Simple realization of the inverse seesaw mechanism

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(Received 18 June 2012; published 8 August 2012)

Differently from the canonical seesaw mechanism, which is grounded in grand unified theories, the inverse seesaw mechanism lacks a special framework that realizes it naturally. In this work we advocate that the 3-3-1 model with right-handed neutrinos has such an appropriate framework to accommodate the inverse seesaw mechanism. We also discuss the smallness of the lepton number violating mass and estimate the branching ratio for the rare lepton flavor violation process $\mu \to e\gamma$.

DOI: 10.1103/PhysRevD.86.035007 PACS numbers: 12.60.Cn, 12.60.Fr, 14.60.St

I. INTRODUCTION

Although experiments in neutrino oscillations have reported that neutrinos are light particles mixed in an unusual way [1],

$$\begin{aligned} \Delta m_{21}^2 &= (7.59 \pm 0.21) \times 10^{-5}\ \mathrm{eV}^2, \\ \Delta m_{31}^2 &= (2.43 \pm 0.13) \times 10^{-3}\ \mathrm{eV}^2, \\ \sin^2(2\theta_{12}) &= 0.861^{+0.026}_{-0.022}, \qquad \sin^2(2\theta_{23}) > 0.92, \\ \sin^2(2\theta_{13}) &= 0.092 \pm 0.016, \end{aligned} \tag{1}$$

from the theoretical side we still miss a definitive understanding of the smallness of the neutrino masses and of the profile of their mixing.

Seesaw mechanisms [2–4] are considered the most elegant way of explaining the smallness of the neutrino masses. Their essence lies in the fact that the lepton number must be explicitly violated at a high-energy scale. As a result, left-handed neutrinos gain small masses through the formula $m_\nu = v_w^2/\Lambda$, where v_w is the electroweak scale and Λ is associated to the lepton number violation scale. In the seesaw mechanisms Λ is generally related to some grand unified theory (GUT) scale. In this way, for $\Lambda = 10^{14}$ GeV, we get neutrino masses at the sub-eV scale. In spite of providing an interesting explanation for the smallness of the neutrino masses, such mechanisms are not phenomenologically testable because the new physics engendered by them will manifest at the 10^{14} GeV scale which is completely out of the range of the current and next accelerator experiments.

A radically different realization of the seesaw mechanism is the so-called inverse seesaw mechanism (ISS) [5], where small neutrino masses arise as a result of new physics at the TeV scale which may be probed at the Large Hadron Collider (LHC) experiments. According to the original idea, the implementation of the ISS mechanism requires the addition of three right-handed neutrinos N_{iR} and three extra standard model singlet neutral fermions, S_{iL}, to the three active neutrinos, ν_{i_L}, with $i = 1, 2, 3$. The mechanism arises when we make use of extra symmetries in order to allow that these nine neutrinos develop exactly the following bilinear terms:

$$\mathcal{L} = -\bar{\nu}_L m_D N_R - \bar{S}_L M N_R - \frac{1}{2}\bar{S}_L \mu S_L^C + \mathrm{H.c.}, \tag{2}$$

where m_D, M, and μ are generic 3×3 complex mass matrices. These terms can be arranged in the following 9×9 neutrino mass matrix in the basis (ν_L, N_L^C, S_L):

$$M_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M^T \\ 0 & M & \mu \end{pmatrix}. \tag{3}$$

On considering the hierarchy $\mu \ll m_D \ll M$, the diagonalization of this 9×9 mass matrix provides the following effective neutrino mass matrix for the standard neutrinos:

$$m_\nu = m_D^T (M^T)^{-1} \mu M^{-1} m_D. \tag{4}$$

The double suppression by the mass scale connected with M makes it possible to have such a scale much below than that one involved in the canonical seesaw mechanism. It happens that standard neutrinos with mass at sub-eV scale are obtained for m_D at the electroweak scale, M at the TeV scale and μ at the keV scale. In this case all the six right-handed neutrinos may develop masses around TeV scale and their mixing with the standard neutrinos is modulated by the ratio $m_D M^{-1}$. The core of the ISS is that the smallness of the neutrino masses is guaranteed by assuming that the μ scale is small and, in order to bring the right-handed neutrino masses down to TeV scale, it has to be at the keV scale [6,7].

