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DK molecule in the Bethe-Salpeter equation approach in the heavy quark limit

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In the heavy quark limit, we establish the Bethe-Salpeter equation for the possible molecular state containing one heavy meson D and one light meson K. With the kernel induced by one-particle-exchange diagrams, we solve the Bethe-Salpeter equation numerically in the ladder and covariant instantaneous approximations and find that the DK bound state may exist. Assuming the observed state $D_{s0}^*(2317)^+$ is an S-wave DK molecular bound state we calculate the strong decay width of $D_{s0}^*(2317)^+$. Our prediction for the decay width $\Gamma(D_{s0}^*(2317)^+ \to D_s^+\pi^0)$ is consistent with the previous investigations and the current experimental data.

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I. INTRODUCTION

In recent years, more and more new heavy hadron states were discovered by various experimental collaborations, e.g., Belle, *BABAR*, CLEO, and CDF. However, the structures and properties of some of these states are not very clear. One of them is the charm-strange hadron state $D_{s0}^*(2317)^+$ which was reported by the *BABAR* Collaboration at SLAC in the invariant mass distribution of $D_s^+\pi^0$, the products of the e^-e^+ annihilation [1]. The measured mass of this state is 2317.8 ± 0.6 MeV and the current results for the quantum numbers of isospin, spin, and parity are $I(J^P) = 0(0^+)$ [2]. Since its decay width is very narrow ($\Gamma < 3.8$ MeV), there are many theoretical works to study its structure with different models, such as a $c\bar{s}$ state [3–6], a four-quark state [7,8], and a $c\bar{s}$ mixed with four-quark state [9]. Because the mass of the state $D_{s0}^*(2317)^+$ lies slightly below the threshold of D and K mesons, $D_{s0}^*(2317)^+$ can also be viewed as an S-wave DK bound state based on the hadronic molecule picture [10–18]. The binding energy is thus $E_b = M_{D_{s0}^*(2317)^+} - M_D - M_K = -45$ MeV, with M_D and M_K being the averaged masses of D and K isospin multiples, respectively.

The Bethe-Salpeter (BS) equation is a formally exact equation to describe the molecular bound state [19–21]. Using the heavy quark effective theory [22], the form of the BS equation for the processes involving heavy quarks can be greatly simplified based on the fact that the dynamics of the bound state is determined by the light constituent particle. The BS equation has been applied to give many theoretical results concerning heavy hadrons [23–26]. In our previous work [17], we assume that the coupling of the vector meson with two heavy pseudoscalar mesons is the same as that with the two light ones containing strange quarks when deriving the interaction kernel. We use the $SU(4)$ symmetry (to be more specific, the symmetry between the charm and strange quarks) and the results of QCD sum rules to deal with the coupling constants involved in the decay process. Since the strange quark is not heavy enough, the symmetry between the charm and strange quarks is not a good one. In this paper, we will work in a more solid framework to describe the interaction between heavy and light mesons with the Lagrangian which respects both the chiral symmetry and the heavy quark symmetry. After establishing and solving numerically the BS equation of the DK bound state, we find that the bound state may exist with reasonable binding energy and form factor cutoff. Then we calculate the decay width of the strong decay $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ including the $\eta - \pi^0$ mixing effect. The result is consistent with the current experimental data.

The remainder of this paper is organized as follows. In Sec. II, we establish the BS equation for the bound state of two pseudoscalar mesons. Then we discuss the interaction kernel and derive the integral equation for the BS wave function. We also discuss the normalization condition of the BS wave function. In Sec. III, we calculate the decay width of $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ including the $\eta - \pi^0$ mixing effect. Finally, Sec. IV is reserved for a summary and discussion.

II. BS EQUATION FOR DK SYSTEM

A. BS equation for two pseudoscalar mesons

In this section we will establish the formalism of the BS equation for the DK bound state. As discussed in the Introduction, the heavy hadron state $D_{s0}^*(2317)^+$ is regarded as an S-wave bound state of D and K mesons. We start by defining the BS wave function for the bound state $|D_{s0}^*(P)\rangle$ as

$$\chi_P(x_1, x_2) = \langle 0|TD(x_1)K(x_2)|D_{s0}^*(P)\rangle, \tag{1}$$

and its conjugate

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$$\bar{\chi}_P(x_1, x_2) = \langle D^*_{s0}(P)|TD^\dagger(x_1)K^\dagger(x_2)|0\rangle, \tag{2}$$

where $D(x_1)$ and $K(x_2)$ are the field operators of the D and K mesons at space coordinates x_1 and x_2, respectively, and $P = Mv$ is the momentum of the bound state with mass M and velocity v.

