (bottom). We show the complete region consistent with positive values for D_e and B_e, the region consistent with the neutrino mass squared differences ratio, the regions consistent with neutrino mixing angles, and the *allowed region* (see text). Note that for Case II and $\tan\beta = 0.3$ the region consistent with mixing angles is basically the same as the allowed region.

$(c_{L1}^Q, c_{L2}^Q, c_{L3}^Q) = (0.7, 0.491253, 0.128276)$, $(c_{R1}^u, c_{R2}^u, c_{R3}^u) = (0.666249, 0.342679, 0.737729)$, $(c_{R1}^d, c_{R2}^d, c_{R3}^d) = (0.690378, 0.468589, 0.591601)$, $(\gamma_{ij}^u) = 1_{3\times3}$, $\gamma_{32}^d = 0.101081$, $\gamma_{33}^d = 2.02488$, and all other γ's equal to 1.

Thus, using dimensionless parameters of order one and dimensionful parameters associated to the scale of the setup, we find that it is possible to reproduce all observed masses and mixing angles in both the quark and lepton sector. Furthermore, we accomplish this with a minimal set of additions and only left-handed neutrinos. It is important to recall that all results presented in this work have been obtained under the ZMA scheme and thus neglect (although safely [17]) possible contributions from KK modes. Yet another possible source of contributions that must be investigated corresponds to contributions from higher-order operators consistent with the gauge and flavor symmetries. In our case, the smallest higher-order operators that could contribute are of the general form $\Phi\Phi\Phi\bar{Q}_L Q_R$ (and similarly for leptons). We find that the gauge- and flavor-invariant contributions, denoted by δm_{ij}, to the zero entries in the mass matrices satisfy $\delta m_{ij}/m_{lk} \sim 10^{-7}$–$10^{-11}$, where the m_{lk} denote the values of the nonzero entries. For the case $ij = lk$, corresponding to contributions to the nonzero entries, we find $\delta m_{ij}/m_{ij} \sim 10^{-4}$. Thus, these contributions can be safely ignored.

Before concluding we make some remarks to be taken into full consideration in a future work. Our model has a potentially interesting scalar phenomenology that requires a complete study, including its vacuum stability and possible collider signals. Another important and related phenomenological issue is the presence of flavor-changing neutral current (FCNC) effects. In our model there are two different possible contributions to FCNC that must be taken into consideration. The first one comes from the fact that we are dealing with a two (flavored) Higgs doublet model in which each Higgs couples to both the up- and down-quark sector. This can result in tree-level FCNC [28,29] that need to be analyzed and will induce constraints on the parameter space in the scalar potential. The second possibility is associated with the presence of mixing between the degrees of freedom of the effective 4D theory and their KK excitations. This is a problem that all extra-dimensional models have to face. One has to guarantee that the corrections induced by this mixing will keep the agreement with observations forcing a lower bound, of order a few TeV, on the KK scale. In our model, the use of a custodial symmetry allows us to consider a lower bound on the first KK mass to that imposed by electroweak precision measurements [19] (see also Refs. [15,16,30]). Furthermore, if the flavor pattern of the Yukawa couplings holds in the 5D theory due to a flavor symmetry, an increased alignment between the 4D fermion mass matrix and the Yukawa and gauge couplings is obtained by suppressing the amount of flavor violation induced by the interactions with KK states. Moreover, as pointed out in

performed a fit to the c-profile parameters and γ's. We find that it is possible to obtain solutions with all parameters of $\mathcal{O}(1)$. In particular, for $\tan\beta = 0.3$, experimental agreement is found for at least the following set of values:

Ref. [17], a bulk flavor symmetry might also induce a cancellation of observable phases; therefore, the dominant new physics contributions to the neutron and electron dipole moments and/or to ϵ'/ϵ_K will vanish.