Differently from the canonical seesaw mechanism that finds its natural place in GUT, the ISS mechanism still lacks a special framework where the six new neutrinos could be a part of some underlying particle content and naturally provide the mass terms in Eq. (9). In this work we show that the $SU(3)_C \times SU(3)_L \times U(1)_N$ model with right-handed neutrinos (331RHN for short) [8] has the appropriate framework to accommodate the ISS mechanism. This is so because this is a model which may manifest at the TeV scale and possesses in its matter content the new six right-handed neutrinos required by the mechanism and easily provides the mass terms in Eq. (9). In addition we develop an explanation for the smallness of the μ parameter and compute the branching ratio for the rare lepton flavor violation process $\mu \to e\gamma$, for which stringent bounds are expected to emerge in future neutrino experiments [9].

In what follows we implement the ISS mechanism in the 331RHN and then we develop a suitable mechanism to explain the smallness of the μ parameter.

II. ISS IN THE 3-3-1 MODEL WITH RIGHT-HANDED NEUTRINOS

We consider the 331RHN [8] whose leptonic sector is composed by

$$f_{aL} = (\nu_{aL} e_{aL} \nu^C_{aL})^T \sim (3, -1/3), \quad e_{a_R} \sim (1, -1), \qquad N_{a_R} \sim (1, 0), \tag{5}$$

where $a = 1, 2, 3$, and the numbers between the parentheses refer to the $SU(3)_L$, $U(1)_N$ transformation properties. In this way we have the minimum matter content needed in the ISS, i.e., nine neutral chiral leptons.

In order to generate mass to all fermions consistently, and also leaving only the electromagnetic symmetry group $U(1)_{\mathrm{em}}$ explicitly realized, we take into account the following three scalar triplets:

$$\begin{aligned} \eta &= (\eta^0 \eta^- \eta'^0)^T \sim (3, -1/3), \\ \chi &= (\chi^0 \chi^- \chi'^0)^T \sim (3, -1/3), \\ \rho &= (\rho^+ \rho^0 \rho'^+)^T \sim (3, 2/3). \end{aligned} \tag{6}$$

The relevant Yukawa Lagrangian for the lepton sector that yields the ISS mechanism for the neutrinos is composed by the following summation of terms:

$$\begin{aligned} \mathcal{L}^Y_{\mathrm{ISS}} &= G_{ab}\epsilon_{ijk}\bar{L}^C_{a_i}\rho^*_j L_{b_k} + G'_{ab}\bar{L}_a \chi N_{b_R} \\ &\quad + \frac{1}{2}\bar{N}^C_R \mu N_R + \mathrm{H.c.} \end{aligned} \tag{7}$$

We assume that the fields η^0, ρ^0 and χ'^0 develop a vacuum expectation value (VEV) according to

$$\langle \eta^0 \rangle = \frac{v_\eta}{\sqrt{2}}, \qquad \langle \rho^0 \rangle = \frac{v_\rho}{\sqrt{2}}, \qquad \langle \chi' \rangle = \frac{v_{\chi'}}{\sqrt{2}}. \tag{8}$$

With this set of VEVs, the Lagrangian above yields the following neutrino mass terms:

$$\mathcal{L}_{\mathrm{mass}} = \bar{\nu}_L m_D \nu_R + \bar{\nu}^C_L M N_R + \frac{1}{2}\bar{N}^C_R \mu N_R + \mathrm{H.c.} \tag{9}$$

