

# Calculating loops without loop calculations: Next-to-leading order computation of pentaquark correlators

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We compute next-to-leading order (NLO) perturbative QCD corrections to the correlators of interpolating pentaquark currents and their absorptive parts. We employ modular techniques in configuration space which saves us from the onus of having to do loop calculations. The modular technique is explained in some detail. We present explicit NLO results for several interpolating pentaquark currents that have been written down in the literature. Our modular approach is easily adapted to the case of NLO corrections to multi-quark correlators with an arbitrary number of quarks/antiquarks.

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## I. INTRODUCTION

The discovery of exotic quark states and bound states of gluons would be another manifestation of QCD, allowing for a quantitative check of its features and finding numerical values of some important QCD parameters. While glueballs are certainly the most searched for states in QCD [1], there is also much interest in exotic states in strong interactions, i.e. states built from quarks within QCD which differ from the simplest valence quark content of mesons or baryons (see e.g. Refs. [2,3]). The theoretical investigation of multi-quark states ( $Q^n \bar{Q}^m$ ,  $n + m > 3$ ) and the experimental search for them may provide important information on the properties of the interaction of quarks and gluons at large distances. Until recently major efforts have been directed to the study of the dibaryon spectrum ( $n = 6$ ,  $m = 0$ ), both theoretically and experimentally [4]. This particular sixfold state is rather peculiar as it is close to the deuteron, building bridges to applications of QCD to medium-energy nuclear physics [5,6]. In Ref. [2], Jaffe predicted that there might exist a stable six-quark  $S$ -wave state—a dihyperyon  $H$ —which is a singlet with respect to both color and flavor  $SU(2)$  (with strangeness  $-2$ ) with the quantum numbers  $J^P = 0^+$  and a mass around 2150 MeV. The quantum numbers of the  $H$  state are identical to the quantum numbers of the  $(\Lambda\Lambda)$  pair of two  $\Lambda(1115)$  hyperons, and its mass is smaller than the sum of the masses of the two  $\Lambda$  hyperons. The  $H$  state is therefore stable with respect to strong interactions and can decay only through weak interactions. To the best of our knowledge, the famous dibaryon state  $H$  is the first state to attract attention in the modern context of QCD. Thereafter there were efforts to identify some mesons in QCD (scalar mesons as a  $KK$  molecule) with a four quark state in order to explain their properties and, in particular, their production and decay patterns [7]. In the intervening years the interest in exotic states has mainly shifted to tetraquarks and pentaquarks.

The study of bound states in QCD is a difficult problem. After almost 40 years of research it is clear that the most promising approach is very likely given by lattice QCD, in particular, since the computer power and computer algorithms have advanced much since the first introduction of lattice QCD in the early seventies of the last century (results are given for instance in Ref. [8]). Besides lattice QCD, model dependent approaches have been used in Refs. [9,10], for example, in the framework of the MIT quark-bag model [11]. It is important to test these model predictions solely on the basis of fundamental principles of QCD. Such a test can be made by means of the method of QCD sum rules, using either the technique of finite-energy sum rules [12] or that of Borel sum rules [13].

The operator product expansion and QCD sum rules serve as a solid testing ground for many calculations in the theory of hadrons. The method of QCD sum rules is based on the fundamental field theoretic principles of QCD, and has proved its effectiveness in calculations of the masses of mesons [13–15] and baryons [16–18]. However, the reliability of perturbative calculations requires a thorough check, in particular, in the uncharted territory of exotic multi-quark states where the collected experimental material is rather small. It is therefore worthwhile to compute some examples in order to get a feeling for the structure of the perturbative series. Work in this direction is under way.

Glueballs have been previously analyzed in the context of QCD sum rules. The perturbative QCD corrections to the sum rules were found to be very large [19]. Exotic mesonic states have been analyzed in Ref. [20]. QCD sum rules for ordinary three quark baryon states have been widely studied. In particular, the correlators of baryonic currents with finite mass heavy quarks have been calculated at next-to-leading order of perturbative QCD, allowing for further improvements in the precision of QCD sum rule predictions [21]. It is known that next-to-leading order

(NLO) perturbative corrections to baryon sum rules are large [15,21,22]. They are expected to be even larger for multi-quark states with  $n > 3$  quarks. Different aspects of such  $n > 3$  multi-quark states in QCD have already been discussed some time ago [23]. One feature of  $n > 3$  multi-quark states is that they have a large internal weight of color states [3].

The immediate purpose of the present investigation is to concentrate on a type of exotic multi-quark state called pentaquarks—states with baryon quantum numbers that contain an additional quark-antiquark pair. These states have been discovered experimentally by different collaborations: LEPS Collaboration (Japan) [24], DIANA Collaboration (Russia) [25], CLAS Collaboration (USA) [26], and SAPHIR Collaboration (Germany) [27]. The results of the present investigation will open the possibility for a high-precision description of these experimental data on pentaquarks. The investigation is also important for further experimental precision studies on these and related states at DESY (HERMES Collaboration [28]) and CERN (NA49 Collaboration [29]). There is also a proposal to launch an experimental study of pentaquark baryons at meson factories [30]. Experimentally these collaborations are using different apparatus and techniques but theoretically the observed states should be understood within QCD. While the first principle numerical computation on the lattice gave rather positive results [31], analytical methods and in particular method of QCD sum rules should definitely be developed for a reliable identification of the new states in the hadronic spectrum.

It appears that the experimental confirmation of these states is problematic at the moment as some collaborations have reconsidered their results and conclusions. However, there is no doubt that such states are possible within QCD and the theoretical study should continue. In case of a definite positive indication from theory the experimental searches could certainly proceed in a much more efficient way.

