

**Inclusive  $b$ -jet and  $b\bar{b}$ -dijet production at the LHC via Reggeized gluons**

V. A. Saleev\*

*Samara State University, Academic Pavlov Street 1, 443011 Samara, Russia*

A. V. Shipilova†

*Institut für Kernphysik, Forschungszentrum Juelich, 52425 Juelich, Germany  
and Samara State University, Academic Pavlov Street 1, 443011 Samara, Russia*

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We study inclusive  $b$ -jet and  $b\bar{b}$ -dijet production at the CERN LHC invoking the hypothesis of gluon Reggeization in  $t$ -channel exchanges at high energy. The  $b$ -jet cross section includes contributions from open  $b$ -quark production and from  $b$ -quark production via gluon-to-bottom-pair fragmentation. The transverse-momentum distributions of inclusive  $b$ -jet production measured with the ATLAS detector at the CERN LHC in different rapidity ranges are calculated both within multi-Regge kinematics and quasi-multi-Regge kinematics. The  $b\bar{b}$ -dijet cross section is calculated within quasi-multi-Regge kinematics as a function of the dijet invariant mass  $M_{jj}$ , the azimuthal angle between the two jets  $\Delta\phi$ , and the angular variable  $\chi$ . At the numerical calculation, we adopt the Kimber-Martin-Ryskin and Blümlein prescriptions to derive unintegrated gluon distribution function of the proton from its collinear counterpart for which we use the Martin-Roberts-Stirling-Thorne set. We find good agreement with measurements by the ATLAS and CMS Collaborations at the LHC at the hadronic c.m. energy of  $\sqrt{S} = 7$  TeV.

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**I. INTRODUCTION**

The study of  $b$ -jet hadroproduction provides an important test of perturbative quantum chromodynamics (QCD) at high energies. The total collision energies,  $\sqrt{S} = 1.8$  TeV and 1.96 TeV in Tevatron runs I and II, respectively, and  $\sqrt{S} = 7$  TeV or 14 TeV at the LHC, sufficiently exceed the characteristic scale  $\mu$  of the relevant hard processes, which is of order of  $b$ -jet transverse momentum  $p_T$ , i.e., we have  $\Lambda_{\text{QCD}} \ll \mu \ll \sqrt{S}$ . In this high-energy regime, so-called “Regge limit,” the contribution of partonic subprocesses involving  $t$ -channel parton (gluon or quark) exchanges to the production cross section can become dominant. Then the transverse momenta of the incoming partons and their off-shell properties can no longer be neglected, and we deal with “Reggeized”  $t$ -channel partons. These  $t$ -channel exchanges obey multi-Regge kinematics (MRK) when the particles produced in the collision are strongly separated in rapidity. If the same situation is realized with groups of particles, then quasi-multi-Regge kinematics (QMRK) is at work. In the case of  $b$ -jet and  $b\bar{b}$ -dijet inclusive production, this means the following:  $b$  jet (MRK) or  $b\bar{b}$  dijet (QMRK) is produced in the central region of rapidity, while other particles are produced with large modula of rapidities.

The parton Reggeization approach [1] is based on the hypothesis of parton Reggeization in  $t$ -channel exchanges at high energy [2]. It was used for the description of a large number of hard processes at the modern hadron colliders

and the obtained results confirm the assumption of a dominant role of MRK or QMRK production mechanisms at high energy. This approach was successfully applied to interpret the production of isolated jets [3], prompt photons [4], diphotons [5], charmed mesons [6], and heavy quarkonia [7–10] measured at the Fermilab Tevatron at the DESY HERA and at the CERN LHC. The theoretical background of a parton Reggeization approach is the effective quantum field theory implemented with the non-Abelian gauge-invariant action including fields of Reggeized gluons [2] and Reggeized quarks [11], which was proposed by Lipatov in 1995 [12]. In this effective theory, Reggeized partons interact with quarks and Yang-Mills gluons in a specific way. Recently, in Ref. [13], the Feynman rules for the effective theory of Reggeized gluons were derived for the induced and some important effective vertices.

Usually it is suggested that the MRK or QMRK production mechanism is the dominant one only at small  $p_T$  values. Our recent study of isolated jet production at the Tevatron collider and LHC (see Ref. [3]) demonstrated that the parton Reggeization approach can be successfully used already in the range of  $x_T = \frac{2p_T}{\sqrt{S}} \lesssim 0.1$ , or at the  $p_T \lesssim 300$ –400 GeV for the energy  $\sqrt{S} = 7$  TeV at the LHC. This result motivates us to apply the parton Reggeization approach for the study of  $b$ -jet and  $b\bar{b}$ -dijet production in the kinematical range of transverse momentum  $20 < p_T < 400$  GeV and rapidity  $|y| < 2.1$ , as it was measured by the ATLAS Collaboration at the CERN LHC [14].

The high-energy factorization scheme with the effective vertices for Reggeized gluons has been used earlier in Refs. [15,16] for description of inclusive open  $b$ -quark [17],  $b$ -jet [18], and  $b\bar{b}$ -dijet [19] production at the

\*saleev@samsu.ru

†alexshipilova@samsu.ru

Tevatron collider. In this paper, we study in the same manner the inclusive  $b$ -jet and  $b\bar{b}$ -dijet production at the CERN LHC invoking the hypothesis of gluon Reggeization in  $t$ -channel exchanges at high energy. We take into account two mechanisms of  $b$ -jet production: the open  $b$ -quark production and “jetlike”  $b$ -quark production via gluon-to-bottom-pair fragmentation [20]. We consider  $b$ -quark jet as an isolated, by the jet-cone condition [21], hadronic jet containing one  $b(\bar{b})$ -quark or  $b\bar{b}$ -quark pair. Thus, the  $b$ -jet production cross section can be written as a sum of two terms. The first one represents a so-called “open  $b$ -quark” production when the  $b$  jet contains  $b(\bar{b})$  quark which is produced directly in the hard partonic subprocess. The second term corresponds to the case of “jetlike” production where a  $b$  jet contains a  $b\bar{b}$ -quark pair which is produced via gluon or light-quark fragmentation. The transverse-momentum distributions of inclusive  $b$ -jet production measured with the ATLAS detector at CERN LHC [14] in the different rapidity ranges are calculated both within multi-Regge kinematics and quasi-multi-Regge kinematics. The  $b\bar{b}$ -dijet cross sections are calculated within quasi-multi-Regge kinematics as functions of the  $b\bar{b}$ -dijet invariant mass  $M_{jj}$ , the azimuthal angle between the two jets  $\Delta\phi$ , and the angular variable  $\chi$ .