In the basis $S_L = (\nu_L, \nu^C_L, N^C_L)$, the mass terms above can be cast in the following manner:

$$\mathcal{L}_{\mathrm{mass}} = \frac{1}{2}\bar{S}^C_L M_\nu S_L + \mathrm{H.c.}, \tag{10}$$

with the mass matrix M_ν having the texture

$$M_\nu = \begin{pmatrix} 0 & m^T_D & 0 \\ m_D & 0 & M^T \\ 0 & M & \mu \end{pmatrix}, \tag{11}$$

where the 3×3 matrices are defined as

$$M_{ab} = G'_{ab}\frac{v_{\chi'}}{\sqrt{2}}, \tag{12}$$

$$m_{Dab} = G_{ab}\frac{v_\rho}{\sqrt{2}}, \tag{13}$$

with M_{ab} and $m_{D_{ab}}$ being Dirac mass matrices, with this last one being antisymmetric. The mass matrix in Eq. (11) is characteristic of the ISS mechanism. We would like to call the attention to the fact that the two energy scales related with the models' gauge symmetry breakdown appear in the mass matrix. Namely, $v_{\chi'}$ in M_{ab} is connected with $SU(3)_L \otimes U(1)_N/SU(2)_L \otimes U(1)_Y$ and could be expected to be at the TeV scale leading to observable effects at the LHC, while v_ρ in m_{Dab} is connected with the electroweak standard model symmetry breakdown scale. Unfortunately, the third scale of energy, μ, characteristic of the mechanism is not a natural outcome of the 331RHN model. For the smallness of μ, we provide, in the next section, an explanation inspired by the one formulated in Ref. [6].

In order to see how M_ν in Eq. (11) can lead to eigenvalues at the eV scale it is useful to define the matrices

$$\mathcal{M}_{D_{6\times3}} = \begin{pmatrix} m_{D_{3\times3}} \\ 0_{3\times3} \end{pmatrix}, \quad \mathcal{M}_{R_{6\times6}} = \begin{pmatrix} 0_{3\times3} & M^T_{3\times3} \\ M_{3\times3} & \mu_{3\times3} \end{pmatrix}, \tag{14}$$

so that we have the following matrix with blocks, where $\mathcal{M}_R$ is supposed invertible matrix:

$$M_{\nu_{9\times9}} = \begin{pmatrix} 0_{3\times3} & \mathcal{M}^T_{D_{3\times6}} \\ \mathcal{M}_{D_{6\times3}} & \mathcal{M}_{R_{6\times6}} \end{pmatrix}. \tag{15}$$

This last matrix can be diagonalized by means of procedures involving block matrices which are presented in Refs. [10,11]. Following these references, we define a diagonalizing matrix, W, such that

$$W^T M_\nu W = \begin{pmatrix} m_{\mathrm{light}_{3\times3}} & 0_{3\times6} \\ 0_{6\times3} & m_{\mathrm{heavy}_{6\times6}} \end{pmatrix}. \tag{16}$$

In this way, the W matrix has the following form:

$$W = \begin{pmatrix} (\sqrt{1 + FF^\dagger})_{3\times3} & F_{3\times6} \\ F^\dagger_{6\times3} & (\sqrt{1 + F^\dagger F})_{6\times6} \end{pmatrix}, \tag{17}$$

where it is understood that

$$\sqrt{1 + FF^\dagger} \equiv 1 - \frac{1}{2}FF^\dagger - \frac{1}{8}FF^\dagger FF^\dagger + \ldots. \tag{18}$$

Under the assumption that F is given as a power series in $\mathcal{M}^{-1}_R$,

$$F = F_1 + F_2 + \ldots, \tag{19}$$

$$F_i \sim (\mathcal{M}_R^{-1})^i, \tag{20}$$

the eigenvalues of $\mathcal{M}_R$ are supposed to be larger than the entries of $\mathcal{M}_D$. This is justified by observing that the entries of M in $\mathcal{M}_R$ are of order $v_{\chi'}$. Then, Eqs. (16) and (17) allow us to determine the blocks m_{light} and m_{heavy} order by order in $\mathcal{M}_R^{-1}$. At lowest order,

$$F \approx F_1 = (\mathcal{M}_D^T \mathcal{M}_R^{-1})^*, \qquad m_{\text{light}} \approx -\mathcal{M}_D^T \mathcal{M}_R^{-1} \mathcal{M}_D,$$
$$m_{\text{heavy}} \approx \mathcal{M}_R. \tag{21}$$