IV. CONCLUSIONS

A five-dimensional warped extension of the Standard Model is considered, where the space-time background is taken to be the Randall-Sundrum model 1, i.e., a warped extra dimension of Planck size in an anti-de Sitter bulk. We consider a zero mode approximation for the four-dimensional effective action one obtains as a dimensional reduction of the higher-dimensional theory and that, in the present case, constitutes an extension of the SM. In the model we study above almost all the fields in the 4D effective action are zero modes of bulk fields; in particular, each left- and right-handed fermion has its own five-dimensional counterpart: only the Higgs fields are purely four-dimensional and live on the IR brane (i.e., on the 4D plane located at the orbifold fixed point $y = \pi R$) and, unlike the bulk fields, have no KK excitations. Bulk fermion fields come about with a (Planck size) mass term parametrized by a real number c which determines the shape of the extra-dimensional profile of the field. The Yukawa interaction between Higgses and fermions is given in terms of overlapping integrals of zero mode profiles for the fields involved and thus depends upon the aforementioned c real parameters along with the numerical coupling constant (Hermitian) matrices (γ's). In order to obtain an NNI-type quark mass matrix, we advocate the presence of a discrete symmetry upon which all fields are charged: the cyclic group Z_4 is the smallest Abelian group consistent with the aforementioned texture. Also, for the charged lepton sector the discrete symmetry fixes the form of the mass matrix in a similar way as for the quark sector. For the quark sector (and similarly for the charged lepton sector) the numerical analysis goes as follows: first we consider the mass matrix parametrized *à la* Harayama in order for it to display NNI form. The diagonalization of the real squared counterpart of the previous mass matrix leaves four parameters that can be fixed using the experimental values for the CKM. In turn, this allows us to fix the entries of the mass matrix itself and, by the comparison of such entries with the ones foreseen by the overlaps of extra-dimensional profiles, one can obtain the compactification parameters (c's) and the Yukawa dimensionless entries (γ's). All these parameters turn out to be of $\mathcal{O}(1)$. For the charged leptons the parametrization is slightly different and the (real) mass matrix depends upon two free parameters constrained only by the reality of the mass matrix: allowed parameter regions satisfying such constraints, as well as all experimental data, are graphically displayed. In the neutrino sector we choose not to introduce right-handed neutrinos in the model and generate neutrino masses radiatively. This is accomplished with the help of

a bulk charged scalar field whose zero mode is peaked on the UV brane (located at the fixed point $y = 0$): such a field also mediates the lepton number violation. With scalar field masses chosen to be $m_\Phi = 0.5$ TeV for the charged Higgses in the loops, and $m_h = M_{\text{Planck}}$ TeV, experimental data taken from 3σ global neutrino data analysis and Daya Bay results are matched by setting all c parameters and γ parameters to values of $\mathcal{O}(1)$. Although a complete phenomenological study of the model is under preparation, some comments regarding the scalar phenomenology of the model, as well as its possible contributions to FCNC, were also briefly discussed.

APPENDIX: ON THE ZERO MODE PROFILES

We briefly review the computation of zero mode profiles for some bulk fields, on the RS1 background considered above: we basically follow what is done in Refs. [13,31]. A bulk field satisfies a second-order differential equation of the form

$$[e^{2\sigma}\eta^{\mu\nu}\partial_\mu\partial_\nu + e^{s\sigma}\partial_y(e^{-s\sigma}\partial_y) - M_\Phi^2]\Phi(x,y) = 0, \quad (\text{A1})$$

where $\Phi = \{A_\mu, \phi, e^{-2\sigma}\Psi_{L,R}\}$, $s = \{2, 4, 1\}$ and $M_\Phi^2 = \{0, ak^2 + b\sigma'', c(c+1)k^2 - c\sigma''\}$: note that compared to Refs. [13,31] we redefined $c \rightarrow -c$ for the right-handed mode. Above, $\sigma(|y|) = k|y|(\text{modulo}2\pi R)$ and thus $\sigma' = k\epsilon(y)$ and $\sigma'' = 2k[\delta(y) - \delta(y - \pi R)]$. Upon Kaluza-Klein decomposition one gets