Let us define $\lambda_1 = m_1/(m_1 + m_2)$ and $\lambda_2 = m_2/(m_1 + m_2)$ with m_1 and m_2 being the masses of the D and K mesons, respectively, and let p be the relative momentum between the two pseudoscalar mesons. In momentum space, the BS wave function can be written as

$$\chi_P(x_1, x_2) = e^{-iP\cdot X}\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\chi_P(p), \tag{3}$$

where X and x are the center-of-mass coordinate and the relative coordinate which are, respectively, defined as

$$X \equiv \lambda_1 x_1 + \lambda_2 x_2, \qquad x \equiv x_1 - x_2. \tag{4}$$

The BS equation for the DK bound state can be written in the following form:

$$\chi_P(p) = S(p_1)\int\frac{d^4q}{(2\pi)^4}G(P, p, q)\chi_P(q)S(p_2), \tag{5}$$

where $S(p_1)$ and $S(p_2)$ are the propagators of D and K mesons, respectively, the momenta of D and K are related to P and p as $p_1 = \lambda_1 P + p$, $p_2 = \lambda_2 P - p$, and $G(P, p, q)$ is the interaction kernel which contains one-particle-exchange diagrams induced by ρ and ω meson exchanges.

For convenience, we use the following forms of longitudinal and transverse relative momenta:

$$p_l = v\cdot p - \lambda_2 M, \qquad p_t = p - (v\cdot p)v. \tag{6}$$

In the leading order of the $1/m_Q$ ($Q = c$) expansion, the propagator of the D meson can be expressed as follows:

$$S(p_1) = \frac{i}{2m_1(p_l + E_b + m_2 + i\epsilon)}, \tag{7}$$

where E_b is the binding energy which is defined as $M = m_1 + m_2 + E_b$.

The propagator of the K meson has the form

$$S(p_2) = \frac{i}{p_l^2 - w_p^2 + i\epsilon}, \tag{8}$$

where $w_p \equiv \sqrt{p_t^2 + m_2^2}$ with $p_t = |p_t|$.

The interaction kernel of the DK system contains a light meson K and a heavy meson D. The interactions corresponding to the K meson are given by [27,28]

$$\mathcal{L}_{KK\rho} = ig_{KK\rho}[\bar{K}\vec{\tau}(\partial_\mu K) - (\partial_\mu\bar{K})\vec{\tau}K]\cdot\vec{\rho}^\mu, \tag{9}$$

$$\mathcal{L}_{KK\omega} = ig_{KK\omega}[\bar{K}(\partial_\mu K) - (\partial_\mu\bar{K})K]\omega^\mu, \tag{10}$$

where

$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \qquad \bar{K} = (\,K^- \quad \bar{K}^0\,), \tag{11}$$

with $\vec{\tau}$ being the usual Pauli matrices and $\vec{\rho}^\mu$ the field operators of ρ mesons. In the $SU(3)_f$ limit, the coupling constants satisfy the relations $g_{KK\rho} = g_{KK\omega} = g_{\rho\pi\pi}/2$, where $g_{\rho\pi\pi}$ is determined by the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation [29], $g_{\rho\pi\pi} \simeq m_\rho/f_\pi \simeq 5.8$ (m_ρ and f_π denote the mass of the ρ meson and the pion weak decay constant, respectively).