In general grounds these results are identical to those obtained from the usual seesaw mechanism. What turns it different is the texture of the matrices $\mathcal{M}_D$ in Eq. (14) and

$$\mathcal{M}_R^{-1} = \begin{pmatrix} -M^{-1}\mu(M^T)^{-1} & M^{-1} \\ (M^T)^{-1} & 0 \end{pmatrix}, \tag{22}$$

which leads to the ISS form for the light Majorana neutrino mass matrix,

$$m_{\text{light}} = m_D^T M^{-1} \mu (M^T)^{-1} m_D. \tag{23}$$

The minimal model we are developing here has the peculiar characteristic that m_D is an antisymmetric matrix. As the three active standard neutrinos masses correspond to the eigenvalues of Eq. (23), there is a prediction that one of them is massless.

A departure from a scenario involving just three active neutrinos, where their mixing is described by an unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, is observed in neutrinos mixing relying on the ISS. It happens that the largest energy scale figuring in Eq. (14) is $M \sim v'_\chi$, supposedly of 1 TeV order. As a consequence description of oscillation involving three active neutrinos will be attached with nonunitary effects modulated by the ratio $v_W^2/v_\chi'^2$. Such effects manifest experimentally through neutrino disappearing in discordance from what is expected when considering unitarity in oscillation phenomena involving the three known neutrinos.

It is worthwhile to review how the nonunitarity aspect is quantified in the ISS mechanism. For obtaining the complete mass eigenstates the matrix in Eq. (16) has still to be transformed to a diagonal form by means of

$$U = \begin{pmatrix} U_0 & 0 \\ 0 & U_1 \end{pmatrix}, \tag{24}$$

where U_0 and U_1 are unitary matrices which turn m_{light} and m_{heavy}, respectively, diagonal.[1] Thus, the matrix which diagonalizes M_ν is then

$$\mathcal{U} = WU = \begin{pmatrix} \sqrt{1 + FF^\dagger}U_0 & FU_1 \\ F^\dagger U_0 & \sqrt{1 + F^\dagger F}U_1 \end{pmatrix}. \tag{25}$$

Let $\mathbf{n}_L$ be the 9×1 vector whose components are neutrino mass eigenstates, where we denote $\mathbf{n}_{iL}^0$, $a = 1, 2, 3$ being the three light mass eigenstates and $\mathbf{n}_{kL}^1$, $k = 1, \ldots, 6$, the six heavy ones, so that

$$\mathbf{n}_L = \begin{pmatrix} \mathbf{n}_L^0 \\ \mathbf{n}_L^1 \end{pmatrix} = \mathcal{U}^\dagger \begin{pmatrix} \nu_L \\ s_L \end{pmatrix}, \tag{26}$$

where

$$s_L = [\, \nu_L^C \quad N_L^C \,]^T$$

is a 6×1 vector. The flavor eigenstates ν_{aL} figuring in charged current are given by the following superposition:

$$\nu_{aL} \approx \left[U_0 - \frac{1}{2} F_1 F_1^\dagger U_0 \right]_{ai} \mathbf{n}_{iL}^0 + (F_1 U_1)_{ak} \mathbf{n}_{kL}^1. \tag{27}$$

The matrix connecting the flavor and light mass eigenstates is given by

$$\mathcal{N} = (1 - \eta) U_0, \tag{28}$$

where η is defined as $\eta \equiv \frac{1}{2} F_1 F_1^\dagger$. $\mathcal{N}$ is nonunitary and replaces the unitary PMNS matrix which parametrizes the mixing in the typical three neutrino scenario. The PMNS matrix is to be identified here with U_0 in Eq. (24). All the nonunitarity effects are characterized by η which is approximately given by

$$\eta \equiv \frac{1}{2} F_1 F_1^\dagger \approx \frac{1}{2} m_D^\dagger (M^{-1})^* (M^{-1})^T m_D. \tag{29}$$

