

θ_{13} and proton lifetime in a minimal $SO(10) \times S_4$ model of flavorP. S. Bhupal Dev,¹ Bhaskar Dutta,² R. N. Mohapatra,¹ and Matthew Sevrerson¹¹*Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA*²*Department of Physics & Astronomy, Mitchell Institute for Fundamental Physics, Texas A & M University, College Station, Texas 77843-4242, USA*

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In a recent paper, a minimal supersymmetric (SUSY) $SO(10) \times S_4$ based unified model of flavor for quarks and leptons was proposed with two **10** and one **126** contributing to fermion masses. An important aspect of this model is that Yukawa couplings emerge dynamically from minimization of the flavon potential, thereby reducing the number of parameters considerably. We make a detailed numerical analysis of this model for fermion mixings including SUSY threshold effects at the TeV scale and type-I corrections to a type-II dominant seesaw for neutrino masses. This is a single-step breaking model with SUSY $SO(10)$ broken at the grand unified theory scale of 2×10^{16} GeV to the minimal supersymmetric standard model. The minimal model has only 11 parameters, and therefore, the charged fermion fits predict the masses (up to an overall scale) and mixings in the neutrino sector. We present correlations for the different predictions in the neutrino mixing parameters. The recent experimental “large” θ_{13} value of $\sim 9^\circ$ can be obtained by a simple extension of the minimal model. We also find that proton decay mode $p \rightarrow K^+ \bar{\nu}_\mu$ has a partial lifetime of $\sim 10^{34}$ yrs, which is within reach of the next round of planned proton decay searches. The successful fit for fermion masses requires the Higgs mass to be below 129 GeV in this model. If the Higgs mass lies between 120 and 128 GeV, as suggested by the recent LHC data, we find a lower limit on the light stop mass of 755 (211) GeV for $\mu > 0 (< 0)$.

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I. INTRODUCTION

The measurement of neutrino masses and mixings during the past decade has provided the first evidence for physics beyond the standard model (SM). It has also raised the possibility that this new knowledge may help to unravel the physics of flavor and also possibly unlock the mystery behind the origin of matter in the universe. An important recent experimental finding in this area has been the announcements by the T2K [1], MINOS [2], and Double CHOOZ [3], and most recently by the Daya Bay and RENO [4,5] experiments that one of the hitherto unknown neutrino mixing angles, namely θ_{13} , is not only nonzero but “large.” Some hints for a large θ_{13} have also been suggested by a global analysis of the existing oscillation data [6,7]. A nonzero θ_{13} has profound implications for our understanding of the physics of neutrino mass, and it is therefore timely to search for the prediction of θ_{13} in various models [8].

A key question before theorists now is as follows: what is the big picture of flavor where known quark and lepton masses and mixings fit together? A framework that suggests itself is grand unified theories (GUT), where all matter and all forces (except gravity) unify at high energy scale. Since single-step coupling unification in these theories requires supersymmetry (SUSY), we will focus on SUSY-GUTs for approaching the flavor problem and use the seesaw mechanism [9] to understand the small neutrino masses. A possible advantage of this framework is that matter unification is expected to reduce the number of free

parameters that determine fermion masses and mixings from 31 parameters in the seesaw extended standard model, so that one can make predictions to make the model testable. The minimal scenario which is predictive for neutrinos is a renormalizable supersymmetric $SO(10)$ model with **10** and **126** Higgs fields contributing to fermion masses [10]. This model embodies both the type-I and type-II [11] seesaw contributions to neutrino masses and has been analyzed in many papers [12–14]. Fitting charged fermion masses in these models leads to predictions for lepton mixing angles and will be tested with higher precision measurement of these angles and the neutrino mass hierarchy. Detailed analysis of the superpotential and symmetry breaking for these models have been carried out in Ref. [15].

The next step with these models is to see if Yukawa couplings can be predicted as consequences of higher-scale symmetries. There have been several such approaches [16–19] which adopt the point of view that Yukawa couplings are dynamical fields, namely the flavon fields, whose vacuum expectation values (vevs) determine the observed Yukawa couplings. If these vevs could be the result of the minimization of simple superpotentials, that would reduce the number of parameters and would indeed be a step further in the search for an understanding of flavor. In a recent paper [17], such a model was presented where the GUT scale $SO(10)$ theory was extended to include three flavon fields with an $SO(10) \times S_4$ symmetry, where the flavon fields, as well as the three matter families, form a three dimensional irreducible representation of S_4 .

It was shown in [17] that the ground state of the $SO(10) \times S_4$ theory has only 11 parameters describing all flavor, i.e., quark and charged lepton masses and quark mixings as well as neutrino masses and lepton mixings. Using pure type-II seesaw contribution, it was argued that the model is in qualitative agreement with observations. In this paper we present a detailed numerical analysis of this model and its predictions in the neutrino sector as well as for proton decay. We find that once we include the SUSY threshold corrections to quark masses and a small type-I contribution to neutrino masses, all existing data in the quark and charged lepton sector can be fitted to a good accuracy; furthermore, the model predicts the solar to atmospheric mass ratio $\Delta m_{\odot}^2/\Delta m_{\text{atm}}^2$ and the solar and atmospheric mixing angles in agreement with the current neutrino oscillation data. However, with the flavon vacuum alignment as in Ref. [17], we find that the fermion fit predicts a reactor mixing angle $\theta_{13} \sim 5^\circ$ which is more than 3σ below the current experimental value [4,5]. We find that a larger value of θ_{13} , consistent with the Daya Bay and RENO results can be obtained by a slightly different flavon vacuum alignment which results due to an additional term in the superpotential allowed by the S_4 symmetry. We will demonstrate the determination of these features, and also note the correlation between the different parameters, which can make it easier to rule out the model.

We also find that in order to get the desired threshold corrections to the b -quark mass to fit observations, we need a large negative μ and/or A -terms in the model. We discuss predictions for Higgs mass for this choice of MSSM parameters, and find that the Higgs mass should be below 129 GeV in order to satisfy the constraints from the fermion sector fit. We also obtain lower limits on the squark masses from the same constraints. In particular, we note that if the Higgs mass is discovered between 120 and 128 GeV, the light stop should be heavier than 755 (211) GeV for $\mu > 0$ ($\mu < 0$) in this model. These features could be used to test the model at the LHC.

