Nucleon structure including high Fock states in AdS/QCD

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We present a detailed analysis of nucleon electromagnetic and axial form factors in a holographic soft-wall model. This approach is based on an action which describes hadrons with broken conformal invariance and incorporates confinement through the presence of a background dilaton field. For $N_c = 3$ we describe the nucleon structure in a superposition of a three-valence quark state with high Fock states including an adjustable number of partons (quarks, antiquarks and gluons) via studying the dynamics of 5D fermion fields of different scaling dimension in anti-de Sitter space. According to the gauge/gravity duality the 5D fermion fields of different scaling dimension correspond to the Fock state components with a specific number of partons. In the present application we restrict to the contribution of 3, 4 and 5 parton components in the nucleon Fock state. With a minimal number of free parameters (dilaton scale parameter, mixing parameters of partial contributions of Fock states, coupling constants in the effective Lagrangian) we achieve a reasonable agreement with data for the nucleon form factors.

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I. INTRODUCTION

Based on the gauge/gravity duality [1], a class of AdS/QCD approaches which model QCD by using methods of extradimensional field theories formulated in anti-de Sitter (AdS) space, was recently successfully developed for describing the phenomenology of hadronic properties (for a recent review see e.g., Ref. [2]). One of the popular formalisms of this kind is the "soft-wall" model [3–24], which uses a soft IR cutoff in the fifth dimension. This procedure can be introduced in the following ways: (i) as a background field (dilaton) in the overall exponential of the action ("dilaton" soft-wall model), (ii) in the warping factor of the AdS metric ("metric" soft-wall model), and (iii) in the effective potential of the action. In Ref. [24] we showed that these three ways of proceeding are equivalent to each other via a redefinition of the bulk fields and by inclusion of extra effective potentials in the action. In our opinion, the "dilaton" form of the soft-wall model is more convenient in performing the calculations.

Applications of the soft-wall model to baryon physics have been worked out in Refs. [9,15,18,20,22,24,25], where the mass spectrum of light and heavy baryons, and electromagnetic and gravitational form factors have been calculated. We should stress that during the last few years significant progress in the understanding of baryon structure using methods of AdS/QCD has been achieved [20,26–32]. In particular, different types of hard-wall models have been suggested and developed in Refs. [20,26–28]. Solitonic approaches, where stable solitons arise from an effective mesonic action which are 5D analogues of 4D skyrmions, have been suggested in Ref. [29]. Direct derivations of holographic solitonic approaches for baryons from string theories have been proposed in Refs. [30,31]. In reference to the 5D soliton AdS/QCD models developed in Refs. [29–31] we view our approach as an effective or phenomenological framework describing baryons in terms of fermion fields. As stressed in Ref. [27] this is not in contradiction with basic principles of QCD, because in 4D QCD the baryons can be described as skyrmions of the chiral meson Lagrangian or equivalently in terms of separate fermion fields coupled to mesons. In our approach the fermion bulk fields are characterized by the 5D mass μ (scaling dimension), which is holographically dual to N—the number of partons in baryons. Both quantities scale in the large N_c expansion as $\mu \sim N \sim N_c$, which means that the baryon is a bound state of N_c quarks. This is consistent with large N_c QCD. On the other hand, keeping in mind that in QCD the number of colors is equal to $N_c = 3$, we in physical applications identify the AdS fermion field of lowest dimension with the baryons containing three quarks. We do not restrict to the three-valence quark picture of baryons and also include higher Fock states involving nonvalence degrees of freedom. The latter are dual to the AdS fermion fields of higher dimension.

Here we present a detailed analysis of the nucleon electromagnetic form factors in a holographic soft-wall model considering the inclusion of higher-dimensional fermion fields. Thus high Fock state contributions are holographically incorporated in the nucleon. This novel approach is based on an action which describes hadrons with broken conformal invariance and which incorporates

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confinement through the presence of a background dilaton field. Notice that the role of higher Fock components in the pion, in the context of holographic QCD, was considered before in Refs. [19,33]. In particular, two Fock components ($q\bar{q}$ and $q\bar{q}q\bar{q}$) were included in the expansion of the pion wave function, which was in turn used in the calculation of pion electromagnetic and $\gamma\gamma^*\pi^0$ transition form factors. It was argued that the components containing gluons (e.g., $q\bar{q}g$) are absent in the confinement potential.

In our framework for $N_c = 3$ nucleons are considered as a superposition of three-valence quark states and high Fock states including an adjustable number of partons (quarks, antiquarks and gluons) by studying the dynamics of the 5D fermion fields of different scaling dimension in AdS space. According to the gauge/gravity duality the 5D fermion fields of different scaling dimension correspond to Fock state components with a specific number of partons. We can sum the bulk fermion actions with an adjustable 5D fermion mass, which is related to the scaling dimension (or the number of partons in the nucleon). This action is consistent with C-, P- and T-invariance. Also, electromagnetic gauge invariance is fulfilled. Therefore, the main advantage of our approach is that it allows to include dynamically any adjustable number of higher Fock states in the nucleon. For this first time we restrict ourselves to the contribution of 3, 4 and 5 parton components in the nucleon Fock state.

The paper is structured as follows. First, in Sec. II, we briefly discuss the basic notions of the approach. In Sec. III, we consider applications of our approach to the electromagnetic properties of the nucleon. Finally, in Sec. IV, we summarize our results.

II. APPROACH

We consider the propagation of a fermion field $\Psi(x, z)$ with spin $J = 1/2$ in five-dimensional AdS space, which contains the contributions of different twist dimensions. In the language of the AdS/QCD dictionary it corresponds to the inclusion of the three-quark and higher-parton states in the nucleon. For this first time we restrict ourselves to the contribution of $3q$, $3q + g$, $3q + q\bar{q}$ and $3q + 2g$ Fock states, where q, $\bar{q}$ and g denote quark, antiquark and gluon, respectively.

The AdS metric is specified by

$$\begin{aligned} ds^2 &= g_{MN}dx^M dx^N = \eta_{ab}e^{2A(z)}dx^a dx^b \\ &= e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2), \qquad (1) \\ \eta_{\mu\nu} &= \mathrm{diag}(1, -1, -1, -1, -1), \end{aligned}$$

where M and $N = 0, 1, \cdots, 4$ are the space-time (base manifold) indices, $a = (\mu, z)$ and $b = (\nu, z)$ are the local Lorentz (tangent) indices, and g_{MN} and η_{ab} are curved and flat metric tensors, which are related by the vielbein $\epsilon^a_M(z) = e^{A(z)}\delta^a_M$ as $g_{MN} = \epsilon^a_M \epsilon^b_N \eta_{ab}$. Here z is the holographic coordinate, R is the AdS radius, and $g = |\det g_{MN}| = e^{10A(z)}$. In the following we restrict ourselves to a conformal-invariant metric with $A(z) = \log(R/z)$.

The main idea for describing the nucleon in AdS/QCD is based on the correspondence (see detailed discussion in Refs. [27,28,32]) between the spinor fields propagating in the bulk space and the QCD interpolating operators creating the nucleons on the boundary of AdS space. The appropriate boundary conditions for the bulk field on the boundary of AdS space ensure that such correspondence is precise due to the equivalence of the functional integrals of both the boundary and bulk theories. In particular, in the boundary theory (QCD) we define the left- and right-handed chiral doublets of nucleons $\mathcal{O}_L = (p_L, n_L)^T$ and $\mathcal{O}_R = (p_R, n_R)^T$, which are fundamental representations of the chiral $SU_L(2)$ and $SU_R(2)$ subgroups. Since the chiral symmetry of the boundary theory is equivalent to the gauge symmetry in the bulk, we need to introduce the pair of bulk fermion fields $\Psi_\pm(x, z)$, which are holographic analogues of the $\mathcal{O}_{R/L}$ operators. In particular, the bulk fields $\Psi_\pm(x, z)$ contain important information about the baryon structure. On one side, their boundary values (non-normalizable solutions) are analogues of the sources for the QCD interpolating operators, which then via the evaluation of the Euclidean generating functionals produce the correlation functions of QCD operators. On the other side, these fields contain normalizable modes (these are regular and therefore are vanishing on the boundary)—profiles in extra dimension, which correspond to the baryon wave functions or expectation values of QCD operators. In our approach the conformal and chiral symmetries are spontaneously broken via the introduction of the background field (dilaton) $\varphi(z)$ in the effective action. We choose the quadratic dependence of the dilaton on the holographic coordinate z, i.e., $\varphi(z) = \kappa^2 z^2$ with κ being a free scale parameter, which scales as $\mathcal{O}(\sqrt{N_c})$ in the large N_c expansion. In particular, later we show that the nucleon (baryon) mass is proportional to the parameter κ, which is consistent with large N_c QCD: $M_N \sim \kappa\sqrt{N_c} \sim N_c$. The dilaton can be considered as the expectation value of the scalar bulk field with dimension 2, which is holographically dual to the dimension-2 gluon operator A^2_μ. Therefore, κ^2 is related to the vacuum expectation value (VEV) $\langle\alpha_s A^2_\mu\rangle \sim N_c$ and scales as $\kappa \sim \sqrt{N_c}$. Note, the dimension-2 gluon operator A^2_μ has been discussed in the literature (see e.g., Refs. [34–38]). The interpretation of the dilaton as the quantity dual to the condensate of the dimension-2 operator has been done in the framework of the soft-wall model [8] where the dilaton was introduced in the warping factor, breaking the conformal-invariant background metric. The main advantage of the dilaton with quadratic profile is the possibility to produce linear Regge-like trajectories for hadron masses. On the other hand, a quadratic form of the dilaton profile is not unique. For example, in the Liu-Tseytlin model (a type of top-down AdS/QCD approach) [39], the conformal invariance is violated by the dilaton, taken in the form $e^{\varphi(z)} = 1 + qz^4$. The parameter q, according to the AdS/QCD dictionary [40], is related to the matrix element of a QCD operator: in particular, the scalar $\langle \alpha_s G^2_{\mu\nu}\rangle \sim N_c$ and pseudoscalar $\langle \alpha_s G_{\mu\nu}\tilde{G}_{\mu\nu}\rangle \sim N_c$ gluon condensates.

An additional source for the breaking of chiral and conformal symmetries is the coupling of the $\Psi_+(x, z)$ and $\Psi_-(x, z)$ fields, which is an essential basic block of hard-wall AdS/QCD approaches. In latter models the conformal invariance is broken by the introduction of an IR brane, cutting the AdS geometry in the z direction. In addition the couplings of the $\Psi_+(x, z)$ and $\Psi_-(x, z)$ fields with the scalar bulk field of dimension-3 are included. Due to the existence of a VEV of this scalar field, the chiral and conformal invariances are broken. In our approach such a mechanism could in principle be included; however it is a higher-order effect since it is generated by operators of dimension higher than 2. In particular, an extra power of the holographic coordinate gives the extra power of $1/\kappa$, which scales as $1/\sqrt{N_c}$. In Appendix A we explicitly demonstrate how the coupling between $\Psi_+(x, z)$ and $\Psi_-(x, z)$ fields modifies our formalism. In the following consideration (including the physical applications) we neglect such a coupling.