In particular, a dedicated experiment has given a negative result in the direct search of the pentaquark state [32]. A review of the present experimental situation can be found in Ref. [33] (see also Ref. [34]). Note that more lattice studies have become available [35], some with a negative outcome as concerns the existence of pentaquark states [36]. This makes the task of the theory even more challenging. Either one has to show that such states do not form for some reason, or to suggest a new mass scale of these states and to identify the appropriate decay modes for their determination [37]. The first task is difficult in as much as one touches on the problem of bound state formation and therefore of the (confined) strong coupling. The latter problem ultimately requires the calculation of perturbative corrections to the operator product expansion used within QCD sum rules.

Sum rule calculations of pentaquarks and corresponding critical analysis' have been presented in numerous papers [38–40]. While the accuracy of the QCD sum rule method is about  $\sim 20\%$  at present, the results obtained agree with experimental claims and model predictions [10]. However, within the QCD sum rule method it is not possible to predict whether the mass of the lowest pentaquark state lies above or below the  $Kp$  threshold (i.e., whether it is stable).

The aim of the present paper is to create a framework for an accurate sum rule analysis of the properties of pentaquark states. The study of pentaquark states within the QCD sum rule method requires a precise knowledge of the absorptive parts of the correlators of the pentaquark interpolating currents. In this paper we present perturbative next-to-leading order calculations of the relevant correlators in QCD.

## II. NLO CORRECTIONS TO THE CORRELATION FUNCTION

According to the QCD sum rule approach to hadron properties, the principal quantity to be analyzed is the correlation function,

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0 | T j(x) \bar{j}(0) | 0 \rangle, \quad (1)$$

where  $j(x)$  is a local current operator with the quantum numbers of the hadron state, termed the interpolating current of the hadron state. The construction of the conjugate operator  $\bar{j}(x)$  depends on whether the hadron is a fermion or a boson. For fermionic states such as the ordinary baryon states or the pentaquark states dealt with in this paper, one has  $\bar{j}(x) = j^\dagger(x) \gamma^0$ . For bosonic states (mesons, tetraquarks, ...) the conjugate operator is just the adjoint operator,  $\bar{j}(x) = j^\dagger(x)$ . The result of the sum rule analysis depends strongly on the choice of the interpolating current as has been shown already in the case of the dibaryon state [23].

In the case of a given pentaquark state the pentaquark current  $j(x)$  is a local scalar current with the quantum numbers of that pentaquark baryon. For instance, take the ground state pentaquark state  $\Theta^+$ . The current is constructed from five quark fields, such that its projection onto the real pentaquark baryon state  $|\Theta^+(p)\rangle$  (within the assumption that this state exists) is nonzero:

$$\langle 0 | j(0) | \Theta^+(p) \rangle = \lambda_{\Theta^+}, \quad p^2 = m_{\Theta^+}^2. \quad (2)$$

Since such a current  $j(x)$  is not unique, the question of its optimal choice arises immediately (see Appendix B for a discussion of this issue). We recall that the problem of choosing the current already arose in the case of baryons [16,17] where the currents are constructed from three quark fields. When the current is constructed from five quark fields as in our case, this problem is much more complicated, since the number of independent currents

with the given quantum numbers is much larger (see also Ref. [40]). The treatment of the current  $j(x)$  in the most general form, i.e. in the form of a linear combination of all the independent local operators with the quantum numbers of  $\theta$ , is a very cumbersome problem. Therefore we confine ourselves to the choice of a few of the simplest currents with the required quantum numbers and analyze the dependence of our results on the properties of these currents. As in the case of mesons and baryons, we shall construct the current  $j(x)$  from quark fields without derivatives.

In accordance with the method of QCD sum rules, we shall calculate the correlation function (1) by means of Wilson's operator expansion, assuming that the vacuum expectation values of the local operators (the so-called condensates) are nonzero. The calculations must be performed in the Euclidean region  $-q^2 \geq 1 \text{ GeV}^2$ . In this region, the effective strong interaction constant  $\alpha_s$  is not very small and one has to calculate the coefficient functions of the operator expansion at least at NLO in perturbation theory. Therefore, the correlator function  $\Pi$  should be calculated at NLO in  $\alpha_s$ . The fact that this may be necessary for calculations of physical quantities in the framework of the sum rule method is confirmed by previous applications of the sum rule method, in particular, by the calculation of baryon masses.

In this paper we explicitly discuss the computational techniques for the unity operator of the operator product expansion. The condensate contributions to the correlation function which have to be incorporated for a consistent NLO analysis are not discussed in this paper but can be calculated along the lines presented here. For instance, the incorporation of the quark condensate requires only minor modifications of the present methods. Quark condensate contributions to baryonic sum rules have been e.g. considered in Ref. [41].

The LO calculation falls into the category of the sunset diagrams (cf. Fig. 1(a)). Sunset diagrams are directly calculable in configuration space [42]. These types of diagrams also appear in the effective low energy gluon correlator for light quarks [43]. The corrections are of two types. The propagator-type corrections depicted in Fig. 1(b) are straightforward and can be easily added with no effort at all. The second type of corrections depicted in Fig. 1(c) correspond to the irreducible diagrams

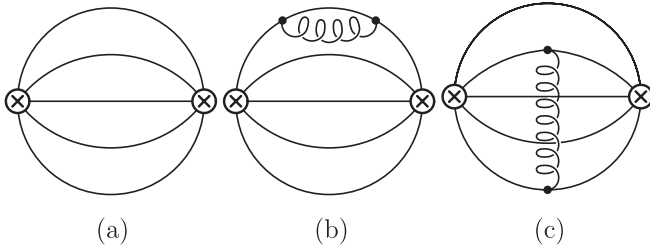


FIG. 1. LO contribution (a) and examples for the NLO propagator (b) and dipropagator corrections (c).

of the fish type. They are rather well known in the massless limit (the more complicated massive case was analyzed in Ref. [44]). In order to deal with the diversity of interpolating currents that have been proposed in the literature for the pentaquark states, we have developed a modular calculation method in configuration space. The modular method reduces the perturbative calculations of the present paper to pure algebraic calculations [45].