In the chiral limit and the heavy quark limit, the effective Lagrangian describing the interactions between heavy and light mesons can be written in the following forms [30]:

$$\begin{aligned}\mathcal{L}_{HHP} &= igTr[H_b\gamma_\mu\mathcal{A}^\mu_{ba}\gamma_5\bar{H}_a],\\ \mathcal{L}_{HHV} &= i\beta Tr[H_b v_\mu(\mathcal{V}_\mu - \rho_\mu)_{ba}\bar{H}_a],\end{aligned} \tag{12}$$

where g and β are coupling constants, and the field H_a represents the heavy pseudoscalar (P_a) and vector (P_a^*) mesons (the subscripts a, $b = 1, 2, 3$ correspond to u, d, s quarks), respectively:

$$\begin{aligned}H_a &= \frac{1+\not{v}}{2}(P^*_{a\mu}\gamma^\mu + iP_a\gamma_5),\\ \bar{H}_a &= \gamma^0 H_a^\dagger\gamma^0 = (P^{*\dagger}_{a\mu}\gamma^\mu + iP_a^\dagger\gamma_5)\frac{1+\not{v}}{2}.\end{aligned} \tag{13}$$

In Eq. (12), the axial-vector current is

$$\mathcal{A}_\mu = \frac{1}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) = \frac{i}{f_\pi}\partial_\mu P + \cdots, \tag{14}$$

and the vector current is

$$\mathcal{V}_\mu = \frac{1}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger) = \frac{1}{2f_\pi^2}[P, \partial_\mu P] + \cdots, \tag{15}$$

where $\xi = \exp(iP/f_\pi)$ and $f_\pi = 132$ MeV; $\rho^\mu_{ba} = ig_V V^\mu_{ba}/\sqrt{2}$; P and V stand for the fields of pseudoscalar and vector mesons, respectively, and they have the following forms:

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \tag{16}$$

and

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \tag{17}$$

According to Eqs. (12)–(17), the effective interaction of the heavy pseudoscalar meson D with a light vector meson can be obtained:

$$\mathcal{L}_{DDV} = -ig_{DDV} P_b^\dagger \overleftrightarrow{\partial}_\mu P_a V^{ba\mu}, \tag{18}$$

where g_{DDV} $(= \beta g_V/\sqrt{2})$ is the coupling constant, and g_V is determined by the relation $g_V = m_\rho / f_\pi (\simeq 5.8)$ [30,31]. The parameter β is fixed by vector meson dominance, $\beta = \sqrt{2} m_V/(g_V f_V) \simeq 0.9$ [32].

The kernel of the BS equation is complicated since the nonperturbative strong interactions are involved. In the ladder approximation, the one-particle-exchange irreducible interaction kernel can be written as

$$G(p_1, p_2; q_1, q_2) = c_I \frac{\beta g_V^2}{2\sqrt{2}} (p_2 + q_2)_\mu (p_1 + q_1)_\nu \Delta^{\mu\nu} \times (k, M_V)(2\pi)^4 \delta^4(q_1 + q_2 - p_1 - p_2), \tag{19}$$

where c_I is the isospin coefficient: $c_I = 3$ for ρ exchange diagram, while $c_I = 1$ for ω exchange diagram. $\Delta_{\mu\nu}(k, M_V)(V = \rho, \omega)$ denotes the massive vector meson propagator and has the following form:

$$\Delta_{\mu\nu}(k, M_V) = \frac{-i}{k^2 - M_V^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_V^2}\right), \tag{20}$$

where M_V is the mass of the exchanged meson and k is its momentum.

To proceed we will simplify the BS equation by applying the covariant instantaneous approximation, $p_l = q_l$, in the kernel as in Refs. [23–25].

In order to reflect the fact that hadrons have finite size, we introduce a form factor in the interaction vertex,

$$F(k_t) = \frac{\Lambda^2 - M_V^2}{\Lambda^2 + k_t^2}, \qquad k_t = p_t - q_t, \tag{21}$$

where Λ is a phenomenological cutoff which will be adjusted to give the solution of the BS equation.

In the heavy quark limit, as a result of the $SU(2)_s \otimes SU(2)_f$ symmetry, the dynamics inside the bound state is controlled by the configuration of the light degrees of freedom. We define

$$\tilde{\chi}_P(p_t^2) = \int \frac{dp_l}{2\pi} \chi_P(p_l, p_t^2). \tag{22}$$

Under the covariant instantaneous approximation, from Eqs. (5), (7), (8), and (19), we obtain the BS equation for $\tilde{\chi}_P(p_t^2)$,

$$\tilde{\chi}_P(p_t^2) = \frac{-c_I \beta g_V^2}{4m_1(-w_p + E_b + m_2)} \int \frac{d^3 q_t}{(2\pi)^3} \times \frac{1}{(p_t - q_t)^2 + M_V^2} F^2(k_t) \tilde{\chi}_P(q_t^2). \tag{23}$$

