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Angular momentum sum rule for spin-one hadronic systems

Swadhin K. Taneja,[1,*] Kunal Kathuria,[2,†] Simonetta Liuti,[2,‡] and Gary R. Goldstein[3,§]
[1]*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11776, USA*
[2]*Physics Department, University of Virginia, 382 McCormick Road, Charlottesville, Virginia 22904, USA*
[3]*Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155, USA*
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We derive a sum rule for the total quark angular momentum of a spin-one hadronic system within a gauge-invariant decomposition of the hadron's spin. We show that the total angular momentum can be measured through deeply virtual Compton scattering experiments using transversely polarized deuterons.

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A crucial, outstanding question in QCD is the proton spin puzzle. A number of experiments performed since the 1980s, including the most recent HERMES, Jefferson Lab and Compass measurements, have confirmed that only about 30% of the proton spin is accounted for by quarks, and that the quark contribution is dominated by the valence component (see review in [1]). Current efforts, both in theory and experiment, are therefore directed towards determining the contributions of the orbital angular momentum (OAM) of the quarks, as well as of the spin and OAM of the gluons. Sum rules were derived that relate the energy-momentum tensor's (EMT) form factors to the nucleon angular momentum [2,3] [4]. In [2], starting from the classical/canonical form of the EMT, it is possible to identify the four contributions from the quark and gluon OAM and spin components. Of these only the quark and gluon spin terms appear among the observables for hard scattering processes. On the other side, the result derived in [3] uses the symmetric, Belinfante form of the EMT and leads to different definitions of the angular momentum components, $J_q = L_q + \Delta\Sigma$, and J_g. These can, in principle, be measured through deeply virtual Compton scattering (DVCS) (see also [8]). However, the interpretation of these components in terms of unintegrated parton angular momentum density distributions is not straightforward. The values of the observables will therefore differ in the two approaches [9].

Motivated by the challenge of the spin puzzle on the one side, and by the feasibility of DVCS-type experiments on the other, we decided to investigate the angular momentum sum rules for hadronic systems of different spin which are provided, in practice, by nuclear targets. In this contribution we present a sum rule for the total angular momentum in a spin-one nucleus; the deuteron. The sum rule is of particular relevance because it involves only one generalized parton distribution (GPD), namely,

$$J_q = \frac{1}{2}\int dx x H_2^q(x, 0, 0). \tag{1}$$

$H_2^q(x, \xi, t)$'s first moment is equal to the deuteron magnetic form factor $G_2(t) \equiv G_M(t)$ [10]. This expression can be compared to the nucleon sum rule [3],

$$J_q = \frac{1}{2}\int dx x[H_q(x, 0, 0) + E_q(x, 0, 0)], \tag{2}$$

where the first moment of the GPD sum $H_q(x, \xi, t) + E_q(x, \xi, t)$ is the nucleon magnetic form factor, $F_1(t) + F_2(t) \equiv G_M(t)$. Similar to the proton GPD E, H_2 does not have a forward partonic limit.

In what follows we outline the fundamental steps of the derivation. We start from the expression for angular momentum in QCD,

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}, \tag{3}$$

where the tensor M^{0ij} is the angular momentum density given in terms of the symmetric, gauge-invariant, and conserved (Belinfante) EMT as $M^{\alpha\mu\nu} = T^{\alpha\nu}x^\mu - T^{\alpha\mu}x^\nu$. Notice that $T^{\mu\nu}$ has separate gauge-invariant contributions from quarks and gluons [3], along with their interaction through the gauge-covariant derivative:

$$\begin{aligned} T^{\mu\nu} &= T_q^{\mu\nu} + T_g^{\mu\nu} \\ &= \frac{1}{2}[\bar{\psi}\gamma^{(\mu} i\overrightarrow{D^{\nu)}}\psi + \bar{\psi}\gamma^{(\mu} i\overleftarrow{D^{\nu)}}\psi] \\ &\quad + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}. \end{aligned} \tag{4}$$

The connection of GPDs to the angular momentum becomes apparent by first writing down the matrix element of $T_{q,g}^{\mu\nu}$, separately for quarks and gluons, for a spin-one system in terms of gravitational form factors as