### III. PRESENTATION OF THE MODULAR METHOD

As already mentioned, the two required main modules of our method are the propagator correction  $S_1(x)$  and the dipropagator correction  $S_2(x)$  which read ( $\not{x} = \gamma^\mu x_\mu$ )

$$\begin{aligned} S_1(x)|_{\text{NLO}} &= S_1(x)|_{\text{LO}} \left\{ 1 - C_F \frac{\alpha_s}{4\pi\epsilon} (\mu_x^2 x^2)^\epsilon \right\} \\ &= S_0(x^2) \left\{ 1 - C_F \frac{\alpha_s}{4\pi\epsilon} (\mu_x^2 x^2)^\epsilon \right\}, \end{aligned} \quad (3)$$

$$\begin{aligned} S_2(x)|_{\text{NLO}} &= S_0(x^2)^2 \left\{ \not{x} \otimes \not{x} - t^a \otimes t^a \frac{\alpha_s}{4\pi} (\mu_x^2 x^2)^\epsilon \right. \\ &\quad \times \left( \gamma^\mu \otimes \gamma^\nu \left[ \left( \frac{1}{\epsilon} + \frac{11}{2} \right) x_\mu x_\nu + \left( \frac{1}{\epsilon} + \frac{1}{2} \right) x^2 g_{\mu\nu} \right] \right. \\ &\quad \left. \left. + \left( \frac{1}{2\epsilon} + \frac{1}{4} \right) \Gamma_3^{\alpha\beta\mu} \otimes \Gamma_{3\alpha\beta}{}^\nu x_\mu x_\nu \right) \right\} \end{aligned} \quad (4)$$

where in the Euclidean domain one has

$$S_0(x^2) = \frac{-\Gamma(2-\epsilon)}{2\pi^{2-\epsilon}(x^2)^{2-\epsilon}}. \quad (5)$$

The renormalization scale  $\mu_x$  is appropriate for calculations in configuration space if one wants to avoid the appearance of  $\ln(4\pi)$  and  $\gamma_E$  terms. The scale  $\mu_x$  is related to the scale  $\bar{\mu}$  of the  $\overline{\text{MS}}$  scheme by

$$\mu_x = \bar{\mu} e^{\gamma_E}/2. \quad (6)$$

The direct product signs “ $\otimes$ ” in the dipropagator correction  $S_2(x)$  serve to distinguish between the two fermion lines involved in the gluon exchange. Finally,

$$\Gamma_3^{\mu\alpha\nu} = \gamma^{[\mu} \gamma^\alpha \gamma^{\nu]} = \frac{1}{2} (\gamma^\mu \gamma^\alpha \gamma^\nu - \gamma^\nu \gamma^\alpha \gamma^\mu) \quad (7)$$

is the totally antisymmetric product of three gamma matrices. Equations (3) and (4) allow one to calculate the corrections to  $n$ -quark/antiquark current correlators of any composition without having to calculate any integrals. In Ref. [45] we have presented results for a model current with five different flavors. In this paper we deal with several interpolating currents suggested in the literature including the equal flavor case. Because of flavors appearing twice or more times in the interpolating current, the Wick contraction will result in a main contribution and different “crossover” contributions.

Before giving our results for the various interpolating currents, we have to deal with renormalization.

Corresponding to the propagator and dipropagator corrections, the correlator function is renormalized by the wave function renormalization factor and the vertex renormalization factor, respectively. Concerning the vertex renormalization factor one has to account for mixing effects. Mixing can occur when gluons are exchanged between quark lines in the pentaquark correlation function. Mixing is taken into account through the subtraction of corresponding vertex divergences generated by an operator that can admix to the initial current. The general formula reads

$$(\psi_i \otimes \psi_j)_R = (\psi_i \otimes \psi_j) - \frac{\alpha_s}{4\pi\varepsilon} \left( 1_{ii'} \otimes 1_{jj'} + \frac{1}{4} \sigma_{ii'}^{\alpha\beta} \otimes \sigma_{jj'}^{\alpha\beta} \right) \times (\psi_{i'} \otimes \psi_{j'}). \quad (8)$$

Here  $\sigma^{\alpha\beta} = i/2[\gamma^\alpha, \gamma^\beta]$  and all numbers are calculated in Feynman gauge. Note that the part proportional to  $\sigma$  is gauge independent. The renormalization within our modular approach follows from the above line of arguments and leads to counterterms which are listed in explicit form in the following.

Once the correlator function is renormalized, we can calculate the spectral density corresponding to the correlator. For this purpose, instead of calculating explicitly via

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0 | T j(x) \bar{j}(0) | 0 \rangle \quad (9)$$

in momentum space, one can use the formulas given in Appendix A.