This paper is organized as follows. In Sec. II, the parton Reggeization approach is briefly reviewed. For our analysis we write down the relevant analytical formulas for squared matrix elements and differential cross sections. In Sec. III, we describe our calculations and present the results obtained. In Sec. IV, the conclusions are summarized.

## II. MODEL

We study  $b$ -jet production in the region of large  $b$ -quark transverse momentum  $p_T \gg m_b$ , where  $m_b$  is a  $b$ -quark mass. At the present time, the conventional approach for calculation of the  $b$ -quark production cross sections is based on the next-to-leading (NLO) approximation in perturbative QCD and collinear parton model [22]. It is well known that fixed-order perturbative QCD calculations are applicable when the transverse momentum  $p_T$  of the produced heavy  $b$  quark is not much larger than its mass  $m_b$ . In the case when the transverse momentum significantly exceeds the mass, the large logarithms of type  $\log(p_T/m_b)$  arise to all orders of  $\alpha_s(\mu)$ , so that a fixed-order approach breaks down [23]. It is possible to resum all these logarithms in the fragmentation approach using the factorization theorem, which states that the cross section for the production process of high- $p_T$   $b$  quark can be written in factorized form as a convolution of the short-distance partonic cross section of parton  $f$  production with the fragmentation function  $D_f^b(z, \mu^2)$  for a formation of a  $b$  quark from the parton  $f$ :

$$\frac{d\hat{\sigma}^{\text{frag}}}{dp_T} = \sum_f \int dz \int dp'_T \frac{d\hat{\sigma}^f}{dp'_T} D_f^b(z, \mu^2) \delta(p_T - zp'_T). \quad (1)$$

The fragmentation functions for heavy quarks in perturbative QCD have been studied at the next-to-leading order QCD approach in Ref. [24].

The experimentally measured transverse energy  $E_T$  (or the transverse momentum  $p_T$ ) of  $b$  jet includes transverse energies (transverse momenta) of all partons inside some jet cone in the rapidity-azimuthal angle plane whose radius is defined as follows:  $R = \sqrt{\Delta y^2 + \Delta\phi^2}$  [21]. It is insignificant which part of the initial parton four-momentum is transferred to the  $b$  quark, and we can simplify Eq. (1) to the form

$$\frac{d\hat{\sigma}^{\text{frag}}}{dp_T} = \sum_f \frac{d\hat{\sigma}^f}{dp_T} n_f(\mu), \quad (2)$$

where  $n_f(\mu) = \int_0^1 D_f^b(z, \mu) dz$  is a  $b$ -quark multiplicity in the  $f$ -parton jet. It is obvious that a  $b$ -quark multiplicity in a gluon-initiated jet greatly exceeds a  $b$ -quark multiplicity in any quark-initiated jets  $n_g(\mu) \gg n_q(\mu)$  with  $q = u, d, s, c$ . We will take into account only the main contribution from the gluon-to-bottom-pair fragmentation  $g \rightarrow b\bar{b}$ . Let us note that in this case the  $b\bar{b}$  pair is considered as a one  $b$ -quark jet.

To describe inclusive  $b$ -jet and  $b\bar{b}$ -jet cross sections in terms of the parton Reggeization approach, in the LO we need to consider gluon fusion subprocesses of open  $b$ -quark and gluon production only, which are to be dominant at the high energy. They are written as

$$R(q_1) + R(q_2) \rightarrow g(p), \quad (3)$$

$$R(q_1) + R(q_2) \rightarrow b(p_1) + \bar{b}(p_2), \quad (4)$$

$$R(q_1) + R(q_2) \rightarrow g(p_1) + g(p_2), \quad (5)$$

where  $R$  is a Reggeized gluon and  $g$  is a Yang-Mills gluon, respectively, with four-momenta indicated in parentheses. The contribution of the partonic subprocess (5) can be neglected in comparison with the contribution of the subprocess (4) because of the strong suppression by the  $g \rightarrow b\bar{b}$  fragmentation ( $n_g \approx 10^{-3}$ ) for both produced gluons. In Ref. [16] it was shown that at the Tevatron energy range the contribution of the subprocesses  $Q + \bar{Q} \rightarrow g$  and  $Q + \bar{Q} \rightarrow b + \bar{b}$  with initial Reggeized quarks is sufficiently smaller compared to the dominant contribution of the subprocesses (3) and (4), and the former becomes sizeable only at the very large  $b$ -jet transverse momentum  $p_T$ . As the LHC energy exceeds by a factor 3.5 the one of the Tevatron collider, we estimate a quark-antiquark annihilation contribution to be even smaller and therefore do not consider it in the present analysis.

Performing a study of high-transverse-momentum  $b$ -quark production ( $p_T \gg m_b$ ) in the collinear parton model, we have an additional  $b$ -quark production mechanism; namely, a production via  $b$ -flavor excitation where  $b(\bar{b})$  quarks are considered as partons in the colliding

protons. For example, this mechanism has been used successfully to describe  $B$ -meson  $p_T$  spectra at the Tevatron and LHC in NLO calculations of the parton model [25]. We have used a similar idea in our previous study of inclusive  $b$ -jet production at the Tevatron within the parton Reggeization approach [16]. In this work we took into account the LO in  $\alpha_s$  contribution from  $2 \rightarrow 1$  partonic subprocess

$$\mathcal{B}R \rightarrow b, \quad (6)$$

where  $\mathcal{B}$  is a Reggeized  $b$  quark. As it is shown in Fig. 1 of Ref. [16], the sum of this contribution and a contribution from the subprocess (4) strongly overestimate the experimental data. In the present analysis we ignore a contribution from the subprocess (6). First, we avoid any chance of double counting between subprocesses (4) and (6). Second, the conception of quark Reggeization for a  $b$  quark inside a proton seems to be wrong.  $b$  quarks are produced preferably at the last step of QCD evolution at the large scale  $\mu \sim p_T$ , and their parton distribution function (PDF) is proportional to a large logarithm  $\log(p_T/m_b)$ . However, the QCD evolution of a Reggeized parton should be valencelike. It means that the Reggeized parton must be a  $t$ -channel parton throughout all steps of QCD evolution in the parton ladder. But a  $b$ -quark conventional collinear PDF, which we take as input for a Kimber-Martin-Ryskin (Blümlein) prescription to obtain a  $b$ -quark unintegrated PDF, satisfies sealike QCD evolution. For this reason we strongly overestimate a value of a  $b$ -quark unintegrated PDF. The more adequate way should be to consider a subprocess  $bR \rightarrow b$  with a collinear  $b$  quark in the initial state, instead of subprocess (6). But even in this case a problem of double counting still exists. That is why in the present study we consider Reggeized-gluon induced contributions like (3) and (4), only.