$$\Phi(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Phi^{(n)}(x)f_n(y), \quad (\text{A2})$$

with $\eta^{\mu\nu}\partial_\mu\partial_\nu\Phi^{(n)}(x) = m_n^2\Phi^{(n)}(x)$, and orthonormality conditions

$$\frac{1}{2\pi R}\int_{-\pi R}^{\pi R} dy\, e^{2l_s\sigma}f_n(y)f_{n'}(y) = \delta_{n,n'},$$
$$l_s = (0, -1, -3/2), \quad (\text{A3})$$

so that

$$[-e^{s\sigma}\partial_y(e^{-s\sigma}\partial_y) + M_\Phi^2]f_n(y) = m_n^2 f_n(y). \quad (\text{A4})$$

Here we only concentrate on the zero modes for which $m_0 = 0$.

1. Vector field

For the vector field it is immediately realized that the only possible zero mode profile is constant, and since the measure is also trivial in this case we simply have $f_0(y) = 1$.

2. Scalar field

For a bulk scalar field we have

$$[-e^{4\sigma}\partial_y(e^{-4\sigma}\partial_y) + ak^2 + b\sigma'']f_0(y) = 0, \quad (A5)$$

that admits solutions

$$f_{0,\pm}(y) = \frac{1}{N_0}e^{b_\pm\sigma}, \qquad b_\pm = 2 \pm \sqrt{4+a}, \quad (A6)$$

and the normalization factor is given by

$$N_{0,\pm} = \left[\frac{e^{2(b_\pm-1)\pi kR}-1}{2\pi kR(b_\pm-1)}\right]^{1/2}$$

$$\approx \frac{1}{\sqrt{2\pi kR|b-1|}}\begin{cases} e^{(b-1)\pi kR}, & b=b_+ \\ 1, & b=b_- \end{cases}, \quad (A7)$$

where in the last expression we took into account that $k\pi R >> 1$ and $b_+ > 1$, $b_- < 1$. Hence, including the measure factor the zero mode profiles read

$$\frac{1}{\sqrt{2\pi R}}e^{-\sigma}f_0(y) \approx \sqrt{|b-1|k}\begin{cases} e^{(b_+-1)k(|y|-\pi R)}, & b=b_+ \\ e^{(b_--1)k|y|}, & b=b_- \end{cases}, \quad (A8)$$

so that for $b=b_+$ the profile is peaked about the IR brane ($y=\pi R$) and for $b=b_-$ the profile is peaked about the UV brane ($y=0$).

3. Fermionic field

For a bulk fermionic field we follow Ref. [13] with the aforementioned renaming of parameter c for the right-handed mode. Hence,

$$[-e^{\sigma}\partial_y(e^{-\sigma}\partial_y) + c(c+1)k^2 - c\sigma'']e^{-2\sigma}f_0(y) = 0, \quad (A9)$$

and

$$f_{0L,R}(y) = \frac{1}{N_0}e^{(2-c)\sigma}, \quad (A10)$$

with

$$N_0 = \left[\frac{e^{2\pi kR(1/2-c)}-1}{2\pi kR(1/2-c)}\right]^{1/2}$$

$$\approx \frac{1}{\sqrt{2\pi kR|1/2-c|}}\begin{cases} e^{(1/2-c)\pi kR}, & c<1/2 \\ 1, & c>1/2 \end{cases}. \quad (A11)$$

Hence, including the measure factor the zero mode profiles read

$$\frac{1}{\sqrt{2\pi R}}e^{-3\sigma/2}f_0(y) \approx \sqrt{|1/2-c|k}\begin{cases} e^{(1/2-c)k(|y|-\pi R)}, & c<1/2 \\ e^{(1/2-c)k|y|}, & c>1/2 \end{cases}, \quad (A12)$$

so that for $c<1/2$ the profile is peaked about the IR brane ($y=\pi R$) whereas for $c>1/2$ the profile is peaked about the UV brane ($y=0$).

