

**Renewed look at  $\eta'$  in medium**Youngshin Kwon,<sup>1,2,\*</sup> Su Houng Lee,<sup>2,†</sup> Kenji Morita,<sup>3,‡</sup> and György Wolf<sup>4,§</sup><sup>1</sup>*Department of Physics, Soongsil University, Seoul 156-743, Korea*<sup>2</sup>*Department of Physics and Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea*<sup>3</sup>*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*<sup>4</sup>*Wigner RCP, RMKI, POB 49, Budapest H-1525, Hungary*

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We revisit the question of whether the  $U_A(1)$  symmetry is effectively restored in hot and dense medium. In particular, by generalizing the Witten-Veneziano formula to finite temperature, we investigate whether the mass of  $\eta'$ -meson will change in medium due to the restoration of chiral symmetry.

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**I. INTRODUCTION**

The breaking of the  $U_A(1)$  symmetry is an operator relation which remains valid even when the spontaneously broken chiral symmetry is restored. However, whether its effect on the  $\eta'$  mass survives even when chiral symmetry is restored is a phenomenological question which has caught the interest of many researchers [1–6]. The question has recently been revived as the RHIC data on the two pion Bose-Einstein correlation at  $\sqrt{s} = 200$  GeV Au + Au collision seems to suggest the quenching of the  $\eta'$  mass in medium [7–9]. Its partial quenching in nuclear medium is also of great interest as such effects could be probed in finer detail in nuclear target experiments [10–12].

The restoration of  $U_A(1)$  symmetry will depend on two important ingredients; its relation to chiral symmetry breaking and the effects from topological configurations. How the former and latter contribute to the  $n$ -point correlation functions has been clarified in Refs. [4–6]. Combining both effects, it was shown [5] that in the chiral limit, with  $N_f$  flavors, the symmetry will effectively be restored in correlation functions composed of up to  $N_f - 1$  points [13]. This means, for example, that for  $N_f = 3$ ,  $U_A(1)$  symmetry will be effectively restored in the two-point functions when chiral symmetry is restored. Still, the argument is based on correlation functions and does not explicitly relate the  $\eta'$  mass to the other pseudoscalar masses. To establish this relation, we revisit the Witten-Veneziano (WV) formula [14,15] for the  $\eta'$  mass in vacuum and generalize it to finite temperature. Although the formula is obtained in the large  $N_c$  limit, we will obtain an explicit relation which relates the  $\eta'$  mass to condensates and two-point correlation functions at finite temperature. Therefore, the symmetry restoration pattern observed in the two-point function will be reflected in the  $\eta'$  mass. For the physical case of  $N_f = 3$  with explicit quark masses,

this result implies that the mass of  $\eta'$  will become degenerate with the other pseudoscalar mesons up to the quark masses when chiral symmetry is restored. An attempt to generalize the WV formula to finite temperature was also reported in Ref. [16].

The paper is organized as follows. In Sec. II, we will revisit the main results in Ref. [5]. We will then review and generalize the WV formula in Sec. III. We discuss the  $\eta'$  mass in Sec. IV. The summary will be given in the last section.

**II. CORRELATION FUNCTIONS AND THE  $U_A(1)$  SYMMETRY**

Here, we start with a brief summary of the main result given in Ref. [5]. The starting point is the Euclidean partition function of QCD:

$$Z[J] = \int D[A] e^{-S_{YM}} \text{Det}[\not{D} + m_q] = \sum_{\nu} Z[J]_{\nu}, \quad (1)$$

where  $S_{YM} = (1/4) \int d^4x G^2$ . The second line writes the partition function in terms of topological configurations with the topological charge  $\nu = (g_s^2/32\pi^2) \int d^4x G\tilde{G}$ . The whole integrand in the first line is a positive definite quantity [4]; we will define  $d\mu \equiv D[A] e^{-S_{YM}} \text{Det}[\not{D} + m_q]$  for later convenience. The topological configurations are always accompanied by  $n_+(n_-)$  number of right-handed (left-handed) fermion zero modes such that  $\nu = n_+ - n_-$ . In such topologically nontrivial configurations, the fermion determinant comes with special chirality such that the partition function with  $\nu = 1$  can be written more explicitly as follows:

$$Z[J]_{\nu=1} = \int D[A]_{\nu=1} e^{-S_{YM}} \text{Det}'[\not{D} + m_q] \times \det\left(\int d^4x \bar{\psi}_0(x) m_q \psi_0(x)\right), \quad (2)$$

where the prime in the fermion determinant means that the chiral zero modes  $\psi_0$  have been explicitly taken out into the second determinant. Therefore, in the chiral limit  $m_q = 0$ , the topological configuration does not contribute