The relevant AdS/QCD action for the description of the nucleon electromagnetic and axial form factors is constructed in terms of the fermion fields $\Psi_{\pm,\tau}(x, z)$ with spin $J = 1/2$ and scaling dimension τ (the isospin index corresponding to the proton and neutron components is suppressed for simplicity), the vector field $V_M(x, z)$ with spin $J = 1$ (holographic analogue of the electromagnetic field) and the axial field $A_M(x, z)$ (holographic analogue of the axial isovector field) [18,20,22,24]:

$$
\begin{aligned}
S &= \int d^4x dz \sqrt{g} e^{-\varphi(z)} \{\mathcal{L}_\Psi(x, z) + \mathcal{L}_{V+A}(x, z) + \mathcal{L}_{\rm int}(x, z)\},\\
\mathcal{L}_\Psi(x, z) &= \sum_{i=+,-}\sum_\tau c_\tau \bar{\Psi}_{i,\tau}(x, z)\hat{\mathcal{D}}_i(z)\Psi_{i,\tau}(x, z),\\
\mathcal{L}_{V+A}(x, z) &= -\frac{1}{4}V_{MN}(x, z)V^{MN}(x, z) - \frac{1}{4}A_{MN}(x, z)A^{MN}(x, z),\\
\mathcal{L}_{\rm int}(x, z) &= \sum_{i=+,-}\sum_\tau c_\tau \bar{\Psi}_{i,\tau}(x, z)\{\hat{\mathcal{V}}_i(x, z) + \hat{\mathcal{A}}_i(x, z)\}\Psi_{i,\tau}(x, z),
\end{aligned}
\tag{2}
$$

where

$$
\begin{aligned}
\hat{\mathcal{D}}_\pm(z) &= \frac{i}{2}\Gamma^M \overleftrightarrow{\partial}_M \mp (\mu + U_F(z)),\\
\hat{\mathcal{V}}_\pm(x, z) &= Q\Gamma^M V_M(x, z) \pm \frac{i}{4}\eta_V[\Gamma^M, \Gamma^N]V_{MN}(x, z) \pm g_V \tau_3 \Gamma^M i\Gamma^z V_M(x, z),\\
\hat{\mathcal{A}}_\pm(x, z) &= \frac{\tau_3}{2}\Big(\mp\Gamma^M A_M(x, z) + \frac{i}{4}\eta_A[\Gamma^M, \Gamma^N]A_{MN}(x, z) + g_A\Gamma^M i\Gamma^z A_M(x, z)\Big).
\end{aligned}
\tag{3}
$$

Here $F_{MN} = \partial_M F_N - \partial_N F_M$ $(F = V, A)$ is the stress tensor of the vector (axial) field, $Q = \mathrm{diag}(1, 0)$ is the nucleon charge matrix, $\tau_3 = \mathrm{diag}(1, -1)$ is the Pauli isospin matrix, $A\overleftrightarrow{\partial}B \equiv A(\partial B) - (\partial A)B$, $\varphi(z) = \kappa^2 z^2$ is the dilaton field with κ being a free scale parameter. $\Gamma^M = \epsilon^M_a \Gamma^a$ and $\Gamma^a = (\gamma^\mu, -i\gamma^5)$ are the five-dimensional Dirac matrices (we use the chiral representation for the γ^μ and γ^5 matrices; see details in Appendix B). Note that the non-Abelian part of the action is irrelevant for the results predicted in the paper: the mass spectrum, the electromagnetic and axial isovector form factors of nucleons. The quantity μ is the bulk fermion mass related to the scaling dimension τ as $m = \mu R = \tau - 3/2$. Notice that the scaling dimension of the AdS fermion field is holographically identified with the scaling dimension of the baryon interpolating operator $\tau = N + L$, where N is the number of partons in the baryon and $L = \max|L_z|$ is the maximal value of the z component of the quark orbital angular momentum in the light-front wave function [4,7]. In the following we restrict to the ground state of nucleons with $L = 0$. $U_F(z) = \varphi(z)/R$ is the dilaton field dependent effective potential. Its presence is necessary due to the following reason. The form of the potential $U_F(z)$ is constrained in order to get solutions of the equations of motion (EOMs) for the fermionic Kaluza-Klein (KK) modes of left and right chirality, and to have the correct asymptotics of the nucleon electromagnetic form factors at large Q^2 [18,20,22].

Notice that the fermion masses m and effective potentials $U_F(z)$ corresponding to the fields Ψ_+ and Ψ_- have opposite signs according to the P-parity transformation (see details in Appendix B). In particular, the absolute sign of the fermion mass is related to the chirality of the boundary operator [27,28]. According to our conventions the QCD operators $\mathcal{O}_R$ and $\mathcal{O}_L$ have positive and negative chirality, and therefore the mass terms of the bulk fields Ψ_+ and Ψ_- have absolute signs "plus" and "minus," respectively. In Refs. [27,28] a different convention for left- and right-handed Weyl spinors was used, which is of course irrelevant for observable properties. In addition to the minimal coupling of the fermion with the vector and the axial fields, we also include other possible (nonminimal) couplings. In particular we introduce (1) a nonminimal coupling of fermion and vector fields in order to generate the Pauli form factors of the nucleon; and (2) a minimal-type coupling, which is absent in four dimensions, but exists in five dimensions. In Appendix B we explicitly demonstrate that these couplings are consistent with P-, C- and T-parity conservation. We will show that these terms do not renormalize the electric charge of the bulk fields and contribute only to the Q^2 dependence or to the slopes of the Dirac nucleon form factors. The coupling g_V is a free parameter which is not constrained by gauge invariance or discrete symmetries (P-, C- or T-parity conservation). We will fix these terms by improving the description of the electromagnetic nucleon radii. The diagonal matrix $\eta_V = \mathrm{diag}\{\eta_V^p, \eta_V^n\}$ contains the coupling constrained by the anomalous magnetic moments $k_{p,n}$ of the nucleons ($k_p = 1.793$ and $k_n = -1.913$ are given in units of nucleon magnetons or n.m.) as $\eta_V^{p,n} \sim k_{p,n} \cdot \kappa/m_N$ where m_N is the nucleon mass.

In the case of the axial field we additionally include (1) the nonminimal coupling of fermion and axial fields, which does not renormalize the axial charge, but gives a nontrivial contribution to the Q^2 dependence and to the slope of the corresponding axial isovector form factor of the nucleon; (2) the axial-type coupling proportional to the nucleon charge, which defines the leading contribution to the isovector axial form factor of the nucleon.

The fields Ψ_τ describe the AdS fermion field with different scaling dimension τ, which in the large N_c expansion scales as $\tau \sim N_c$. Restricting to a finite number of colors $N_c = 3$, we use $\tau = 3, 4, 5$, etc. In this paper we restrict to the three leading contributions $\tau = 3$, 4 and 5. According to the AdS/QCD dictionary the fermion field $\Psi_{\tau=3}$ is the holographic analogue of the nucleon interpolating operator with twist dimension 3, which means that the corresponding nucleon Fock state contains three valence quarks. The fermion field $\Psi_{\tau=4}$ effectively models the nucleon operator with twist-4 (the corresponding Fock state contains 4 partons—3 valence quarks plus a gluon field). Finally, the fermion field $\Psi_{\tau=5}$ models the nucleon operators with twist-5. The corresponding Fock states contain 5 partons: (1) 3 valence quarks plus a $q\bar{q}$ pair of sea quarks or (2) 3 valence quarks plus 2 gluons. Therefore, the coefficients c_τ are a set of parameters which take into account the mixing of AdS fermion fields with different scaling dimension τ. The set of mixing parameters c_τ is constrained by the correct normalization of the kinetic term of the four-dimensional spinor field and by charge conservation as $\sum_\tau c_\tau = 1$ (see details below). In the consideration of the vector (axial) field we apply the axial gauge $V(A)_z(x, z) = 0$.

In Figs. 1–3 we give an illustration for the inclusion of twist-3 (Fig. 1), twist-4 (Fig. 2) and twist-5 (Fig. 3) partonic Fock states in the description of electromagnetic transition between nucleons. Due to the gauge/gravity duality we identify the respective sets of QCD diagrams to the corresponding vector-current transition matrix elements involving the fermion field of corresponding twist dimension. One should stress that AdS/QCD gives a unique possibility to describe a set of QCD diagrams just by one graph (for each partonic content of the nucleon) and obtain predictions for hadronic observables in analytical form.

Finalizing our discussion of the 5D effective action (2) we would like to point out again that it obeys P-, C- and T-invariance. This action further contains new terms describing the interaction of vector and axial fields with fermions, which were not considered before in the context of AdS/QCD. These new terms do not renormalize the charge (i.e., vanish at $Q^2 = 0$), but they contribute to the Q^2 dependence and the slopes of the corresponding form factors. Their relevance for giving a sufficient description of the data will be shown further on.

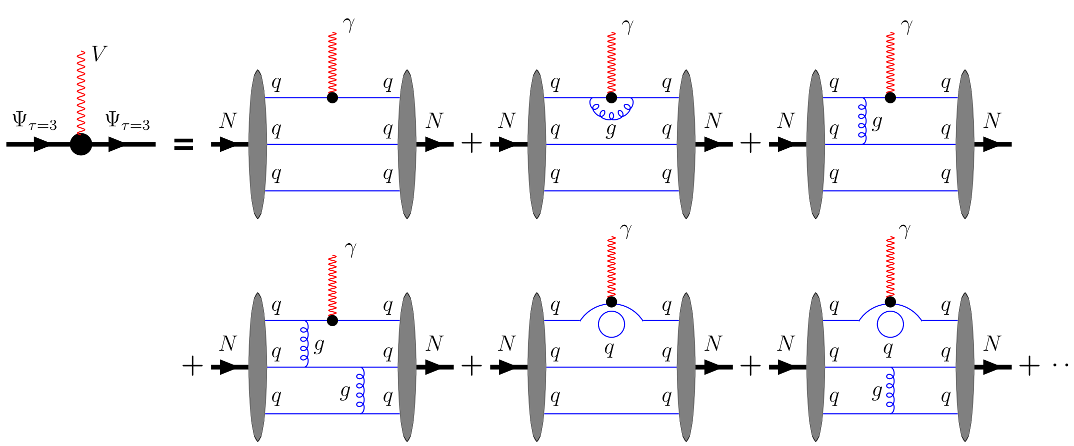

FIG. 1 (color online). Gauge/gravity duality between the vector-current transition matrix element involving twist dimension-3 fermion fields in AdS and the electromagnetic matrix elements involving twist-3 partonic Fock states in nucleons.