After performing the azimuthal integration, the BS equation (23) takes on the following form:

$$\begin{aligned}\tilde{\chi}_P(p_t^2) = {}& \frac{-\beta g_V^2/2}{8\sqrt{2} m_1 w_p(-w_p + E_b + m_2)} \int \frac{dq_t q_t^2}{(2\pi)^2} \Bigg\{\Bigg[4w_p(M - w_p) + p_t^2 + q_t^2 + \frac{(p_t^2 - q_t^2)^2}{M_V^2}\Bigg] \\ & \times \Bigg[\frac{1}{2p_t q_t} \ln\frac{(p_t + q_t)^2 + M_V^2}{(p_t - q_t)^2 + M_V^2} - \frac{1}{2p_t q_t} \ln\frac{(p_t + q_t)^2 + \Lambda^2}{(p_t - q_t)^2 + \Lambda^2} - \frac{2(\Lambda^2 - M_V^2)}{[(p_t + q_t)^2 + \Lambda^2][(p_t - q_t)^2 + \Lambda^2]}\Bigg] \\ & - \frac{(p_t^2 + q_t^2 + M_V^2)}{2p_t q_t} \ln\frac{(p_t - q_t)^2 + M_V^2}{(p_t + q_t)^2 + M_V^2} + \frac{(p_t^2 + q_t^2 + \Lambda^2)}{2p_t q_t} \ln\frac{(p_t - q_t)^2 + \Lambda^2}{(p_t + q_t)^2 + \Lambda^2} \\ & - (\Lambda^2 - M_V^2)\Bigg[\frac{2(p_t^2 + q_t^2 + \Lambda^2)}{(p_t^2 + q_t^2 + \Lambda^2)^2 - 4p_t^2 q_t^2} + \frac{1}{2p_t q_t} \ln\frac{(p_t - q_t)^2 + \Lambda^2}{(p_t + q_t)^2 + \Lambda^2}\Bigg]\Bigg\} \tilde{\chi}_P(q_t^2),\end{aligned} \tag{24}$$

where $p_t = |p_t|$, $q_t = |q_t|$.

B. Structure of the heavy hadron state $D_{s0}^*(2317)^+$

As discussed in the previous paper [17], for the DK system the isoscalar bound state can be written as

$$|D_{s0}^*(P)\rangle_{(0,0)} = \frac{1}{\sqrt{2}}(|D^+ K^0\rangle + |D^0 K^+\rangle), \tag{25}$$

where the subscript (0,0) refers to the isospin and its third component $(I, I_3) = (0, 0)$. The BS wave function of the DK bound state can be written as

$$\langle 0|T D_i(x_1) K_j(x_2)|D_{s0}^*(P)\rangle_{(I,I_3)} = C_{(I,I_3)}^{ij} \chi_P(x_1, x_2), \tag{26}$$

where $D_i(x_1)$ and $K_j(x_2)$ $(i, j = 1, 2)$ are the following field operators:

$$D_1 = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{D^0}}} (a_{D^0} e^{-ipx} + a^{\dagger}_{\bar{D}^0} e^{ipx}), \qquad D_2 = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{D^\pm}}} (a_{D^+} e^{-ipx} + a^{\dagger}_{D^-} e^{ipx}),$$
$$K_1 = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{K^\pm}}} (a_{K^+} e^{-ipx} + a^{\dagger}_{K^-} e^{ipx}), \qquad K_2 = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_{K^0}}} (a_{K^0} e^{-ipx} + a^{\dagger}_{\bar{K}^0} e^{ipx}), \tag{27}$$

and $C^{ij}_{(I,I_3)}$ is the isospin coefficient. For the isoscalar state only the case of $i = j$ contributes with $C^{11}_{(0,0)} = C^{22}_{(0,0)} = 1/\sqrt{2}$.

C. Normalization condition of the BS wave function

In general, the normalization condition of the BS wave function can be written as in Refs. [20,26]:

$$i \int \frac{d^4 p d^4 q}{(2\pi)^8} \bar{\chi}(p) \frac{\partial}{\partial P^0} [I_P(p, q)] \chi(q) = 1, \tag{28}$$

where $I_P(p, q) = (2\pi)^4 \delta^4(p - q) S^{-1}(p_1) S^{-1}(p_2)$.