The squared amplitude of subprocess (3) reads [7,16]

$$|\overline{\mathcal{M}(R + R \rightarrow g)}|^2 = \frac{3}{2} \pi \alpha_s p_T^2, \quad (7)$$

where  $p_T^2 = t_1 + t_2 + 2\sqrt{t_1 t_2} \cos \phi_{12}$ ,  $t_1 = -q_1^2 = \vec{q}_{1T}^2$ ,  $t_2 = -q_2^2 = \vec{q}_{2T}^2$ , with  $\vec{q}_{1T}$  and  $\vec{q}_{2T}$  representing the transverse momenta of initial Reggeized gluons, and  $\phi_{12}$  is the azimuthal angle enclosed between them.

The squared amplitude of subprocess (4) was obtained in Ref. [26] using the effective Feynman rules of the parton Reggeization approach. It coincides with the previous result of Ref. [27] which is expressed in the alternative form. The answers of Refs. [26,27] can be written down as a linear combination of an Abelian and a non-Abelian term as

$$|\overline{\mathcal{M}(R + R \rightarrow b + \bar{b})}|^2 = 256 \pi^2 \alpha_s^2 \left[ \frac{1}{2N_c} \mathcal{M}_A + \frac{N_c}{2(N_c^2 - 1)} \mathcal{M}_{NA} \right], \quad (8)$$

where

$$\begin{aligned} \mathcal{M}_A &= \frac{t_1 t_2}{\tilde{t} \tilde{u}} - \left( 1 + \frac{\alpha_1 \beta_2 S}{\tilde{u}} + \frac{\alpha_2 \beta_1 S}{\tilde{t}} \right)^2, \\ \mathcal{M}_{NA} &= \frac{2}{S^2} \left( \frac{\alpha_1 \beta_2 S^2}{\tilde{u}} + \frac{S}{2} + \frac{\Delta}{\hat{s}} \right) \left( \frac{\alpha_2 \beta_1 S^2}{\tilde{t}} + \frac{S}{2} - \frac{\Delta}{\hat{s}} \right) \\ &\quad - \frac{t_1 t_2}{x_1 x_2 \hat{s}} \left[ \left( \frac{1}{\tilde{t}} - \frac{1}{\tilde{u}} \right) (\alpha_1 \beta_2 - \alpha_2 \beta_1) + \frac{x_1 x_2 \hat{s}}{\tilde{t} \tilde{u}} - \frac{2}{S} \right], \\ \Delta &= \frac{S}{2} \left[ \tilde{u} - \tilde{t} + 2S(\alpha_1 \beta_2 - \alpha_2 \beta_1) \right. \\ &\quad \left. + t_1 \frac{\beta_1 - \beta_2}{\beta_1 + \beta_2} - t_2 \frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} \right], \end{aligned} \quad (9)$$

$\tilde{t} = \hat{t} - m_b^2$ ,  $\tilde{u} = \hat{u} - m_b^2$ ,  $\alpha_1 = 2(p_1 \cdot P_2)/S$ ,  $\alpha_2 = 2(p_2 \cdot P_2)/S$ ,  $\beta_1 = 2(p_1 \cdot P_1)/S$ , and  $\beta_2 = 2(p_2 \cdot P_1)/S$ . Here, the Mandelstam variables are defined as  $\hat{s} = (q_1 + q_2)^2$ ,  $\hat{t} = (q_1 - p_1)^2$ ,  $\hat{u} = (q_2 - p_1)^2$ ,  $S = (P_1 + P_2)^2$ , where  $P_1$  and  $P_2$  denote the four-momenta of the incoming protons.

Exploiting the hypothesis of high-energy factorization, we express the hadronic cross sections  $d\sigma$  as convolutions of partonic cross sections  $d\hat{\sigma}$  with unintegrated PDFs  $\Phi_g^h$  of Reggeized gluon in the hadrons  $h$ . For the processes under consideration here, we have

$$\begin{aligned} d\sigma(pp \rightarrow gX) &= \int \frac{dx_1}{x_1} \int \frac{d^2 q_{1T}}{\pi} \int \frac{dx_2}{x_2} \\ &\quad \times \int \frac{d^2 q_{2T}}{\pi} \Phi_g^p(x_1, t_1, \mu^2) \Phi_g^p(x_2, t_2, \mu^2) \\ &\quad \times d\hat{\sigma}(RR \rightarrow g), \end{aligned} \quad (10)$$

$$\begin{aligned} d\sigma(pp \rightarrow b\bar{b}X) &= \int \frac{dx_1}{x_1} \int \frac{d^2 q_{1T}}{\pi} \int \frac{dx_2}{x_2} \\ &\quad \times \int \frac{d^2 q_{2T}}{\pi} \Phi_g^p(x_1, t_1, \mu^2) \Phi_g^p(x_2, t_2, \mu^2) \\ &\quad \times d\hat{\sigma}(RR \rightarrow b\bar{b}). \end{aligned} \quad (11)$$

The unintegrated PDFs  $\Phi_g^h(x, t, \mu^2)$  are related to their collinear counterparts  $F_g^h(x, \mu^2)$  by the normalization condition

$$xF_a^h(x, \mu^2) = \int^{\mu^2} dt \Phi_a^h(x, t, \mu^2), \quad (12)$$

which yields the correct transition from formulas in the parton Reggeization approach to those in the collinear parton model, where the transverse momenta of the partons are neglected.

In our numerical analysis, we adopt as our default the prescription proposed by Kimber *et al.* [28] to obtain unintegrated gluon PDF of the proton from the conventional integrated one, as implemented in Watt's code [29]. The precise analysis of Kimber-Martin-Ryskin (KMR) gluon unintegrated PDF had been performed in Ref. [30], including an accurate study of the dependence on the choice of collinear input. As is well known [31], other