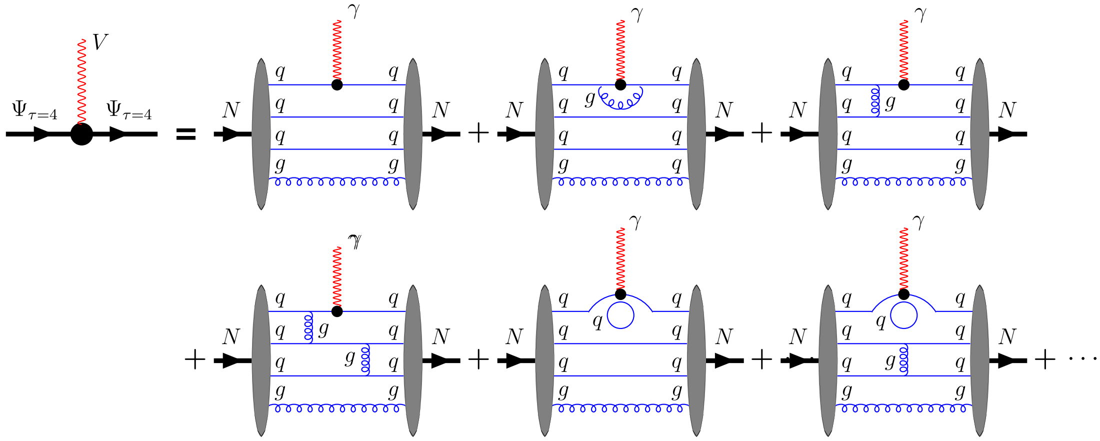

FIG. 2 (color online). Gauge/gravity duality between the vector-current transition matrix element involving twist dimension-4 fermion fields in AdS and the electromagnetic matrix elements involving twist-4 partonic Fock states in nucleons.

A. Mass spectrum

One advantage of the soft-wall AdS/QCD model is that most of the calculations can be done analytically. In a first step, we show how in this approach the baryon spectrum and wave functions are generated. We follow the procedure pursued in Refs. [18,20,22,24]. Dropping the vector and axial fields, and rescaling the fermionic fields as

$$\Psi_{i,\tau}(x,z) = e^{\varphi(z)/2}\psi_{i,\tau}(x,z), \tag{4}$$

we remove the dilaton field from the overall exponential. In terms of the field $\psi_\tau(x,z)$ the modified action in the Lorentzian signature reads as

$$S_0 = \int d^4x dz e^{4A(z)} \sum_{i=+,-}\sum_\tau c_\tau \bar\psi_{i,\tau}(x,z)\Big\{ i\not\partial + \gamma^5\partial_z + 2A'(z)\gamma^5 - \delta_i \frac{e^{A(z)}}{R}(m+\varphi(z))\Big\}\psi_{i,\tau}(x,z), \tag{5}$$

where $\not\partial = \gamma^\mu\partial_\mu$, $\delta_\pm = \pm 1$ and the fermion field $\psi_{i,\tau}(x,z)$ satisfies the following EOM [18,20,22,24]:

$$\Big[i\not\partial + \gamma^5\partial_z + 2A'(z)\gamma^5 \mp \frac{e^{A(z)}}{R}(m+\varphi(z))\Big]\psi_{\pm,\tau}(x,z) = 0. \tag{6}$$

Based on these solutions the fermionic action should be extended by an extra term in the UV boundary (see details in Refs. [20,32]) in order to guarantee the gauge/gravity correspondence—equivalence between the AdS functional integral and the generating functional for correlation functions in QCD.

Next we split the fermion field into left- and right-chirality components

$$\begin{aligned} \psi_{i,\tau}(x,z) &= \psi^L_{i,\tau}(x,z) + \psi^R_{i,\tau}(x,z), \\ \psi^{L/R}_{i,\tau}(x,z) &= \frac{1\mp\gamma^5}{2}\psi_{i,\tau}(x,z), \\ \gamma^5\psi^{L/R}_{i,\tau}(x,z) &= \mp\psi^{L/R}_{i,\tau}(x,z). \end{aligned} \tag{7}$$

and perform a KK expansion for the $\psi^{L/R}_{i,\tau}(x,z)$ fields:

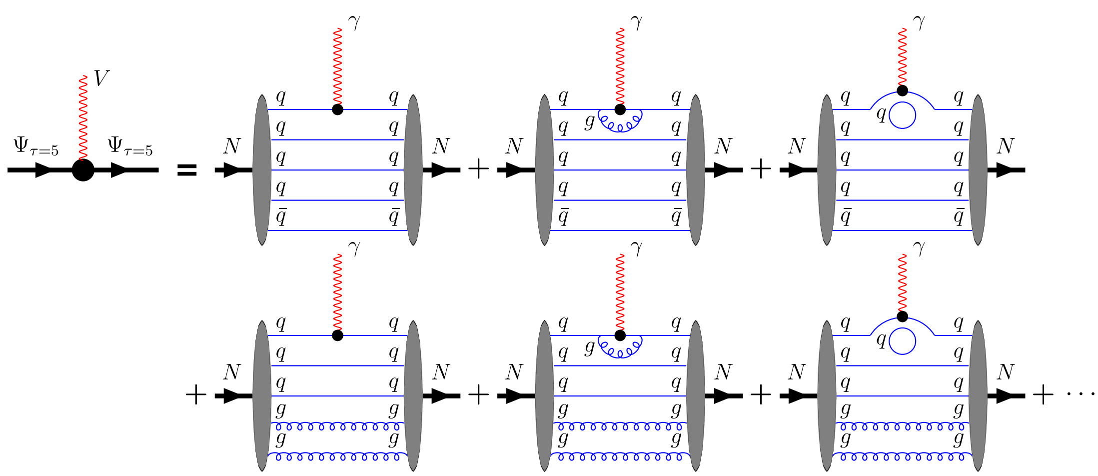

FIG. 3 (color online). Gauge/gravity duality between the vector-current transition matrix element involving twist dimension-5 fermion bulk fields in AdS and the electromagnetic matrix elements involving twist-5 partonic Fock states in nucleons.

$$\psi^{L/R}_{i,\tau}(x,z) = \frac{1}{\sqrt{2}}\sum_n \psi^{L/R}_n(x) F^{L/R}_{i,\tau,n}(z), \tag{8}$$

where $\psi^{L/R}_n(x)$ are the four-dimensional boundary fields (KK modes). These are Weyl spinors forming the Dirac bispinors $\psi_n(x) = \psi^L_n(x) + \psi^R_n(x)$, and $F^{L/R}_{i,\tau,n}(z)$ are the normalizable profile functions. Due to four-dimensional P- and C-parity invariance the bulk profiles are related as (see details in Appendix B)

$$F^R_{\pm,\tau,n}(z) = \mp F^L_{\mp,\tau,n}(z). \tag{9}$$

Using this constraint in the following we use the simplified notations

$$\begin{aligned} F^R_{\tau,n}(z) &\equiv F^R_{+,\tau,n}(z) = -F^L_{-,\tau,n}(z),\\ F^L_{\tau,n}(z) &\equiv F^L_{+,\tau,n}(z) = F^R_{-,\tau,n}(z). \end{aligned} \tag{10}$$

Note that the profiles $F^{L/R}_{\tau,n}(z)$ are the holographic analogues of the nucleon wave functions with specific radial quantum number n and twist dimension τ (the latter corresponds to the specific partonic content of the nucleon Fock component), which satisfy the two coupled one-dimensional EOMs [18,20,22]:

$$\left[\partial_z \pm \frac{e^A}{R}(m+\varphi) + 2A'\right] F^{L/R}_{\tau,n}(z) = \pm M_{n\tau} F^{R/L}_{\tau,n}(z). \tag{11}$$

Therefore, our main idea is to find the solutions for the bulk profiles of the AdS field in the z direction, and then calculate the physical properties of hadrons. After straightforward algebra one can obtain the decoupled EOMs:

$$\left[-\partial_z^2 - 4A'\partial_z + \frac{e^{2A}}{R^2}(m+\varphi)^2 \mp \frac{e^A}{R}(A'(m+\varphi)+\varphi') - 4A'^2 - 2A''\right] F^{L/R}_{\tau,n}(z) = M^2_{n\tau} F^{L/R}_{\tau,n}(z). \tag{12}$$

Doing the substitution

$$F^{L/R}_{\tau,n}(z) = e^{-2A(z)} f^{L/R}_{\tau,n}(z) \tag{13}$$

we derive the Schrödinger-type EOM for $f^{L/R}_{\tau,n}(z)$

$$\left[-\partial_z^2 + \frac{e^{2A}}{R^2}(m+\varphi)^2 \mp \frac{e^A}{R}(A'(m+\varphi)+\varphi')\right] f^{L/R}_{\tau,n}(z) = M^2_{n\tau} f^{L/R}_{\tau,n}(z). \tag{14}$$

For $A(z) = \log(R/z)$, $\varphi(z) = \kappa^2 z^2$ we get

$$\left[-\partial_z^2 + \kappa^4 z^2 + 2\kappa^2\left(m \mp \frac{1}{2}\right) + \frac{m(m\pm 1)}{z^2}\right] f^{L/R}_{\tau,n}(z) = M^2_{n\tau} f^{L/R}_{\tau,n}(z), \tag{15}$$

where

$$f^L_{\tau,n}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau)}}\kappa^\tau z^{\tau-1/2} e^{-\kappa^2 z^2/2} L_n^{\tau-1}(\kappa^2 z^2), \tag{16}$$

$$f^R_{\tau,n}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}}\kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2} L_n^{\tau-2}(\kappa^2 z^2) \tag{17}$$

and

$$M^2_{n\tau} = 4\kappa^2(n+\tau-1) \tag{18}$$

with

$$\int_0^\infty dz f^{L/R}_{\tau,n_1}(z) f^{L/R}_{\tau,n_2}(z) = \delta_{n_1 n_2}. \tag{19}$$

Here

$$L_n^\tau(x) = \frac{x^{-\tau}e^x}{n!}\frac{d^n}{dx^n}(e^{-x}x^{\tau+n}) \tag{20}$$

are the generalized Laguerre polynomials. In above formulas we substituted $m = \tau - 3/2$.

One can see that the functions $F^{L/R}_{\tau,n}(z) = e^{-2A(z)} f^{L/R}_{\tau,n}(z)$ have the correct scaling behavior for small z

$$F^L_{\tau,n}(z) \sim z^{\tau+3/2}, \qquad F^R_{\tau,n}(z) \sim z^{\tau+1/2} \tag{21}$$

when identified with the corresponding nucleon wave functions with twist τ, and vanish at large z (confinement). Below, in the discussion of the electromagnetic properties of the nucleon, we explicitly demonstrate that the Dirac and Pauli form factors have the correct scaling dependence at large Q^2.