After carrying out the integration over the longitudinal momentum p_l, we obtain the normalization equation of the BS wave function as follows:

$$\int \frac{d^3 p_t}{(2\pi)^3} \bigg\{ \lambda_1 \frac{(M - w_p)[\phi_1 + 4w_p(M - w_p)\phi_2]^2}{4m_1^2 w_p (m_2 + E_b - w_p)^2} + \lambda_2 [\phi_1 + 4w_p(M - w_p)\phi_2] \times \frac{[\phi_1 + 4(M - 2w_p)(2m_2 + 2E_b - w_p) + 4w_p^2]}{4m_1^2 w_p (m_2 + E_b - w_p)^2} \bigg\} = 1, \tag{29}$$

where

$$\phi_1(p_t) = c_I \frac{\beta g_V^2}{2\sqrt{2}} \int \frac{d^3 q_t}{(2\pi)^3} \bigg[\frac{p_t^2 + q_t^2 + (p_t^2 - q_t^2)^2/M_V^2}{(p_t - q_t)^2 + M_V^2} + \frac{2p_t \cdot q_t}{(p_t - q_t)^2 + M_V^2} \bigg] F^2(k_t) \phi_P(q_t),$$
$$\phi_2(p_t) = c_I \frac{\beta g_V^2}{2\sqrt{2}} \int \frac{d^3 q_t}{(2\pi)^3} \frac{1}{(p_t - q_t)^2 + M_V^2} F^2(k_t) \phi_P(q_t). \tag{30}$$

In the BS equation (24) there is only one free parameter, the cutoff Λ, which contains the information about the nonpoint interaction due to the structure of hadrons at the interaction vertices. In practice, there are large uncertainties for the value of Λ. In our calculation the hadronic structure of ρ, K, and D mesons are involved. To discuss the range of Λ we begin with the simplest situation where ρ is replaced by a photon. From fitting the data for the electromagnetic form factors of π and K mesons (which are obtained from the hadronic total cross sections for the $e^+e^- \to \pi\pi$ and $e^+e^- \to KK$ processes) [33] with the form of Eq. (21) with M_V being put to be zero, we obtain the values of Λ as 0.72 and 1 GeV for π and K mesons, respectively. When considering the vertices of a gluon and light ud diquarks, the value of Λ is 1.27 GeV [23,34]. Fitting the results which are obtained in the BS equation approach [35], we obtain the values of Λ as 1.35, 1.96, and 2.63 GeV corresponding to cc, bc, and bb heavy diquarks, respectively. It is easy to see that for a heavier meson (or diquark) at the vertex, the value of Λ in the form factor is larger. This is a reasonable trend since with the increase of the mass of the meson (diquark) the radius of the meson (diquark) becomes smaller, and hence the corresponding Λ becomes bigger. We expect that this trend still holds for the more complicated vertices of the ρ meson and other mesons. For meson scattering processes the value of Λ is taken to be 1.5 GeV for the $\rho\pi\pi$ vertex [36], while for the ρKK interaction Λ varies in the range of 1.5–4.5 GeV [27,37]. For the $K\bar{K}$ bound state the value of Λ is chosen to be between 1.17 and 4.5 GeV [38]. For the mesons with heavy quarks, the value of Λ can be as large as 3 GeV [28]. From the above discussion, we can see that there are large uncertainties of Λ in present studies. Since we do not have experimental data which can be used to obtain the value of Λ for the ρDD vertex, we let the value of Λ vary in the range 1–4 GeV in our study of the DK bound state.

The BS wave function in Eq. (24) satisfies a homogeneous integral equation, and we can discretize the integration region $(0, \infty)$ into n pieces (n is large enough) by the n-point Gauss quadrature rule. The BS equation then becomes an eigenvalue equation. In the calculation, we choose to work in the rest frame of the bound state in which $P = (M, 0)$. Fixing the binding energy of the DK bound state to be -45 MeV, we vary the cutoff Λ in the range 1–4 GeV to find the solutions of the eigenvalue equation corresponding to the BS equation (24) with the eigenvalue to be 1. We find that there are two values of Λ, 2.97 and 3.73 GeV, from which we can get the solution of the BS equation.