popular prescriptions, such as those by Blümlein [32] or by Jung and Salam [33], produce unintegrated PDFs with distinctly different  $t$  dependences. In our analysis we do not evaluate the unintegrated gluon PDF after Jung and Salam [33] because this PDF had been tabulated only in a range of  $t$ ,  $\mu^2 \leq 10^4$  GeV<sup>2</sup>. It is not enough to calculate  $b$ -jet production cross sections up to  $p_T = 400$  GeV in accordance with measurements of the relevant experiments. In fact, we had to use the unintegrated gluon PDF up to  $t$ ,  $\mu^2 \leq 10^6$  GeV<sup>2</sup>. In order to assess the resulting theoretical uncertainty, we also evaluate the unintegrated gluon PDF using the Blümlein approach, which resums small- $x$  effects according to the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [2]. As input for these procedures, we use the LO set of the Martin-Roberts-Stirling-Thorne [34] proton PDF as our default. The relevant theoretical study of particle production in the high-energy factorization scheme using KMR and Blümlein unintegrated gluon PDFs (Refs. [4–9]) demonstrates that both unintegrated PDFs lead to a similar behavior of production spectra at the nonlarge particle transverse momentum ( $p_T \leq 20$  GeV). In the case of high transverse momentum production of isolated jets and prompt photons [3], the theoretical predictions obtained with these PDFs are different. Although we take identical collinear inputs for both KMR and Blümlein approaches, the relevant kernels of integrand transformation between collinear and unintegrated PDFs differ. The KMR approach is based on the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation, while the Blümlein approach is based on the BFKL evolution equation. As the BFKL approach seems to be preferable in the region of very small  $x \ll 1$ , which corresponds to nonlarge  $p_T$  at fixed  $\sqrt{S}$ , the KMR unintegrated gluon PDF should be more suitable to describe the experimental data at large  $p_T$ .

Throughout our analysis the renormalization and factorization scales are identified and chosen to be  $\mu = \xi p_T$ , where  $\xi$  is varied between 1/2 and 2 about its default value 1 to estimate the theoretical uncertainty due to the freedom in the choice of scales. The resulting errors are indicated as shaded bands in the figures.

The master formula for the doubly differential cross section of inclusive  $b$ -jet production via gluon-to-bottom-pair fragmentation at the  $p_T \gg m_b$  reads as follows:

$$\begin{aligned} \frac{d\sigma^{\text{frag}}(pp \rightarrow bX)}{dp_T dy} &= \frac{1}{p_T^3} \int d\phi_1 \int dt_1 \Phi_g^p(x_1, t_1, \mu^2) \\ &\quad \times \Phi_g^p(x_2, t_2, \mu^2) \\ &\quad \times n_g(\mu) |\overline{\mathcal{M}(RR \rightarrow g)}|^2, \end{aligned} \quad (13)$$

where  $y$  is the rapidity of  $b$  quark, and  $\phi_1$  is the azimuthal angle enclosed between the vectors  $\vec{q}_{1T}$  and  $\vec{p}_T$ ,

$$x_{1,2} = \frac{p_T \exp(\pm y)}{\sqrt{S}}, \quad t_2 = t_1 + p_T^2 - 2\sqrt{t_1} p_T \cos(\phi_1).$$

In the case of  $b\bar{b}$ -dijet production via the partonic subprocess (4) we get the differential cross section in the following form:

$$\begin{aligned} \frac{d\sigma^{\text{open}}(pp \rightarrow b\bar{b}X)}{dp_{1T} dy_1 dp_{2T} dy_2 d\Delta\phi} &= \frac{p_{1T} p_{2T}}{16\pi^3} \int dt_1 \int d\phi_1 \Phi_g^p(x_1, t_1, \mu^2) \\ &\quad \times \Phi_g^p(x_2, t_2, \mu^2) \frac{|\overline{\mathcal{M}(RR \rightarrow b\bar{b})}|^2}{(x_1 x_2 S)^2}, \end{aligned} \quad (14)$$

where  $p_{1,2T}$  and  $y_{1,2}$  are  $b$ -quark and  $\bar{b}$ -antiquark transverse momenta and rapidities, respectively, and  $\Delta\phi$  is the azimuthal angle enclosed between the vectors  $\vec{p}_{1T}$  and  $\vec{p}_{2T}$ ,

$$\begin{aligned} x_1 &= (p_1^0 + p_2^0 + p_1^z + p_2^z)/\sqrt{S}, \\ x_2 &= (p_1^0 + p_2^0 - p_1^z - p_2^z)/\sqrt{S}, \\ p_{1,2}^0 &= \frac{p_{1,2T}}{2} [\exp(y_{1,2}) + \exp(-y_{1,2})], \\ p_{1,2}^z &= \frac{p_{1,2T}}{2} [\exp(y_{1,2}) - \exp(-y_{1,2})]. \end{aligned}$$

The inclusive  $b$ -jet transverse-momentum spectrum can be presented in the following form:

$$\begin{aligned} \frac{d\sigma^{\text{bjet}}}{dp_T} &= \frac{d\sigma^{\text{frag}}}{dp_T} + 2 \times \frac{d\sigma^{\text{open}}}{dp_T} \theta(R_{b\bar{b}} - R) \\ &\quad + \frac{d\sigma^{\text{open}}}{dp_T} \theta(R - R_{b\bar{b}}), \end{aligned} \quad (15)$$

where  $R_{b\bar{b}} = \sqrt{(y_b - y_{\bar{b}})^2 + (\phi_b - \phi_{\bar{b}})^2}$ ,  $R$  is the experimentally fixed jet radius parameter, and  $\theta(x)$  is the unit step function. In such a way, the subprocess (4) of open  $b$ -quark production contributes two separate  $b$ -quark jets while  $R_{b\bar{b}} > R$ , and only one  $b$ -quark jet while  $R_{b\bar{b}} < R$ .

### III. RESULTS

Recently, the ATLAS Collaboration presented data on inclusive and dijet production cross sections which have been measured for jets containing  $b$  hadrons ( $b$  jets) in proton-proton collisions at a center-of-mass energy of  $\sqrt{S} = 7$  TeV [14]. The inclusive  $b$ -jet cross section was measured as a function of transverse momentum in the range  $20 < p_T < 400$  GeV and rapidity in the range  $|y| < 2.1$ . The  $b\bar{b}$ -dijet cross section was measured as a function of the dijet invariant mass in the range  $110 < M_{jj} < 760$  GeV, the azimuthal angle difference between the two jets  $\Delta\phi$ , and the angular variable  $\chi$  in two dijet mass regions. Jets were reconstructed with jet radius parameter  $R = 0.4$ . The angular variable  $\chi$  is defined as follows:  $\chi = \exp|y_1 - y_2|$ . To measure the cross section as a function of  $\chi$ , an additional acceptance requirement was used that restricts the boost of the dijet system to  $|y_{\text{boost}}| = 0.5|y_1 + y_2| < 1.1$ .