*Present address: Department of Physics, Dalhousie University, Halifax NS, B3H 3J5, Canada.
taneja@skipper.physics.sunysb.edu
†kk7t@virginia.edu
‡sl4y@virginia.edu
§gary.goldstein@tufts.edu

$$\langle p'|T^{\mu\nu}|p\rangle = -\frac{1}{2}P^\mu P^\nu(\epsilon'^*\epsilon)\mathcal{G}_1(t) - \frac{1}{4}P^\mu P^\nu\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_2(t) - \frac{1}{2}[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2](\epsilon'^*\epsilon)\mathcal{G}_3(t)$$
$$-\frac{1}{4}[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_4(t) + \frac{1}{4}[(\epsilon'^{*\mu}(\epsilon P) + \epsilon^\mu(\epsilon'^* P))P^\nu + \mu\leftrightarrow\nu]\mathcal{G}_5(t)$$
$$+\frac{1}{4}[(\epsilon'^{*\mu}(\epsilon P) - \epsilon^\mu(\epsilon'^* P))\Delta^\nu + \mu\leftrightarrow\nu + 2g_{\mu\nu}(\epsilon P)(\epsilon'^* P) - (\epsilon'^{*\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu)\Delta^2]\mathcal{G}_6(t)$$
$$+\frac{1}{2}[\epsilon^{*\prime\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu]\mathcal{G}_7(t) + g^{\mu\nu}(\epsilon'^*\epsilon)M^2\mathcal{G}_8(t), \quad (5)$$

where $t = \Delta^2$, $P = p + p'$ and $\Delta = p' - p$, and ϵ, ϵ' are the polarization vectors of the deuteron in the initial and final helicity states, respectively. There are seven conserved independent form factors, $\mathcal{G}_i(t)$, $i = 1, 7$, and an additional nonconserved term, $g^{\mu\nu}(\epsilon'^*\epsilon)M^2\mathcal{G}_8(t)$. In analogy with the nucleon case [11,12], the enumeration of the independent deuteron EMT form factors, as well as its Lorentz structure, was obtained using the partial wave formalism and crossing symmetry [details on our method for counting the form factors are presented in [13] (nucleon) and in an upcoming paper [14] (deuteron)].

The energy-momentum tensor was constructed by considering all independent scalar, vector, axial-vector, and tensor components formed with the polarization and momentum four-vectors. In particular, the $\mathcal{G}_7$ term in our formula corresponds to the irreducible second-rank tensor component. From an independent analysis of the J^{PC} quantum numbers for a spin-one system in the t channel, we find that seven form factors describe the $n = 2$ moments of the vector operator. These are: $\mathcal{G}_1 \ldots \mathcal{G}_7$. Notice that the $\mathcal{G}_7$ term is at variance with $\mathcal{G}_6$ where an irreducible tensor component also appears, since in this case the tensor is multiplied by momenta to guarantee conservation. The coefficient of $\mathcal{G}_7$ is conserved in the forward limit. This sets it apart from the $\mathcal{G}_8$ coefficient which cannot be conserved. Therefore, it turns out that $\mathcal{G}_8$ does not contribute to the sum rules while $\mathcal{G}_7$ does. A more elaborate and detailed explanation will be given in a forthcoming paper. We emphasize, however, that the detailed treatment of this point does not affect the spin sum rule, which is the main focus, and the original result in this paper.

Following a point raised in Ref. [8], we carefully used a wave-packet approach to derive the relation between J^z and the EMT [2]. From Eq. (3), and using Eq. (5) for a spin-one system,

$$J^z_{q,g} = \frac{1}{2}\mathcal{G}_5(0). \quad (6)$$

One can now connect the gravitational form factors with the coefficients of the correlator for (unpolarized) DVCS. For a spin-one system one can write this in terms of five unpolarized GPDs (from the Lorentz symmetric part of the hadronic tensor) [10],

$$\int\frac{d\kappa}{2\pi}e^{ix\kappa Pn}\langle p', \lambda'|\bar\psi(-\kappa n)\gamma n\psi(\kappa n)|p, \lambda\rangle$$
$$= -(\epsilon'^*\epsilon)H_1 + \frac{(\epsilon n)(\epsilon'^* P) + (\epsilon'^* n)(\epsilon P)}{Pn}H_2$$
$$-\frac{(\epsilon P)(\epsilon'^* P)}{2M^2}H_3 + \frac{(\epsilon n)(\epsilon'^* P) - (\epsilon'^* n)(\epsilon P)}{Pn}H_4$$
$$+\left\{4M^2\frac{(\epsilon n)(\epsilon'^* n)}{(Pn)^2} + \frac{1}{3}(\epsilon'^*\epsilon)\right\}H_5, \quad (7)$$

where n is a light-like vector. It follows that by expanding the matrix element on the left hand side of Eq. (7) and taking the second moment with respect to x one can find the following relation between the second moments of the GPDs H_i and the form factors $\mathcal{G}_i$:

$$2\int dxx[H_1(x,\xi,t) - \frac{1}{3}H_5(x,\xi,t)] = \mathcal{G}_1(t) + \xi^2\mathcal{G}_3(t), \quad (8)$$

$$2\int dxxH_2(x,\xi,t) = \mathcal{G}_5(t), \quad (9)$$

$$2\int dxxH_3(x,\xi,t) = \mathcal{G}_2(t) + \xi^2\mathcal{G}_4(t), \quad (10)$$

$$-4\int dxxH_4(x,\xi,t) = \xi\mathcal{G}_6(t), \quad (11)$$

$$\int dxxH_5(x,\xi,t) = -\frac{t}{8M_D^2}\mathcal{G}_6(t) + \frac{1}{2}\mathcal{G}_7(t). \quad (12)$$

For $t = 0$ then one finds the sum rule relation between the deuteron GPD H_2, and the angular momentum $J_{q,g}$, shown in Eq. (1):

$$J_{q,g} = \frac{1}{2}\int dxxH_2^{q,g}(x,0,0). \quad (13)$$

This sum rule, which was derived following the same steps as for the spin-1/2 case, is both the main result and the starting point of our paper. We now ask the questions: (i) what is the parton content of H_2, and (ii) can H_2 be extracted from experiment with sufficient accuracy? In order to explain the partonic sharing of angular momentum in the deuteron we start from a picture in terms of bound nucleons. Equation (7) can be written in terms of "quark-nucleus" helicity amplitudes that depend on ξ, t and Q^2 while implicitly convoluting over the unobserved quark and nucleon momenta,

$$C_{\Lambda'\lambda'_q,\Lambda\lambda_q} = \sum_{\lambda_N,\lambda'_N} B_{\Lambda'\lambda'_N,\Lambda\lambda_N} \otimes A_{\lambda'_N\lambda'_q,\lambda_N\lambda_q}, \tag{14}$$

where $A_{\lambda'_N\lambda'_q;\lambda_N,\lambda_q}$ and $B_{\lambda',\lambda'_N;\lambda,\lambda_N}$ are the quark-nucleon [15] and nucleon-deuteron helicity amplitudes, respectively, Λ, λ_N, λ_q, being the deuteron, nucleon, and quark helicities. H_2 can be explicitly evaluated from Eq. (14) using the convolution formalism that was developed in [16], taking care of the angular structure for the deuteron [17]. For $H_2(x,0,0) = H_2$, only the $\{\Lambda',\Lambda\} \equiv \{1,1\},\{0,1\}$ deuteron helicity components contribute [14,17],

$$\begin{aligned} H_2 &= 2\sum_{\lambda_q}\left(C_{1\lambda_q,1\lambda_q} - \frac{1}{\sqrt{2\tau_D}}C_{1\lambda_q,0\lambda_q}\right) \\ &\approx \int_0^{M_D/M} dz f^{\mathbf{1,1}}(z) H_N(x/z,0,0) \\ &\quad + f^{\mathbf{0,1}}(z) E_N(x/z,0,0), \end{aligned} \tag{15}$$

where $H_N = H_u + H_d$, $E_N = E_u + E_d$ are the isoscalar nucleon GPDs, the kinematical variables, $x = k^+/(P_D^+/2)$, $z = p^+/(P_D^+/2)$, $p = |\mathbf{p}|$, $\tau_D = (t_0 - t)/2M_D^2$, with $t_0 = -4\xi^2 M_D^2/(1-\xi^2)$, involve the quark, nucleon and deuteron four-momenta, k_μ, p_μ, and $P_{D,\mu}$, respectively,

$$f^{\mathbf{1,1}}(z) = 2\pi M \int_{p_{\min}(z)}^{\infty} dp\, p \sum_{\lambda} \chi_{\mathbf{1}}^{*\lambda'_{N_1}\lambda_{N_2}}(z,p)\chi_{\mathbf{1}}^{\lambda_{N_1}\lambda_{N_2}}(z,p), \tag{16a}$$

$$f^{\mathbf{0,1}}(z) = 4\pi M \int_{p_{\min}(z)}^{\infty} dp\, p \sum_{\lambda} \chi_{\mathbf{0}}^{*\lambda'_{N_1}\lambda_{N_2}}(z,p)\chi_{\mathbf{1}}^{\lambda_{N_1}\lambda_{N_2}}(z,p), \tag{16b}$$