\*ykwon@ssu.ac.kr

†suhoung@yonsei.ac.kr

‡kmorita@yukawa.kyoto-u.ac.jp

§wolf.gyorgy@wigner.mta.hu

to the partition function as the fermion determinant gives zero. However, these terms do contribute in the correlation functions and select out the  $\eta'$  from the other pseudoscalars. Higher topological configurations will contribute at higher point functions when there are sufficiently many external legs to saturate the zero modes.

To see this, consider a two-point function of a generic quark bilinear:

$$\begin{aligned} \Pi_{\Gamma}(x) &= \langle \bar{q}(x)\Gamma q(x), \bar{q}(0)\Gamma q(0) \rangle \\ &= \frac{1}{Z} \int d\mu [-\text{Tr}[S_A(x, 0)\Gamma S_A(0, x)\Gamma] \\ &\quad + \text{Tr}[S_A(x, x)\Gamma] \text{Tr}[S_A(0, 0)\Gamma] + (\text{zero mode})], \end{aligned} \quad (3)$$

the first, second and third term being the connected, disconnected terms and possible zero mode contribution, respectively.  $S_A$  is the quark propagator in the presence of the gauge field. The traces are over the Dirac and flavor indices of the quarks. Hence, when  $\Gamma$  contains a flavor matrix, the contributions from the disconnected diagrams are identically zero.

The results in Refs. [4,5] can be summarized as follows. When one takes the difference between the two-point functions of chiral partners, the difference vanishes when chiral symmetry is restored. When the difference is taken between those composed of currents which are related by a chiral transformation and an extra  $U_A(1)$  transformation, there will be an extra contribution from the zero modes. As an example, the difference between a pseudoscalar and  $\eta'$  is given as follows in SU(2):

$$\begin{aligned} \Pi_{\pi}(x) - \Pi_{\eta'}(x) &= \frac{1}{Z} \int d\mu [\text{Tr}[S_A(x, x)\gamma_5] \text{Tr}[S_A(0, 0)\gamma_5]] \\ &\quad + \frac{1}{Z} \int d\mu_{\nu=\pm 1} [4\bar{\psi}_0(x)\psi_0(x)\bar{\psi}_0(0)\psi_0(0)]. \end{aligned} \quad (4)$$

In Ref. [4], it was shown that the first term goes to zero in the chiral limit when chiral symmetry is restored. This result is in fact independent of the number of flavors and also valid when the  $\gamma_5$  inside the trace is replaced by other gamma matrices such as  $\gamma_{\mu}$  or  $\gamma_{\mu}\gamma_5$ .

The zero-mode contributions appearing in the second term of Eq. (4) come from the topological configuration in Eq. (2) and are responsible for the appearance of the  $U_A(1)$  effect. However, when  $N_f > 2$ , the second determinant in Eq. (2) will have  $2N_f$  zero mode lines, and hence the zero-mode contributions in Eq. (4) will be proportional to  $O(m_q^{N_f-2})$  and vanish in the chiral limit. Therefore, when chiral symmetry is restored, the  $U_A(1)$  breaking effect will not appear in the two-point functions. However, this does not necessarily mean that the  $\eta'$  mass will become degenerate with the other pseudoscalar mesons because the coupling of the currents to the  $\eta'$  might just go to zero.

Therefore, let us look at a relation which directly relates the mass of  $\eta'$  to the chiral order parameters.