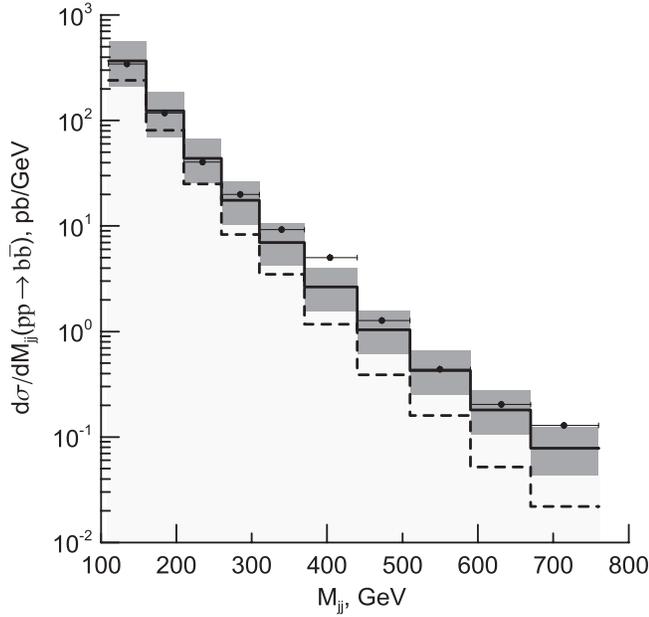


FIG. 1. The  $b\bar{b}$ -dijet cross section as a function of dijet invariant mass  $M_{jj}$  for  $b$  jets with  $p_T > 40$  GeV,  $|y| < 2.1$ . The data are from the ATLAS Collaboration [14]; the solid polyline corresponds to KMR unintegrated PDF and the dashed one to Blümlein PDF. The shaded bands indicate the theoretical uncertainties in the case of KMR unintegrated PDF.

The  $b\bar{b}$ -dijet cross section as a function of dijet invariant mass  $M_{jj}$  for  $b$  jets with  $p_T > 40$  GeV and  $|y| < 2.1$  is shown in Fig. 1. The data are compared to LO parton Reggeization approach predictions; the solid polyline corresponds to KMR unintegrated PDF [28] and the dashed one to Blümlein PDF [32]. We observe nice agreement between data and theoretical prediction obtained with the KMR unintegrated PDF. In the case of Blümlein PDF, the theoretical histogram lies about factor 2 lower than the experimental data and this difference increases towards the high values of dijet invariant mass.

In Fig. 2 the  $b\bar{b}$ -dijet cross section as a function of the azimuthal angle difference  $\Delta\phi$  between the two jets for  $b$  jets with  $p_T > 40$  GeV,  $|y| < 2.1$ , and a dijet invariant mass of  $M_{jj} > 110$  GeV is presented. The normalized to the total cross section data are compared to LO parton Reggeization approach predictions; the solid polyline corresponds to KMR unintegrated PDF and the dashed line to Blümlein PDF. For both unintegrated PDFs our predictions lie within the experimental uncertainty interval of data except only one point at the  $\Delta\phi \approx 2$ . We need to mention that in the case of CDF measurements at the Tevatron [19], the azimuthal-separation-angle distribution of inclusive  $b\bar{b}$ -dijet production is well described using the parton Reggeization approach formalism at all values of the azimuthal angle difference  $0 < \Delta\phi < \pi$  (see Fig. 4 in Ref. [16]).

The  $b\bar{b}$ -dijet cross section as a function of angular variable  $\chi$  for  $b$  jets with  $p_T > 40$  GeV,  $|y| < 2.1$ , and

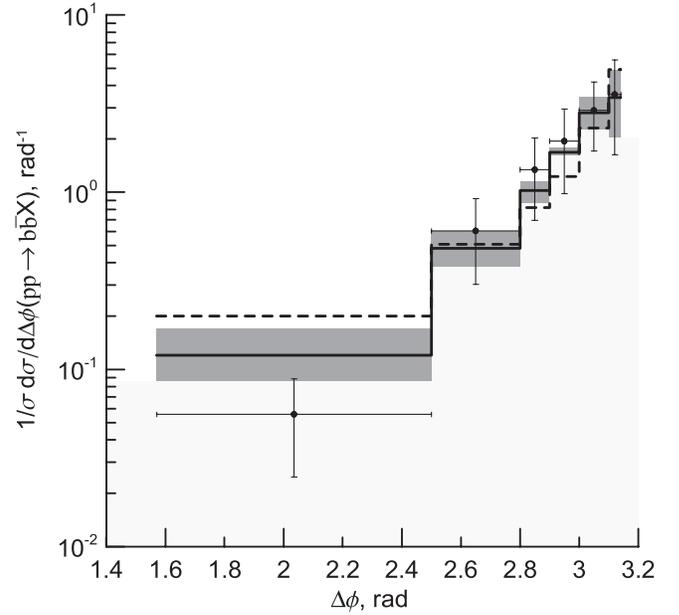


FIG. 2. The  $b\bar{b}$ -dijet cross section as a function of the azimuthal angle difference between the two jets for  $b$  jets with  $p_T > 40$  GeV,  $|y| < 2.1$  and a dijet invariant mass of  $M_{jj} < 110$  GeV. The data are from the ATLAS Collaboration [14]; the solid polyline corresponds to KMR unintegrated PDF and the dashed one to Blümlein PDF. The shaded bands indicate the theoretical uncertainties in the case of KMR unintegrated PDF.

$|y_{\text{boost}}| = \frac{1}{2}|y_1 + y_2| < 1.1$ , for dijet invariant mass ranges  $110 < M_{jj} < 370$  GeV and  $370 < M_{jj} < 850$  GeV are shown in Figs. 3 and 4, correspondingly. The normalized to the total cross section data are compared to our LO parton Reggeization approach predictions. In the range of  $110 < M_{jj} < 370$  GeV, the polylines corresponding to KMR and to Blümlein unintegrated PDFs coincide. In the region of large invariant masses  $370 < M_{jj} < 850$  GeV, the prediction obtained with the Blümlein unintegrated PDF lies about factor 2 lower than the data. On the contrary, the calculations with the KMR unintegrated gluon PDF are found to be in a good agreement with the data.

To calculate inclusive  $b$ -jet transverse-momentum production spectra we need to take into account gluon-to-bottom-pair production mechanism and use the function of  $b\bar{b}$ -pair multiplicity  $n_g(\mu)$  in a gluon jet. Because the existing theoretical predictions (see, for example, Ref. [35]) contain large uncertainties, we consider  $n_g(\mu)$  as a free phenomenological parameter, which is extracted from the experimental data from the ATLAS Collaboration [14] for the inclusive  $b$ -jet cross sections.

In Fig. 5, the inclusive differential  $b$ -jet cross section as a function of  $p_T$  for  $b$  jets with  $|y| < 2.1$  is compared with our LO predictions of the parton Reggeization approach. The contribution of QMRK subprocess (4) and the contribution of MRK subprocess (3) are shown separately.