Finally, we discuss proton lifetime predictions in this model. We find that in order to suppress the $RRRR$ contributions to proton lifetime, we need to work in the low $\tan\beta$ regime, which is the reason we need a large A -term for the threshold correction, as noted above. We find that the B -violating $LLLL$ terms contribute dominantly to $p \rightarrow K^+ \bar{\nu}_\mu$ decay mode for which we find a partial lifetime of $\sim 10^{34}$ yrs. We emphasize that proton lifetime predictions as well as the predictions for neutrino mixings can be used to test the model.

The paper is organized as follows: in Sec. II, we present the essential points of the minimal $SO(10) \times S_4$ model; in Sec. III, we discuss the fermion mass fits and the predictions for the neutrino sector; in Sec. IV, we present a slightly different vacuum alignment which predicts a large θ_{13} while being consistent with the rest of the fermion sector. In Sec. V, we discuss the SUSY threshold correction

required to fit quark masses and its implications for gluino, stop and the Higgs masses in the model. In Sec. VI, we discuss our predictions for proton lifetime and in Sec. VII, we summarize our results.

II. DETAILS OF THE MODEL

The class of $SO(10)$ models we are interested in here have two **10** Higgs fields (denoted by H, H') and one **$\bar{126}$** (accompanied by **126**, denoted by $\Delta + \bar{\Delta}$). The $SO(10)$ -invariant Yukawa couplings of the model are given by:

$$\mathcal{L}_Y = h\psi\psi H + h'\psi\psi H' + f\psi\psi\bar{\Delta} \quad (1)$$

where ψ 's denote the **16** dimensional spinors of $SO(10)$, which contain all the matter fields of each generation; there are, of course, three such fields, though we have suppressed the generation indices. The Yukawa couplings are therefore 3×3 matrices in generation space.

The effective Yukawa couplings f, h, h' are assumed to have descended from a higher scale theory which has an S_4 symmetry broken by flavon fields ϕ_i determined by the minimum of the flavon potential. The alignment of the vev of the flavon fields as given in Ref. [17] are

$$\phi_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \quad (2)$$

As noted in Ref. [17], in order to get the desired Yukawa couplings naturally from the high scale theory, we supplement the S_4 symmetry group by an Z_n group, and the corresponding effective superpotential is given by

$$W = (\phi_1\psi)(\phi_1\psi)H + (\phi_2\psi)(\phi_2\psi)\bar{\Delta} + (\phi_3\psi\psi)\bar{\Delta} + (\phi_2\psi\psi)H' \quad (3)$$

where the brackets stand for the S_4 singlet contraction of flavor index [20].

The fermion mass matrices are derived from the Yukawa interaction as follows: after GUT symmetry breaking, two linear combinations of the SM doublets remain light, denoted by H_u and H_d . Typically, $H_{u(d)} = \sum_\alpha U_{u(d)\alpha} H_{u(d)\alpha}$, where $H_{u(d)\alpha}$ are the up(down)-type SM doublets in the $(H, H', \bar{\Delta})$. The effective H_u coupling at the MSSM scale is then given by $QH_u u^c (hU_{uH} + fU_{u\Delta} + h'U_{uH'})$, and similarly for H_d . The fermion mass matrices can be written in terms of these couplings as

$$\begin{aligned} \mathcal{M}_u &= \bar{h} + r_2\bar{f} + r_3\bar{h}' & \mathcal{M}_d &= \frac{r_1}{\tan\beta}(\bar{h} + \bar{f} + \bar{h}') \\ \mathcal{M}_\ell &= \frac{r_1}{\tan\beta}(\bar{h} - 3\bar{f} + \bar{h}') & \mathcal{M}_{\nu_D} &= \bar{h} - 3r_2\bar{f} + r_3\bar{h}', \end{aligned} \quad (4)$$

where we have absorbed the mixings ($U_{u\alpha}, U_{d\alpha}$) and the vevs $\langle H_{u,d} \rangle = \kappa_{u,d}$, with $\tan\beta = \kappa_u/\kappa_d$, to redefine the Yukawa coupling matrices as follows:

$$\bar{h} = \kappa_u U_{uH} h, \quad \bar{f} = \frac{\kappa_u U_{d\Delta}}{r_1} f, \quad \bar{h}' = \frac{\kappa_u U_{dH'}}{r_1} h', \quad (5)$$

with the ratios

$$r_1 = \frac{U_{dH}}{U_{uH}}, \quad r_2 = r_1 \frac{U_{u\Delta}}{U_{d\Delta}}, \quad r_3 = r_1 \frac{U_{uH'}}{U_{dH'}}. \quad (6)$$

These effective coupling matrices determined by the flavon sector are of the form

$$\bar{h} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M \end{pmatrix}, \quad (7)$$

$$\bar{f} = \begin{pmatrix} 0 & m_1 & m_1 \\ m_1 & m_0 & m_1 - m_0 \\ m_1 & m_1 - m_0 & m_0 \end{pmatrix}, \quad (8)$$

$$\bar{h}' = \begin{pmatrix} 0 & \delta & -\delta \\ \delta & 0 & 0 \\ -\delta & 0 & 0 \end{pmatrix}. \quad (9)$$

It was shown in Ref. [17] that S_4 symmetry constrains \bar{h} to have the above rank-one form. The parameters m_0, m_1, δ are chosen to be complex, giving a total of 10 parameters in the charged-fermion sector.