The nucleon mass is identified with the expression

$$M_n = \sum_\tau c_\tau M_{n\tau} = 2\kappa\sum_\tau c_\tau\sqrt{n+\tau-1}. \tag{22}$$

Due to the dilaton the chiral and conformal symmetries are spontaneously broken in our approach. Switching off the dilaton field (which corresponds to the limit $\kappa = 0$) leads to the restoration of the chiral and conformal symmetries. In particular, the masses of the bulk profiles $f^{L/R}_{\tau,n}(z)$ and the nucleon mass vanish in this limit. As we stressed before, the nucleon (baryon) mass is proportional to the parameter κ and this is consistent with large N_c QCD: $M_N \sim \kappa\sqrt{\tau} \sim N_c$, where $\tau \sim N_c$. On the other hand, it is known that the nucleon (baryon) mass is proportional to the quark condensate $|\langle\bar{q}q\rangle|$ (so-called Ioffe formula) [41]. It means that our soft-wall model indicates that there could be a relation between condensates of the dimension-2 gluon operator $\mathcal{O}_{A^2} = A^2_\mu$ and dimension-3 scalar quark-antiquark operator $\mathcal{O}_{\bar{q}q} = \bar{q}q$. As we show in Appendix A the contribution of the condensate of the dimension-3 scalar bulk field into the nucleon mass is suppressed by one order of N_c. It means that in the dilaton type of the soft-wall model the dilaton gives the leading contribution to the spontaneous breaking of chiral symmetry (and therefore to the nucleon mass) in comparison with the dimension-3 scalar bulk field.

Integration over the holographic coordinate z, with the use of the normalization condition (19) for the profile functions $f^{L/R}_{\tau,n}(z)$, gives a four-dimensional action for the fermion field $\psi_n(x) = \psi^L_n(x) + \psi^R_n(x)$:

$$S_0 = \sum_n \int d^4x \bar{\psi}_n(x)[i\not\partial - M_n]\psi_n(x). \tag{23}$$

This last equation is a manifestation of the gauge-gravity duality. It explicitly demonstrates that effective actions for conventional hadrons in four dimensions can be generated from actions for bulk fields propagating in five-dimensional AdS space. The effect of the extra dimension is encoded in the hadronic mass squared (in our case in the nucleon mass M_n, where n is the radial quantum number), which is the superposition of the solutions of the Schrödinger equation (14) for the KK profiles in the extra dimension. Notice that the constraint $\sum_\tau c_\tau = 1$ for the mixing parameters c_τ was essential in order to get the correct normalization of the kinetic term $\bar{\psi}_n(x)i\not\partial\psi_n(x)$ of the four-dimensional spinor field.

B. Electromagnetic structure of nucleons

The nucleon electromagnetic form factors F^N_1 and F^N_2 ($N = p$, n correspond to proton and neutron) are conventionally defined by the matrix element of the electromagnetic current as

$$\langle p'|J^\mu(0)|p\rangle = \bar{u}(p')[\gamma^\mu F^N_1(t) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F^N_2(t)]u(p), \tag{24}$$

where $q = p' - p$ is the momentum transfer and $t = q^2$; m_N is the nucleon mass; and F^N_1 and F^N_2 are the Dirac and Pauli form factors, which are normalized to the electric charge e_N and anomalous magnetic moment k_N of the corresponding nucleon: $F^N_1(0) = e_N$ and $F^N_2(0) = k_N$.

In our approach the nucleon form factors are generated by the action

$$S^V_{\rm int} = \int d^4x dz \sqrt{g} e^{-\varphi(z)} \mathcal{L}^V_{\rm int}(x, z) \tag{25}$$

containing the minimal and nonminimal couplings of fermion and vector AdS fields. The expressions for the Dirac and Pauli nucleon form factors are given by

$$\begin{aligned} F^p_1(Q^2) &= C_1(Q^2) + g_V C_2(Q^2) + \eta^p_V C_3(Q^2),\\ F^p_2(Q^2) &= \eta^p_V C_4(Q^2),\\ F^n_1(Q^2) &= -g_V C_2(Q^2) + \eta^n_V C_3(Q^2),\\ F^n_2(Q^2) &= \eta^n_V C_4(Q^2), \end{aligned} \tag{26}$$

where $Q^2 = -t$ and $C_i(Q^2)$ are the structure integrals:

$$\begin{aligned} C_1(Q^2) &= \frac{1}{2}\int_0^\infty dz V(Q,z)\sum_\tau c_\tau([f^L_\tau(z)]^2 + [f^R_\tau(z)]^2),\\ C_2(Q^2) &= \frac{1}{2}\int_0^\infty dz V(Q,z)\sum_\tau c_\tau([f^R_\tau(z)]^2 - [f^L_\tau(z)]^2),\\ C_3(Q^2) &= \frac{1}{2}\int_0^\infty dz z\partial_z V(Q,z)\sum_\tau c_\tau([f^L_\tau(z)]^2 - [f^R_\tau(z)]^2),\\ C_4(Q^2) &= 2m_N\int_0^\infty dz z V(Q,z)\sum_\tau c_\tau f^L_\tau(z) f^R_\tau(z). \end{aligned} \tag{27}$$

The functions $f^{R/L}_\tau(z) \equiv f^{R/L}_{\tau,n=0}(z)$ are the bulk profiles of fermions with $n = 0$ (corresponding to the ground-state nucleon with radial quantum number $n = 0$) found in the previous subsection:

$$f^L_\tau(z) = \sqrt{\frac{2}{\Gamma(\tau)}}\kappa^\tau z^{\tau-1/2} e^{-\kappa^2 z^2/2}, \tag{28}$$

$$f^R_\tau(z) = \sqrt{\frac{2}{\Gamma(\tau-1)}}\kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2}. \tag{29}$$

$V(Q, z)$ is the bulk-to-boundary propagator of the transverse massless vector bulk field (the holographic analogue of the electromagnetic field) defined as

$$V_\mu(x, z) = \int \frac{d^4q}{(2\pi)^4} e^{-iqx} V_\mu(q) V(q, z) \tag{30}$$

and obeys the following EOM:

$$\partial_z\left(\frac{e^{-\varphi(z)}}{z}\partial_z V(q, z)\right) + q^2 \frac{e^{-\varphi(z)}}{z} V(q, z) = 0, \tag{31}$$

which is derived from the action

$$S_V = \int d^4x dz \sqrt{g} e^{-\varphi(z)} \mathcal{L}_V(x, z). \tag{32}$$

In the soft-wall model the solution for $V(Q, z)$ is given in analytical form in terms of the Gamma $\Gamma(n)$ and Tricomi $U(a, b, z)$ functions:

$$V(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right). \tag{33}$$

The bulk-to-boundary propagator $V(Q, z)$ obeys the normalization condition $V(0, z) = 1$ consistent with gauge invariance and fulfills the following ultraviolet (UV) and infrared (IR) boundary conditions:

$$V(Q, 0) = 1, \qquad V(Q, \infty) = 0. \tag{34}$$

The UV boundary condition corresponds to the local (structureless) coupling of the electromagnetic field to matter fields, while the IR boundary condition implies that the vector field vanishes at $z = \infty$.

In order to obtain analytical expressions for the functions $C_i(Q^2)$ (see Appendix C) it is convenient to use the integral representation for $V(Q, z)$ introduced in Ref. [11]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{(Q^2/4\kappa^2)} e^{-((\kappa^2 z^2 x/(1-x))}, \tag{35}$$

where the variable x is equivalent to the light-cone momentum fraction [5].

There are a few very important properties of the $C_i(Q^2)$ functions. At $Q^2 = 0$ they are normalized as

$$C_1(0) = 1, \qquad C_2(0) = C_3(0) = 0, \qquad C_4(0) = \frac{2m_N}{\kappa} \sum_\tau c_\tau \sqrt{\tau - 1}. \tag{36}$$

The normalizations of C_i ($i = 1, 2, 3$) are consistent with gauge invariance (the charge normalization for nucleons). In particular, this means that the proton and neutron Dirac form factors are normalized accordingly:

$$F_1^p(0) = 1, \qquad F_1^n(0) = 0. \tag{37}$$

Here we take into account the constraint $\sum_\tau c_\tau = 1$ of the mixing parameters c_τ, which is also essential to get the correct normalization of the kinetic term of the four-dimensional spinor field on the boundary of AdS space. The anomalous magnetic moments of the nucleons $N = p$, n are given by

$$\kappa_N = \eta_V^N C_4(0) = \frac{2\eta_V^N m_N}{\kappa} \sum_\tau c_\tau \sqrt{\tau - 1}. \tag{38}$$

In the analysis of the electromagnetic form factors we will use the dipole formula

$$G_D(Q^2) = \frac{1}{(1 + Q^2/\Lambda^2)^2}, \tag{39}$$

where $\Lambda^2 = 0.71\ \mathrm{GeV}^2$.

C. Axial isovector form factor of nucleons

The nucleon isovector axial form factor $G_A(t)$ is conventionally defined by the matrix element of the axial isovector current as

$$\langle p' | A_3^\mu(0) | p \rangle = \bar{u}(p') \left[\gamma^\mu G_A(t) + \frac{q^\mu}{2m_N} G_P(t) \right] \gamma^5 \frac{\tau_3}{2} u(p), \tag{40}$$

which is normalized to the nucleon axial charge $G_A(0) = g_A$.

In our approach the $G_A(Q^2)$ form factor is generated by the action

$$S_{\mathrm{int}}^A = \int d^4x dz \sqrt{g} e^{-\varphi(z)} \mathcal{L}_{\mathrm{int}}^A(x,z) \tag{41}$$

containing the minimal and nonminimal couplings of fermion and axial-vector AdS fields. The expression for the axial isovector nucleon form factor is given by

$$G_A(Q^2) = g_A D_1(Q^2) + D_2(Q^2) + \eta_A D_3(Q^2), \tag{42}$$

where $D_i(Q^2)$ are the structure integrals:

$$\begin{aligned} D_1(Q^2) &= \frac{1}{2} \int_0^\infty dz A(Q,z) \sum_\tau c_\tau ([f_\tau^L(z)]^2 + [f_\tau^R(z)]^2), \\ D_2(Q^2) &= \frac{1}{2} \int_0^\infty dz A(Q,z) \sum_\tau c_\tau ([f_\tau^L(z)]^2 - [f_\tau^R(z)]^2), \\ D_3(Q^2) &= -\frac{1}{2} \int_0^\infty dz z \partial_z A(Q,z) \sum_\tau c_\tau ([f_\tau^L(z)]^2 + [f_\tau^R(z)]^2). \end{aligned} \tag{43}$$

Now $A(Q,z)$ is the bulk-to-boundary propagator of the transverse massless axial bulk field (the holographic analogue of the axial isovector field). In our approximation it coincides with $V(Q,z)$. The functions $D_i(Q^2)$ at $Q^2 = 0$ are normalized as

$$D_1(0) = 1, \qquad D_2(0) = D_3(0) = 0. \tag{44}$$

As in the case of the C_i functions these results are based on the normalization properties of the bulk profiles $f^{R/L}(z)$ and the constraint condition $\sum_\tau c_\tau = 1$.

Our prediction for the form factor $G_A(Q^2)$ will be compared to the dipole fit formula

$$G_A^D(Q^2) = \frac{g_A}{(1 + Q^2/M_A^2)^2} \tag{45}$$

extracted from neutrino scattering experiments, where $M_A = 1.026 \pm 0.021$ GeV [42].

III. RESULTS

In this section we present the related numerical analysis of nucleon properties: magnetic moments ($\mu_p = 1 + \kappa_p$, $\mu_n = \kappa_n$), electromagnetic and axial radii (r_E^p, $\langle r_E^2 \rangle^n$, r_M^p, r_M^n, r_A), isovector axial and electromagnetic Dirac, Pauli, and Sachs form factors and their ratios in the Euclidean region.