### III. WITTEN-VENEZIANO FORMULA

#### A. WV formula at zero temperature

As a first step of this study, we review the derivation of the WV mass formula [14]. We start with the gluonic correlation function defined in a pure glue theory:

$$U(k) = i \int d^4x e^{ik \cdot x} \langle \mathcal{T} G\tilde{G}(x)G\tilde{G}(0) \rangle. \quad (5)$$

One should note that, in the large  $N_c$  limit [17], Eq. (5) scales as order  $N_c^2$ . There is also a well-known low-energy theorem for the correlation function at zero external momentum  $U(k=0) \neq 0$ , to whose value we will come back in the next section.

However, when massless quarks are added to the theory, the low-energy theorem leads to the vanishing correlation function  $U_{lq}(k=0) = 0$ , where the subscript  $lq$  means the presence of light quarks, through the anomaly relation which relates the pseudoscalar gluon current to axial current  $\sum_q \partial_{\mu} \bar{q} \gamma_{\mu} \gamma_5 q = N_f \frac{\alpha_s}{4\pi} G\tilde{G}$ , where  $\tilde{G}_{\mu\nu}^a = 1/2 \epsilon_{\mu\nu\alpha\beta} G_{\alpha\beta}^a$ . This seems a little odd, because quark effects are suppressed in large  $N_c$ , but for the low-energy theorem, the leading  $N_c$  effect seems to be canceled by a subleading  $N_c$  effect. The answer to this question led to the WV formula.

In terms of the physical states, the correlation functions looks as follows when light quarks are added:

$$\begin{aligned} U_{lq}(k) &= - \sum_n \frac{|\langle 0|G\tilde{G}|n^{\text{th}} \text{glueball} \rangle|^2}{k^2 - M_n^2} \\ &\quad - \sum_n \frac{|\langle 0|G\tilde{G}|n^{\text{th}} \text{meson} \rangle|^2}{k^2 - m_n^2} \\ &\equiv U_0(k) + U_1(k). \end{aligned} \quad (6)$$

In the spectral form, Eq. (6), the first term in the right-hand side indicates contributions from glueballs, while the second term shows those from the mesons composed of light quarks. One can show that the residue of the first term is of order  $N_c^2$  whereas the quark effects are of order  $N_c$  [18]:

$$\begin{aligned} \langle 0|G\tilde{G}|n^{\text{th}} \text{glueball} \rangle &= N_c a_n, \\ \langle 0|G\tilde{G}|n^{\text{th}} \text{meson} \rangle &= \sqrt{N_c} c_n. \end{aligned} \quad (7)$$

Since all the meson masses should have a smooth large  $N_c$  limit  $O(1)$ , the terms of quark effects,  $U_1$ , are suppressed in  $1/N_c$  as expected. The resolution to the seemingly inconsistent result comes from noting the existence of the  $\eta'$ -meson whose mass squared scales as order  $1/N_c$ . One then recovers the consistent low-energy theorem  $U_{lq}(0) = U_0(0) + U_1(0) = 0$ , if the second term is saturated by the  $\eta'$ -meson and scales as  $N_c^2$ . From this condition, one finds

$$U_0(0) = -\frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{m_{\eta'}^2} = -\frac{N_c c_{\eta'}^2}{m_{\eta'}^2}. \quad (8)$$

By using the  $U(1)_A$  anomaly,

$$\langle 0|G\tilde{G}|\eta'\rangle = \frac{4\pi}{\alpha_s} \frac{1}{N_f} \langle 0|\partial_\mu J_5^\mu|\eta'\rangle = \frac{4\pi}{\alpha_s} \frac{1}{N_f} \sqrt{N_f} m_{\eta'}^2 f_\pi. \quad (9)$$

Equation (8) becomes as follows:

$$U_0(0) = -\frac{1}{N_f} m_{\eta'}^2 f_\pi^2 \left(\frac{4\pi}{\alpha_s}\right)^2, \quad (10)$$

where  $N_f$  is the number of light flavors. In Eq. (9), we made use of  $f_{\eta'} = f_\pi$  to lowest order in  $N_c$ . Equation (10) is the celebrated WV formula.

### B. WV formula at finite temperature

Consider the correction to Eq. (10) at finite temperature. As mentioned before, the correlation function in Eq. (5) is order  $N_c^2$ , as can be seen by the two loops representing two gluon lines in Fig. 1(a). At finite temperature, the thermal correction could come from the thermal gluon or quark interactions. Figures 1(b) and 1(c) show the thermal corrections to  $U_0(k)$  while Figs. 1(d) and 1(e) show those to  $U_1(k)$ .