#### IV. RESULTS FOR PENTAQUARKS OF THE FIRST KIND

The correlators in this section are

$$\langle 0 | T j(x) \bar{j}(0) | 0 \rangle = S_0(x^2)^5 (x^2)^2 \not{x} \Pi_j(x^2). \quad (10)$$

We start with different interpolating currents proposed for the lowest pentaquark state  $\Theta^+$  at 1530 MeV with quantum numbers  $J^P = 1/2^+$  and  $S = 1$ . Reference [38] gives an overview over pentaquarks which are built up by a diquark, a meson and a single quark. In the following these currents will be called pentaquark currents of the first kind. The interpolating current with isospin  $I = 0$  is given by

$$\eta_0(x) = \frac{1}{\sqrt{2}} \epsilon_{abc} [u_a^T(x) C \gamma_5 d_b(x)] \{ u_c(x) \bar{s}_e(x) i \gamma_5 d_c(x) - (u \leftrightarrow d) \}. \quad (11)$$

Because of the two parts of the interpolating current, there are two diagonal and two mixed bare contributions,

$$\begin{aligned} \Pi_{\eta_0 B}^{11}(x^2) &= 180 \left\{ 1 + \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( \frac{1}{\varepsilon} + \frac{13}{3} \right) \right\} \\ &\quad - 12 \left\{ 1 + \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( \frac{3}{\varepsilon} + 3 \right) \right\} \\ &\quad - 3 \left\{ 1 + \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( -\frac{1}{\varepsilon} + \frac{17}{3} \right) \right\} \\ &= \Pi_{\eta_0 B}^{22}(x^2), \\ \Pi_{\eta_0 B}^{12}(x^2) &= 18 \left\{ 1 + \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( \frac{7}{\varepsilon} + \frac{1}{3} \right) \right\} \\ &\quad + 3 \left\{ 1 + \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( -\frac{1}{\varepsilon} + \frac{17}{3} \right) \right\} \\ &= \Pi_{\eta_0 B}^{21}(x^2). \end{aligned} \quad (12)$$

The counterterms for the current read

$$\begin{aligned} \Delta \Pi_{\eta_0}^{11} &= -180 \frac{\alpha_s}{\pi} \left( \frac{1}{\varepsilon} \right) + 12 \frac{\alpha_s}{\pi} \left( \frac{3}{\varepsilon} - \frac{7}{3} \right) - 3 \frac{\alpha_s}{\pi} \left( \frac{1}{\varepsilon} \right) = \Delta \Pi_{\eta_0}^{22}, \\ \Delta \Pi_{\eta_0}^{12} &= -18 \frac{\alpha_s}{\pi} \left( \frac{7}{\varepsilon} - \frac{14}{3} \right) + 3 \frac{\alpha_s}{\pi} \left( \frac{1}{\varepsilon} \right) = \Delta \Pi_{\eta_0}^{21}. \end{aligned} \quad (13)$$

The singularities cancel in the renormalized results which reads

$$\begin{aligned} \Pi_{\eta_0 R}^{11}(x^2) &= 180 \left\{ 1 + \frac{\alpha_s}{\pi} \left( \frac{13}{3} + \ln(\mu_x^2 x^2) \right) \right\} \\ &\quad - 12 \left\{ 1 + \frac{\alpha_s}{\pi} \left( \frac{16}{3} + 3 \ln(\mu_x^2 x^2) \right) \right\} \\ &\quad - 3 \left\{ 1 + \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( \frac{17}{3} - \ln(\mu_x^2 x^2) \right) \right\} \\ &= \Pi_{\eta_0 R}^{22}(x^2), \\ \Pi_{\eta_0 R}^{12}(x^2) &= 18 \left\{ 1 + \frac{\alpha_s}{\pi} (5 + 7 \ln(\mu_x^2 x^2)) \right\} \\ &\quad + 3 \left\{ 1 + \frac{\alpha_s}{\pi} \left( \frac{17}{3} - \ln(\mu_x^2 x^2) \right) \right\} \\ &= \Pi_{\eta_0 R}^{21}(x^2). \end{aligned} \quad (14)$$

In order to calculate the spectral density we have to treat the first order correction and the counterterm separately. The reason is that these two contributions have different  $x^2$  powers. The general procedure for the calculation of the spectral density is left to Appendix A. The result for the spectral density reads

$$\rho(s) = \frac{s^5}{604800(4\pi)^8} \times \left\{ A_0 + \frac{\alpha_s}{\pi} \left( B_1 + C_1 + \frac{512}{105} B_0 + B_0 \ln \left( \frac{\bar{\mu}^2}{s} \right) \right) \right\}, \quad (15)$$

where  $A_0$  is the LO contribution,  $B_0$  and  $B_1$  are the singular and finite parts of the NLO result, respectively, and  $C_0 (= -B_0)$  and  $C_1$  are the singular and finite parts of the counterterm. Collecting all contributions one obtains

$$\begin{aligned}\Pi_{\eta_0 B} &= 372 + 60 \frac{\alpha_s}{\pi} \left( \frac{9}{\varepsilon} + 25 \right) = A_0 + \frac{\alpha_s}{\pi} \left( \frac{B_0}{\varepsilon} + B_1 \right), \\ \Delta \Pi_{\eta_0} &= 60 \frac{\alpha_s}{\pi} \left( -\frac{9}{\varepsilon} + \frac{28}{15} \right) = \frac{\alpha_s}{\pi} \left( \frac{C_0}{\varepsilon} + C_1 \right).\end{aligned}\quad (16)$$

The spectral density finally reads

$$\begin{aligned}\rho_{\eta_0}(s) &= \frac{s^5}{604800(4\pi)^8} \left\{ 372 + 60 \frac{\alpha_s}{\pi} \left( 9 \ln \left( \frac{\bar{\mu}^2}{s} \right) + \frac{7429}{105} \right) \right\} \\ &= \frac{31s^5}{50400(4\pi)^8} \left\{ 1 + \frac{\alpha_s}{\pi} \left( \frac{45}{31} \ln \left( \frac{\bar{\mu}^2}{s} \right) + \frac{7429}{651} \right) \right\}.\end{aligned}\quad (17)$$