We first want to recall the definitions of the Sachs form factors $G_{E/M}(Q^2)$, the electromagnetic $\langle r_{E/M}^2 \rangle^N$ and isovector axial $\langle r_A^2 \rangle$ radii:

$$\begin{aligned} G_E^N(Q^2) &= F_1^N(Q^2) - \frac{Q^2}{4m_N^2} F_2^N(Q^2), \\ G_M^N(Q^2) &= F_1^N(Q^2) + F_2^N(Q^2), \\ \langle r_E^2 \rangle^N &= -6 \frac{dG_N^E(Q^2)}{dQ^2} \bigg|_{Q^2=0}, \\ \langle r_M^2 \rangle^N &= -\frac{6}{G_M^N(0)} \frac{dG_M^N(Q^2)}{dQ^2} \bigg|_{Q^2=0}, \\ \langle r_A^2 \rangle &= -\frac{6}{G_A(0)} \frac{dG_A(Q^2)}{dQ^2} \bigg|_{Q^2=0}, \end{aligned} \tag{46}$$

with $G_M^N(0) \equiv \mu_N$ and $G_A(0) \equiv g_A$.

The five free parameters κ, c_3, c_4, g_V and η_A are fixed to the values

$$\kappa = 383\ \text{MeV}, \qquad c_3 = 1.25, \qquad c_4 = 0.16, \qquad g_V = 0.3, \qquad \eta_A = 0.5. \tag{47}$$

Note that the parameter c_5 is expressed through c_3 and c_4 as

$$c_5 = 1 - c_3 - c_4 = -0.41. \tag{48}$$

Here the parameters c_3, c_4 are constrained by the nucleon mass. The parameter κ is fixed by the nucleon mass and the electromagnetic radii. The parameters g_V and η_A are fitted by fine-tuning of the neutron electromagnetic and nucleon axial radius, respectively. Notice also that the other parameters are fixed by the magnetic moments and the axial charge of nucleons and should not be counted as free parameters:

$$g_A = 1.270, \qquad \eta_V^p = \frac{\kappa(\mu_p - 1)}{2m_N C_0} = 0.30,$$
$$\eta_V^n = \frac{\kappa \mu_n}{2m_N C_0} = -0.32, \qquad C_0 = \sqrt{2}c_3 + \sqrt{3}c_4 + 2c_5. \tag{49}$$

In Table I we present the results for the nucleon mass and the electroweak properties of nucleons. Results for the nucleon electromagnetic form factors in comparison to known data are shown in Figs. 4–14. In particular, in Figs. 4, 6, and 7 we present the ratios of proton charge and nucleon magnetic form factors to the dipole form factor G_D. In Fig. 5 we present the results for the ratio $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$. In Figs. 8 and 9 we present the prediction for the charge neutron form factor and the ratio $G_E^n(Q^2)/G_M^n(Q^2)$. In Figs. 10 and 11 we present the predictions for the Dirac nucleon form factors multiplied by Q^4. Figures 12 and 13 show the ratios of the Pauli and Dirac form factors of the proton multiplied with Q^2 and with $Q^2/\log^2(Q^2/\Lambda^2)$, where $\Lambda = 0.3$ GeV. Finally, in Fig. 14 we present the predictions for the ratio of the nucleon axial isovector form factor to the dipole form factor $G_A^D(Q^2)$.

We demonstrated that the soft-wall holographic model in the semiclassical approximation reproduces the main features of the electromagnetic structure of the nucleon. In particular, we achieved the following results: the analytical power scaling of the elastic nucleon form factors at large momentum transfers in accordance with quark-counting rules; reproduction of experimental data for magnetic moments and electromagnetic radii.

One can see that with a minimal number of free parameters (five parameters) we obtain a reasonable description of

TABLE I. Mass and electromagnetic properties of nucleons.

Quantity	Our results	Data [42]
m_p (GeV)	0.938 27	0.938 27
μ_p (in n.m.)	2.793	2.793
μ_n (in n.m.)	−1.913	−1.913
g_A	1.270	1.2701
r_E^p (fm)	0.840	0.8768 ± 0.0069
$\langle r_E^2\rangle^n$ (fm^2)	−0.117	−0.1161 ± 0.0022
r_M^p (fm)	0.785	0.777 ± 0.013 ± 0.010
r_M^n (fm)	0.792	$0.862^{+0.009}_{-0.008}$
r_A (fm)	0.667	0.67 ± 0.01

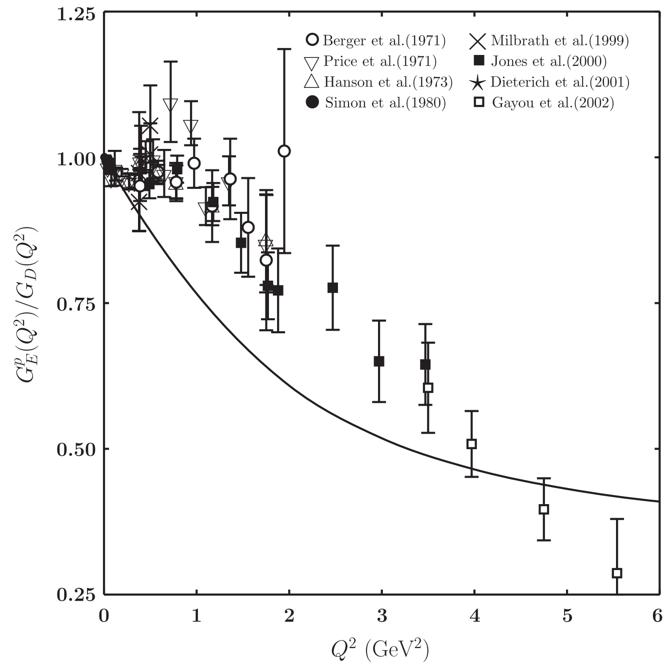

FIG. 4. Ratio $G_E^p(Q^2)/G_D(Q^2)$. Experimental data are taken from Refs. [44–51].

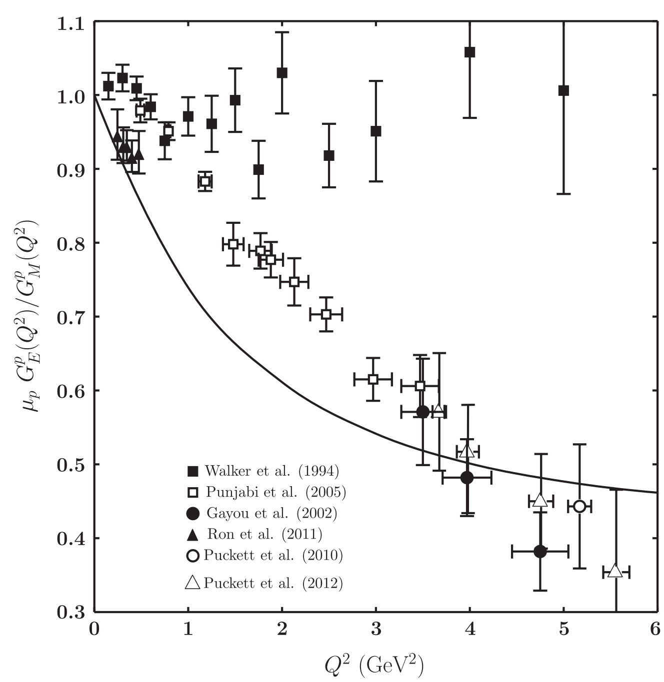

FIG. 5. Ratio $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$ in comparison to the experimental data taken from Refs. [51–55].

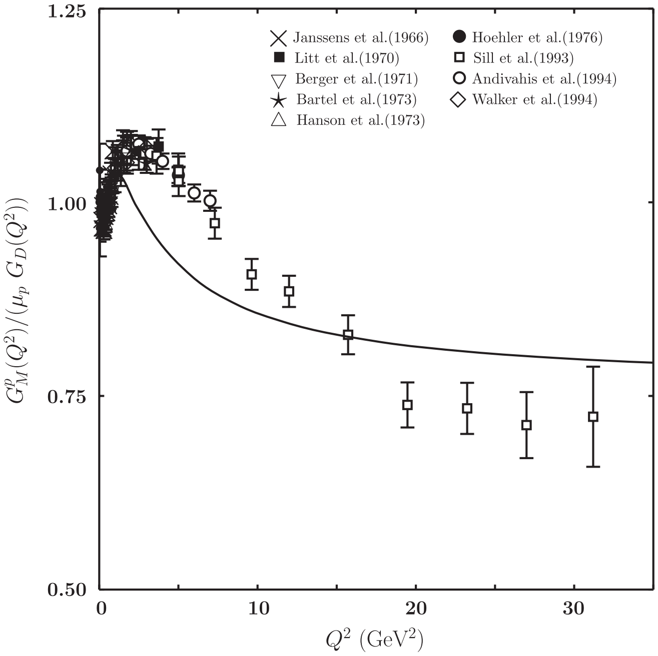

FIG. 6. Ratio $G_M^p(Q^2)/(\mu_p G_D(Q^2))$. Experimental data are taken from Refs. [51–53].

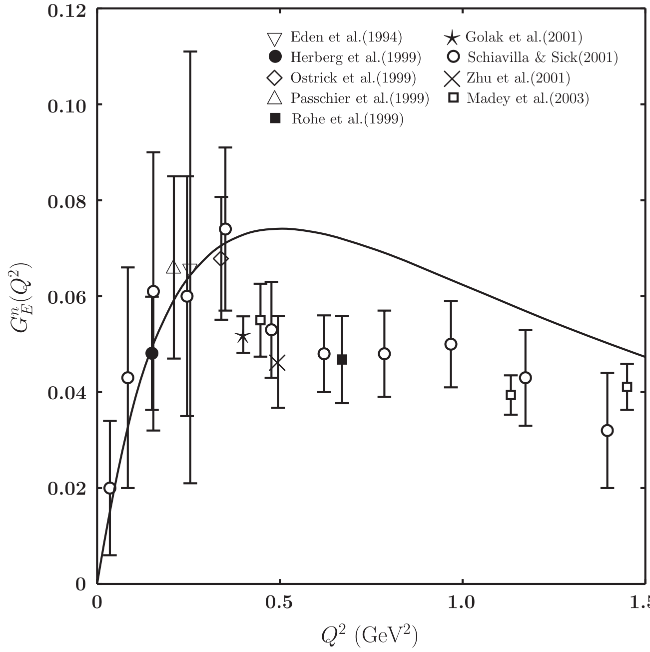

FIG. 8. The charge neutron form factor $G_E^n(Q^2)$. Experimental data are taken from Refs. [66–74].

the nucleon electromagnetic and axial-vector form factors including the correct power scaling at large Q^2. It demonstrates that the soft-wall model successfully describes nucleon structure at any resolution scale. In a next step, one can include effects of quark masses and extend the approach to nucleon resonances, light baryons with higher spins, strange and heavy baryons.