The neutrino mass matrix is, in general, given by a combination of the type-I and II seesaw mechanism:

$$\mathcal{M}_\nu = \nu_{L,R} - \mathcal{M}_{\nu_D} (\nu_{Rf})^{-1} (\mathcal{M}_{\nu_D})^T, \quad (10)$$

where $\nu_{L,R}$ are the vevs of the SM triplet Higgs fields $\Delta_{L,R}$ in **126**. If we assume type-II dominance, imposed by the ratio of ν_L and ν_R , and the magnitude of the coupling f , the neutrino mass matrix \mathcal{M}_ν takes an approximate tribimaximal (TBM) [21] form via the form of f given in Eq. (8), which is diagonalized by

$$V_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \quad (11)$$

Note that the full neutrino mixing matrix, $U_{\text{PMNS}} = V_\ell^\dagger V_\nu$, where V_ℓ and V_ν are the unitary matrices that diagonalize the charged lepton and neutrino mass matrices, respectively. Hence, we will necessarily have corrections to the TBM mixing given by Eq. (11) coming from the charged lepton mass matrix as well as the type-I contribution. Note further that since the f matrix also contributes to the quark and charged lepton masses, neutrino masses and quark masses are connected, thus making the model predictive.

III. PREDICTIONS IN THE NEUTRINO SECTOR

We diagonalize the mass matrices given by Eqs. (4) and use the MINUIT2 tool library [22] to minimize the sum of

TABLE I. Best fit values for the model parameters at the GUT scale.

M (GeV)	83.06
m_0 (GeV)	$1.201 - 0.9007i$
m_1 (GeV)	$0.2033 - 0.01170i$
δ (GeV)	$0.2129 + 0.08201i$
$r_1/\tan\beta$	0.01624
r_2	-0.1382
r_3	0.1358
α	5.0°

chi-squares for the mass eigenvalues and CKM mixing in the charged-fermion sector as well as the mass-squared differences Δm_\odot^2 and Δm_{atm}^2 in the neutrino sector. We note that a small but nonzero type-I contribution is required in the neutrino mass matrix given by Eq. (10), in order to have a consistent fit with the correct mass squared ratio $\Delta m_\odot^2/\Delta m_{\text{atm}}^2$; on the other hand, a large type-I contribution will spoil the TBM structure given by Eq. (8) and hence results in too small mixing angles. A balancing of the two is required in order to satisfy the observed neutrino oscillation data. We also note that SUSY threshold corrections to the down quark mass matrix [23] must be included in order to get a consistent fit for the charged fermion sector. The details of this analysis are given in Sec. V. Also, as

TABLE II. Best fit values for the charged fermion masses and the most relevant quark mixing parameters, as well as the solar-to-atmospheric mass squared ratio. The 1σ experimental values [24], with masses and mixings extrapolated by MSSM renormalization group (RG) equations to the GUT scale, are also shown for comparison. Note that the values of the bottom quark mass and the CKM mixing parameters involving the third generation quoted here include the SUSY-threshold corrections (see Sec. V).

	Best fit	Exp value
m_e (MeV)	0.3585	$0.3585_{-0.003}^{+0.0003}$
m_μ (MeV)	75.6717	$75.6715_{-0.0501}^{+0.0578}$
m_τ (GeV)	1.2922	$1.2922_{-0.0012}^{+0.0013}$
m_d (MeV)	2.0034	$1.5036_{-0.2304}^{+0.4235}$
m_s (MeV)	23.4494	$29.9454_{-4.5444}^{+4.3001}$
m_b (GeV)	1.0335	$1.0636_{-0.0865}^{+0.1414}$
m_u (MeV)	0.8192	$0.7238_{-0.1467}^{+0.1365}$
m_c (MeV)	207.4990	$210.3273_{-21.2264}^{+19.0036}$
m_t (GeV)	82.8964	$82.4333_{-14.7686}^{+30.2676}$
V_{us}	0.2245	0.2243 ± 0.0016
V_{ub}	0.0034	0.0032 ± 0.0005
V_{cb}	0.0351	0.0351 ± 0.0013
J	2.052×10^{-5}	$(2.2 \pm 0.6) \times 10^{-5}$
$\Delta m_\odot^2/\Delta m_{\text{atm}}^2$	0.0311	0.0320 ± 0.0025
χ^2	3.39	

discussed in Sec. VI, the proton decay constraints are satisfied only for low $\tan\beta$ in this model, as the Yukawa couplings responsible for the proton decay rates grow with $\tan\beta$. Hence, we have chosen $\tan\beta = 10$ for our numerical analysis given below.

The fit results are displayed in Tables I and II; Table I gives the numerical values of the model parameters yielding the best fit values shown in Table II. Here, α is the mixing angle for the third generation matter fermion ψ with the vector-like field ψ_V specific to the model [17]; the limit $\alpha = 0$ gives the form for the mass matrices dictated by S_4 symmetry, as given by Eqs. (8) exactly, and the fit value of $\alpha = 5^\circ$ approximates this limit. With this in mind, note that the top quark mass in the model is given by

$$m_t \simeq U_{uH} \kappa_u h_{33} \cos^2 \alpha. \quad (12)$$

In the neutrino sector, as noted earlier, the correct mass squared ratio $\Delta m_{\odot}^2 / \Delta m_{\text{atm}}^2$ as well as large solar and atmospheric mixing angles fix the relative size between the type-I and type-II contributions, and then the overall scale is determined from the largest mass eigenvalue, assuming a normal hierarchy. We find that for the best fit parameters shown in Table I, and for $v_R = 2.0 \times 10^{16}$ GeV (same as the GUT scale), $v_L = 6.810$ eV yields the right neutrino mass scale with $m_3 \sim 0.05$ eV. For estimating the proton decay rates as well as for the neutrino masses and mixing, we must extract the magnitudes of the raw Yukawa couplings h ,

$$h = \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1.677 \end{pmatrix}$$

$$f = \begin{pmatrix} 0 & (-5.628 + 0.3238i) \times 10^{-4} & (-5.628 + 0.3238i) \times 10^{-4} \\ (-5.628 + 0.3238i) \times 10^{-4} & (-3.326 + 2.494i) \times 10^{-3} & (2.763 - 2.461i) \times 10^{-3} \\ (-5.628 + 0.3238i) \times 10^{-4} & (2.763 - 2.461i) \times 10^{-3} & (-3.326 + 2.494i) \times 10^{-3} \end{pmatrix} \quad (13)$$

$$h' = \begin{pmatrix} 0 & (3.243 + 1.250i) \times 10^{-4} & (-3.243 - 1.250i) \times 10^{-4} \\ (3.243 + 1.250i) \times 10^{-4} & 0 & 0 \\ (-3.243 - 1.250i) \times 10^{-4} & 0 & 0 \end{pmatrix}.$$