The perturbative correction can be seen to be rather large, cf. 7429/651( $\alpha_s/\pi$ ). For the remaining currents proposed in Ref. [38], one has

$$\begin{aligned}\eta_1(x) &= \frac{1}{\sqrt{2}} \epsilon_{abc} [u_a^T(x) C \gamma_5 d_b(x)] \{ u_e(x) \bar{s}_e(x) i \gamma_5 d_c(x) \\ &\quad + (u \leftrightarrow d) \},\end{aligned}\quad (18)$$

$$\begin{aligned}\eta'_1(x) &= \frac{1}{\sqrt{2}} \epsilon_{abc} [u_a^T(x) C \gamma^\mu d_b(x)] \{ \gamma_\mu \gamma_5 u_e(x) \bar{s}_e(x) i \gamma_5 d_c(x) \\ &\quad - (u \leftrightarrow d) \},\end{aligned}\quad (19)$$

$$\begin{aligned}\eta_2(x) &= \frac{1}{\sqrt{2}} \epsilon_{abc} \{ [u_a^T(x) C \gamma^\mu u_b(x)] \gamma_\mu \gamma_5 d_e(x) \bar{s}_e(x) i \gamma_5 d_c(x) \\ &\quad + (u \leftrightarrow d) \},\end{aligned}\quad (20)$$

$$\eta'_2(x) = \epsilon_{abc} [u_a^T(x) C \gamma^\mu u_b(x)] \gamma_\mu \gamma_5 u_e(x) \bar{s}_e(x) i \gamma_5 u_c(x).\quad (21)$$

Again we only give results for the spectral densities. They read

$$\rho_{\eta_1}(s) = \frac{s^5}{2100(4\pi)^8} \left\{ 1 + \frac{\alpha_s}{\pi} \left( \frac{1}{6} \ln \left( \frac{\bar{\mu}^2}{s} \right) + \frac{517}{105} \right) \right\},\quad (22)$$

$$\rho_{\eta'_1}(s) = \frac{17s^5}{6300(4\pi)^8} \left\{ 1 + \frac{\alpha_s}{\pi} \left( \frac{2255}{408} \right) \right\},\quad (23)$$

$$\rho_{\eta_2}(s) = \frac{s^5}{525(4\pi)^8} \left\{ 1 + \frac{\alpha_s}{\pi} \left( -\frac{7}{6} \ln \left( \frac{\bar{\mu}^2}{s} \right) + \frac{377}{360} \right) \right\},\quad (24)$$

$$\rho_{\eta'_2}(s) = \frac{s^5}{525(4\pi)^8} \left\{ 1 + \frac{\alpha_s}{\pi} \left( -\frac{11}{6} \ln \left( \frac{\bar{\mu}^2}{s} \right) - \frac{15877}{10080} \right) \right\}.\quad (25)$$

The perturbative corrections can become as large as 2255/408( $\alpha_s/\pi$ ).

## V. PENTAQUARKS OF THE SECOND KIND

Pentaquarks of the second kind consist of two diquarks and one antiquark. Three possible choices are given

in Ref. [31], and, with some small deviations, in Refs. [46,47]. They read

$$\begin{aligned}\theta_1^+(x) &= \epsilon_{abc} \epsilon_{aef} \epsilon_{bgh} [u_e^T(x) C d_f(x)] \\ &\quad \times [u_g^T(x) C \gamma_5 d_h(x)] C \bar{s}_c^T(x),\end{aligned}\quad (26)$$

$$\begin{aligned}\theta_2^{+, \mu}(x) &= \epsilon_{abc} \epsilon_{aef} \epsilon_{bgh} [u_e^T(x) C \gamma_5 d_f(x)] \\ &\quad \times [u_g^T(x) C \gamma^\mu \gamma_5 d_h(x)] C \bar{s}_c^T(x),\end{aligned}\quad (27)$$

$$\begin{aligned}\theta_3^{+, \mu}(x) &= \epsilon_{abc} \epsilon_{aef} \epsilon_{bgh} [u_e^T(x) C d_f(x)] \\ &\quad \times [u_g^T(x) C \gamma^\mu \gamma_5 d_h(x)] \gamma_5 C \bar{s}_c^T(x).\end{aligned}\quad (28)$$

First we consider the case when the Lorentz index  $\mu$  in the correlator is contracted. One then has

$$\rho_{\theta_1}(s) = \frac{s^5}{1575(4\pi)^8} \left\{ 1 + \frac{\alpha_s}{\pi} \left( 2 \ln \left( \frac{\bar{\mu}^2}{s} \right) + \frac{1021}{70} \right) \right\},\quad (29)$$

$$\rho_{\theta_2}(s) = \frac{s^5}{1575(4\pi)^8} \left\{ 1 + \frac{\alpha_s}{\pi} \left( \frac{5}{8} \ln \left( \frac{\bar{\mu}^2}{s} \right) + \frac{2963}{336} \right) \right\} = \rho_{\theta_3}(s).\quad (30)$$