IV. SUMMARY

We presented a soft-wall model which allows us to include higher Fock states in the analysis of the nucleon structure. This approach is based on an action which describes hadrons with broken conformal invariance and incorporates confinement through the presence of a background dilaton field. For $N_c = 3$ the nucleon is described

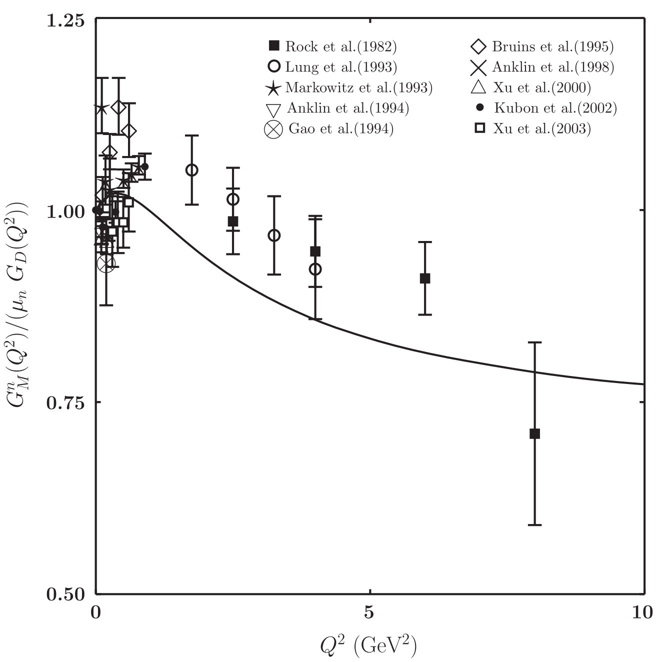

FIG. 7. Ratio $G_M^n(Q^2)/(\mu_n G_D(Q^2))$. Experimental data are taken from Refs. [56–65].

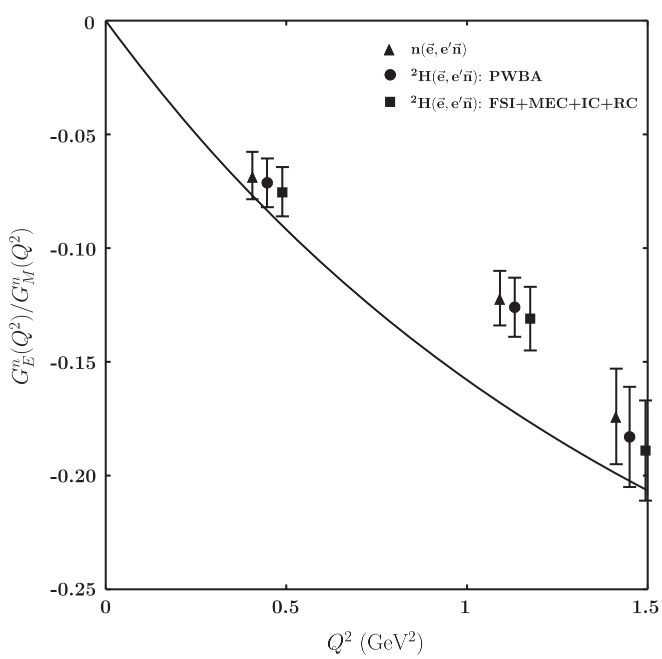

FIG. 9. Ratio $G_E^n(Q^2)/G_M^n(Q^2)$. Experimental data are taken from Refs. [75].

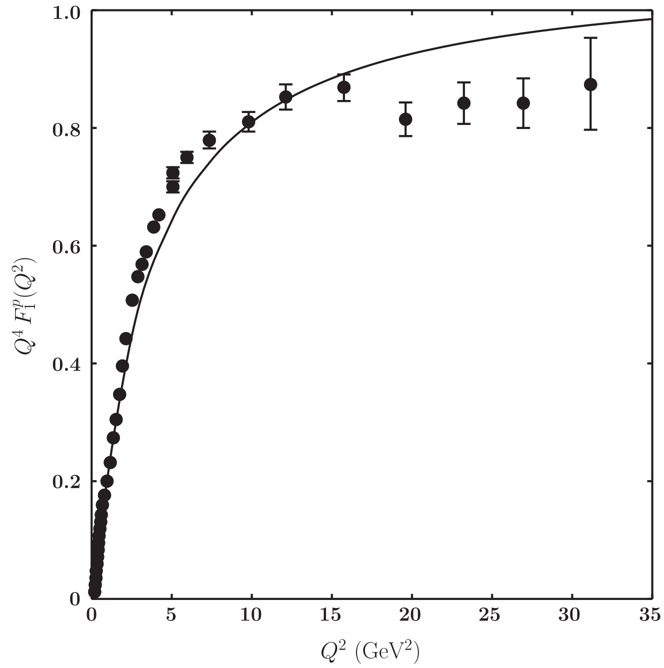

FIG. 10. Proton Dirac form factor multiplied with Q^4. Experimental data are taken from Ref. [76].

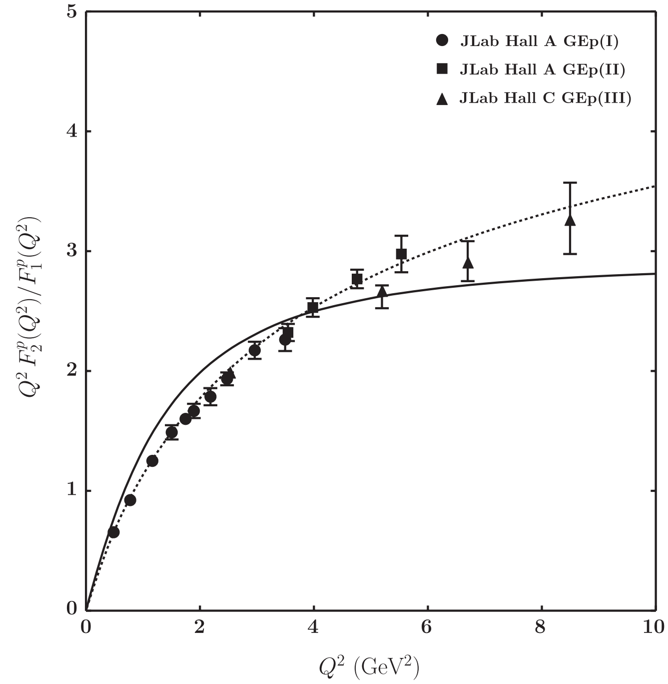

FIG. 12. Results for $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$. The solid line is the prediction of the soft-wall AdS/QCD model and the dashed line is the approximation of data suggested in Ref. [77]. Experimental data are taken from Refs. [48,50,53,55,78–82].

in terms of a superposition of the three-valence quark state with high Fock states with an adjustable number of partons (quarks, antiquarks and gluons). Its structure is determined by studying the dynamics of 5D fermion fields of different scaling dimension in AdS space. According to the gauge/gravity duality the 5D fermion fields of different scaling dimension correspond to Fock state components with a specific number of partons. For the first application we restrict ourselves to the contribution of 3, 4, and 5 parton components in the nucleon Fock state. The role of higher

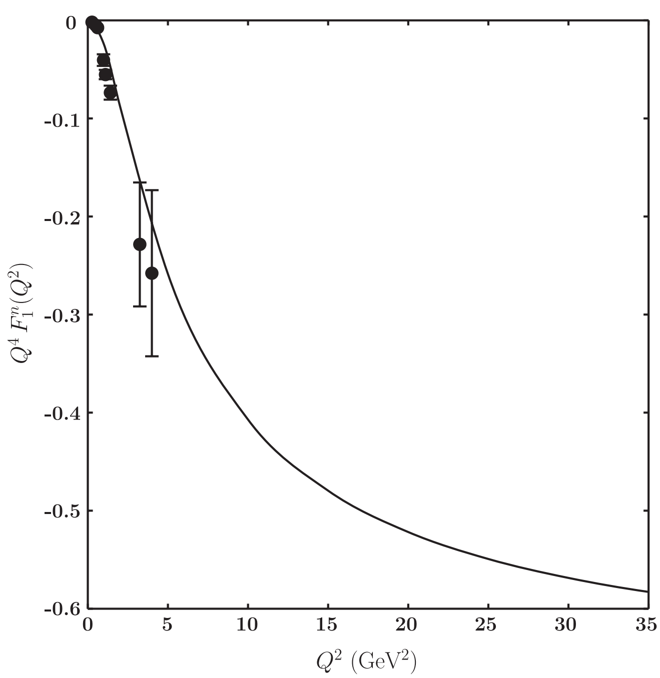

FIG. 11. Neutron Dirac form factor multiplied with Q^4. Experimental data are taken from Ref. [76].

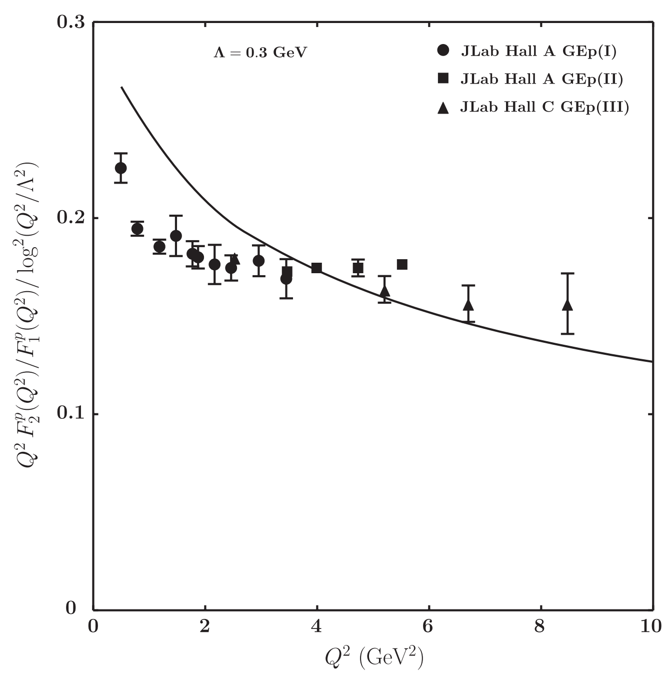

FIG. 13. Results for $Q^2 F_2^p(Q^2)/F_1^p(Q^2)/\log^2(Q^2/\Lambda^2)$ at $\Lambda = 0.3$ GeV. Experimental data are taken from Refs. [48,50,53,55,78–82].