The predicted values for the neutrino mixing parameters corresponding to the best fit parameter values in the model given in Table I are summarized in Table III. We find that a consistent fermion sector fit in this model predicts the reactor mixing angle θ_{13} to be nonzero and within a very narrow range 4.5° – 5.5° , which is well within the 3σ lower bound of many recent experimental results [1–3], but is only marginally consistent with the latest result from Daya Bay [4] and RENO [5]. We show in Sec. IV that a large θ_{13} value consistent with the Daya Bay and RENO results can be obtained in this model with

TABLE III. The model predictions for the neutrino mixing angles for the best fit parameter values given in Table I. We also show the 3σ range of values from the updated global neutrino data analysis [6], and for θ_{13} , we show in square brackets the most recent Daya Bay result [4]. Note that the predicted value is only marginally consistent with the 3σ value of the Daya Bay result.

	Predicted value	3σ exp range
θ_{12}	32.34°	$(30.6\text{--}36.8)^\circ$
θ_{23}	49.41°	$(35.7\text{--}53.1)^\circ$
θ_{13}	5.13°	$(1.8\text{--}12.1)^\circ$ [(5.9–11.6)°]
δ_D	144.4°	

f , h' , which can be done using the expressions in Eq. (5). However, these couplings depend on the vev mixing parameters $U_{q\alpha}$, and hence, there is some freedom in their determination, although the unitarity constraints on the U 's, $\sum_\alpha |U_{q\alpha}|^2 \leq 1$, and the top-quark mass relation in this model given by Eq. (12) provide some restriction on these mixing parameters. The values chosen for the up-type mixings are $U_{uH} = 0.40$, $U_{u\Delta} = 0.4033$, and $U_{uH'} = 0.72$, and using the fit values for the r 's from Table I and Eq. (6), the resulting values for the down-type mixings are $U_{dH} = 0.06497$, $U_{d\Delta} = -0.4739$, and $U_{dH'} = 0.8611$. Given these values and the running vevs $\kappa_u = 123.8$, $\kappa_d = 17.9$ GeV at GUT-scale [24], the resulting dimensionless couplings are found to be

a slightly different vacuum alignment than that given in Ref. [17].

IV. AN IMPROVED FIT WITH A DIFFERENT VACUUM ALIGNMENT

In this section, we discuss a different flavon vacuum alignment than that presented earlier [cf. Eq. (2)]. This requires us to choose a specific value of n for the Z_n symmetry of the superpotential given by Eq. (3). With this assumption, we can add the S_4 -singlet part of a linear term like $\phi_2 \phi_3$ to the superpotential in Eq. (3) which

TABLE IV. The improved best fit values for the model parameters at the GUT scale.

M (GeV)	84.33
m_0 (GeV)	$2.607 - 0.3277i$
m_1 (GeV)	$-0.3052 - 0.03412i$
δ (GeV)	$-0.1937 - 0.2719i$
$r_1/\tan\beta$	0.01591
r_2	-0.1388
r_3	0.05867
α	18.5°

upon minimization results in the following vacuum structure:

$$\phi_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \epsilon \\ a \\ b \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \quad (14)$$

One set of values for the components given above are $(\epsilon, a, b) = (-0.080, -0.752, 0.692)$, and $(x, y, z) = (0.937, 0.928, 0.936)$. Given this shifted flavon vacuum alignment, our mass matrix couplings in comparison to Eqs. (8) and (9) become

$$\bar{f} = m_0 \begin{pmatrix} \epsilon^2 & \epsilon a & \epsilon b \\ a\epsilon & a^2 & ab \\ b\epsilon & ba & b^2 \end{pmatrix} + m_1 \begin{pmatrix} 0 & z & y \\ z & 0 & x \\ y & x & 0 \end{pmatrix}, \quad (15)$$

$$\bar{h}' = \delta \begin{pmatrix} 0 & b & a \\ b & 0 & \epsilon \\ a & \epsilon & 0 \end{pmatrix}, \quad (16)$$

with no change to the h coupling. Performing the χ^2 -minimization again with these new couplings gives a fit with no substantial changes in the charged sector, but with important improvements to the neutrino sector predictions. The resulting parameter values for this fit are

$$h = \begin{pmatrix} 0 \\ 0 \\ 1.946 \end{pmatrix}$$

$$f = \begin{pmatrix} (-3.067 + 0.3855i) \times 10^{-5} & (2.245 + 0.9247i) \times 10^{-4} & (7.649 + 0.2352i) \times 10^{-4} \\ (2.245 + 0.9247i) \times 10^{-4} & (-2.623 + 0.3297i) \times 10^{-3} & (2.923 - 0.2465i) \times 10^{-3} \\ (7.649 + 0.2352i) \times 10^{-4} & (2.923 - 0.2465i) \times 10^{-3} & (-2.221 + 0.2792i) \times 10^{-3} \end{pmatrix} \quad (17)$$

$$h' = \begin{pmatrix} 0 & (2.541 - 3.567i) \times 10^{-4} & (-2.762 + 3.877i) \times 10^{-4} \\ (2.541 - 3.567i) \times 10^{-4} & 0 & (-2.986 + 4.192i) \times 10^{-5} \\ (-2.762 + 3.877i) \times 10^{-4} & (-2.986 + 4.192i) \times 10^{-5} & 0 \end{pmatrix}.$$

The predicted values for the neutrino mixing parameters corresponding to the best fit parameter values in the model given in Table IV are summarized in Table VI; the correlations between the different parameters in the neutrino sector while satisfying the charged fermion constraints are

TABLE V. The improved best fit values for the charged fermion masses and the most relevant quark mixing parameters, as well as the solar-to-atmospheric mass squared ratio.