The dominant thermal gluonic contribution to  $U_0(k)$  comes from Fig. 1(b) and scales as  $N_c^2$  as in the vacuum scaling. The scaling comes as follows:

$$(N_c) \times \left(\frac{1}{\sqrt{N_c}}\right)^2 (N_c)^2 = O(N_c^2), \quad (11)$$

which comes from the internal loop, two coupling and the number of external thermal gluons, respectively. The contributions where the gluons couple directly to the currents scale the same as in Fig. 1(b). On the other hand, the contributions from thermal quarks to  $U_0(k)$  scales as follows:

$$(N_c) \times \left(\frac{1}{\sqrt{N_c}}\right)^2 (N_c) = O(N_c), \quad (12)$$

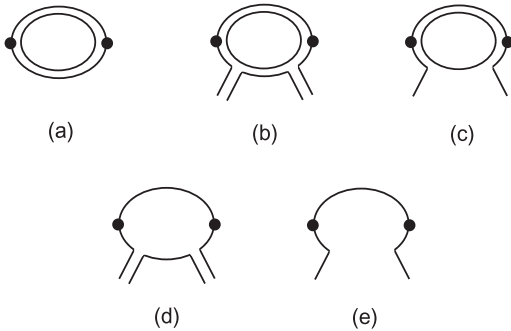


FIG. 1. Two sets of diagrams.

where the factors are the same as before except for the last factor, which comes from the number of external quark lines, as can be seen from Fig. 1(c). Therefore, in the large  $N_c$  limit, the thermal gluonic effect scales the same as in the leading vacuum scaling and will contribute to modifying  $U_0(k)$ .

As for the modification in the quark loops  $U_1(k)$ , the thermal gluonic effects are shown in Fig. 1(d), and the thermal quark effects in Fig. 1(e). Both scale as  $O(N_c)$ , and can thus be neglected in the leading-order correction.

If the system is in the confined state, the hadronic side will be saturated by color singlet glueballs, mesons and nucleons. Here, the dominant contribution comes from the nucleons. One can show that the contribution from the nucleons to all figures in Fig. 1 scale as  $O(N_c)$  because the nucleon contains  $N_c$  quarks. On the other hand, the contributions from mesons or glueballs are suppressed in  $1/N_c$  as the number of constituents are finite. Hence, hadron effects can be neglected until near the crossover point where the density of states increases, after which one can use the quark and gluon degrees of freedom.

Therefore, same arguments hold as in the vacuum. Namely, the addition of quarks somehow has to cancel the leading  $N_c$  behavior at  $k = 0$ . This cancellation cannot be done by collective states, as quark collective states are also suppressed in the large  $N_c$  limit, and hence has to come from a modified  $\eta'$  contribution. All in all, a similar equation to Eq. (8) will hold at finite temperature, with  $|\langle 0|G\tilde{G}|\eta'\rangle|$  now defined at finite temperature at  $\eta'$  momentum zero. Moreover, it should be noted that the  $\eta'$  mass we are discussing now could be different from that of the pole mass as we are discussing the scalar part of the mass, which survives at  $k^\mu \rightarrow 0$ . A simplified example would be to assume that the small energy and momentum self-energy has the following form, with  $a(T)$  and  $b(T)$  being the small corrections:

$$\Sigma_{\eta'} = a(T)k_0^2 + b(T)\vec{k}^2 + m^2(T). \quad (13)$$

The pole mass at  $\vec{k} \rightarrow 0$  would be  $\sqrt{m^2 + m^2(T)}/\sqrt{1 - a(T)}$ . But the mass we are talking about is  $\sqrt{m^2 + m^2(T)}$ .

Nevertheless,  $U_0$  has a nontrivial correction at finite temperature as we will see in the following sections.