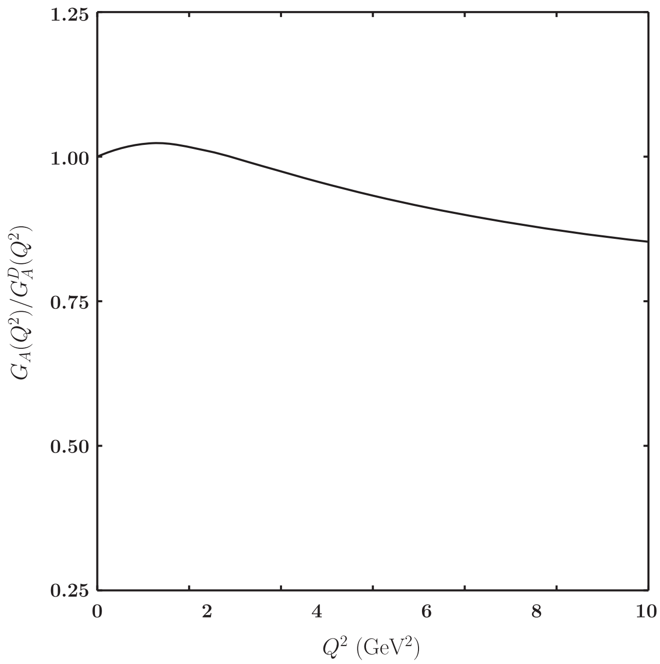

FIG. 14. Results for $G_A(Q^2)/G_A^D(Q^2)$.

Fock components in the context of holographic QCD has been already considered in the case of the pion [19,33]. In particular, two components ($q\bar{q}$ and $q\bar{q}q\bar{q}$) were included in the expansion of the pion wave function, which was then used in the calculation of pion electromagnetic and $\gamma\gamma^*\pi^0$ transition form factors. It was further argued that the components containing gluons (e.g., $q\bar{q}g$) are absent in the confinement potential. In our case the contribution of the twist-4 component containing three constituent quarks and a single gluon is not zero, but is suppressed, which is partially in line with the conclusion of Refs. [19,33]. On the other hand, our mechanism generating the inclusion of higher Fock states is different from the one suggested in Refs. [19,33]. Additionally, the pion and the nucleon are quite different hadronic bound states, and therefore, the role of Fock states containing gluons could be different. We think that this issue requires further investigation.

We presented a detailed analysis of nucleon electromagnetic and axial form factors. With a minimal number of free parameters (dilaton scale parameter, mixing parameters of the partial contributions of Fock states, and a few coupling constants in the effective Lagrangian) we achieved a reasonable agreement with data for the nucleon electromagnetic and axial isovector form factors. Note that all form factors have the correct scaling at large Q^2. As next applications we plan to extend our approach to nucleon resonances (e.g., Roper) and baryons with strangeness. There is also a possibility to study nuclear systems using methods of AdS/QCD via studying dynamics of 5D fields of higher dimensions, which holographically correspond to nuclei with a specific number of nucleons and electrons.

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APPENDIX A: COUPLING OF $\Psi_+(x, z)$ AND $\Psi_-(x, z)$ FERMION FIELDS

Following Ref. [27] we introduce the Yukawa-type coupling of $\Psi_+(x, z)$ and $\Psi_-(x, z)$ fields with the bulk scalar field $X(x, z)$ dual to the dimension-3 quark operator $\mathcal{O}_q = \bar{q}q$:

$$\mathcal{L}_Y(x, z) = -\frac{g}{R}(\bar{\Psi}_-(x, z)X(x, z)\Psi_+(x, z) + \bar{\Psi}_+(x, z)X^\dagger(x, z)\Psi_-(x, z)), \qquad (A1)$$

where g is the coupling constant, which scales in large N_c expansion as $g \sim \sqrt{N_c}$. As originally was shown, the VEV of the scalar bulk field $X_0(z)$ is the linear combination of two solutions (see e.g., Ref. [3]), which for asymptotically small z behaves as

$$X_0(z) \to \frac{1}{2}\hat{m}z + \frac{1}{2}\Sigma z^3, \qquad (A2)$$

where $\hat{m}$ is the current quark mass and $\Sigma = |\langle\bar{q}q\rangle|$ is quark condensate in the chiral limit $\hat{m} \to 0$. On the other hand, in the original soft-wall model the IR asymptotics $z \to \infty$ dictates that Σ is simply proportional to $\hat{m}$ which is in contradiction with QCD. It was suggested to include in the effective action the potential containing higher-order terms in the scalar field to resolve the problem $\Sigma \sim \hat{m}$. Later in Ref. [13] this idea was realized by adding a quartic term $\sim X^4(x, z)$ in the effective action. At the same time in Ref. [43] it was noticed that the scalar operator $\mathcal{O}_q$ and its source J_q can always be rescaled by a constant a

$$\mathcal{O}_q \to a\mathcal{O}_q, \qquad J_q \to J_q/a \qquad (A3)$$

keeping the product $J_q\mathcal{O}_q$ unchanged. Then using the arguments of large N_c QCD it was shown that the constant a must scale as $a \sim 1/\sqrt{N_c}$. Therefore, according to the large N_c QCD the VEV of the scalar field must obey the following expansion:

$$X_0(z) \to \frac{\sqrt{N_c}}{2}\hat{m}z + \frac{1}{2\sqrt{N_c}}\Sigma z^3, \qquad (A4)$$

where $\Sigma \sim N_c$. It means that the contribution of the VEV of the scalar field $gX_0(z)$ scales as $\mathcal{O}(N_c)$. Inclusion of ψ_+ and ψ_- mixing modifies the EOM for the fermion fields (6) as

$$\left[i\not\partial + \gamma^5\partial_z + 2A'(z)\gamma^5 \mp \frac{e^{A(z)}}{R}(m + \varphi(z))\right]\psi_{\pm,\tau}(x,z) = gX_0(z)\psi_{\mp,\tau}(x,z). \tag{A5}$$

After straightforward calculations we get the following EOMs for the $f^{L/R}_{\tau,n}(z)$ profiles:

$$\left[-\partial_z^2 + \frac{e^{2A}}{R^2}(m+\varphi)^2 \mp \frac{e^A}{R}(A'(m+\varphi)+\varphi') \mp \frac{gX_0(z)}{\tilde{M}_{n\tau} \mp gX_0(z)} + g^2X_0^2(z)\right]\tilde{f}^{L/R}_{\tau,n}(z) = \tilde{M}^2_{n\tau}\tilde{f}^{L/R}_{\tau,n}(z). \tag{A6}$$

Here the symbol "tilde" on the top of the solutions $\tilde{f}^{L/R}_{\tau,n}(z)$ and $\tilde{M}_{n\tau}$ means that they differ from the set $f^{L/R}_{\tau,n}(z)$ and $M_{n\tau}$ in the case when we neglect the mixing of Ψ_+ and Ψ_- fermion fields. One can see that inclusion of the $gX_0(z)$ term gives a deviation of the mass spectra from a Regge-like trajectory. Let us estimate the contribution of the $gX_0(z)$ term perturbatively using the solutions obtained for the case of a pure dilaton contribution [see Eq. (16)]. In particular, the nucleon mass shift ΔM_N due to the $gX_0(z)$ term is given by

$$\Delta M_N = \sum_\tau c_\tau \Delta M_{n\tau}, \tag{A7}$$

where

$$\Delta M_{n\tau} = \frac{g}{2}\int_0^\infty \frac{dz}{z} X_0(z)[(f^L_{\tau,n}(z))^2 - (f^R_{\tau,n}(z))^2] = \frac{g}{\sqrt{N_c}}\frac{\Sigma}{4\kappa^2}. \tag{A8}$$

One can see that the term with current quark mass exactly vanishes and $\Delta M_{n\tau}$ does not depend on twist τ. Therefore, using our condition $\sum_\tau c_\tau = 1$ we get for the shift of the nucleon mass

$$\Delta M_N = \frac{g}{\sqrt{N_c}}\frac{\Sigma}{4\kappa^2} \sim N_c^0. \tag{A9}$$

As we stressed before, the contribution of the VEV scalar field with dimension 3 is suppressed in comparison to the dilaton contribution encoding the VEV of the scalar field with dimension 2 by a factor $1/N_c$.

Using a typical value for the quark condensate $\Sigma = (0.225\ \text{GeV})^3$ and the value of $\kappa = 0.383$ GeV we get the estimate of

$$\Delta M_N = \frac{g}{\sqrt{N_c}} 0.02\ \text{GeV}. \tag{A10}$$

Then taking the typical values of the coupling $g \simeq 10$ used in hard-wall approaches [27] we finally get $\Delta M_N \simeq 115$ MeV for $N_c = 3$.

APPENDIX B: *P*-, *C*- AND *T*-PARITY TRANSFORMATIONS OF BULK FIELDS IN ADS SPACE

We use the chiral representation for the four-dimensional Dirac matrices γ^μ and γ^5:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{B1}$$

The $(\frac{1}{2},0)$ left- and $(0,\frac{1}{2})$ right-handed Weyl spinors $\psi^{L/R}(x) = \frac{1\mp\gamma^5}{2}\psi(x)$ are eigenstates of the chirality operator

$$\gamma^5\psi^{L/R} = \mp\psi^{L/R} \tag{B2}$$

and they form the $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ Dirac bispinor $\psi = (\psi^L, \psi^R)^T$.

The *P*-parity transformations of the fermion fields $\psi^{L/R}$, ψ are defined as (here U_P stands for the unitary operator of *P*-parity transformation):

$$\begin{aligned} U_P^{-1}\psi^{L/R}(t,\vec{x})U_P &= \gamma^0\psi^{R/L}(t,-\vec{x}) \quad \text{and} \\ U_P^{-1}\psi(t,\vec{x})U_P &= \gamma^0\psi(t,-\vec{x}), \\ U_P^{-1}\bar{\psi}^{L/R}(t,\vec{x})U_P &= \bar{\psi}^{R/L}(t,-\vec{x})\gamma^0 \quad \text{and} \\ U_P^{-1}\bar{\psi}(t,\vec{x})U_P &= \bar{\psi}(t,-\vec{x})\gamma^0. \end{aligned} \tag{B3}$$

Next we define the *P*-parity transformations of the five-dimensional bulk fields (1/2-fermion, vector and axial fields; in case of 1/2-fermion fields we drop the summation over radial quantum number):

$$\begin{aligned} U_P^{-1}\Psi_{\tau,\pm}(t,\vec{x},z)U_P &= \pm\gamma^0\gamma^5\Psi_{\tau,\mp}(t,-\vec{x},z), \\ U_P^{-1}\bar{\Psi}_{\tau,\pm}(t,\vec{x},z)U_P &= \pm\bar{\Psi}_{\tau,\mp}(t,-\vec{x},z)\gamma^0\gamma^5, \\ U_P^{-1}(V^0(t,\vec{x},z), V^i(t,\vec{x},z), 0)U_P &= (V^0(t,-\vec{x},z), -V^i(t,-\vec{x},z), 0), \\ U_P^{-1}(A^0(t,\vec{x},z), A^i(t,\vec{x},z), 0)U_P &= -(A^0(t,-\vec{x},z), -A^i(t,-\vec{x},z), 0). \end{aligned} \tag{B4}$$