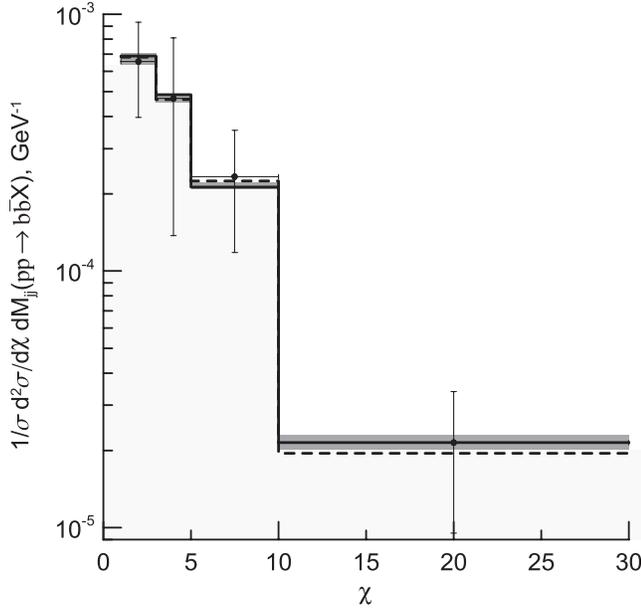


FIG. 3. The  $b\bar{b}$ -dijet cross section as a function of  $\chi$  for  $b$  jets with  $p_T > 40$  GeV,  $|y| < 2.1$ , and  $|y_{\text{boost}}| = \frac{1}{2}|y_1 + y_2| < 1.1$ , for dijet invariant mass range  $110 < M_{jj} < 370$  GeV. The data are from the ATLAS Collaboration [14]; the solid polyline corresponds to KMR unintegrated PDF and the dashed one to Blümlein PDF. The shaded bands indicate the theoretical uncertainties in the case of KMR unintegrated PDF.

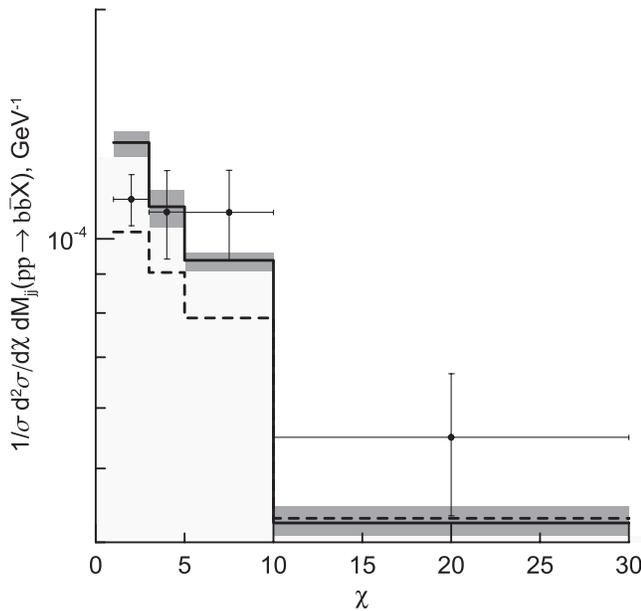


FIG. 4. The  $b\bar{b}$ -dijet cross section as a function of  $\chi$  for  $b$  jets with  $p_T > 40$  GeV,  $|y| < 2.1$ , and  $|y_{\text{boost}}| = \frac{1}{2}|y_1 + y_2| < 1.1$ , for dijet invariant mass range  $370 < M_{jj} < 850$  GeV. The data are from the ATLAS Collaboration [14]; the solid polyline corresponds to KMR unintegrated PDF and the dashed one to Blümlein PDF. The shaded bands indicate the theoretical uncertainties in the case of KMR unintegrated PDF.

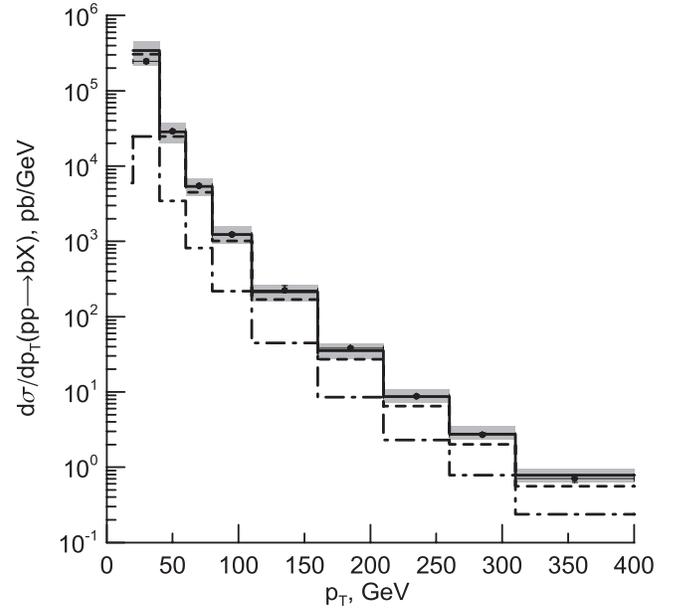


FIG. 5. Inclusive differential  $b$ -jet cross section as a function of  $p_T$  for  $b$  jets with  $|y| < 2.1$ . The data are from the ATLAS Collaboration [14]. The dashed polyline corresponds to contribution of the open  $b$ -quark production, the dash-dotted one to the gluon-to-bottom-pair fragmentation, and the solid to the sum of them all. The calculation is done with the KMR unintegrated PDF.