III. THE DECAY WIDTH AND THE NUMERICAL RESULTS

In this section, we will calculate the decay width of the process $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ through exchanging D^* and K^* mesons. As pointed out in the Introduction, this decay is an isospin violating process. The authors in Refs. [13,39] originally proposed that there are two mechanisms in the decay process $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ when considering the effects of isospin violation: one originates from the direct transition $D_{s0}^*(2317)^+ \to D_s^+\pi^0$, while the other originates from the indirect transition $D_{s0}^*(2317)^+ \to D_s^+\eta \to D_s^+\pi^0$ via $\eta - \pi^0$ mixing. In Ref. [13] the authors calculated the direct and indirect contributions separately, while in Ref. [39] the two mechanisms were combined together into a new form. Following the method used in Ref. [39], in Ref. [40] we introduced the mixing effect by modifying the η and π^0 fields into a new form [30,41],

$$\pi^0 \to \pi^0\cos\epsilon - \eta\sin\epsilon, \qquad \eta \to \pi^0\sin\epsilon + \eta\cos\epsilon, \tag{31}$$

where ϵ is the mixing angle

$$\tan 2\epsilon = \frac{\sqrt{3}}{2}\frac{m_d - m_u}{m_s - \hat{m}}, \tag{32}$$

with $\hat{m} = (m_d + m_u)/2$.

Then one can include the direct and indirect contributions by applying the pure π-coupling form with the modified flavor structure in Eq. (31). In other words, we replace $\pi^0\tau_3$ by $\pi^0(\tau_3\cos\epsilon + I\sin\epsilon/\sqrt{3})$ for the coupling to DD^*, and by $\pi^0(\tau_3\cos\epsilon + I\sin\epsilon\sqrt{3})$ for the coupling to KK^*.

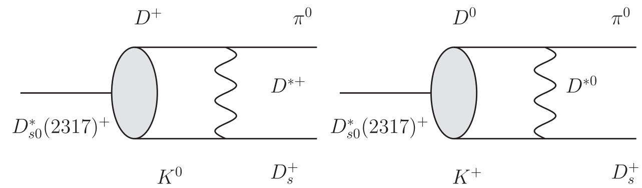

FIG. 1. Feynman diagrams for the strong decay $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ induced by D^* exchange.

The interactions concerning the decay process read [27,30]

$$\begin{aligned}\mathcal{L}_{D^*DP} &= -ig_{D^*DP}(D_b D_{\mu a}^{*\dagger} - D_{\mu b}^* D_a^\dagger)\partial^\mu P_{ba},\\ \mathcal{L}_{K^*K\pi} &= ig_{K^*K\pi}\{[\partial_\mu K^\dagger \hat{\pi}_K K^{*\mu} - K^\dagger(\partial_\mu\hat{\pi}_K)K^{*\mu}]\\ &\quad - [K^{*\mu\dagger}\hat{\pi}_K\partial_\mu K - K^{*\mu\dagger}(\partial_\mu\hat{\pi}_K)K]\},\end{aligned} \tag{33}$$

where the coupling constant $g_{D^*DP} = 2g\sqrt{m_D m_D^*}/f_\pi$, the parameter g is obtained from the full decay width of D^{*+}, $g = 0.59$ [42], the coupling constant $g_{K^*K\pi}$ is related to $g_{\rho\pi\pi}$ with the form $g_{K^*K\pi} = g_{\rho\pi\pi}/2$, $\hat{\pi}_K = \pi_1\tau_1 + \pi_2\tau_2 + \pi_3(\tau_3\cos\epsilon + I\sin\epsilon\sqrt{3})$($\pi_1$ and π_2 are related to π^+ and π^-, π_3 refers to π^0, I is the identity matrix), and the doublets of D^* and K^* are as follows:

$$D^* = \begin{pmatrix} D^{*0} \\ D^{*+}\end{pmatrix}, \qquad K^* = \begin{pmatrix} K^{*+} \\ K^{*0}\end{pmatrix}. \tag{34}$$