From the above equations we get the following conditions between the bulk profiles of fermion fields:

$$F^R_\pm(z) = \mp F^L_\mp(z). \tag{B5}$$

Using the transformation of bulk fields it is easy to demonstrate that the effective Lagrangian/action of our model is P-parity invariant (some terms transform among themselves). In particular, we get

$$\begin{aligned}
&U_P^{-1}\bar\Psi_{\pm,\tau}(t,\vec x,z)\Psi_{\pm,\tau}(t,\vec x,z)U_P = -\bar\Psi_{\mp,\tau}(t,-\vec x,z)\Psi_{\mp,\tau}(t,\vec x,z),\\
&U_P^{-1}\bar\Psi_{\pm,\tau}(t,\vec x,z)\hat{\mathcal D}_\pm(z)\Psi_{\pm,\tau}(t,\vec x,z)U_P = \bar\Psi_{\mp,\tau}(t,-\vec x,z)\hat{\mathcal D}_\mp(z)\Psi_{\mp,\tau}(t,\vec x,z),\\
&U_P^{-1}\bar\Psi_{\pm,\tau}(t,\vec x,z)\hat{\mathcal V}_\pm(t,\vec x,z)\Psi_{\pm,\tau}(t,\vec x,z)U_P = \bar\Psi_{\mp,\tau}(t,-\vec x,z)\hat{\mathcal V}_\mp(t,-\vec x,z)\Psi_{\pm,\tau}(t,-\vec x,z),\\
&U_P^{-1}\bar\Psi_{\pm,\tau}(t,\vec x,z)\hat{\mathcal A}_\pm(t,\vec x,z)\Psi_{\pm,\tau}(t,\vec x,z)U_P = \bar\Psi_{\mp,\tau}(t,-\vec x,z)\hat{\mathcal A}_\mp(t,-\vec x,z)\Psi_{\pm,\tau}(t,-\vec x,z),
\end{aligned} \tag{B6}$$

and therefore

$$\begin{aligned}
&U_P^{-1}\mathcal L_\Psi(t,\vec x,z)U_P = \mathcal L_\Psi(t,-\vec x,z), \qquad U_P^{-1}\mathcal L_{V+A}(t,\vec x,z)U_P = \mathcal L_{V+A}(t,-\vec x,z),\\
&U_P^{-1}\mathcal L_{\rm int}(t,\vec x,z)U_P = \mathcal L_{\rm int}(t,-\vec x,z), \qquad U_P^{-1}SU_P = S,
\end{aligned} \tag{B7}$$

where S is the effective action of our approach. Note, in the consideration of the vector (axial) field we apply the axial gauge $V(A)_z(x,z) = 0$.

Charge conjugation of four-dimensional spinors, vector and axial fields is defined with the use of the corresponding unitary operator U_C as

$$\begin{aligned}
&U_C^{-1}\psi(x)U_C = C\bar\psi^T(x), \qquad U_C^{-1}\bar\psi(x)U_C = \psi^T(x)C, \qquad U_C^{-1}\psi_{L/R}(x)U_C = C\bar\psi^T_{R/L}(x),\\
&U_C^{-1}\bar\psi_{L/R}(x)U_C = \psi^T_{R/L}(x)C, \qquad U_C^{-1}V_\mu(x)U_C = -V_\mu(x), \qquad U_C^{-1}A_\mu(x)U_C = A_\mu(x),
\end{aligned} \tag{B8}$$

where

$$C = i\gamma^0\gamma^2, \qquad C^T = C^\dagger = C^{-1} = -C. \tag{B9}$$

C transformations of AdS fields read as

$$\begin{aligned}
&U_C^{-1}\psi_\pm(x,z)U_C = \mp C\gamma^5\bar\psi^T_\mp(x,z), \qquad U_C^{-1}\bar\psi_\pm(x,z)U_C = \pm\psi^T_\mp(x,z)\gamma^5 C,\\
&U_C^{-1}V_M(x,z)U_C = -V_M(x,z), \qquad U_C^{-1}A_M(x,z)U_C = A_M(x,z).
\end{aligned} \tag{B10}$$

Therefore, one can straightforwardly prove that all terms of the effective action are C-invariant when the relation for the bulk profiles (B5) holds, e.g.,

$$\begin{aligned}
U_C^{-1}\bar\psi_\pm(x,z)\psi_\pm(x,z)U_C &= -\bar\psi_\mp(x,z)\psi_\mp(x,z),\\
U_C^{-1}\bar\psi_\pm(x,z)\Gamma^M V_M(x,z)\psi_\pm(x,z)U_C &= \bar\psi_\mp(x,z)\Gamma^M V_M(x,z)\psi_\mp(x,z),\\
U_C^{-1}\bar\psi_\pm(x,z)\Gamma^M A_M(x,z)\psi_\pm(x,z)U_C &= -\bar\psi_\mp(x,z)\Gamma^M A_M(x,z)\psi_\mp(x,z),\\
U_C^{-1}\bar\psi_\pm(x,z)\Gamma^M i\Gamma^z V_M(x,z)\psi_\pm(x,z)U_C &= -\bar\psi_\mp(x,z)\Gamma^M i\Gamma^z V_M(x,z)\psi_\mp(x,z),\\
U_C^{-1}\bar\psi_\pm(x,z)\Gamma^M i\Gamma^z A_M(x,z)\psi_\pm(x,z)U_C &= \bar\psi_\mp(x,z)\Gamma^M i\Gamma^z A_M(x,z)\psi_\mp(x,z),
\end{aligned} \tag{B11}$$

etc. We therefore have $U_C^{-1}SU_C = S$.

The T-parity transformation of four-dimensional spinors, vector and axial fields is defined with the use of corresponding antiunitary operator U_T as

$$\begin{aligned}
&U_T^{-1}\psi(t,\vec x)U_T = T\psi(-t,\vec x), \qquad U_T^{-1}\bar\psi(t,\vec x)U_T = -\bar\psi(-t,\vec x)T,\\
&U_T^{-1}\psi_{L/R}(t,\vec x)U_T = T\psi_{L/R}(-t,\vec x), \qquad U_T^{-1}\bar\psi_{L/R}(t,\vec x)U_T = -\bar\psi_{L/R}(-t,\vec x)T,\\
&U_T^{-1}(V^0(t,\vec x),V^i(t,\vec x))U_T = (V^0(-t,\vec x),-V^i(-t,\vec x)), \qquad U_T^{-1}(A^0(t,\vec x),A^i(t,\vec x))U_T = (A^0(-t,\vec x),-A^i(-t,\vec x)),
\end{aligned} \tag{B12}$$

where

$$T = -\gamma^1\gamma^3, \qquad T^T = T^\dagger = T^{-1} = -T. \tag{B13}$$

The T transformations of the AdS fields read as

$$\begin{aligned} &U_T^{-1}\psi_\pm(t,\vec{x},z)U_T = T\psi_\pm(-t,\vec{x},z), \qquad U_T^{-1}\bar{\psi}_\pm(t,\vec{x},z)U_T = -\bar{\psi}_\pm(-t,\vec{x},z)T,\\ &U_T^{-1}(V^0(t,\vec{x},z),V^i(t,\vec{x}),0)U_T = (V^0(-t,\vec{x},z),-V^i(-t,\vec{x},z),0),\\ &U_T^{-1}(A^0(t,\vec{x},z),A^i(t,\vec{x},z),0)U_T = (A^0(-t,\vec{x},z),-A^i(-t,\vec{x},z),0). \end{aligned} \tag{B14}$$

Therefore, one can straightforwardly prove that all terms of the effective action are separately T-invariant and $U_T^{-1}SU_T = S$.

APPENDIX C: STRUCTURE INTEGRALS IN THE ADS/QCD MODEL

The C_i and D_i functions defining the nucleon form factors are given by

$$\begin{aligned} &C_1(Q^2)=D_1(Q^2)=\sum_\tau c_\tau B(a+1,\tau)\Big(\tau+\frac{a}{2}\Big), \quad C_2(Q^2)=-D_2(Q^2)=\frac{a}{2}\sum_\tau c_\tau B(a+1,\tau),\\ &C_3(Q^2)=a\sum_\tau c_\tau B(a+1,\tau+1)\frac{a(\tau-1)-1}{\tau}, \quad C_4(Q^2)=\frac{2m_N}{\kappa}\sum_\tau c_\tau(a+1+\tau)B(a+1,\tau+1)\sqrt{\tau-1},\\ &D_3(Q^2)=a\sum_\tau c_\tau B(a+1,\tau+1)\frac{a(\tau-1)+2\tau^2-1}{\tau}, \end{aligned} \tag{C1}$$

where $a = Q^2/(4\kappa^2)$ and

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \tag{C2}$$

is the Beta function.

The slopes

$$C(D)_i'(0) = \frac{dC(D)_i(Q^2)}{dQ^2}\bigg|_{Q^2=0} \tag{C3}$$

of the $C(D)_i$ functions are given by

$$\begin{aligned} &C_1'(0)=D_1'(0)=-\frac{1}{8\kappa^2}\sum_\tau c_\tau\Big[\frac{1}{\tau}+2(\psi(\tau)-\psi(1))\Big], \quad C_2'(0)=-D_2'(0)=\frac{1}{8\kappa^2}\sum_\tau\frac{c_\tau}{\tau}, \quad C_3'(0)=-\frac{1}{4\kappa^2}\sum_\tau\frac{c_\tau}{\tau(\tau+1)},\\ &C_4'(0)=-\frac{m_N}{2\kappa^3}\sum_\tau c_\tau\sqrt{\tau-1}\Big[\frac{1}{\tau}+\psi(\tau)-\psi(1)\Big], \quad D_3'(0)=\frac{1}{4\kappa^2}\sum_\tau c_\tau\frac{2\tau^2-1}{\tau(\tau+1)}, \end{aligned} \tag{C4}$$

where

$$\psi(\tau) = \frac{d\log(\Gamma(\tau))}{d\tau} = \frac{1}{\Gamma(\tau)}\frac{d\Gamma(\tau)}{d\tau} \tag{C5}$$

is the Digamma function obeying the recurrence formula $\psi(\tau+1) = \psi(\tau) + 1/\tau$.

It is easy to see that the functions $C_i(Q^2)$ and $D_i(Q^2)$ scale at $Q^2 \to \infty$ as

$$\begin{aligned} &C_1^{\rm asym}(Q^2) = C_2^{\rm asym}(Q^2) = D_1^{\rm asym}(Q^2) = -D_2^{\rm asym}(Q^2) = \frac{1}{2}\sum_\tau c_\tau\frac{\Gamma(\tau)}{a^{\tau-1}}, \qquad C_3^{\rm asym}(Q^2) = D_3^{\rm asym}(Q^2) = -\sum_\tau c_\tau\frac{\Gamma(\tau)(\tau-1)}{a^{\tau-1}},\\ &C_4^{\rm asym}(Q^2) = \frac{2m_N}{\kappa}\sum_\tau c_\tau\frac{\Gamma(\tau+1)}{a^\tau}\sqrt{\tau-1}. \end{aligned} \tag{C6}$$

From these considerations it is clear that the leading twist $\tau = 3$ contributions to the nucleon form factors scale as

$$F_1^N(Q^2)\sim\frac{1}{Q^4}, \quad F_2^N(Q^2)\sim\frac{1}{Q^6}, \quad G_A(Q^2)\sim\frac{1}{Q^4}. \tag{C7}$$

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