We see that open  $b$ -quark production mechanism does not describe the data, especially at the large  $p_T$ , and some contribution from gluon-to-bottom-pair fragmentation mechanism is needed. We have obtained a good description of the data using  $n_g(\mu)$  as a free parameter. In Fig. 6, the  $b\bar{b}$ -pair multiplicity  $n_g(\mu)$  in a gluon jet as a function of  $p_T$  extracted from the ATLAS data for the inclusive  $b$ -jet production spectra [14] is shown. The open circles and dashed fitting line correspond to Blümlein unintegrated PDF, and the black circles and solid fitting line correspond to KMR unintegrated PDF. The general theoretical consideration [35] leads to the following analytical approximation for the  $b\bar{b}$ -pair multiplicity in a gluon jet:

$$n_g(\mu) = A \ln \frac{\mu^2}{m_b^2}, \quad (16)$$

where we fixed  $m_b = 4.75$  GeV and  $\mu = p_T/4$  [36] and found that  $A_{\text{KMR}} = 0.0012$  in the case of KMR unintegrated PDF, and  $A_B = 0.0027$  in the case of Blümlein unintegrated PDF. At the scale  $\mu \simeq m_Z/4$ , which corresponds gluon-to-bottom-pair fragmentation of secondary gluon in the  $Z$ -boson decay ( $Z \rightarrow q\bar{q} \rightarrow q\bar{q}b\bar{b}$ ), our approximation (16) yields  $n_g \simeq 0.002$ – $0.004$ , that is in agreement with the measurements at the LEP collider:  $n_g = (3.3 \pm 1.8) \times 10^{-3}$  from the DELPHI Collaboration [37], and  $n_g = (2.44 \pm 0.93) \times 10^{-3}$  from the SLD Collaboration [38]. The difference in obtained  $b\bar{b}$ -pair multiplicities  $n_g(\mu)$  with the KMR and Blümlein

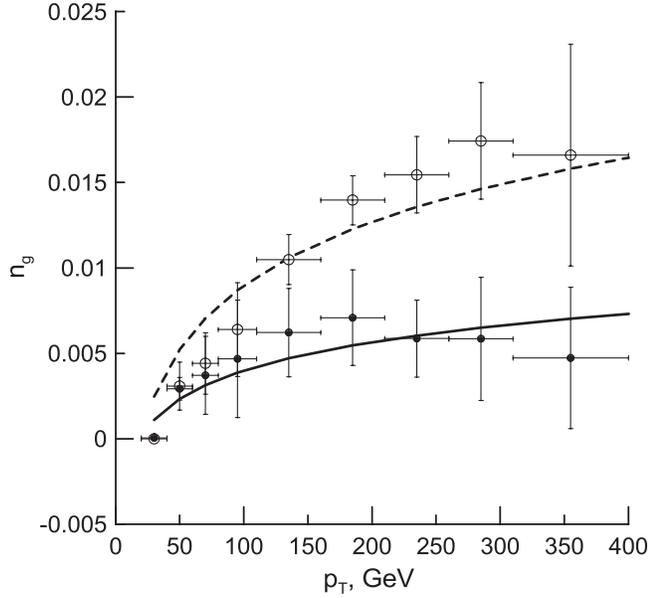


FIG. 6. The  $b\bar{b}$ -pair multiplicity  $n_g$  in a gluon jet as a function of  $p_T$  extracted from the ATLAS data for the inclusive  $b$ -jet production spectra [14]. The open circles and dashed fitting line correspond to Blümlein unintegrated PDF; the black circles and solid fitting line correspond to KMR unintegrated PDF.

unintegrated PDFs should be used to distinguish the last ones. We conclude that KMR unintegrated PDF looks preferably to describe  $b$ -jet production cross sections. Opposite this conclusion, we found recently [3] that Blümlein unintegrated PDF is better to describe all-flavor inclusive jet production spectra [39].

The measured by ATLAS Collaboration [14] inclusive double-differential  $b$ -jet cross sections as functions of  $p_T$  for the different rapidity ranges (1)  $|y| < 0.3$  ( $\times 10^6$ ), (2)  $0.3 < |y| < 0.8$  ( $\times 10^4$ ), (3)  $0.8 < |y| < 1.2$  ( $\times 10^2$ ), and (4)  $1.2 < |y| < 2.1$  are shown in Fig. 7. Here, our theoretical predictions are obtained taking into account both contributions: open  $b$ -quark production and fragmentation production with  $n_g(\mu)$  as in (16) and with KMR unintegrated PDF. We demonstrate good agreement with the data in all rapidity intervals.

To test the universality of the approach as well as the universality of the obtained function  $n_g(\mu)$  we compare our prediction with experimental data for transverse-momentum  $b$ -jet spectra from the CMS Collaboration at the CERN LHC [40] (Fig. 8) and the CDF Collaboration at the Fermilab Tevatron [18] (Fig. 9). In both cases we find a good agreement between the theoretical predictions and experimental data.

Looking at Figs. 5–9, we find that contribution of the gluon-to-bottom-pair fragmentation in inclusive  $b$ -jet production  $p_T$  spectra increases from 10%–15% at the  $p_T \approx 50$  GeV up to 30%–40% at the  $p_T \approx 350$  GeV. This conclusion contradicts the prediction of the NLO calculations in the collinear parton model in which the

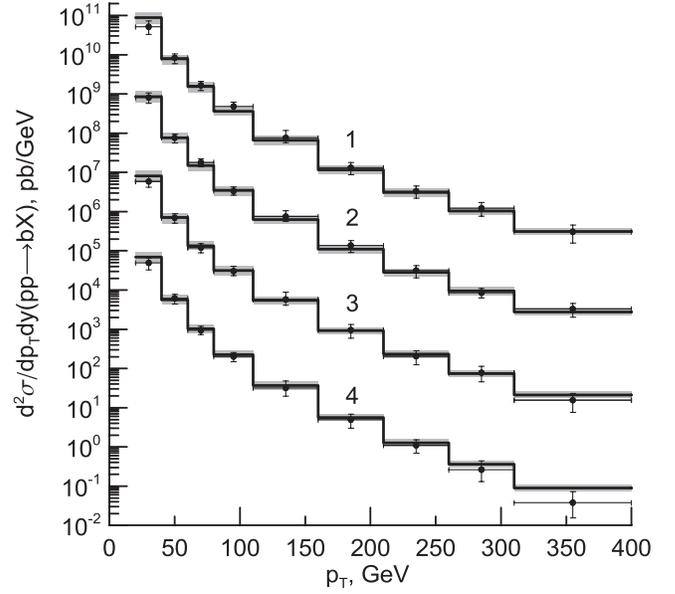


FIG. 7. Inclusive double-differential  $b$ -jet cross sections as functions of  $p_T$  for the different rapidity ranges: (1)  $|y| < 0.3$  ( $\times 10^6$ ), (2)  $0.3 < |y| < 0.8$  ( $\times 10^4$ ), (3)  $0.8 < |y| < 1.2$  ( $\times 10^2$ ), and (4)  $1.2 < |y| < 2.1$ . The data are from ATLAS Collaboration [14]. The solid polylines correspond to the sum of all contributions (15) and KMR unintegrated PDF.

gluon-to-bottom-pair fragmentation mechanism would be dominant at the large- $p_T$  region at the LHC and it would be about 50% at the Tevatron collider [36,41].