	Best fit	Exp value
m_e (MeV)	0.3585	$0.3585^{+0.0003}_{-0.003}$
m_μ (MeV)	75.6719	$75.6715^{+0.0578}_{-0.0501}$
m_τ (GeV)	1.2922	$1.2922^{+0.0013}_{-0.0012}$
m_d (MeV)	0.8960	$1.5036^{+0.4235}_{-0.2304}$
m_s (MeV)	21.9535	$29.9454^{+4.3001}_{-4.5444}$
m_b (GeV)	1.0627	$1.0636^{+0.1414}_{-0.0865}$
m_u (MeV)	0.7284	$0.7238^{+0.1365}_{-0.1467}$
m_c (MeV)	209.8979	$210.3273^{+19.0036}_{-21.2264}$
m_t (GeV)	84.1739	$82.4333^{+30.2676}_{-14.7686}$
V_{us}	0.2243	0.2243 ± 0.0016
V_{ub}	0.0033	0.0032 ± 0.0005
V_{cb}	0.0351	0.0351 ± 0.0013
J	2.19×10^{-5}	$(2.2 \pm 0.6) \times 10^{-5}$
$\Delta m_\odot^2/\Delta m_{\text{atm}}^2$	0.0321	0.0320 ± 0.0025
χ^2	4.05	

given in Table IV, and the best fit values for the masses and mixings are given in Table V.

The value v_R for this fit is 0.7×10^{16} GeV, the value of v_L was taken as 8.921 eV, and the values for the up-type Higgs mixings were chosen to be $U_{uH} = 0.35$, $U_{u\Delta} = 0.63$, and $U_{uH'} = 0.25$; given the fit values for the r 's from Table IV, the resulting values for the down-type mixings are $U_{dH} = 0.05568$, $U_{d\Delta} = -0.7223$, and $U_{dH'} = 0.6779$. Using these values and the same prescription as in the previous section but with the new f and h' coupling definitions, the resulting dimensionless couplings are now found to be

shown in the scatter plots of Fig. 1. Notice that in addition to small improvements to the predicted values for θ_{12} and θ_{23} compared to those given in Table III, the value for θ_{13} is now larger and consistent within 1σ of the Daya Bay central value of $\sim 8.8^\circ$ [4].

TABLE VI. The model predictions for the neutrino mixing angles for the best fit parameter values given in Table IV. We also show the 3σ range of values from the updated global neutrino data analysis [6], and for θ_{13} , we show in square brackets the most recent Daya Bay result [4]. Note that the predicted value now is consistent with the Daya Bay result.

	Predicted value	3σ exp range
θ_{12}	33.77°	$(30.6\text{--}36.8)^\circ$
θ_{23}	44.82°	$(35.7\text{--}53.1)^\circ$
θ_{13}	9.02°	$(1.8\text{--}12.1)^\circ$ [(5.9–11.6)°]
δ_D	-165.28°	

V. THRESHOLD CORRECTION AND LOW ENERGY MASS SPECTRUM

In order to compare the fermion masses and mixing values obtained from the model at the GUT scale with the experimental values at the weak scale, we must take into account the SUSY threshold correction effects [23,25]. There are two main contributions to the SUSY threshold

correction to the fermion masses: one coming from the gluino loop and another from the chargino loop. The largest correction is for the bottom mass, which is given by [25]

$$\frac{\delta m_b}{m_b} \simeq \epsilon_1 + \epsilon_2 |V_{tb}|^2, \quad (18)$$

where

$$\epsilon_1 = \frac{2\alpha_s}{3\pi} \mu m_{\tilde{g}} \tan\beta I_3(m_{\tilde{g}}^2, m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2), \quad (19)$$

$$\epsilon_2 = \frac{1}{16\pi^2} \mu A_t y_t^2 \tan\beta I_3(\mu^2, m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \quad (20)$$

and the function I_3 is given by

$$I_3(a, b, c) = \frac{ab \log(\frac{a}{b}) + bc \log(\frac{b}{c}) + ca \log(\frac{c}{a})}{(a-b)(b-c)(a-c)}. \quad (21)$$

Similarly, if we do not add any off-diagonal threshold corrections, the CKM mixings involving the third generation receive corrections as follows [25]:

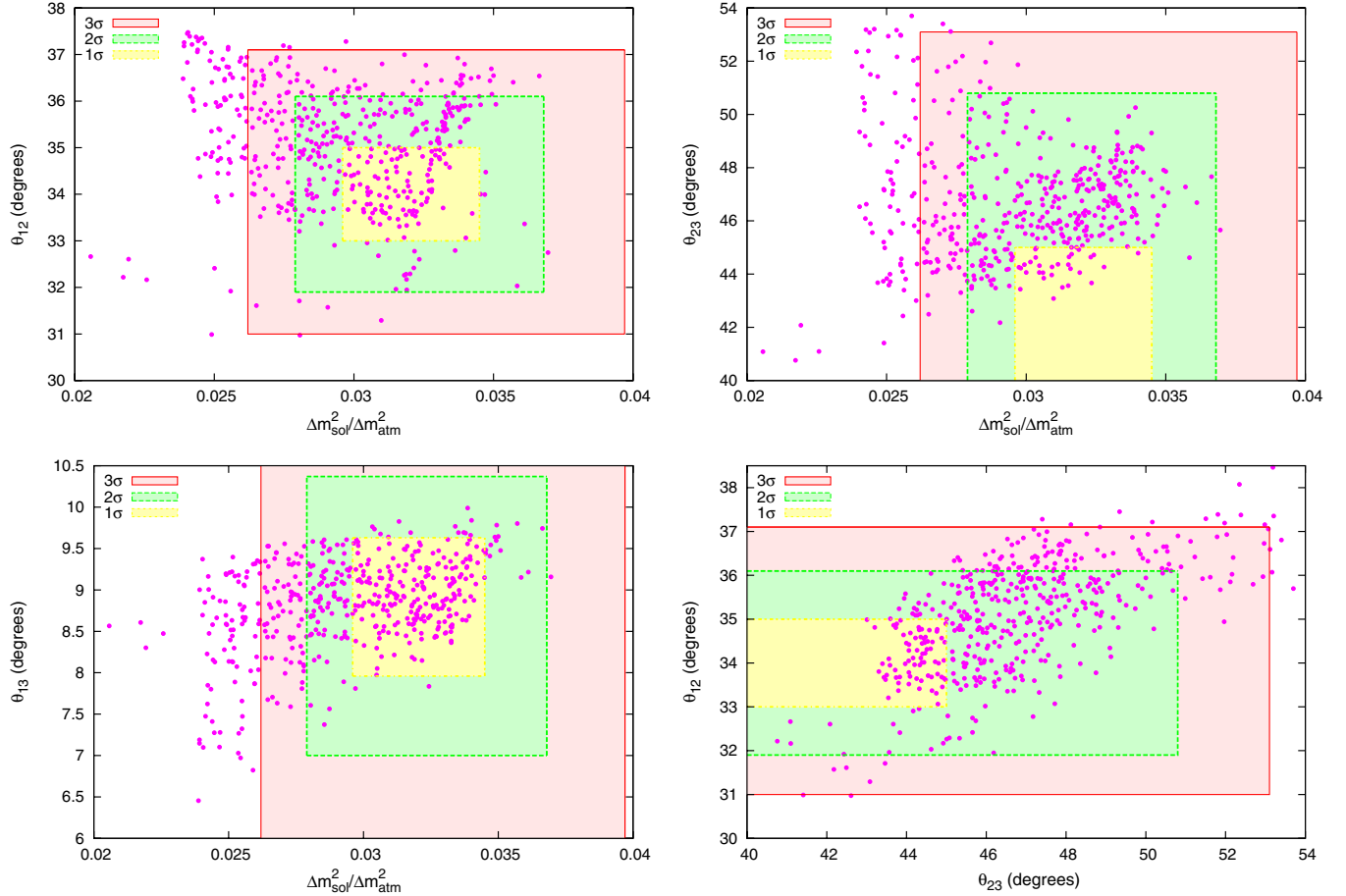


FIG. 1 (color online). The predicted correlation between the neutrino sector observables, namely $\Delta m_{\odot}^2/\Delta m_{\text{atm}}^2$ and the mixing angles θ_{12} , θ_{23} , θ_{13} , based on the fermion sector fit in the model. The 1, 2 and 3σ experimental limits for θ_{12} , θ_{23} and $\Delta m_{\odot}^2/\Delta m_{\text{atm}}^2$ are also shown (shaded regions).

$$\frac{\delta V_{ub}}{V_{ub}} \simeq \frac{\delta V_{cb}}{V_{cb}} \simeq \frac{\delta V_{td}}{V_{td}} \simeq \frac{\delta V_{ts}}{V_{ts}} \approx -\epsilon_2. \quad (22)$$

However, once off diagonal threshold corrections are included, there are further changes to CKM mixing, which we take into account in our numerical analysis.

From the numerical fit, we find that at the GUT scale, without including the threshold corrections, some of the best-fit values predicted by the model do not agree with experimental values extrapolated to the GUT scale (see Table II). In particular, we find

$$\begin{aligned} m_b &= 1.37 \text{ GeV}, & |V_{ub}| &= 0.0015, \\ |V_{cb}| &= 0.0160, & |V_{td}| &= 0.0047, & |V_{ts}| &= 0.0153. \end{aligned} \quad (23)$$

Comparing these values with the experimental values, we note that large negative threshold corrections are required for the model to have a consistent fermion-sector fit. We parametrize the SUSY threshold corrections at the GUT scale by modifying the third generation elements of the down-quark mass matrix as follows:

$$\begin{aligned} \mathcal{M}'_d &= \mathcal{M}_d + \delta \mathcal{M}_d, \\ \text{where } \delta \mathcal{M}_d &= \frac{r_1}{\tan \beta} \begin{pmatrix} 0 & 0 & \delta_{13} \\ 0 & 0 & \delta_{23} \\ \delta_{13} & \delta_{23} & \delta_{33} \end{pmatrix} \end{aligned} \quad (24)$$

and \mathcal{M}_d is given by Eq. (4). The required threshold corrections at the GUT scale are

$$\begin{aligned} \delta_{13} &= 0.09 \text{ GeV}, & \delta_{23} &= -0.96 \text{ GeV}, \\ \delta_{33} &= -20.68 \text{ GeV}. \end{aligned} \quad (25)$$

Note that these threshold corrections, when extrapolated down to the weak scale and added to the RG-extrapolated values of the b -quark mass and the CKM mixing parameters, yield results within 1σ range of the experimental values at M_Z .

At the weak scale, it is clear from Eqs. (19) and (20) that for the large negative threshold corrections given by

$$\mathcal{M}_t^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + m_Z^2 \cos 2\beta \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right), & m_t(-A_t + \mu \cot \beta) \\ m_t(-A_t + \mu \cot \beta) & m_{\tilde{t}_R}^2 + m_t^2 + m_Z^2 \cos 2\beta \left(\frac{2}{3} \sin^2 \theta_W \right) \end{pmatrix}.$$

Note that the same mass parameters, namely the gluino and third generation squark masses as well as the trilinear term A_t , appear in the threshold correction and Higgs mass correction. Given that this model requires a large threshold correction to have a consistent fit in the fermion sector, we expect some correlation between the two corrections. To show it quantitatively, we have chosen the simplest case of minimal supergravity-type GUT scale spectrum, as an illustration, although our results do not depend on the assumption of minimal supergravity. Here we scan over the

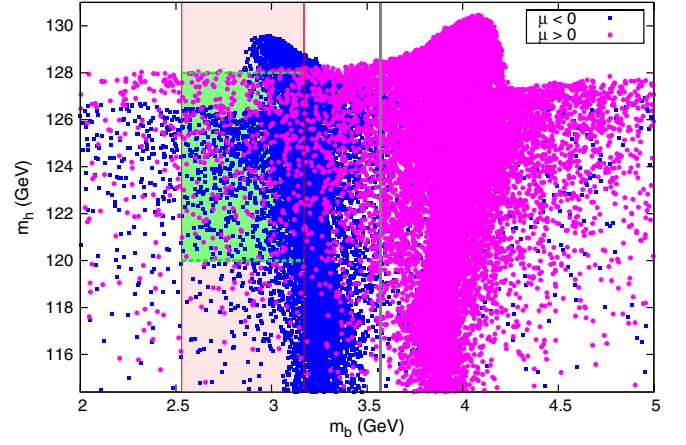


FIG. 2 (color online). Higgs mass prediction in the model required by the bottom quark mass fit, including the threshold correction effects. The dashed vertical line is the best fit value for $m_b(M_Z)$ without the threshold corrections. The vertical (red) shaded region is the 1σ experimental range of $m_b(M_Z) = (2.85 \pm 0.32)$ GeV, obtained from the LEP data [34]. The horizontal (green) shaded region shows the range in which $\sim 3\sigma$ excess of events for SM-like Higgs were observed recently at the LHC [28].