According to Eq. (33), the decay $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ induced by the D^* meson is shown in Fig. 1. The amplitude takes the following form:

$$\begin{aligned}\mathcal{M}_{D^*} &= i\sqrt{2M_{D_s}M_D}M_{D^*}\frac{g^2}{f_\pi^2}\sqrt{M}(\tau_3\cos\epsilon + I\sin\epsilon/\sqrt{3})\int\frac{d^4p}{(2\pi)^4}\frac{1}{(k^2 - m_D^{*2} + i\epsilon)}\\ &\quad\times\left[q_2\cdot(q_1 - k) - \frac{q_2\cdot k(q_1 - k)\cdot k}{m_D^{*2}}\right]\bigg|_{k=p-\lambda_2 q_2+\lambda_1 q_1}\left(\frac{\Lambda^2 - M_D^{*2}}{\Lambda^2 + M_{D_s}^2 + p_t^2 + 2M_{D_s}v'\cdot p_t}\right)^2\phi_P(p),\end{aligned} \tag{35}$$

where $q_1(q_2)$ is the momentum of the meson D_s^+ (π^0), and v' is the velocity of D_s^+.

Similarly, the diagrams for $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ through exchanging the K^* meson are shown in Fig. 2. One can write the amplitude as

$$\begin{aligned}\mathcal{M}_{K^*} &= i\sqrt{\frac{M_{D_s}}{M_D}}\frac{\beta g_V^2}{4}\sqrt{M}(\tau_3\cos\epsilon + I\sin\epsilon\sqrt{3})\int\frac{d^4p}{(2\pi)^4}\frac{1}{(k^2 - M_{K^*}^2 + i\epsilon)}\\ &\quad\times\left[(2q_1 - k)\cdot(2q_2 + k) - \frac{k\cdot(2q_1 - k)k\cdot(2q_2 + k)}{M_{K^*}^2}\right]\bigg|_{k=\lambda_2 q_1+\lambda_1 q_2-p}\left(\frac{\Lambda^2 - M_{K^*}^2}{\Lambda^2 + M_{D_s}^2 + p_t^2 - 2M_{D_s}v'\cdot p_t}\right)^2\phi_P(p).\end{aligned} \tag{36}$$

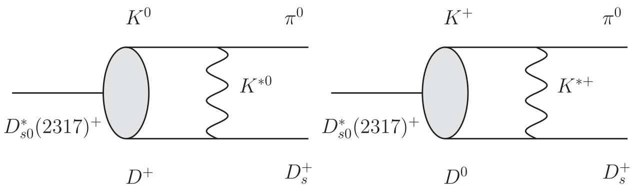

FIG. 2. Feynman diagrams for the strong decay $D^*_{s0}(2317)^+ \to D^+_s \pi^0$ induced by K^* exchange.

In the limit of isospin symmetry ($m_u = m_d$), there will be no contribution from the D^* and K^* exchange diagrams, since the masses of D^0 and D^+, D^{*0} and D^{*+}, K^0 and K^+, K^{*0} and K^{*+} are the same, respectively.

The total decay amplitude can be written in the form

$$\mathcal{M} = \mathcal{M}_{D^*} + \mathcal{M}_{K^*}. \tag{37}$$

The decay width for $D^*_{s0}(2317)^+ \to D^+_s \pi^0$ reads

$$d\Gamma = \frac{1}{32\pi^2} \left| \mathcal{M} \right|^2 \frac{|\mathbf{q}|^2}{M^2} d\Omega, \tag{38}$$

where $|\mathbf{q}|$ is the absolute value of the three-momentum of the particles in the final state in the rest frame of $D^*_{s0}(2317)^+$, which has the following form:

$$|\mathbf{q}| = \frac{\sqrt{[M^2 - (M_{\pi^0} + M_{D^+_s})^2][M^2 - (M_{\pi^0} - M_{D^+_s})^2]}}{2M}. \tag{39}$$

In the calculation, we use the following input parameters [2]:

$$\begin{aligned} M_{D^0} &= 1864.80\ \text{MeV}, & M_{D^+} &= 1869.57\ \text{MeV}, \\ M_{D^{*0}} &= 2006.93\ \text{MeV}, & M_{D^{*+}} &= 2010.22\ \text{MeV}, \\ M_{K^0} &= 497.61\ \text{MeV}, & M_{K^+} &= 493.68\ \text{MeV}, \\ M_{K^{*0}} &= 895.94\ \text{MeV}, & M_{K^{*+}} &= 891.66\ \text{MeV}, \\ M_{\pi^0} &= 134.98\ \text{MeV}, & M_{D^+_s} &= 1968.45\ \text{MeV}. \end{aligned} \tag{40}$$