Observation of oscillation phenomena involves charged currents interactions. Since the three left-handed neutrinos entering in such interactions are now a superposition of the nine mass eigenstates, as given by Eq. (27), we have the following charged current Lagrangian:

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \bar{l}_{aL} \gamma^\mu \nu_{aL} W_\mu^- + \text{H.c.}$$
$$\approx -\frac{g}{\sqrt{2}} \bar{l}_{aL} \gamma^\mu \{ \mathcal{N}_{ai} \mathbf{n}_{iL}^0 + \mathcal{K}_{ak} \mathbf{n}_{kL}^1 \} W_\mu^- + \text{H.c.}, \tag{30}$$

where

$$\mathcal{K}_{ak} = (F_1 \quad U_1)_{ak}.$$

For the term in Eq. (30) involving the heavy neutrinos $\mathbf{n}_{kL}^1$ there is a suppression coming from elements of

$$(F_1 \quad U_1)_{ak}$$

which are expected at least of order v_ρ / v_χ. It can be estimated, by taking $v_\rho \approx 10^2$ GeV and $v_\chi \approx 10^3$ GeV, to be of order 10^{-1}. This gives a sizable mixing among left-handed and right-handed neutrinos which can be probed through the rare lepton flavor violating processes that we are going to address in a moment.

[1] Any negative mass eigenvalue can be turned in a positive one by defining a diagonal matrix K such that UK leads to a diagonal form for Eq. (16) with all entries nonnegative, but it can be omitted without further consequences.

Returning to m_{light}, on substituting $m_D = G v_\rho$, $M = G' v_{\chi'}$, we obtain

$$m_{\text{light}} = (G^T (G'^T)^{-1} \mu (G')^{-1} G) \frac{v_\rho^2}{v_{\chi'}^2}. \tag{31}$$

Remember that G is an antisymmetric matrix, implying that one eigenvalue of the neutrino mass matrix in Eq. (31) is null. Then, automatically the squared mass difference in Eq. (1) provides, necessarily, the following neutrino mass spectrum:

$$m_1 = 0, \qquad m_2 \approx 4.8 \times 10^{-2}\ \text{eV}, \qquad m_3 \approx 8.7 \times 10^{-3}\ \text{eV}. \tag{32}$$

In other words, in the framework of the ISS mechanism developed here, the solar and atmospheric neutrino oscillation experiments provide the absolute mass of the neutrinos.

In view of this, let us check if m_ν in Eq. (31) is capable of providing the mass spectrum given in Eq. (32) and the correct mixing matrix. For this we have to diagonalize m_ν in Eq. (31). However, notice that it involves many free parameters in the form of Yukawa couplings, the components of G and G' and, unfortunately, there are no constraints over them.

With such a large set of free parameters, there is a great deal of possible solutions that lead to the correct neutrino mass spectrum and mixing. However, due to the nonunitarity of the mixing matrix $\mathcal{N}$, any set of values for the entries in G and G' that do the job must obey the following constraints [12]:

$$|\eta| < \begin{pmatrix} 2.0 \times 10^{-3} & 3.5 \times 10^{-5} & 8.0 \times 10^{-3} \\ 3.5 \times 10^{-5} & 8.0 \times 10^{-4} & 5.1 \times 10^{-3} \\ 8.0 \times 10^{-3} & 5.1 \times 10^{-3} & 2.7 \times 10^{-3} \end{pmatrix}. \tag{33}$$

In what concerns the energy parameters that appear in m_ν above, we consider $v_\eta = v_\rho = v$, thus the constraint $v_\eta^2 + v_\rho^2 = (246\ \text{GeV})^2$ imposes $v = 174$ GeV. The natural value of $v_{\chi'}$ is around TeV, but since we are interested only in its order of magnitude, we consider exactly 1 TeV. On the other hand, the usual scale of μ is around keV. Here, we consider $\mu = 0.3\mathbf{I}$ keV where $\mathbf{I}$ is the identity matrix.