Eq. (25), we must have $\mu < 0$ if the gluino term is dominant, or opposite signs for μ and A_t if the chargino contribution is dominant. These observations have important consequences for the MSSM light Higgs mass as well as the sparticle spectrum, as shown below.

The one-loop radiative correction to the MSSM light Higgs mass is given by [26]

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta + \frac{3g_2^2 m_t^4}{16\pi^2 m_W^2} \log \left[\left(1 + \frac{m_{\tilde{t}_1}^2}{m_t^2} \right) \left(1 + \frac{m_{\tilde{t}_2}^2}{m_t^2} \right) \right] \quad (26)$$

where $m_{\tilde{t}_{1,2}}$ are the stop mass eigenvalues, which are obtained by diagonalizing the matrix

parameter space for $200 \text{ GeV} \leq m_0 \leq 2 \text{ TeV}$, $200 \text{ GeV} \leq m_{1/2} \leq 2 \text{ TeV}$, $-5 \text{ TeV} \leq A_0 \leq 5 \text{ TeV}$, and $\tan \beta = 10$, using the ISAJET package [27]. The results are shown in Fig. 2, from which we find that the lightest Higgs mass is required to be below 129 (128) GeV for $\mu < 0$ (> 0) in order to have the right amount of threshold correction to satisfy the bottom-quark mass constraint (vertical red shaded region) in this model. The horizontal green shaded region shows the range of Higgs mass in which a mild excess of events has been recently reported at the LHC [28].

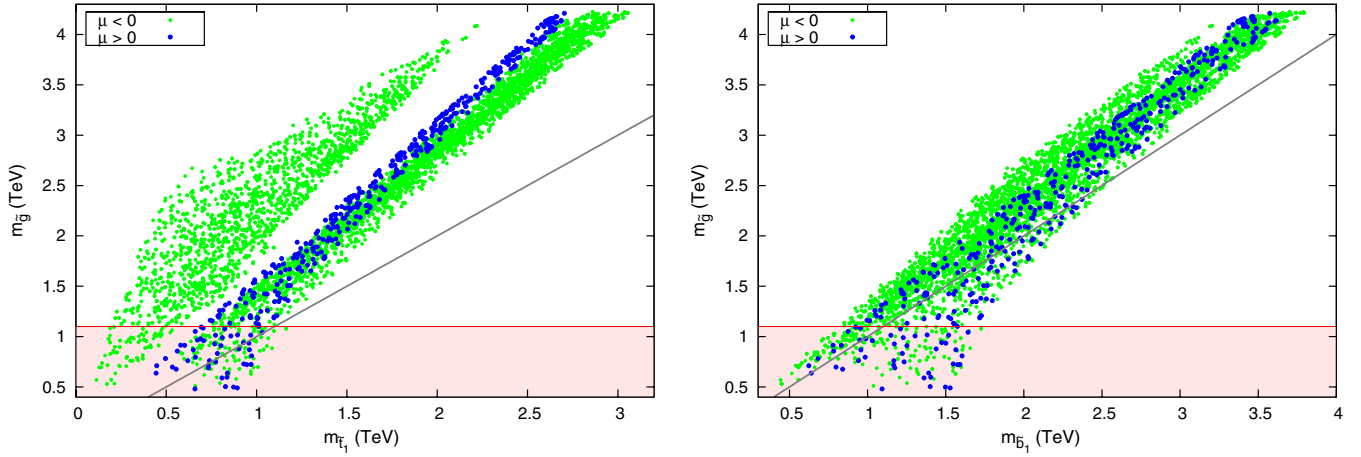


FIG. 3 (color online). The correlation between the gluino mass and the light stop and sbottom masses, corresponding to the shaded region in Fig. 2. The dashed line is for gluino mass equal to the relevant squark mass in the plot. The red shaded region is ruled out by the current LHC data [29].

The correlations between the gluino mass and the light stop and sbottom masses for the required threshold corrections (shaded region in Fig. 2) are shown in Fig. 3. We find that the large threshold correction requirement in this model requires the gluino to be always heavier than the light stop, but not necessarily heavier than the light sbottom. Moreover, for gluino masses satisfying the current LHC lower bound of 1.1 TeV [29] and for Higgs mass between 120 and 128 GeV, we find a lower limit for the stop mass of 755 (211) GeV for $\mu > 0$ (< 0) and similarly for the sbottom mass of 1013 (895) GeV. The milder limit on the squark masses for $\mu < 0$ is because of the fact that in this case, the required negative threshold corrections can be obtained from both gluino and chargino contributions [cf. Eqs. (19) and (20)], thus allowing for the A_t, A_b values necessary for light stop and sbottom masses, respectively. However, for $\mu > 0$ case, the gluino contribution is of the wrong sign, and hence we must have very large negative A_t values to obtain the required threshold corrections.

VI. EXPECTATIONS FOR PROTON DECAY

We now turn to a discussion of proton lifetime in our model. As experimental limits on proton lifetimes keep increasing, many simple SUSY-GUT models have either been ruled out or become more and more constrained. It is therefore important to ensure that any GUT model for neutrino masses is consistent with those limits. As is well known, the dominant contributions to proton decay comes from color triplet Higgsino exchange in these modes [30] and can in general lead to large proton decay amplitudes [31]. It was suggested in Ref. [32] that one way to suppress this amplitude without invoking cancellations is to choose appropriate flavor structure for the Yukawa couplings. The current model falls into this category where the existence of small elements in h, h' and f matrices leads to the expectation that we should be able to satisfy the proton

decay constraints without any cancellation. Let us now see how this occurs using the fit for Yukawa couplings we obtained in the previous section.