Comparing as a whole our results with the theoretical predictions obtained in the NLO of a parton model, which

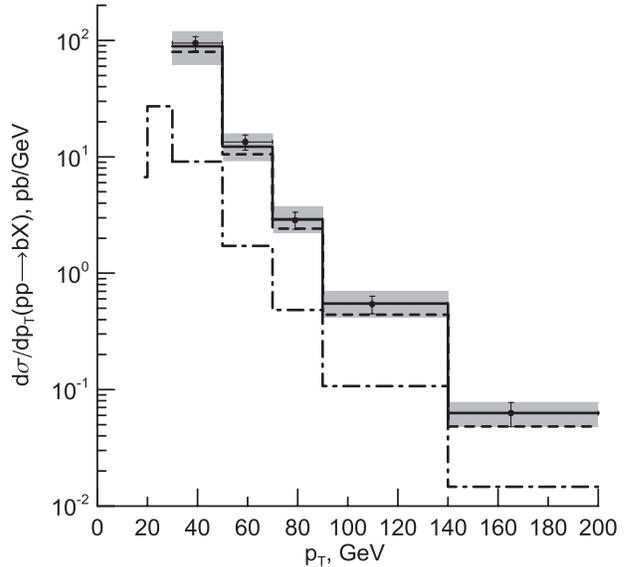


FIG. 8. Inclusive differential  $b$ -jet cross section as a function of  $p_T$  for  $b$  jets with  $|y| < 2.4$ . The data are from CMS Collaboration [40]. The dashed polyline corresponds to contribution of the open  $b$ -quark production, the dash-dotted one to the gluon-to-bottom-pair fragmentation, the solid to the sum of them all. The calculation is done with the KMR unintegrated PDF.

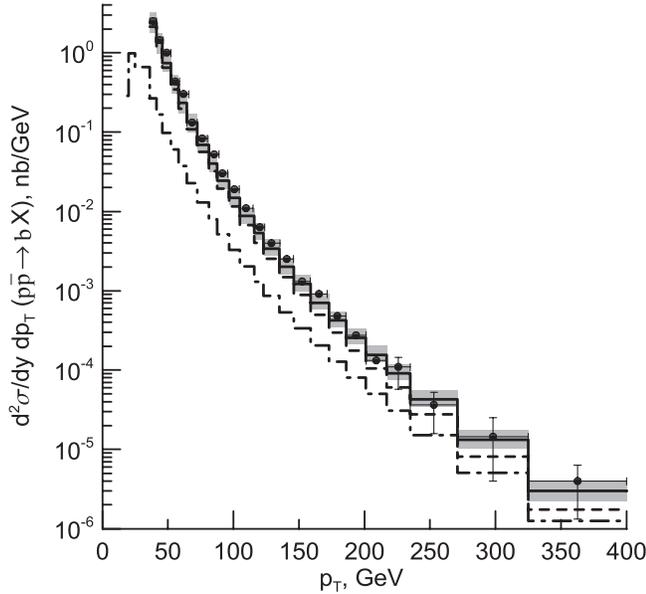


FIG. 9. Inclusive differential  $b$ -jet cross section as a function of  $p_T$  for  $b$  jets with  $|y| < 0.7$ . The data are from CDF Collaboration [18]. The dashed polyline corresponds to contribution of the open  $b$ -quark production, the dash-dotted one to the gluon-to-bottom-pair fragmentation, the solid to the sum of them all. The calculation is done with the KMR unintegrated PDF.

also describe ATLAS data for  $b$ -jet production [14], we would like to pay attention to difficulties of the fixed order collinear calculations. At first, the  $K$  factor between LO and NLO calculations is very large at the high  $p_T$ . The scale uncertainty decreases from LO calculation to NLO calculation, but it still remains large. The last one can be a signal on large NNLO contributions, which are not taken into account. Second, to describe data at nonlarge  $p_T \leq 50$  GeV at the energy of  $\sqrt{s} = 2-7$  TeV in the collinear parton model it is needed to add the soft gluon resummation procedure, which is far from the application field of the DGLAP evolution equation and should be considered as a phenomenological trick rather than a rigorous approach. Both of these difficulties are solved in the PRA by introducing the off-shell LO Reggeized parton amplitudes and unintegrated gluon PDF, which take into account large logarithmic contributions in all orders in  $\alpha_s$ :  $(\alpha_s \ln(\mu^2/\Lambda_{\text{QCD}}^2))^n$  and  $(\alpha_s \ln(1/x))^n$ .

In such a way, we have obtained the self-coordinated description in the PRA of  $b\bar{b}$ -dijet cross sections where the

open  $b\bar{b}$ -quark pair production works solely, and the  $b$ -jet inclusive cross sections where open  $b\bar{b}$ -quark pair production is the main contribution, while the gluon-to-bottom-pair fragmentation production is also important.

#### IV. CONCLUSIONS

The CERN LHC is currently probing particle physics at the terascale c.m. energies  $\sqrt{s}$ , so that the hierarchy  $\Lambda_{\text{QCD}} \ll \mu \ll \sqrt{s}$ , which defines the MRK and QMRK regimes, is satisfied for processes of heavy quark ( $c$  or  $b$ ) production in the central region of rapidity, where  $\mu$  is of order of their transverse momentum. In this paper, we studied QCD processes of particular interest, namely inclusive  $b$ -jet and  $b\bar{b}$ -dijet hadroproduction, at LOs in the parton Reggeization approach, in which they are mediated by  $2 \rightarrow 1$  and  $2 \rightarrow 2$  partonic subprocesses initiated by Reggeized gluon collisions.

We describe well the recent LHC data measured by the ATLAS Collaboration [14] at the whole presented range of the  $b\bar{b}$ -jet transverse momenta, the  $b\bar{b}$ -jet rapidity, the  $b\bar{b}$ -dijet invariant mass  $M_{jj}$ , the azimuthal angle between the two jets  $\Delta\phi$ , and the angular variable  $\chi$ . We show that the gluon-to-bottom-pair fragmentation component [24], which takes into account effects of large logarithms  $\log(p_T/m_b)$ , increases in inclusive  $b$ -jet production at the high transverse momenta  $p_T$  up to 30%–40% of sum of all contributions. The  $b\bar{b}$ -pair multiplicity which we extracted fitting the ATLAS data [14] is in agreement with the previous measurements at the LEP collider [37,38]. Comparing different unintegrated gluon PDFs, we have found that the agreement with the data has been obtained when we used KMR PDF [28], and the calculations with Blümlein PDF [32] regularly underestimate data, approximately by factor 2, in the region of large  $b$ -jet  $p_T$  and the large  $b\bar{b}$ -dijet invariant mass.

#### ACKNOWLEDGMENTS

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