## IV. $\eta'$ MASS AT FINITE TEMPERATURE

### A. Low-energy theorem

Now,  $U_0(0)$  can be obtained from the low-energy theorem. Here, we use the derivation using the heavy quark expansion [19]. For technical reasons, we start from a slightly different definition of the correlation function

$$P(k) = i \int d^4x e^{ik \cdot x} \left\langle \mathcal{T} \frac{3\alpha_s}{4\pi} G\tilde{G}(x) \frac{3\alpha_s}{4\pi} G\tilde{G}(0) \right\rangle. \quad (14)$$

It can be shown [19] that in the absence of light quarks,

$$P(k=0) = -\frac{2}{32\pi^2} \frac{d}{d(-1/4g_s^2)} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \quad (15)$$

from which we can obtain  $U_0(0) = \left(\frac{4\pi}{3\alpha_s}\right)^2 P(0)$ .

Now, any matrix element with canonical dimension  $d$  should be proportional to the  $d$ th power of the scale

$$\Lambda = M_0 \exp\left(-\frac{8\pi^2}{bg_s^2}\right), \quad (16)$$

where  $b = 11 - \frac{2}{3}N_f$  and  $M_0$  is the ultraviolet cutoff [20]. Hence, the gluon condensate at finite temperature and density should be of the following form:

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{T,\mu} = \Lambda^d f\left(\frac{T}{\Lambda}, \frac{\mu}{\Lambda}\right), \quad (17)$$

where  $d = 4$  and  $f$  is a generic function specifying the temperature and density dependence of the gluon condensate. Then Eq. (15) becomes

$$P(k=0) = -\frac{2}{b} \left( d - T \frac{\partial}{\partial T} - \mu \frac{\partial}{\partial \mu} \right) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{T,\mu}. \quad (18)$$

Now, combining Eqs. (8) and (18), one finds,

$$\left(\frac{3\alpha_s}{4\pi}\right)^2 \frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{m_{\eta'}^2} = \frac{2}{b} \left( d - T \frac{\partial}{\partial T} - \mu \frac{\partial}{\partial \mu} \right) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{T,\mu}. \quad (19)$$

But now since we can make the identification of the left- and right-hand side only in the large  $N_c$  limit, the right-hand side should be calculated in the quenched approximation. This means that one should just read off the temperature dependence of the gluon condensate from the lattice calculation for pure gauge theory, and also take  $b = 11$ . Thus, the  $\eta'$  mass is given by

$$m_{\eta'}^2 = \left(\frac{3\alpha_s}{4\pi}\right)^2 \frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{\frac{2}{b} \left( d - T \frac{\partial}{\partial T} \right) \langle \frac{\alpha_s}{\pi} G^2 \rangle_{T,\text{pure gauge}}}. \quad (20)$$

## B. Gluonic part

In order to evaluate the in-medium  $\eta'$  mass from Eq. (20), all we need are the temperature dependence of the gluon condensate and the coupling of  $G\tilde{G}$  to  $\eta'$ . First, let us consider the denominator of Eq. (20). It has been known for a long time that the gluon condensate has contribution from the perturbative and nonperturbative parts. Moreover, it was also known that at the critical temperature, the nonperturbative contribution changes abruptly, but does not vanish completely, and retains more than half of its nonperturbative value [21–23].

The effect of subtracting out the second term in the denominator of Eq. (20) is to get rid of the perturbative correction, or the seemingly scale-breaking effect which is

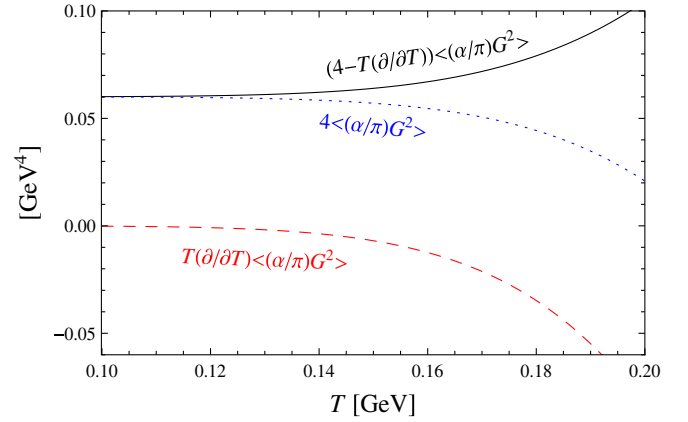


FIG. 2 (color online).  $T$ -dependence of gluon condensate and its derivative fitted to Wuppertal-Budapest lattice data in full QCD.