The colored Higgs triplets, $\phi_T + \phi_{\bar{T}}$: $((3, 1, -1/3) + \text{c.c.})$ responsible for proton decay in our model arise from $\mathbf{10} + \mathbf{10}' + \mathbf{126} + \overline{\mathbf{126}} + \mathbf{210}$ multiplets. Once the triplet fields ϕ_T and $\phi_{\bar{T}}$ are integrated out, both $LLLL(C_L)$ and $RRRR(C_R)$ operators leading to proton decay emerge:

$$W_5 = \frac{1}{2} C_L^{ijkl} \ell_k q_l q_i q_j + C_R^{ijkl} e_k^c u_l^c u_i^c d_j^c. \quad (27)$$

The color triplet $\phi_{T,\bar{T}}$ fields are linear combinations of six fields, two of them arising from two $\mathbf{10}$'s, three of them arising from $\mathbf{126} + \overline{\mathbf{126}}$ and one from $\mathbf{210}$. This leads to a 6×6 dimensional mass matrix for the triplets: $(\phi_{\bar{T}})_a (M_T)_{ab} (\phi_T)_b$. One can write the dimension five operators in terms of the couplings h, h' and f as follows:

$$\begin{aligned} C_L^{ijkl} = & ch_{ij}h_{kl} + x_1 h_{ij}h'_{kl} + x_2 h'_{ij}h_{kl} + x_3 h'_{ij}h'_{kl} \\ & + x_4 f_{ij}f_{kl} + x_5 f_{ij}h_{kl} + x_6 h_{ij}f_{kl} \\ & + x_7 h'_{ij}f_{kl} + x_8 f_{ij}h'_{kl}. \end{aligned} \quad (28)$$

Similarly, we can write C_{RRRR} operator (and change x_i 's to different coefficients y_i). The coefficient c is $(M_T^{-1})_{11}$ and the coefficients x_i and y_i are also given by the components of M_T^{-1} . The proton decay amplitude can be written as

$$A = \frac{\alpha_2 \beta_p \tilde{A}}{4\pi M_T} \frac{m_{\tilde{W}}}{m_{\tilde{q}}^2}, \quad (29)$$

where $\tilde{A} = c\tilde{A}_{hh} + x_1\tilde{A}_{hf} + \dots$ using the c 's and x_i 's given in Eq. (28), and β_p is the nucleon matrix element of the three-quark operator. In our calculation we use $m_{\tilde{q}} = 1.3$ TeV as a typical value for the first two generation squark masses (to satisfy the LHC lower bound), and

similarly $m_{\tilde{W}} = 200$ GeV as a typical wino mass. The proton decay width can be written simply as

$$\Gamma \simeq (2.7 \times 10^{-50} \text{ GeV}) |C|^2 |f(F, D)|^2 \left(\frac{2 \times 10^{16} \text{ GeV}}{M_T} \right)^2 \times \left(\frac{m_{\tilde{W}}}{200 \text{ GeV}} \right)^2 \left(\frac{1 \text{ TeV}}{m_{\tilde{q}}} \right)^4, \quad (30)$$

where $f(F, D)$ are the hadronic form factors, typically of $\mathcal{O}(1)$. Assuming that the colored Higgs from **10**'s are the lightest, we find the largest contribution to the $p \rightarrow \bar{\nu}_\mu K^+$ to be arising from h'_{12} . Using the values of h' from the Eq. (13) and varying M_T within a factor 5 of the GUT scale, we find that the partial lifetime of proton in this mode can be as large as 7×10^{33} years. If however, we lower $\tan\beta$ to 5 (h'_{12} will be smaller in this case), or raise the squark mass to 1.5 TeV, the lifetime can be as large as 10^{34} years. The partial lifetimes for other flavors of ν are found to be larger. Similarly, the partial lifetimes for other decay modes, e.g., $n \rightarrow \bar{\nu}_\mu \pi^0$ are found to be much larger of $\mathcal{O}(10^{38})$ years. We should emphasize here that, in order to generate these numbers, we did not invoke any cancellation. The smallness of the elements of the Yukawa coupling matrices are sufficient in suppressing the decay rates. We note that the lifetime of the mode $p \rightarrow \bar{\nu}_\mu K^+$ is within the search limit of the proposed Hyper-Kamiokande experiment [33].

VII. SUMMARY

We have analyzed the predictions of a minimal $SO(10) \times S_4$ model of flavor with dominant type-II seesaw form for neutrino masses, where the forms of the Yukawa

couplings for the two **10** Higgs fields and one **126** Higgs field are determined dynamically by flavon vevs at the minimum of the S_4 -invariant flavon potential with an additional Z_n symmetry. The model has eleven parameters including complex phases and is, thus, a relatively economical one when compared to other models discussed in the literature. It gives a very good fit to the charged fermion masses and the CKM parameters, and it also predicts the neutrino mixing angles θ_{12} , θ_{23} as well as $\Delta m_{\odot}^2 / \Delta m_{\text{atm}}^2$ in agreement with observation. Furthermore, it predicts a nonzero value for θ_{13} between 6° – 10° , which is in the current experimentally preferred range. With more accurate determination of θ_{13} and its correlation with θ_{23} , the model could be tested in the near future. The model also predicts a normal hierarchy for the neutrinos and hence an effective neutrino mass in neutrinoless double beta decay which is a few milli-electron-volts and is thus not observable in the current round of the searches for this process. The proton lifetime for $p \rightarrow \bar{\nu}_\mu k^+$ decay mode can be 10^{34} years. Finally, the successful fit for fermion masses require the Higgs mass to be below 129 GeV in this model and put lower bounds on the third generation squark masses which are well within the reach of LHC.

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