The perturbative correction to  $\rho_{\theta_1}$  is the largest of all the cases treated in this paper with 1021/70( $\alpha_s/\pi$ ). When the Lorentz index is not contracted, we obtain an ordinary and a crossover contribution for the correlators

$$\Pi_{\theta_i}^{\mu\nu}(x) = \langle 0 | T \theta_i^{+, \mu}(x) \bar{\theta}_i^{+, \nu}(0) | 0 \rangle, \quad i = 2, 3, \quad (31)$$

which are the same for both currents, namely,

$$\begin{aligned}\Pi_{\theta_i B}^{\mu\nu, o}(x) &= -384 x^2 \not{x} (x^2 g^{\mu\nu} - 2x^\mu x^\nu) S_0(x^2)^5 \\ &\quad \times \left\{ 1 + \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( \frac{10}{3\varepsilon} + 15 \right) \right\} \\ &\quad + 256 x^4 \gamma^\mu \not{x} \gamma^\nu S_0(x^2)^5 \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( \frac{1}{\varepsilon} + \frac{1}{2} \right),\end{aligned}\quad (32)$$

$$\begin{aligned}\Pi_{\theta_i B}^{\mu\nu, x}(x) &= 384 x^2 \not{x} (x^2 g^{\mu\nu} - 2x^\mu x^\nu) S_0(x^2)^5 \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( \frac{1}{\varepsilon} - \frac{1}{2} \right) \\ &\quad - 192 x^4 \gamma^\mu \not{x} \gamma^\nu S_0(x^2)^5 \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( \frac{1}{\varepsilon} + \frac{1}{2} \right).\end{aligned}\quad (33)$$

The total contribution is

$$\begin{aligned}\Pi_{\theta_i B}^{\mu\nu}(x) &= -384 x^2 \not{x} (x^2 g^{\mu\nu} - 2x^\mu x^\nu) S_0(x^2)^5 \\ &\quad \times \left\{ 1 + \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( \frac{8}{3\varepsilon} + \frac{31}{2} \right) \right\} \\ &\quad + 64 x^4 \gamma^\mu \not{x} \gamma^\nu S_0(x^2)^5 \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon \left( \frac{1}{\varepsilon} + \frac{1}{2} \right).\end{aligned}\quad (34)$$

The counterterm reads

$$\Delta\Pi_{\theta_i}^{\mu\nu}(x) = \frac{\alpha_s}{\pi} \frac{1}{\varepsilon} S_0(x^2)^5 [896x^2 \not{x}(x^2 g^{\mu\nu} - 2x^\mu x^\nu) - 64x^4 \gamma^\mu \not{x} \gamma^\nu]. \quad (35)$$

For the calculation of the absorptive part of (34) and (35) related to  $x^2 \not{x}(x^2 g^{\mu\nu} - 2x^\mu x^\nu)$  one has to extend the considerations of Appendix A to tensors of rank 3, resulting in a spectral density

$$\rho_{\theta,1} = \frac{s^4}{2520(4\pi)^8} \left\{ 1 + \frac{\alpha_s}{\pi} \left( \frac{7}{3} \ln\left(\frac{\bar{\mu}^2}{s}\right) + \frac{9613}{360} \right) \right\}. \quad (36)$$

For the calculation of the absorptive part related to  $x^4 \gamma^\mu \not{x} \gamma^\nu$  one can use Appendix A directly to obtain

$$\rho_{\theta,2} = \frac{s^5}{9450(4\pi)^8} \left\{ \frac{\alpha_s}{\pi} \left( \ln\left(\frac{\bar{\mu}^2}{s}\right) + \frac{1129}{210} \right) \right\}. \quad (37)$$

## VI. CONCLUSION

The quark condensate  $\langle \bar{q}q \rangle$  is the only dimensionful quantity number that appears in the sum rule analysis since we assume factorization for the vacuum expectation value of the six quark operators [48]). Inclusion of terms  $\sim m_s$  should not substantially change the quantitative results as the mass of the strange quark is small [49]. It therefore follows from dimensional arguments that the mass of the pentaquark baryon is  $m_\theta \sim (|\langle \bar{q}q \rangle|)^{1/3}$  as long as power corrections determine the mass. Consequently,  $m_\theta$  should not change by more than  $\sim 10\%$  if  $\langle \bar{q}q \rangle$  varies by 30%. Such a variation is quite possible because of the uncertainties in the light quark masses determined from the numerical value of the light quark condensate as calculated from the partially conserved axial current relation for the pion. The analogous expression for the strange quark condensate obtains some corrections due to the  $s$ -quark mass which are well under control [50]. Nevertheless, this still leaves the uncertainty whether the pentaquark state is above or below the threshold.

We recall in this respect that the high accuracy of the MIT quark-bag model permitted Jaffe to conclude that the dibaryon state  $H$  lies below the  $\Lambda\Lambda$  threshold and is therefore stable. The same conclusion was drawn from a model calculation based on chiral solitons in Ref. [10]. However, for a model independent approach, the relatively low accuracy of the method in the determination of the mass ( $\sim 15\%$ ) does not make it possible to draw any conclusion about whether the mass of the exotic baryon lies below or above the  $KN$  threshold.

In this paper we have calculated NLO perturbative corrections to the correlator of various pentaquark currents and their absorptive parts. We have shown that such a calculation can be done by purely algebraic means for any given interpolating current using the modular methods developed by us in detail. As it turns out, the NLO corrections to the correlators are large. As the coupling constant is large at the relevant energy scale [51], the large perturbative  $\alpha_s$

corrections will heavily change the relative weight of the perturbative and the nonperturbative condensate terms. It would be interesting to find out how the large NLO corrections to the absorptive parts of the current correlators affect the sum rule analysis of pentaquark states. This would form the subject of a separate publication.