where λ_{N_1} (λ'_{N_1}) are the initial (returning) nucleons' helicities, λ_{N_2} is the spectator nucleon one, the sum index is $\lambda = \{\lambda_{N_1}, \lambda'_{N_1}, \lambda_{N_2}\}$; $\chi_\Lambda^{\lambda_{N_1},\lambda_{N_2}}(z,p)$ is the deuteron wave function [18,19],

$$\begin{aligned} \chi_\Lambda^{\lambda_{N_1},\lambda_{N_2}}(z,p) &= \mathcal{N} \sum_{L,m_L,m_S} \begin{pmatrix} j_1 & j_2 & 1 \\ \lambda_{N_1} & \lambda_{N_2} & m_S \end{pmatrix} \\ &\quad \times \begin{pmatrix} L & S & J \\ m_L & m_S & \Lambda \end{pmatrix} Y_{Lm_L}\left(\frac{\mathbf{p}}{p}\right) u_L(p). \end{aligned} \tag{17}$$

In Eq. (17), $j_1 = j_2 = 1/2$, $S = J = 1$; Y_{Lm_L} depends on $\cos\theta = [M(1-z) - E]/p$, M being the nucleon mass and E the deuteron's binding energy, which is consistent with the formalism for describing deep inelastic processes from nuclear targets [20] in the approximation where the quarks' $k_\perp$ dependence is trivially integrated over, and no off-shell effects are considered [16].

Our results are shown in Figs. 1 and 2. In Fig. 1 we present the proton u and d quarks components of both the total angular momentum density (upper panel), and the orbital angular momentum density (lower panel),

$$L_q(x) = J_q(x) - \frac{1}{2}\Delta q(x), \tag{18}$$

$\Delta q(x)$ being the quark polarized density, and $J_q(x)$ being the integrand in Eq. (2). Both the unpolarized and polarized u and d quarks GPDs used in the calculation are from the parametrization of Ref. [15]. The importance of perturbative QCD evolution is evident from the comparison of results at an initial low scale used e.g. in spectator models, $Q^2 = \mu^2 \approx 0.1$ GeV^2, and evolved to $Q^2 = 4$ GeV^2 (see discussion in [21]). As a consequence of the Regge behavior of Δq, the OAM density is peaked at low x. Our values for the proton's angular momentum components are: $J_u = 0.286 \pm 0.011$, $J_d = -0.049 \pm 0.007$, $L_u = -0.104 \pm 0.087$, $L_d = 0.088 \pm 0.031$ at $Q^2 = 4$ GeV^2.

The total angular momentum density of quarks in the deuteron is compared to the nucleon one in Fig. 2. The upper panel shows the isoscalar combination, $J_N(x) = J_u(x) + J_d(x)$ at $Q^2 = 4$ GeV^2. In the absence of nuclear effects, i.e. if the deuteron were treated as two independently moving nucleons, in Eq. (15), $f^{11}(z) = f^{01}(z) = \delta(1-z)$, and $H_2 = H + E$. Even including nuclear

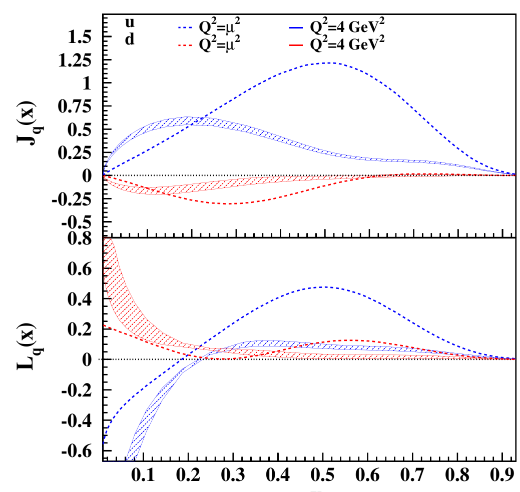

FIG. 1 (color online). Upper panel: Total angular momentum density distributions, J_q, $q = u, d$, calculated using the GPD parametrization of Ref. [15]. Theoretical error bands are included. Lower panel: Orbital angular momentum density distributions, L_q, $q = u, d$, obtained from Eq. (18), using the parametrizations from [15] (J_q) and [24] (Δq). In both panels the dashed lines correspond to the scale $\mu^2 \approx 0.1$ GeV^2 where spectator models are evaluated [9]; the full lines from our fit results are calculated at $Q^2 = 4$ GeV^2. Blue: u quarks; red: d quarks.