Regarding the Yukawa couplings entries in G and G', notice that G is antisymmetric matrix. Thus, it has only three independent free parameters. In view of this we cannot make the common assumption of considering M as being diagonal and having a degenerate mass matrix once we are at risk of having less free parameters than necessary to give the correct pattern of neutrino masses and mixing. Thus, the simplest scenario here is one where M is diagonal but nondegenerate. But even in this case it is not possible to uniquely fix the parameters in G and G'. In what follows we present a particular solution of the diagonalization of the mass matrix m_ν above which involves the following set of values for G and G' entries:

$$G = \begin{pmatrix} 0 & 0.02 & 0.012 \\ -0.02 & 0.0 & 0.01 \\ -0.012 & -0.01 & 0.0 \end{pmatrix}, \quad G' = \begin{pmatrix} 0.32 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.0 \\ 0.0 & 0.0 & 0.9 \end{pmatrix}. \tag{34}$$

With these G, G' and the values for the VEVs v, $v_{\chi'}$ and μ presented above, the diagonalization of the mass matrix m_{light} in Eq. (31) furnishes the desired eigenvalues in Eq. (32) and, in addition, yields a standard PMNS mixing matrix given by

$$U_{\text{PMNS}} = \begin{pmatrix} 0.802987 & 0.583404 & 0.121869 \\ -0.485344 & 0.521409 & 0.701836 \\ 0.34591 & -0.622714 & 0.701836 \end{pmatrix}. \tag{35}$$

This U_{PMNS} implies in the following mixing angles $\theta_{12} = 36°$, $\theta_{23} = 45°$ and $\theta_{13} = 7°$.

This set of values for the entries in G and G' yields

$$|\eta| = \begin{pmatrix} 1.4 \times 10^{-5} & -5.0 \times 10^{-10} & 4.7 \times 10^{-6} \\ -5.0 \times 10^{-10} & 3.6 \times 10^{-5} & 3.9 \times 10^{-5} \\ 4.7 \times 10^{-6} & 3.8 \times 10^{-5} & 4.2 \times 10^{-5} \end{pmatrix}, \tag{36}$$

which respect the bounds in Eq. (33).

Let us see the prediction for the six right-handed neutrinos masses that such a set of values for the parameters in G and G' can provide. On diagonalizing $m_{\text{heavy}} = \mathcal{M}_R$ in Eq. (14), we obtain two eigenvalues around 900 GeV, another two around 800 GeV, and two more around 320 GeV. With these masses such heavy neutrinos may be probed in the LHC through the process $pp \to l^\pm l^\pm l^\mp \nu(\bar{\nu})$ [13] or in future neutrino experiments through rare lepton decays like $\mu \to e\gamma$.

We focus now on the rare lepton flavor violation (LFV) process $\mu \to e\gamma$. Such a process is allowed by the second coupling in Eq. (30). The branching ratio for the process mediated by these six heavy neutrinos is given by [14]

$$\text{BR}(\mu \to e\gamma) \approx \frac{\alpha_W^3 \sin^2(\theta_W) m_\mu^5}{256 \pi^2 m_W^4 \Gamma_\mu} \times \left| \sum_{1=1}^{6} \mathcal{K}_{\alpha i} \mathcal{K}_{\beta i} I\left(\frac{m_{N_i}^2}{m_W^2}\right) \right|^2, \tag{37}$$

where

$$I(x) = -\frac{2x^3 + 5x^2 - x}{4(1-x)^3} - \frac{3x^3 \ln x}{2(1-x)^4}. \tag{38}$$

In the above branching ratio $\alpha_W = \frac{g^2}{4\pi}$ with g being the weak coupling, θ_W is the electroweak mixing angle, m_μ is the muon mass, m_W is the $W^\pm$ mass, and Γ_μ is the muon total decay width. The present values of these parameters are found in Ref. [15]. In order to obtain $\mathcal{K}$, we get F_1 from Eq. (21) and diagonalize $\mathcal{M}_R$ in Eq. (14) to obtain U_1.