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## APPENDIX A: EXPLICIT DERIVATION OF THE SPECTRAL DENSITY

The momentum space representation of the correlator function can be obtained from the configuration space representation by using the integration formula

$$\Pi(p) = 2\pi^{\lambda+1} \int_0^\infty \left(\frac{px}{2}\right)^{-\lambda} J_\lambda(px) \Pi(x) x^{2\lambda+1} dx, \quad (A1)$$

where  $\lambda = 1 - \varepsilon$  and  $J_\lambda(z)$  is the Bessel function of the first kind. In the case that the correlation function  $\Pi(x) = (x^2)^{-\alpha}$  is a simple power in  $x^2$ , the integral can be obtained analytically. The result is

$$\Pi_\alpha(p^2) = \pi^{\lambda+1} \left(\frac{p^2}{4}\right)^{\alpha-\lambda-1} \frac{\Gamma(\lambda+1-\alpha)}{\Gamma(\alpha)}. \quad (A2)$$

The corresponding spectral density is given by the discontinuity of the correlation function where the discontinuity of the correlation function (in the Euclidean domain!) lies along the negative real axis. One obtains

$$\rho_\alpha(s) = \frac{1}{2\pi i} \text{Disc} \Pi_\alpha(-s) = \pi^{\lambda+1} \left(\frac{s}{4}\right)^{\alpha-\lambda-1} \frac{1}{\Gamma(\alpha)\Gamma(\alpha-\lambda)}. \quad (A3)$$

In order to calculate the spectral density, we have to use the scalar correlation function. The vector-type correlation function of the pentaquarks (as well as those of all states composed of fermions) can be obtained from the derivative of this scalar correlation function  $F(x^2)$ . One has

$$\partial_\mu F(x^2) = 2x_\mu \frac{\partial F(x^2)}{\partial x^2} = x_\mu f(x^2). \quad (A4)$$

Given the function  $f(x^2)$ , the scalar correlation function is obtained by integrating over  $x^2/2$ . In case of pentaquarks, we have

$$f(x^2) = (S_0(x^2))^5 (x^2)^2 \left\{ A + \frac{\alpha_s}{\pi} (\mu_x^2 x^2)^\varepsilon B \right\}, \quad (A5)$$

where  $A$  contains the LO contribution and the counterterm and where  $B$  contains the NLO contribution. Recalling the  $x^2$  dependence of  $S_0(x^2)$  in Eq. (5), one obtains

$$\begin{aligned} F(x^2) &= \frac{1}{2} \left( \frac{-\Gamma(2-\varepsilon)}{2\pi^{2-\varepsilon}} \right)^5 \int \left\{ (x^2)^{5\varepsilon-8} A + \frac{\alpha_s}{\pi} (\mu_x^2)^\varepsilon (x^2)^{6\varepsilon-8} B \right\} dx^2 \\ &= \frac{1}{2} \left( \frac{-\Gamma(2-\varepsilon)}{2\pi^{2-\varepsilon}} \right)^5 \left\{ \frac{(x^2)^{5\varepsilon-7}}{5\varepsilon-7} A + \frac{\alpha_s}{\pi} (\mu_x^2)^\varepsilon \frac{(x^2)^{6\varepsilon-7}}{6\varepsilon-7} B \right\}. \end{aligned} \quad (\text{A6})$$

For the corresponding spectral density one has

$$\begin{aligned} \rho_F(s) &= \frac{1}{2} \left( \frac{-\Gamma(2-\varepsilon)}{2\pi^{2-\varepsilon}} \right)^5 \pi^{2-\varepsilon} \times \left\{ \frac{(s/4)^{5-4\varepsilon} A}{(5\varepsilon-7)\Gamma(7-5\varepsilon)\Gamma(6-4\varepsilon)} + \frac{\alpha_s}{\pi} (\mu_x^2)^\varepsilon \frac{(s/4)^{5-5\varepsilon} B}{(6\varepsilon-7)\Gamma(7-6\varepsilon)\Gamma(6-5\varepsilon)} \right\} \\ &= \frac{-\pi^{2-\varepsilon} (s/4)^{5-4\varepsilon}}{2\Gamma(8-5\varepsilon)\Gamma(6-4\varepsilon)} \left( \frac{-\Gamma(2-\varepsilon)}{2\pi^{2-\varepsilon}} \right)^5 \left\{ A + \frac{\alpha_s}{\pi} \left( \frac{\bar{\mu}^2}{s} \right)^\varepsilon B (1 + (\psi(8) + \psi(6) + 2\gamma_E)\varepsilon) \right\} \\ &= \frac{(s/4)^{5-4\varepsilon} \Gamma(2-\varepsilon)^5}{64\pi^{8-4\varepsilon} \Gamma(8-5\varepsilon)\Gamma(6-4\varepsilon)} \left\{ A + \frac{\alpha_s}{\pi} \left( \frac{\bar{\mu}^2}{s} \right)^\varepsilon B \left( 1 + \frac{512}{105} \varepsilon \right) \right\}, \end{aligned} \quad (\text{A7})$$

where we have made use of the expansion  $\Gamma(a-\varepsilon) = \Gamma(a)(1 - \varepsilon\psi(a) + O(\varepsilon^2))$  and where we have incorporated the scale change  $\mu_x = e^{\gamma_E} \bar{\mu}/2$ . Here  $\psi(a) = \Gamma'(a)/\Gamma(a)$  is the polygamma function. We then use

$$A = A_0 + \frac{\alpha_s}{\pi} \left( \frac{C_0}{\varepsilon} + C_1 \right), \quad B = \frac{B_0}{\varepsilon} + B_1, \quad (\text{A8})$$

where  $A_0$  is the LO contribution,  $B_0$  and  $B_1$  are the singular respectively finite NLO contribution, and  $C_0$  and  $C_1$  are the singular respectively finite contribution of the counterterm ( $C_0 = -B_0$ ). One finally obtains