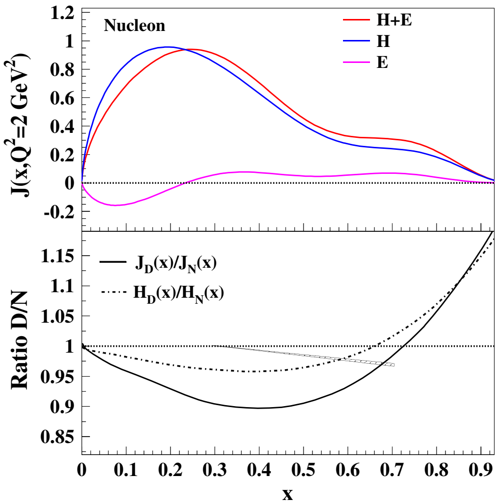

FIG. 2 (color online). Upper panel: Contributions $H + E$, H, and E, to the integrand in the angular momentum sum rule, Eq. (1). All curves were calculated at the scale $Q^2 = 4\ \mathrm{GeV}^2$, using the parametrization from Ref. [15] for the free nucleon. Lower panel: Ratio of deuteron to nucleon contributions to angular momentum, $H_2/(H_N + E_N) \equiv J_D(x)/J_N(x)$ (full curve), calculated using Eq. (15) for the deuteron plotted along with the ratio of unpolarized deuteron to nucleon deep inelastic structure functions H_D/H_N (dashes). The small hatched area represents the experimental results from Ref. [25]. Red: $H + E$; blue: H; magenta: E.

effects, the deuteron angular momentum is dominated by the GPD H. The separate dependences of the various components in the deuteron, and their impact on angular momentum are illustrated in the lower panel of Fig. 2, representing the ratio of the nuclear to nucleon contributions, H_D/H_N, for the unpolarized deep inelastic structure functions, and $H_2/(H_N + E_N) \equiv J_D(x)/J_N(x)$. As in the forward case [22], we find that the distinct angular dependence of the D-wave component plays a nontrivial role (more details will be given in [14]), producing a most striking effect through the GPD E. Its impact is however suppressed. A similar effect also can be shown for $H_5(x, 0, 0) \equiv b_1$, in agreement with the model calculations of [22].

How does this affect the spin sum rule? On one side, in a deuteron target, we observe that the angular momentum is dominated by the GPD H. If the nuclear effects were found to be small, as predicted within a "standard" nuclear model—nucleons bound by exchanged mesons—the deuteron target would provide an easier access to total angular momentum. On the other side, any deviation from the standard nuclear model predictions presented here would signal a different origin of OAM, perhaps related to gluon components, and would therefore be extremely interesting. The question of whether the quarks' OAM can actually be measured for a deuteron target is therefore mandatory. While observables were presented in [23] that contain several deuteron GPDs, none of them is sensitive to H_2. Here we suggest the measurement of the deuteron target transverse spin asymmetry, A_{UT}, which we derive in terms of GPDs as

$$A_{UT} \approx -\frac{4\sqrt{\tau_0}}{\Sigma}\Im m \times\left[\mathcal{H}_1^*\mathcal{H}_5 + \left(\mathcal{H}_1^* + \frac{1}{6}\mathcal{H}_5^*\right)(\mathcal{H}_2 - \mathcal{H}_4)\right], \quad (19)$$

where $\tau_0 = \tau(\xi = 0)$, Σ is the sum of the transversely polarized target cross sections, and $\mathcal{H}_i$, are the Compton form factors for the corresponding GPDs. One can see that the term containing $\mathcal{H}_2$ should dominate the asymmetry, given the expected smallness of $\mathcal{H}_5$ [10,17].

In conclusion, we analyzed the question of OAM in a spin-one hadronic system. We derived a sum rule whereby the second moment of the GPD H_2 gives the total angular momentum, H_2 being the same GPD whose first moment gives the magnetic moment. Nuclear effects evaluated within a standard model for the deuteron give $H_2 \approx H + E$; that is, the quarks' angular momenta in the deuteron, and hence their OAM, are predicted to be similar to the sum of the neutron plus proton taken alone. This cancellation is consistent with the smallness of the deuteron magnetic moment, reflecting the approximate cancellation between the proton and neutron magnetic moments. If found in experiment, deviations from this standard behavior, which is calculable to high precision and under control, could be a signal of other degrees of freedom, such as six quark components or $k_\perp$-dependent re-interactions beyond the collinear convolution considered here. In either situation studying spin-one hadronic systems might shed light on the elusive gluon angular momentum components. Finally, we showed that measuring angular momentum in the deuteron can be at reach in future experimental facilities with high enough energy and luminosity, through transverse spin observables.

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