Considering all this, we obtain the approximate value for $\text{BR}(\mu \to e\gamma)$,

$$\text{BR}(\mu \to e\gamma) \approx 3 \times 10^{-14}. \tag{39}$$

The current upper bound on this branching ratio is $\text{BR}(\mu \to e\gamma) < 4.9 \times 10^{-11}$ [15]. Our result for this branching ratio respects the upper bound and, interestingly, falls inside the sensitivity of future neutrino experiments [9], which will be able to probe a branching ratio up to 10^{-18}, representing an additional test of our proposal.

POSSIBLE REALIZATION OF A SMALL μ

In this section we develop a dynamical explanation to the smallness of the parameter μ. Basically, we adapt the mechanism developed in Ref. [6] to our context.

We first remark that the 331RHN model with only an additional discrete symmetry does not work in the appropriate way as to furnish the correct entries in the mass matrix Eq. (3). In view of this we thought of a minimal modification of this model and added a scalar singlet $\sigma \sim (1, 0)$ to its scalar content in Eq. (6). In order to avoid unpleasant terms in the Lagrangian of the model, we can then impose a Z_3 symmetry with only the following fields transforming nontrivially according to the following assignment:

$$\begin{aligned} N_{aR} \to e^{2i\pi/3} N_{aR}, \quad L_a = e^{4i\pi/3} L_a, \quad \sigma \to e^{2i\pi/3}\sigma, \\ \chi \to e^{2i\pi/3}\chi, \quad \rho = e^{4i\pi/3}\rho. \end{aligned} \tag{40}$$

This discrete symmetry has a twofold importance. It restricts the Yukawa interaction terms leading, after spontaneous symmetry breaking, to mass terms needed for producing the texture like that in Eq. (3). Also, the Z_3 symmetry plays a role in the scalar potential allowing terms, like the trilinear one, which guarantees a safe spectrum of scalar fields (meaning, no extremely light scalar).

The Yukawa Lagrangian of interest to the implementation of the ISS mechanism involving scalars and leptons, invariant by Z_3, is composed by the following sum of terms:

$$\begin{aligned} \mathcal{L}_Y = {} & g_{ab}\epsilon_{ijk}\overline{(L_{a_i})^c} L_{b_j}\rho_k + g'_{ab}\bar{L}_a \chi N_{b_R} \\ & + \frac{\lambda_{ab}}{2}\sigma^0 \bar{N}^c_{aL} N_{bR} + \text{H.c.} \end{aligned} \tag{41}$$

Let us assume the following shift on the neutral scalars:

$$\begin{aligned} & \eta^0, \eta', \rho^0, \chi', \sigma^0 \\ & \to \frac{1}{\sqrt{2}}(v_{\eta,\eta',\rho,\chi',\sigma} + R_{\eta,\eta',\rho,\chi',\sigma} + iI_{\eta,\eta',\rho,\chi',\sigma}). \end{aligned} \tag{42}$$

With these VEVs the Yukawa terms in Eq. (41) provide the neutrino mass terms in Eq. (9) with μ being recognized as $\mu = \frac{\lambda v_\sigma}{\sqrt{2}}$. In this case a small μ requires a small v_σ. In order to achieve this we evoke a kind of type II seesaw mechanism [3] over v_σ, built from the scalar potential that obeys the extra Z_3 symmetry,