$$\begin{aligned} \rho_F(s) &= \frac{(s/4)^{5-4\varepsilon} \Gamma(2-\varepsilon)^5}{64\pi^{8-4\varepsilon} \Gamma(8-5\varepsilon)\Gamma(6-4\varepsilon)} \times \left\{ A_0 + \frac{\alpha_s}{\pi} \left( \frac{B_0 + C_0}{\varepsilon} + B_0 \ln \left( \frac{\bar{\mu}^2}{s} \right) + \frac{512}{105} B_0 + B_1 + C_1 \right) \right\} \\ &= \frac{s^5}{604800(4\pi)^8} \left\{ A_0 + \frac{\alpha_s}{\pi} \left( B_0 \ln \left( \frac{\bar{\mu}^2}{s} \right) + \frac{512}{105} B_0 + B_1 + C_1 \right) \right\}. \end{aligned} \quad (\text{A9})$$

Because the singularities cancel one can set  $\varepsilon = 0$  in the last step.

## APPENDIX B: ON THE CHOICE FOR THE INTERPOLATING CURRENT

Formally, the QCD sum rule method dictates *a priori* the only condition for the choice of the current: it must possess the required quantum numbers. However, *a posteriori* (after the fit) positive results can be obtained only for “physical” currents. In particular, the two requirements formulated below are usually necessary. This may be an indication that there exists a criterion which makes it possible to select the optimal (physical) current from the set of currents with the quantum numbers of the given channel. This argument was also given in Ref. [5], in which the deuteron mass was calculated by the same method.

The choice of the interpolating current is crucial in this respect and has to be considered very carefully. The physical current  $\Theta$  satisfies the following two requirements. First, there exists a nonzero nonrelativistic limit for it [i.e. if the quark field  $\psi(x)$  is represented in the standard manner in terms of the small and large components, the term containing only the large component will be nonzero]. We note that in Refs. [16,52] it was already pointed out that

the existence of a nonrelativistic limit is a desirable property for the construction of currents when employing the QCD sum rule method. The demand for the existence of such a limit is quite natural as the results should be reproducible in some effective potential model of constituent quarks. Second, the color (and flavor) structure is important. This will be explained in some detail in the following.

According to Ref. [31], “the  $qqqq\bar{q}$  state can be decomposed into a pair of color singlet states as  $qqq$  and  $q\bar{q}$  [...]”. For instance, one can start a study with a simple minded local operator for the  $\Theta^+(1540)$ , which is constructed from the product of a neutron operator and a  $K^+$  operator such as  $\Theta = \epsilon_{abc}(d_a^T C \gamma_5 u_b) d_c (\bar{s}_e \gamma_5 u_e)$ . The two-point correlation function composed of this operator, in general, couples not only to the  $\Theta$  state (single hadron) but also to the two-hadron states such as an interacting  $KN$  system. Even worse, when the mass of the  $qqqq\bar{q}$  state is higher than the threshold of the hadronic two-body system, the two-point function should be dominated by the two-hadron states. Thus, a specific operator with as little overlap with the hadronic two-body states as possible is desired in order

to identify the signal of the pentaquark states [...]” And following Ref. [38], “isospin and color structure guarantee that these currents will never couple to a  $K^+n$  molecule or any other  $K^+n$  intermediate state [...]” This is the reason to use a nontrivial color structure in the previous paper.

We briefly comment also on a second choice of currents with nontrivial flavor structure which was not considered in this paper. If we represent the current  $\Theta$  in the form of a product “singlet  $\otimes$  singlet” with respect to color, i.e., in the form  $\Theta = \Psi(x)\Gamma\Phi(x)$ , where each of the operators  $\Psi$ ,  $\Phi$  is a color singlet and  $\Gamma$  is a string of Dirac  $\gamma$  matrices, we can choose

$$\Theta = \Psi_{A_3}^{A_4}(x)\Phi_{A_4}^{A_3}(x), \quad (\text{B1})$$

where the color-singlet operator  $\Psi$  is a flavor octet,

$$\begin{aligned} (\Psi_{A_3}^{A_4})^\alpha &= \epsilon_{a_1 a_2 a_3} (\psi_{A_1}^{a_1} C \gamma_5 \psi_{A_2}^{a_2}) (\psi_{B_1}^{a_3})^\alpha \\ &\times \frac{1}{2} \epsilon^{A_1 A_2 B_2} \left( \delta_{A_3}^{B_1} \delta_{B_2}^{A_4} - \frac{1}{3} \delta_{B_2}^{B_1} \delta_{A_3}^{A_4} \right). \end{aligned} \quad (\text{B2})$$

Here  $\alpha$  is a spinor index. The flavor octet (**B2**) has the quantum numbers of the baryon octet and has e.g. been used in Refs. [16,17] to calculate the properties of light baryons within the QCD sum rule method. Thus, if the current  $\Theta$  is represented in the form “singlet  $\otimes$  singlet” with respect to color, it has to have the structure “octet  $\otimes$  octet” with respect to flavor (each color singlet is a flavor octet). Physically, the following picture emerges. The colorless state  $\Theta$  splits into two colorless clusters which separate at large distances to become a meson and a baryon. As a result, we conclude that the current  $\Theta$  is the most physical current in the sense that it has a nonrelativistic limit and that it is constructed as a “octet  $\otimes$  octet” state with respect to flavor.

One can introduce interpolating currents including space-time derivatives of the field operators. This is, for instance, needed for the description of the orbital excitations of hadrons. The calculational techniques developed in this paper can also be applied to these cases. However, in this paper we have restricted our discussion to interpolating currents without derivatives.

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