[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] G. C. Branco, D. Emmanuel-Costa, and C. Simoes, Phys. Lett. B **690**, 62 (2010).

[3] K. Harayama and N. Okamura, Phys. Lett. B **387**, 614 (1996).

[4] T. Ito, N. Okamura, and M. Tanimoto, Phys. Rev. D **58**, 077301 (1998); T. Ito and M. Tanimoto, Phys. Rev. D **55**, 1509 (1997); E. Takasugi and M. Yoshimura, Prog. Theor. Phys. **98**, 1313 (1997); Z. Berezhiani and A. Rossi, J. High Energy Phys. 03 (1999) 002; G. C. Branco, M. N. Rebelo, and J. I. Silva-Marcos, Phys. Lett. B **597**, 155 (2004); K. S. Babu, arXiv:0910.2948; G. C. Branco, L. Lavoura, and F. Mota, Phys. Rev. D **39**, 3443 (1989); G. C. Branco, D. Emmanuel-Costa, R. G. Felipe, and H. Serodio, Phys. Lett. B **670**, 340 (2009); G. C. Branco, D. Emmanuel-Costa and R. G. Felipe, Phys. Lett. B **477**, 147 (2000); H. Fritzsch, Z.-z. Xing, and Y.-L. Zhou, Phys. Lett. B **697**, 357 (2011).

[5] H. Fritzsch, Phys. Lett. B **73**, 317 (1978).

[6] G. Altarelli and F. Feruglio, Rev. Mod. Phys. **82**, 2701 (2010).

[7] H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. Shimizu, and M. Tanimoto, Prog. Theor. Phys. Suppl. **183**, 1 (2010).

[8] G. Altarelli, Proc. Sci., HRMS2010 (2010) 022.

[9] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[10] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000).

[11] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61**, 033005 (2000).

[12] T. Gherghetta and A. Pomarol, Nucl. Phys. **B586**, 141 (2000).

[13] S. J. Huber and Q. Shafi, Phys. Lett. B **498**, 256 (2001).

[14] S. J. Huber, Nucl. Phys. **B666**, 269 (2003).

[15] M.-C. Chen and H.-B. Yu, Phys. Lett. B **672**, 253 (2009).

[16] M. Bauer, R. Malm, and M. Neubert, Phys. Rev. Lett. **108**, 081603 (2012).

[17] A. Kadosh and E. Pallante, J. High Energy Phys. 08 (2010) 115.

[18] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990).

[19] K. Agashe, A. Delgado, M. J. May, and R. Sundrum, J. High Energy Phys. 08 (2003) 050.

[20] A. Aranda, C. Bonilla, and A. D. Rojas, Phys. Rev. D **85**, 036004 (2012).

[21] A. Zee, Phys. Lett. **93B**, 389 (1980); **95B**, 461 (1980).

[22] K. S. Babu and V. S. Mathur, Phys. Rev. D **38**, 3550 (1988).

[23] T. Schwetz, M. Tortola, and J. W. F. Valle, New J. Phys. **13**, 109401 (2011).

[24] F. P. An *et al.* (DAYA-BAY Collaboration), Phys. Rev. Lett. **108**, 171803 (2012).

[25] S. Riemer-Sorensen *et al.*, Phys. Rev. D **85**, 081101 (2012).

[26] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010), and 2011 partial update for the 2012 edition (pdg.lbl.gov).

[27] A. Aranda, C. Bonilla, R. Ramos, and A. D. Rojas, Phys. Rev. D **84**, 016009 (2011).

[28] D. Atwood, L. Reina, and A. Soni, Phys. Rev. D **55**, 3156 (1997).

[29] A. Azatov, M. Toharia, and L. Zhu, Phys. Rev. D **80**, 035016 (2009).

[30] S. Casagrande, F. Goertz, U. Haisch, M. Neubert, and T. Pfoh, J. High Energy Phys. 09 (2010) 014.

[31] T. Gherghetta and A. Pomarol, Nucl. Phys. **B586**, 141 (2000).