$$\begin{aligned} V = {} & (\mu_1^2 + \lambda_1|\eta|^2)|\eta|^2 + (\mu_2^2 + \lambda_2|\rho|^2)|\rho|^2 + (\mu_3^2 \\ & + \lambda_3|\chi|^2)|\chi|^2 + (\mu_4^2 + \lambda_4|\sigma|^2)|\sigma|^2 + \lambda_5|\eta|^2|\rho|^2 \\ & + \lambda_6|\eta|^2|\chi|^2 + \lambda_7|\rho|^2|\chi|^2 + \lambda_8|\eta^\dagger\rho|^2 + \lambda_9|\eta^\dagger\chi|^2 \\ & + \lambda_{10}|\rho^\dagger\chi|^2 + (\lambda_{11}|\eta|^2 + \lambda_{12}|\rho|^2 + \lambda_{13}|\chi|^2)|\sigma|^2 \\ & + \left(\frac{f_1}{\sqrt{2}}\epsilon^{ijk}\eta_i\rho_j\chi_k + \frac{f_2}{\sqrt{2}}\chi^\dagger\eta\sigma + \frac{f_3}{\sqrt{2}}\sigma^3 + \text{H.c.}\right). \end{aligned} \tag{43}$$

The existence of a minimum of the potential in Eq. (43) requires its first derivatives, with respect to the neutral scalar fields developing VEVs, to vanish, which leads to a set of five constraint equations. However, for our proposal here the only constraint equation that matters is the one related to the neutral scalar field σ^0,

$$\begin{aligned} & v_\sigma[2\mu_\sigma^2 + 2\lambda_4 v_\sigma^2 + \lambda_{11}(v_\eta^2 + v_{\eta'}^2) \\ & \quad + \lambda_{12}v_\rho^2 + \lambda_{13}v_{\chi'}^2 + 3f_3 v_\sigma] + f_2 v_\chi v_{\eta'} = 0. \end{aligned} \tag{44}$$

The traditional assumption here is that the scalar σ^0 is very heavy belonging to a GUT scale [3]. In this case on assuming that μ_σ is the dominant energy parameter in the constraint equation above, we obtain

$$v_\sigma \approx \frac{f_2 v_{\eta'} v_{\chi'}}{\mu_\sigma^2}. \tag{45}$$

As f_2 is related to a term that explicitly violates the lepton number, it is also usual to assume that it belongs to the GUT scale too. Assuming that the GUT scale is Λ we must have $\Lambda = \mu_\sigma = f_2$. In this case, we obtain

$$v_\sigma \approx \frac{v_{\eta'} v_{\chi'}}{\Lambda}. \tag{46}$$

There is an upper bound over $v_{\eta'} < 40$ GeV derived in Ref. [16]. On assuming reasonable values for the VEVs of the model, $v_{\eta'} = 10$ GeV, $v_{\chi'} = 10^3$ GeV and $\Lambda = 10^{10-11}$ GeV, we obtain $v_\sigma \approx 0.1$–1 KeV, which implies μ around KeV.

IV. CONCLUSIONS

The appealing point behind the ISS mechanism is the fact that it is a phenomenological seesaw mechanism whose signatures are right-handed neutrinos at the TeV scale which may be probed at LHC and in future neutrino experiments through the rare LFV process.

Although the ISS mechanism is a phenomenologically feasible seesaw mechanism, it lacks a natural underlying framework, namely one that accommodates right-handed neutrinos at the TeV scale. In this work we implemented the ISS mechanism in the 331RHN.[2] We showed that the model possesses the appropriate neutrino content and the energy scales as required by the mechanism. We also provided a concrete example which recovered the neutrino physics involved in oscillation neutrino experiments, and evaluated the rare LFV decay $\mu \to e\gamma$ whose prediction is around $\mathrm{BR}(\mu \to e\gamma) \approx 3 \times 10^{-14}$. Such a robust value for this branching ratio may be probed in future neutrino experiments and represents a further means of testing our proposal. Finally, we developed a scheme where the model can be suitably modified to provide a natural explanation of the smallness of the characteristic ISS parameter μ (keV scale).

In view of all these results, it seems that the 3331RHN model is an interesting framework for realizing the ISS mechanism.

[2]We remark that when we concluded this manuscript a similar attempt was proposed in Ref. [17].

ACKNOWLEDGMENTS

This work was supported by Conselho Nacional de Pesquisa e Desenvolvimento Científico- CNPq (C. A. S. P., P. S. R. S. and A. G. D.) and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior—CAPES (A. S.). A. G. D. also thanks FAPESP for supporting this work.

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