not related to scale breaking but due to the introduction of an external scale parameter  $T$ . The leading perturbative correction to the gluon condensate is proportional to  $g_s^4 T^4$  [24,25]. Therefore, assuming that the temperature dependence is of the form

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_T = G_0(T) + ag_s^4 T^4, \quad (21)$$

we find

$$\left( d - T \frac{\partial}{\partial T} \right) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_T = \left( d - T \frac{\partial}{\partial T} \right) G_0(T), \quad (22)$$

if the temperature dependence of  $g_s$  is neglected [26]. The only temperature dependence which survives is  $G_0(T)$ , whose scale dependence is coming from dimensional transmutation and not from the external temperature only. It is the nonperturbative part which dominates the behavior of the right-hand side of Eq. (20).

In the dotted line of Fig. 2, we show a resonance gas model result for the gluon condensate [27], which has been fit to reproduce the Wuppertal-Budapest's full QCD data of the trace anomaly [28]: The gluon condensate part is obtained by taking the chiral limit in the resonance gas model, which corresponds to subtracting out the fermion part in the trace anomaly. The dashed and solid lines are obtained by operating on the temperature-dependent operators as needed in Eq. (20).

As we discussed before, for the gluon condensate, one has to use the lattice result obtained in the pure-gauge theory calculation. The critical temperature in such a calculation  $T_{\text{pure-gauge}} \sim 260$  MeV is known to be around 100 MeV larger than that from a full QCD calculation  $T_{\text{QCD}}$  [25,29]. On the other hand, while the expected change of the gluon condensate is more abrupt in the pure-gauge calculation, the actual change in the condensate value itself at the critical temperature is found to be similar to the full calculation [30]. This means that the



change of the gluon condensate can be effectively neglected up to temperatures near  $T_{\text{QCD}}$ , and using the full lattice calculation as it appears in Fig. 2 will be an estimate of the maximum reducing effect to the  $\eta'$  mass expected from the denominator of Eq. (20). As we will see in the next section, the dominating impact will come from the change in the coupling  $\langle 0|G\tilde{G}|\eta'\rangle$ .

### C. Coupling to $\eta'$

The final step in obtaining the mass of  $\eta'$  using Eq. (20), when chiral symmetry is restored, is estimating the change of the coupling  $\langle 0|G\tilde{G}|\eta'\rangle$ . For that purpose, let us consider  $U_{lq}(k)$  in Eq. (5) in the full theory, but rewrite it in terms of the quark axial current using the anomaly relation.

$$\begin{aligned} U(k) &= i \int d^4x e^{ik \cdot x} \langle \mathcal{T} G\tilde{G}(x) G\tilde{G}(0) \rangle \\ &= k^\mu k^\nu i \int d^4x e^{ik \cdot x} \left( \frac{4\pi}{\alpha_s N_f} \right)^2 \\ &\quad \times [\langle \mathcal{T} \bar{q} i \gamma_\mu \gamma_5 q(x) \bar{q} i \gamma_\nu \gamma_5 q(0) \rangle \\ &\quad - \langle \mathcal{T} \bar{q} \gamma_\mu q(x) \bar{q} \gamma_\nu q(0) \rangle], \end{aligned} \quad (23)$$

where we have subtracted out the contribution from the conserved vector current. Using the previous terminology, when chiral symmetry is restored, the connected piece will cancel, as they are the same as the difference between flavored chiral partners, and only the disconnected pieces will remain. Assuming that the spectral sum starts from the  $\eta'$ , we find Eq. (23) can then be written as follows:

$$\begin{aligned} U(k) &= - \frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{k^2 - m_{\eta'}^2} - \dots \rightarrow k^\mu k^\nu \int \left( \frac{4\pi}{\alpha_s N_f} \right)^2 \\ &\quad \times \frac{1}{Z} \int d\mu [\text{Tr}[S_A(x, x) i \gamma_\mu \gamma_5] \text{Tr}[S_A(0, 0) i \gamma_\nu \gamma_5] \\ &\quad - \text{Tr}[S_A(x, x) \gamma_\mu] \text{Tr}[S_A(0, 0) \gamma_\nu]]. \end{aligned} \quad (24)$$