Combining this with the numerical solution of the BS wave function, we calculate the decay width of the strong decay $D^*_{s0}(2317)^+ \to D^+_s \pi^0$ including the $\eta - \pi^0$ mixing effect. The result reads as follows:

$$\Gamma(D^*_{s0}(2317)^+ \to D^+_s \pi^0) = \begin{cases} 147\ \text{keV} & \Lambda = 2.97\ \text{GeV}, \\ 387\ \text{keV} & \Lambda = 3.73\ \text{GeV}. \end{cases} \tag{41}$$

There are some other predictions for the decay rate of $D^*_{s0}(2317)^+ \to D^+_s \pi^0$ in different approaches. With the DK molecular bound state picture, the authors in Ref. [15] give $180 \pm 40 \pm 100$ keV, and in Ref. [14] the decay rate is found to be 76 and 140 keV at leading and next-to-leading order chiral Lagrangian, respectively, while in Ref. [13] the value is 46.7–111.9 keV. In the quark-antiquark [3] and tetraquark [8] approaches, the magnitudes of the decay widths of $D^*_{s0}(2317)^+ \to D^+_s \pi^0$ are smaller than the predictions in the molecular picture, except that Ref. [5] gives 129 ± 43 keV and 109 ± 16 keV when the final momenta are 320 and 459 MeV, respectively. All the theoretical results for the decay width in various approaches are within the range of the current experimental data.

IV. SUMMARY AND CONCLUSION

In the heavy quark limit, we studied the possible DK molecular bound state in the BS equation formalism. Since the mass of the observed heavy state $D^*_{s0}(2317)^+$ is slightly below the threshold of D and K mesons, we assumed that $D^*_{s0}(2317)^+$ is an S-wave DK bound state. The kernel is induced by one-particle-exchange diagrams. When deriving the interaction kernel, we used the Lagrangian, which respects the chiral symmetry and the heavy quark symmetry, to describe interactions between heavy and light mesons. We also used $SU(3)$ symmetry to deal with the coupling constants concerning light mesons. This framework is more solid than that in our previous work. Then we established the BS equation for the DK bound state. Using the Gauss quadrature rule to calculate the integral, the BS equation (24) becomes an eigenvalue equation. The numerical result for the BS wave function is obtained by solving the eigenvalue equation when the binding energy is -45 MeV and the values of Λ are 2.97 and 3.73 GeV, respectively. As discussed in Sec. II, the value of Λ for the ρKK vertex has large uncertainties, and we do not have experimental data which can be used to give the value of Λ for the ρDD vertex. In the calculation, we took the same value of Λ for the ρKK and ρDD vertices and allowed the value of Λ in our study to vary in the range 1–4 GeV. The DK bound state will exist if Λ can indeed take the values in our article. We expect that more experimental data will be collected in future experiments which can constrain the range of Λ more precisely.

Furthermore, we applied the obtained numerical result for the BS wave function to calculate the decay width of the strong decay process $D^*_{s0}(2317)^+ \to D^+_s \pi^0$. The $\eta - \pi^0$ mixing effect is introduced by modifying the η and π^0 fields to a new form, $\pi^0 \to \pi^0 \cos\epsilon - \eta \sin\epsilon$, $\eta \to \pi^0 \sin\epsilon + \eta \cos\epsilon$. Our result shows that the decay width of $D^*_{s0}(2317)^+ \to D^+_s \pi^0$ is 147 and 387 keV when the values of Λ are 2.97 and 3.73 GeV, respectively. The decay rate in our previous work is 29.39–37.65 keV [17]. This large difference is mainly because there is no symmetry between the charm and strange quarks. We have worked in the chiral limit and the heavy quark limit. $1/m_Q$ and $1/\Lambda_{\chi SB}$ corrections also contribute to the decay rate. This will be studied in the future. We also compared our result with those in other models. Generally speaking, predictions from different models are all within the range of the current experimental resolution of the detector, 3.8 MeV.

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