However, the disconnected pieces are all of the same order in  $m_q$  when chiral symmetry is restored.

$$\text{Tr}[S_A(x, x)] \sim \text{Tr}[S_A(x, x)\Gamma] \sim O(m_q), \quad (25)$$

where  $\Gamma$  is a Hermitian gamma matrix [4]. This is so because the chiral order parameter can be written as  $\langle \bar{q}q \rangle = -\frac{1}{Z} \int d\mu \text{Tr}[S_A(0, 0)] = -\pi\rho(\lambda = 0)$ , the density of zero eigenvalues of the Dirac operator in the presence of the gauge field [31]. Hence, when chiral symmetry is restored and the density of zero eigenvalues vanishes, both Eqs. (24) and (25) disappear. Since Eq. (24) is valid for any  $k$  other than zero for which the low-energy theorem is obtained, for both the leading and subleading  $N_c$  terms, we find that

$$\langle 0|G\tilde{G}|\eta'\rangle \sim O(m_q), \quad (26)$$

when chiral symmetry is restored. Therefore, going back to Eq. (20) and making use of the previous discussions, we find that when chiral symmetry is restored,

$$m_{\eta'}^2 \xrightarrow{\langle \bar{q}q \rangle \rightarrow 0} 0, \quad (27)$$

in the chiral limit. One concludes that in the large  $N_c$  limit of QCD,  $\eta'$  mass will become degenerate with the other Goldstone bosons. It is interesting to note that a similar conclusion was obtained in Ref. [16]: that the anomalous  $U_A(1)$   $\eta'$  mass squared vanishes at high  $T$  as the chiral quark condensate  $\langle \bar{q}q \rangle$ .

## V. CONCLUSIONS

It should be noted that the  $\eta'$  mass which is being quenched [in Eq. (27)] is the part of the mass which comes from the breaking of the  $U_A(1)$  symmetry. Going back to Eq. (10) and substituting the vacuum value of Eq. (18), one finds

$$m_{\eta'} = \sqrt{\frac{8}{33}} \frac{1}{f_\pi} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle^{1/2} \approx 464 \text{ MeV}, \quad (28)$$

where we have used  $f_\pi = 130 \text{ MeV}$  and  $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.35 \text{ GeV})^4$  [32]. This is smaller than the vacuum value of the  $\eta'$  mass as expected. Assuming that the pseudoscalar mesons do not change their mass toward the crossover point, it is this extra  $U_A(1)$  mass of  $\eta'$  which is going to be quenched in the chiral symmetry restored phase. Naively, an analogous relation to Eq. (28) can be obtained at finite temperature by substituting Eq. (9) to Eq. (20). However, it should be noted that Eq. (9) is not valid at finite temperature as the time and space component of the pion decay constant are no longer identical in the medium [34]. Moreover, no quenched lattice calculation exists for both decay constants near the phase transition point. That is why we derived a general relation between the coupling of the pseudoscalar gluon operator to the  $\eta'$  and the chiral order parameter.

A few remarks are in order. First, in the quenched approximation, the changes of order parameters take place only near the phase transition point. This suggests that the effect of mass quenching might only be visible when the hadronization temperature is close to the crossover point as in the case of RHIC or LHC energies, for example. Second, it is hard to make a quantitative estimate on how much of this mass is quenched in the nuclear medium, as the effects of density are subleading in the large  $N_c$  limit. However, assuming Eq. (20) is an exact relation, we can use Eq. (24) to approximate  $\langle 0|G\tilde{G}|\eta'\rangle \propto \text{Tr}[S_A(x, x)] \propto \langle \bar{q}q \rangle$  and then use it in Eq. (20) to deduce  $m_{\eta'} \propto \langle \bar{q}q \rangle$ , assuming that the change in the gluon condensate is small in nuclear medium. Therefore, if the chiral order parameter reduces by 20% in nuclear medium, the  $U_A(1)$  breaking part of the  $\eta'$  mass will also reduce by the same fraction.

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